RD Spillovers, Innovation, and Entry

Khazabi, Massoud and Quyen, Nguyen

University of Ottawa

1 July 2011

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Massoud Khazabi[1]

University of Ottawa

The paper extends a theoratical framework to analyze the impact of R&D spillovers on entry and the resulting equilibrium market structure. It is shown that the degree of spillovers plays a fundamental role on the number of firms entering the market, their R&D activities, and social welfare. The analysis suggests that social welfare is maximized at some intermediate degree of spillovers. The policy implication of this result is that neither complete protection of intellectual property right nor lax enforcement of patent laws is socially optimal.

[1] Massoud Khazabi, University of Ottawa and Government of Canada, mkhaz034@uottawa.ca

1 Introduction

In the economic literature, innovation – also called R&D – is often classified as product innovation or process innovation. A firm carries out a program of product innovation to find a new product that it hopes will generate new demand and lead to large profits. Process innovation, on the other hand, aims at finding a new process to reduce the production cost of a product. A lower production cost, which is the desired outcome of the R&D program, gives the firm a cost advantage over its rivals. Whether a program of innovation will be carried out or not depends on the cost of R&D and the market structure in which the firm finds itself. Knowledge and benefits obtained by a firm from its R&D activities typically leak out to other firms, to consumers and, eventually, to other countries. These leakages – called R&D spillovers – mean that a firm cannot appropriate all the fruit of its R&D activities, especially when spillovers flow to its competitors in the same industry. However, from society's point of view, spillovers represent positive externalities in the sense that they reduce the production costs of other firms, with the ensuing consequence of lower prices for consumers. In a review of the literature on R&D aimed at providing guidelines for recent efforts to include R&D in the national income accounts, Sveikauskas (2007) indicated that perhaps the private rate of return to R&D is 25%, while it is 65% for social returns.

In light of the positive externalities generated by R&D activities, the authorities charged with competition policy in Europe and Japan have adopted a rather permissive anti-trust attitude toward R&D cooperation for quite some time. The research – both theoretical and empirical – received the needed impetus in 1984 when the US passed the National Cooperation Act in 1984, allowing firms to cooperate in R&D, but not in product markets. Over the last two decades, the economics of R&D spillovers has been one of the most active fields of research in industrial economics.

The theoretical literature on competition and cooperation in R&D with technological spillovers can be said to begin with the pioneering work of d'Aspremont and Jacquemin (1988) (AJ hereafter), who formulated a two-stage duopoly game of R&D spillovers in which the two firms behave in a non-cooperative manner in the second (production) stage, but can either cooperate or behave in a non-cooperative manner in the first (R&D) stage. Now when the two firms behave cooperatively in the R&D stage, it is reasonable to expect that the cooperation will lead to a lower level of total R&D expenditures made by the two firms because of less wasteful duplications and a lower level of total output resulted from the monopoly power. AJ demonstrated that these expectations are far from being fulfilled in a simple two-stage linear-quadratic game – linear demand curve, linear total cost, and quadratic R&D costs. Three different scenarios of competition are considered by AJ. Under the first scenario, the two firms act non-cooperatively in both stages of the game. Under the second scenario, the two firms cooperate in the first stage, but behave non-cooperatively in the second stage. Under the third scenario, the two firms behave jointly like a single integrated firm in both stages of the game. AJ also considered the problem faced by the central planner. The linear-quadratic structure of the model makes it possible to obtain a closed-form solution for each of the problems and allows for a comparison of the solutions – R&D expenditures and welfare – of the four models. In the analysis carried out by AJ, the degree of R&D spillovers plays a critical role. AJ showed that when the degree of R&D spillovers is high, the level of R&D expenditures and total output are higher under the second scenario than under the first scenario. Otherwise, the opposite results hold. For a high degree of R&D spillovers, the R&D expenditures under the social optimum are highest to be followed – in descending order – successively by the R&D expenditures under the scenario that the two firms act like a single integrated firm, the scenario that the two firms cooperate in R&D, but act non-cooperatively in the production stage, and the scenario that the two firms behave non-cooperatively in both stages of the game.

The results on the comparative performance of non-cooperative and cooperative R&D derived by AJ have received – in the duopoly context – a thorough generalization by Amir et al. (2003). In particular, when the two firms cooperate in R&D, these researchers allowed the firms to determine jointly their R&D expenditures and the degree of R&D spillovers. The R&D degree of spillovers is thus endogenous, and can be chosen to maximize joint profits net of R&D costs.

The AJ model was extended by Suzumura (1992) to the case of many firms, general demand, and general cost conditions. The more general model of Suzumura precludes the possibility of computing the equilibria of the two-stage games for various specifications of R&D, and it is no longer possible to compare these equilibria directly. This researcher resolved this difficulty by trying to answer the question of starting from an equilibrium – under non-cooperative R&D or cooperative R&D – can social welfare be raised by marginally increasing R&D expenditures? Two measures of social welfare are used in these exercises: the first-best social optimum and the second-best social optimum. According to the first-best measure of social welfare, the sum of consumer and producer surplus is maximized, and the marginal cost pricing rule, which underlines the first-best solution, can be enforced by the authorities. According to the second-best measure of social welfare, the firms are allowed to compete according to the Cournot model of competition. Suzumura demonstrated that when the degree of spillovers is high, starting from the equilibrium level of R&D expenditures under the scenario that the firms act non-cooperatively in both stages of the game, first-best social welfare can be raised by marginally increasing R&D expenditures. The opposite result holds if there are no spillovers. As for the equilibrium under cooperative R&D, first-best social welfare can be raised by marginally increasing R&D expenditures, whether the degree of spillovers is high or low. If one uses the second-best measure of social welfare, then starting from the equilibrium level of R&D expenditures under the scenario that the firms act non-cooperatively in both stages of the game, social welfare can be raised by marginally increasing R&D expenditures if the degree of spillovers is high. The opposite result holds if there are no R&D spillovers and if the number of firms is sufficiently large. As for the case of cooperative R&D, social welfare can be raised by marginally increasing R&D expenditures, whether the degree of spillovers is high or low.

The welfare results of Suzumura are obtained under two extreme assumptions – high and low degrees of spillovers. Yi (1996) completed the analysis of Suzumura by considering the intermediate case of neither high nor low degrees of spillovers. More specifically, Yi established the following results. First, cooperative R&D lowers both R&D expenditures and social welfare for intermediate degrees of spillovers. Second, cooperative R&D lowers R&D expenditures, but has an ambiguous effect on social welfare for low degrees of spillovers. Third, as the elasticity of the slope of the inverse market demand curve rises, cooperative R&D raises social welfare for a larger set of degrees of spillovers, and in the limit, is socially beneficial for all degrees of spillovers.

The AJ's model has also been extended by other researchers, such as Kamien et al. (1992), Kamien and Zang (1993), Poyago-Theotoky (1996), and Atallah (2000) to study the issue of R&D cartelization and research joint ventures. In the models formulated by theses authors a subset of the firms in the industry might get together and form a Research Joint Venture. A survey of the main results of the literature on spillovers and innovative activities is provided by De Bondt (1996). The predictions of the AJ model, especially the important question of whether spillovers increase firms' incentives to cooperate in R&D, has been addressed by a number of empirical studies with mixed results; see, for example, Cassiman and Veugelers (2002) and Sustens (2004).

In the literature on R&D spillovers and process innovation, efforts are mostly focused on the comparative R&D expenditures and the relative social welfare between non-cooperative and cooperative R&D. The question of how innovation is affected when there is more competition, i.e., when the number of firms rises, is ignored by most researchers, except for De Bondt et al. (1992), who discussed how the number of firms affects innovation. However, the question of how R&D spillovers affect entry was not addressed by these researchers. In this paper, we attempt to fill part of this lacuna by endogenizing the number of firms. More specifically, our model addresses the following questions. First, how does the degree of spillovers affect the equilibrium number of firms? Second, how does the degree of spillovers affect the equilibrium market structure? Third, when is the equilibrium symmetric and when is it not symmetric, and in the case of asymmetric equilibria, how many firms choose to incur a positive amount of own R&D cost and how many firms choose to free ride on the R&D activities of others?[2]

The model we formulate to analyze the influence of R&D spillovers on entry is a two-stage game played by a number of firms producing a homogeneous good. In the first stage of the game, the firms carry out R&D activities to lower their production cost. It is assumed that before innovation, all the firms have the same marginal cost. In the second stage, the firms compete in the product market according to the Cournot model of competition. All the firms act non-cooperatively in both stages of the game. In the R&D stage, each firm runs its own research lab, and takes into account the natural spillovers that flow among firms in a strategic manner. In modeling the horizontal spillovers among firms, we follow the pioneering work of Ruff (1969), who analyzed a stylized growth model in which firms compete according to the Cournot model

[2]See Dasgupta and Stiglitz (1980), Atallah (2007), Vives (2008), and Erkal and Piccinin (2010) for more information.

of competition, and in which firms undertake R&D activities by employing research workers. In Ruff's model, a firm recognizes a potential transmission of knowledge from other firms, and the transmission of knowledge is modeled by assuming that the *effective input* in R&D of a firm consists of its own input plus part of the inputs of all the other firms. Ruff's analytical treatment of R&D spillovers has been adopted by later researchers, such as Spence (1984), Kamien et al. (1992), and Amir et al. (2003). On the other hand, in the AJ-type models, the R&D process is represented by a cost function, which gives the R&D cost (the dependent variable) as a quadratic function of the desired level of cost reduction (the independent variable). In these models, a firm chooses its own level of cost reduction, and the spillovers take the form of R&D output spillovers in the sense that the *effective cost reduction* of a firm is the firm's own chosen cost reduction plus a fraction of the cost reductions chosen by all the other firms. At first sight, one might think that the two ways of modeling R&D spillovers are equivalent, and should yield the same results. However, this presumption is not correct because for some questions the results obtained by the two approaches diverge. For example, Amir et al. (2003) found that a firm's *effective R&D expenditure* is a strictly decreasing function of the degree of spillovers while De Bondt et al. (1992) found an inverted-U relationship between the degree of spillovers and the *effective cost reduction* of each firm.

Our findings can be described as follows. If the degree of spillovers is low, only a finite number of firms enter the market, and after entry – all the firms expend the same amount of their own resources on R&D. We refer to this type of equilibrium as a symmetric equilibrium with innovation. The intuition behind this result is not hard to understand. When the degree of spillovers is low, a firm cannot rely on the R&D externalities generated by the other firms to lower its own marginal cost. If the degree of spillovers is high, an infinite number of firms enter the market, and after entry none of the firms chooses to expend any of its own resource on R&D. The equilibrium is a perfectly competitive equilibrium without innovation. For intermediate values of the degree of spillovers, the equilibrium is an asymmetric equilibrium under which some firms choose to expend their own resources on R&D, while others choose not to do so. Furthermore, all the firms that choose to expend their own resources on R&D choose the same amount of own expenditure on R&D. In the literature on R&D spillovers and innovation, the number of firms in the market is taken as exogenous, and the equilibrium is presumed to be symmetric. In our model, the number of firms that enter the market is endogenous, and varies according to the degree of spillovers. The endogenization of the number of firms yields different types of equilibrium market structures – symmetric equilibrium with innovation, asymmetric equilibrium with innovation, and perfect competition without innovation – and these are novel results in the field of R&D spillovers and innovation.

Our welfare analysis suggests that social welfare rises with the degree of spillovers when it is low, reaches a maximum when the degree of spillovers enters its intermediate range, and then declines to the level associated with the competitive equilibrium without innovation. The policy implication of this result is that the intellectual property right should not be fully protected and the enforcement of patent laws should not be too lax. The optimal degree of protection should reflect the right trade-off between allocative and dynamic efficiency.

The paper is organized as follows. In Section 2, the general features of the model are presented. In Section 3, the post-innovation equilibrium is discussed. The equilibrium in the innovation stage is analyzed in Section 4. In Section 5, entry is discussed. Section 6 presents the main properties of the equilibrium. In Section 7, a numerical simulation of the model is presented. The simulation illustrates the main properties of the model. Some concluding remarks are given in Section 8. The annex contains some technical arguments used to support the theoretical arguments in Section 6.

2 The general features of the model

Consider the market for a homogenous commodity in which there are n symmetric firms. These firms play a two-stage game, which can be described as follows. In the first stage, each firm carries out an R&D program to reduce its production cost. In the second stage, the firms – with lower marginal costs brought about by the process innovation programs carried out in the first stage – compete in the product market. Let

(1) $\quad p = a - bQ$

be the inverse market demand curve for the product, where Q is the industry output, and p is the market price. Also, a and b are two positive parameters.

We assume that in both stages of the game the firms act non-cooperatively and that in the second stage they compete according to the Cournot model of competition. In the first stage, and before the process innovation, the firms are symmetric in the sense that all the firms start with the same initial marginal cost, say $c^0, 0 < c^0 < a$. To model the R&D process, we follow Ruff (1969), and suppose that research workers constitute the only input in a program of process innovation. Furthermore, assuming that the wage received by a worker is the numéraire, we can identify the number of research workers with the R&D expenditure. We shall let $f[X]$ denote the reduction in marginal cost yielded by a program of process innovation when X is the firms *effective R&D expenditure*, with the effective R&D expenditure being the sum of the firm's own R&D expenditure plus the spillovers from the other innovation programs. The R&D production function is assumed to be continuously differentiable, strictly increasing, and strictly concave. Furthermore, $f[0]=0, 0<f'[0]<\infty$, and $sup_X f[X] \leq c^0$. The condition $sup_X f[X] \leq c^0$ ensures that the cost reduction is strictly less than c^0, regardless of the level of effective R&D input. Also, we shall assume that $f'[0]$ is not too low to discourage a firm, when it is the only firm in the market, from investing in R&D to reduce its marginal cost. Note that the Inada condition $\ell im_{X\to\infty} f'[X]=0$ follows from the concavity and the boundedness of the R&D production function. An R&D technology that satisfies these assumptions is

(2) $$f[X] = c^0(1 - e^{-\gamma X}),$$

where $\gamma > 0$ is a parameter that characterizes the productivity of the R&D technology.[3]

[3] In the AJ model, the R&D cost needed to achieve a level of marginal cost reduction is assumed to be a quadratic function of the R&D output. The production function that corresponds to such a cost function has the functional form $f[X] = 2\sqrt{\frac{X}{\gamma}}$, where $\gamma > 0$ is a parameter that characterizes the productivity of the R&D process, with a lower value of γ representing a more productive technology. Although it makes the computation of an equilibrium simple, the R&D technology of the AJ model has some undesirable features. First, when the effective R&D expenditure is large, the reduction in marginal cost will exceed the initial marginal cost, rendering the post-innovation marginal cost negative, and this is absurd. Second, given that the initial marginal cost was the outcome of past R&D activities and given the presumed diminishing returns involved in R&D activities, the reduction in marginal cost yielded by new R&D activities must necessarily be bounded at the margin. That is, the Inada condition $\ell im_{X\to 0} f'[X] = \infty$

The cost reduction obtained by firm i is assumed to be given by $f[x_i + \beta\sum_{j\neq i} x_j]$, where $0 \leq \beta \leq 1$, is a parameter that represents the degree of spillovers from the R&D activities of all the other firms, and $x_j, j = 1,...,n$, is firm $j's$ own R&D expenditure. The expression $\beta\sum_{j\neq i} x_j$ represents the spillovers to firm i from the R&D activities of all the other firms. The sum $X_i = x_i + \beta\sum_{j\neq i} x_j$ thus represents the effective R&D expenditure of firm i in its own program of process innovation. When $\beta = 0$, there are no spillovers, and when $\beta = 1$, there are full spillovers. The intermediate case $0 < \beta < 1$ corresponds to the situation of partial spillovers.

3 The post-innovation equilibrium

When each firm carries out its own process innovation program, the reduction in the marginal cost of a firm, say firm i, is $f[x_i + \beta\sum_{j\neq i} x_j]$, and its post-innovation marginal cost is given by

(3) $$c_i^1 = c^0 - f[x_i + \beta\sum_{j\neq i} x_j].$$

Let q_i be the output of firm i. The profit obtained by firm i in the production stage and under the strategy profile $(q_1, \dots, q_i, \dots, q_n)$ is

$$\varphi_i[q_1, \dots, q_i, \dots, q_n] = q_i(a - b(q_1 + \cdots + q_i + \cdots + q_n) - c_i^1).$$

In the production stage, firm i solves the following profit maximization problem:

(4) $$max_{q_i}\varphi_i[q_1, \dots, q_i, \dots, q_n]. \qquad (i = 1, \dots, n).$$

If the post-innovation cost of firm i is high, it will not be able to compete with the other firms and will choose not to produce, with the ensuing consequence that it makes zero profit in the production stage. On the other hand, if its post-innovation marginal cost is not too high, firm i

exhibited by the R&D technology in the AJ model is difficult to defend. Finally, the quadratic R&D cost function yields a reduction in marginal cost that is proportional to the difference between the choke price and the initial marginal cost (D'Aspremont and Jacquemin (1988), page 1114). This result means that the higher is the initial marginal cost, the lower will be the marginal cost reduction. In particular, when the initial marginal cost is so high to be equal to the choke price, the marginal cost reduction will be zero, a result that is clearly unsatisfactory.

will be able to produce a positive level of output and earn positive profits in the production stage. Thus, the first-order condition that characterizes the best response of firm i to $\left(q_j\right)_{j\neq i}$ is

$$(5)\qquad a - b(q_1 + \cdots + q_i + \cdots + q_n) - c_i^1 - bq_i \leq 0, \qquad (i = 1, \dots, n),$$

with equality holding if $q_i > 0$. The first-order condition (5) is also sufficient because $\varphi_i[q_1, \dots, q_i, \dots, q_n]$ is strictly concave in q_i.

The n first-order conditions (5) can be used to compute the equilibrium output of each firm, as a function of its post-innovation marginal cost. If we let $q_i[c_1^1, \dots, c_n^1]$ denote the equilibrium output of firm i in the production stage, then the profit it earns in this stage is given by

$$(6)\qquad \pi_i[c_1^1, \dots, c_n^1] = \varphi_i[q_1[c_1^1, \dots, c_n^1], \dots, q_i[c_1^1, \dots, c_n^1], \dots, q_n[c_1^1, \dots, c_n^1]].$$

Note that $\pi_i[c_1^1, \dots, c_n^1] \geq 0$, with strict inequality holding if c_i^1 is not too high.

Let I denote the subset of firms that produce a positive level of output and a fortiori earns a positive level of profit in the production stage. For each $i \in I$, the first-order condition (5) holds with equality, and summing these first-order conditions over $i \in I$, we obtain

$$(7)\qquad |I|a - |I|bQ - \textstyle\sum_{i\in I} c_i^1 - bQ \leq 0,$$

where $Q = q_1 + \cdots + q_i + \cdots + q_n$ denotes the industry output, and $|I|$ denotes the number of elements in I, ie., the number of firms that produce a positive level of output.

It follows from (7) that

$$(8)\qquad Q = \frac{|I|a - \sum_{i\in I} c_i^1}{(|I|+1)b}.$$

Using (8), we obtain the following expression for the equilibrium market price

$$(9)\qquad p = a - bQ = \frac{a + \sum_{i\in I} c_i^1}{|I|+1}.$$

Using (8) in (5), we obtain the following expression for the output of a firm, say i, which produces a positive level of output in the production stage:

$$(10)\quad q_i = \frac{a-|I|c_i^1+\sum_{j\in I, j\neq i} c_j^1}{(|I|+1)b}, \qquad (i \in I).$$

The profit made by firm i in the post-innovation stage is then given by

$$(11)\quad \pi_i[c_1^1, \dots, c_n^1] = q_i(p - c_i^1)$$

$$= \begin{cases} \frac{\left(a-|I|c_i^1+\sum_{j\in I, j\neq i} c_j^1\right)^2}{(|I|+1)^2 b}, \ if\ i \in I \\ o,\ otherwise. \end{cases}$$

4 The equilibrium in the innovation stage

Let $x_j, j = 1, \dots, n$, denote the own R&D expenditure made by firm j in the first stage. Because the profit made by a firm in the production stage is bounded above, it is not optimal for any firm to spend a large amount of its own resources on R&D. Thus, we shall assume that the own R&D made by each firm is constrained to belong to a closed bounded interval, say $0 \leq x_j \leq K, j = 1, \dots, n$, where K is a finite positive number. Given the list $(x_1, \dots, x_n)$ of own R&D expenditures, the profit – net of R&D costs – earned by firm i over the two stages of the game is given by

$$(12)\quad \phi_i[x_1, \dots, x_i, \dots, x_n] = -x_i + \pi_i[c_1^1, \dots, c_i^1, \dots, c_n^1] =$$

$$-x_i + \pi_i\left[c^0 - f[x_1 + \beta \textstyle\sum_{j\neq 1} x_j], \dots, c^0 - f[x_i + \beta \sum_{j\neq i} x_j], \dots, c^0 - f[x_n + \beta \sum_{j\neq n} x_j]\right].$$

The first-order condition that characterizes the best response of firm i to $(x_j)_{j\neq i}$ is

$$(13)\quad \frac{\partial \phi_i[x_1, \dots, x_i, \dots, x_n]}{\partial x_i} = -1 - \frac{\partial \pi_i[c_1^1, \dots, c_i^1, \dots, c_n^1]}{\partial c_i^1} f'\left[x_i + \beta \textstyle\sum_{j\neq i} x_j\right]$$

$$-\beta \sum_{j\neq i} \frac{\partial \pi_i[c_1^1, \dots, c_i^1, \dots, c_n^1]}{\partial c_j^1} f'\left[x_j + \beta \textstyle\sum_{j'\neq j} x_{j'}\right] \leq 0,$$

with equality holding when $x_i > 0$.

Note that it is not optimal for a firm to spend a positive amount of its own resources on R&D and then chooses not to produce a positive level of output in the production stage. Furthermore, a firm might find it profitable to produce a positive level of output even when it did not spend any of its own resources on R&D in the first stage. Under this scenario, it takes advantage of the spillovers from the R&D activities of all the firms which choose to spend a positive amount of their own resources on their R&D activities to lower its own marginal cost.

When the own R&D expenditure of firm i is positive, the following second-order condition must also be satisfied:

(14) $\frac{\partial^2 \phi_i[x_1,\dots,x_i,\dots,x_n]}{\partial x_i^2} < 0.$

Let $\zeta_i[x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n]$ denote firm $i's$ best response – presumed to be unique – to $(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n), i = 1, \dots, n$. It is simple to show that the map

$$\zeta_i: (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n) \to \zeta_i[x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n]$$

is continuous. Thus, the map

$$(x_1, \dots, x_i, \dots, x_n) \to (\zeta_i[x_1, \dots, x_i, \dots, x_n])_{i=1}^n$$

is a continuous map from the convex compact subset $[0, K]^n$ of the n-dimensional Euclidean space into itself, and thus, according to the Brouwer's fixed point theorem, will have a fixed point. The fixed point, which we presume to be unique and denote by $(x_1[n,\beta], \dots, x_n[n,\beta])$, represents the equilibrium list of own R&D expenditures made by the n firms. Note that the argument just presented establishes the existence of an equilibrium for any number of firms. Furthermore, for an arbitrary number of firms in the market, some firms might make zero net profit (net of own R&D cost). In particular, when the number of firms is large, the equilibrium might involve some firms being not active in the production stage, which necessarily means that these firms do not expend any of their own resources on R&D, and thus can be dropped from the game without affecting the equilibrium generated by the remaining firms. In other words, these firms will choose not to enter the market.

For an equilibrium under which each firm spends a positive amount of its own resources on R&D, (12) takes on the following more specific form:

(15) $\phi_i[x_1, \dots, x_i, \dots, x_n]$

$$= -x_i + \frac{1}{b(n+1)^2}\begin{pmatrix} a - n\left(c^0 - f\left[x_i + \beta \sum_{j\neq i} x_j\right]\right) \\ + \sum_{j\neq i}\left(c^0 - f\left[x_j + \beta\left(x_i + \sum_{j'\neq i, j'\neq j} x_{j'}\right)\right]\right) \end{pmatrix}^2.$$

Furthermore, the first-order condition (13) becomes

$$(16) \qquad \frac{\partial \phi_i[x_1,\dots,x_i,\dots,x_n]}{\partial x_i} = -1 + \frac{2}{b(n+1)^2}\begin{pmatrix} a - n\left(c^0 - f\left[x_i + \beta \sum_{j\neq i} x_j\right]\right) \\ + \sum_{j\neq i}\left(c^0 - f\left[x_j + \beta\left(x_i + \sum_{j'\neq i, j'\neq j} x_{j'}\right)\right]\right) \end{pmatrix} \times \begin{pmatrix} nf'\left[x_i + \beta \sum_{j\neq i} x_j\right] \\ -\beta \sum_{j\neq i} f'\left[x_j + \beta\left(x_i + \sum_{j'\neq i, j'\neq j} x_{j'}\right)\right] \end{pmatrix} = 0, \ \ (i = 1, \dots, n),$$

and the second-order condition (14) becomes

$$(17) \qquad \frac{\partial^2 \phi_i[x_1,\dots,x_i,\dots,x_n]}{\partial x_i^2} = \begin{pmatrix} a - n\left(c^0 - f\left[x_i + \beta \sum_{j\neq i} x_j\right]\right) \\ + \sum_{j\neq i}\left(c^0 - f\left[x_j + \beta\left(x_i + \sum_{j'\neq i, j'\neq j} x_{j'}\right)\right]\right) \end{pmatrix} \times \begin{pmatrix} nf''\left[x_i + \beta \sum_{j\neq i} x_j\right] \\ -\beta^2 \sum_{j\neq i} f''\left[x_j + \beta\left(x_i + \sum_{j'\neq i, j'\neq j} x_{j'}\right)\right] \end{pmatrix} + \begin{pmatrix} nf'\left[x_i + \beta \sum_{j\neq i} x_j\right] \\ -\beta \sum_{j\neq i} f'\left[x_j + \beta\left(x_i + \sum_{j'\neq i, j'\neq j} x_{j'}\right)\right] \end{pmatrix}^2 < 0.$$

Note that if each firm makes a positive own expenditure on R&D, i.e., if $x_i[n,\beta] > 0$ for all $i = 1, \dots, n$, then the n first-order conditions in (16) are symmetric, and it is necessary that all the own R&D expenditures are the same, i.e., $x_1[n,\beta] = \cdots = x_n[n,\beta] = x[n,\beta]$, where we have used $x[n,\beta]$ to denote their common own R&D expenditure. In this case, the equilibrium is a symmetric equilibrium under which each firm spends a positive amount of its own resources on R&D, and we refer to such an equilibrium as a *symmetric equilibrium with innovation*. It might also happen that in equilibrium some firms incur a positive level of own R&D cost while some other firms choose not to do so. In this case, the equilibrium is *an asymmetric equilibrium*, with one proper subset of the firms incurring a positive level of own R&D cost while all the firms outside this subset choose not to do so. Because the first-order conditions in (16) that characterize the own R&D expenditures of the firms that choose to spend a positive amount of their own resources on R&D are symmetric, the own R&D expenditures of these firms must be the same. Finally, it might happen that in equilibrium none of the firms chooses to spend any of its own resources on R&D. In this case, the equilibrium is a symmetric equilibrium without innovation.

For a symmetric equilibrium with innovation, the effective R&D expenditure of each firm is given by

$$X_1[n,\beta] = \cdots = X_n[n,\beta] = X[n,\beta] = x[n,\beta](1+(n-1)\beta),$$

where $X[n,\beta]$ denote the firms' common effective expenditure. Furthermore, the first-order condition (16) is then reduced to

$$(18) \qquad -1 + \frac{2(n-(n-1)\beta)}{b(n+1)^2}(a - c^0 + f[X[n,\beta]])f'\big[X[n,\beta]\big] = 0,$$

and the second-order condition (17) is reduced to

$$(19) \qquad \frac{n-(n-1)\beta^2}{n-(n-1)\beta}\big(a - c^0 + f[X[n,\beta]]\big)f''\big[X[n,\beta]\big] + \big(f'[X[n,\beta]]\big)^2 < 0.$$

Because

$$\frac{2(n-(n-1)\beta)}{b(n+1)^2}(a - c^0 + f[X[n,\beta]])f'\big[X[n,\beta]\big] < 1$$

when n is large, the first-order condition (18) will fail to hold and a fortiori no symmetric equilibrium with innovation will exist.

For the case $n = 1$, the first-order condition (18) is reduced to

$$(20) \qquad -1 + \frac{1}{2b}(a - c^0 + f[X[1,\beta]])f'\big[X[1,\beta]\big],$$

which is the first-order condition for maximizing (15). As for the second-order condition (19), it is reduced to

$$(21) \qquad \big(a - c^0 + f\big[X[1,\beta]\big]\big)f''\big[X[1,\beta]\big] + \big(f'\big[X[1,\beta]\big]\big)^2 < 0,$$

which is the second-order condition for maximizing (15), namely the second-order condition for monopoly profit maximization.

We note in passing that if the curve $X \to (a - c^0 + f[X])f'[X],\ X \geq 0$, is downward-sloping, a main assumption in the model of Kamien et al. (1992), then the solution of the first-order condition (18) is unique, and a symmetric equilibrium with innovation, if it exists, is necessarily unique. If more than one value of X satisfies the first-order condition (18), then the second-order condition (19) must be used to eliminate the inappropriate value of X that satisfies this first-order condition. Another possibility is that there might be a value of X that satisfies both the first order condition (18) and the second-order condition (19), but such a value of X leads to a negative net profit for each firm when we set $x_1 = \cdots = x_n = \frac{X}{1+(n-1)\beta}$ in (15). Under such a scenario, there is no symmetric equilibrium under which X constitutes the effective R&D expenditure of each firm in the market.

The product price under a symmetric equilibrium with innovation when there are n firms in the market is given by

(22) $$p[n,\beta] = \frac{a+n\left(c^0-f[X[n,\beta]]\right)}{n+1}.$$

The net profit made by a firm under a symmetric equilibrium with innovation, when there are n firms in the market, is given by

(23) $$-\frac{X[n,\beta]}{1+(n-1)\beta} + \frac{\left(a-\left(c^0-f[X[n,\beta]]\right)\right)^2}{b(n+1)^2}.$$

An example: Suppose that the R&D production function is given by (2). For a symmetric equilibrium with innovation, the first-order condition (18) becomes

(24) $$-1 + \frac{2(n-(n-1)\beta)}{b(n+1)^2}(a - c^0 e^{-\gamma X})\gamma c^0 e^{-\gamma X} = 0.$$

The slope of the curve

(25) $$\phi: X \to \phi[X] = -1 + \frac{2(n-(n-1)\beta)}{b(n+1)^2}(a - c^0 e^{-\gamma X})\gamma c^0 e^{-\gamma X}, X \geq 0,$$

is

(26) $$\phi'[X] = \frac{2(n-(n-1)\beta)\gamma^2 c^0}{b(n+1)^2} e^{-2\gamma X}(2c^0 - a e^{\gamma X}).$$

Observe that if $c^0 \leq \frac{a}{2}$, then $\phi'[X] < 0$ is negative for all $X > 0$, and the curve $\phi: X \to \phi[X]$ is downward-sloping. In this case, the solution of the first-order condition (18), if it exists, is unique. On the other hand, if $c^0 > \frac{a}{2}$, then the curve $\phi: X \to \phi[X]$ has the shape of an inverted U:

rising at first, attaining its global maximum at $X = \frac{1}{\gamma} Log\left[\frac{2c^0}{a}\right]$, and then strictly declining to $-\infty$ when $X \to \infty$. For the case $c^0 > \frac{a}{2}$, if the curve $\phi: X \to \phi[X]$ does not cross the horizontal axis on its rising part, then a symmetric equilibrium with innovation does not exist. On the other hand, if the curve $\phi: X \to \phi[X]$ crosses the horizontal axis on its rising part, then on its declining part it crosses the horizontal axis again. In this case, there exist two values of X that satisfy the first-order condition (18). At the first crossing the expression on the left side of (19) is equal to $(n-1)(1-\beta)\beta c^0 > 0$, and this means that the second-order condition (19) is not satisfied at the first crossing. Thus, a symmetric equilibrium with innovation, if it exists, must occur at the second crossing.

If we let $Z = e^{-\gamma X}$, then the first-order condition (24) becomes

(27) $\quad -1 + \frac{2(n-(n-1)\beta)}{b(n+1)^2}(a - c^0 Z)\gamma c^0 Z = 0,$

which is a quadratic equation in Z. The two roots of (27) are of the same sign, and are given by

(28) $\quad \left\{Z \to \frac{a\eta c^0 - \sqrt{\eta(-2b(1+n)^2 + a^2\eta)(c^0)^2}}{2\eta(c^0)^2}, Z \to \frac{a\eta c^0 + \sqrt{\eta(-2b(1+n)^2 + a^2\eta)(c^0)^2}}{2\eta(c^0)^2}\right\},$

where we have let $\eta = (n(1-\beta) + \beta)\gamma$.

As can be seen from (28), the second root is positive. Hence the first root is also positive. If the second root is greater than 1, then it must be rejected because $Z = e^{-\gamma X} < 1$. On the other hand, if the second root is less than 1, then the first root is also less than 1. Furthermore, if the curve $\phi: X \to \phi[X]$ crosses the horizontal axis twice, then the first-order condition (24) has two roots, and the larger root corresponds to the second crossing, and this means that the smaller root of (27) is the correct value of Z for the effective R&D expenditure under the symmetric equilibrium with innovation. Thus, the effective R&D expenditure of a firm under a symmetric equilibrium with innovation is given by

(29) $\quad X[n,\beta] = -\frac{1}{\gamma} Log\big[Z[n,\beta]\big],$

where we have let

(30) $\quad Z[n,\beta] = \frac{1}{2c^0}\left(a - \sqrt{\frac{a^2\gamma(n(1-\beta)+\beta) - 2b(1+n)^2}{\gamma(n(1-\beta)+\beta)}}\right) < 1.$

5 Entry

To fix ideas about the entry process, we shall assume that a firm will enter the market only if it can earn positive net profits. For any value of β, there are two possible scenarios to consider. Under the first scenario, there is a positive integer m, such that (i) when there are m firms in the market, each firm earns positive net profit in equilibrium, and (ii) when there are more than are m firms in the market, at least one firm earns zero net profit in equilibrium. Because we assume that a firm only enters the market if it earn positive net profit, exactly m firms will enter the market under this scenario, and m then represents the equilibrium number of firms. Under the second scenario, for any positive integer m, there exists a positive integer $n \geq m$, such that under the equilibrium with n firms in the market each firm earns positive net profit. In this case, the entry process goes on indefinitely, and in the limit, the equilibrium number of firms is infinite. In what follows, we shall denote by $n[\beta]$ the equilibrium number of firms. Under the first scenario, $n[\beta] < \infty$, while under the second scenario, $n[\beta] = \infty$.

6 The properties of the equilibrium

Proposition 1: *Under any equilibrium, the number of firms that expend a positive amount of their own resources on R&D is finite.*

PROOF: The proof is by *reductio ad absurdum.* Suppose that there exists an equilibrium under which an infinite number of firms choose to expend a positive amount of their own resources on R&D. The competition among these firms will drive the product price down to their post-innovation marginal cost, and each of them will earn zero profit in the production stage. The profit net of own R&D cost of each of these firms will then be negative, and this cannot occur in equilibrium. ■

In what follows, we denote by $n^{+}[\beta]$ the number of firms in the market that choose to spend a positive amount of their own resources on R&D under the equilibrium with β as the degree of spillovers. Note that $0 \leq n^{+}[\beta] \leq n[\beta]$. Without any loss of generality, we can assume that the first $n^{+}[\beta]$ firms are exactly the firms that choose to expend a positive amount of their own resources on R&D and that the firms which choose not to expend any of their own resources on R&D are the last $(n[\beta] - n^{+}[\beta])$ firms.

Also, for any β, let $\underline{n}[\beta]$ denote the critical number of firms such that for any positive integer $n \leq \underline{n}[\beta]$, there exists a symmetric equilibrium with innovation when n firms enter the market, but no symmetric equilibrium with innovation when there are $\underline{n}[\beta] + 1$ firms in the market. It is clear that $\underline{n}[\beta] \leq n[\beta]$. Note that when $\underline{n}[\beta] = n[\beta]$, the equilibrium is a symmetric equilibrium with innovation under which exactly $\underline{n}[\beta]$ firms will enter the market.

When the degree of spillovers is low, a firm cannot rely on the spillovers generated by the other firms to lower its own marginal cost. To obtain any desired reduction in marginal cost, a firm must bear most of the costs needed to run its own research lab, and this discourages entry. Furthermore, once a firm has entered the market, it must expend a substantial amount of its own resources to generate a given level of cost reduction. Hence when the degree of spillovers is low, we can expect a small number of firms to enter the market, and once a firm has entered the market, it will expend a positive amount of its own resource on R&D. These intuitive results are confirmed in Proposition 2.

Proposition 2: *When the degree of spillovers is low and the initial common marginal cost is close to the choke price, the number of firms that enter the market is* $\underline{n}[\beta]$, *and the equilibrium is a symmetric equilibrium with innovation.*

PROOF: Under the symmetric equilibrium with innovation that prevails after $\underline{n}[\beta]$ firms have entered the market, the product price, according to (20), is given by

$$(31) \quad p\left[\underline{n}[\beta], \beta\right] = \frac{a + n\left(c^{0} - f[X[\underline{n}[\beta], \beta]]\right)}{n+1}.$$

Observe that $p\left[\underline{n}[\beta], \beta\right]$ will much lower than c^{0} if c^{0} is close to the choke price a. Now if the first $\underline{n}[\beta]$ firms continue to use the strategies associated with the symmetric equilibrium with $\underline{n}[\beta]$ firms, then firm $\underline{n}[\beta] + 1$, which does not spend any of its own resources on R&D, will not manage to lower its cost below $p\left[\underline{n}[\beta], \beta\right]$ because of the low degree of spillovers, and thus will not be able to produce any positive output in order to earn positive profits. Thus, no more entry will take place after $\underline{n}[\beta]$ firms have entered the market. ■

The following lemma asserts that at some stage during the entry process, if an equilibrium under which none of the firms chooses to expend its own resource on R&D is reached, then the entry process will continue indefinitely, and in the limit the equilibrium is the competitive equilibrium without innovation.

Lemma 1: *For any value of β, if there exists a positive integer n such that under the equilibrium with β as the degree of spillovers and n as the number of firms in the market no firm chooses to expend its own resource on R&D, then the entry process continues indefinitely, and in the limit, the resulting equilibrium is the competitive equilibrium without innovation.*

PROOF: Suppose that n firms have already entered the market. Let us consider the problem faced by firm $n+1$, which is contemplating entering the market. Let us imagine that the marginal cost of firm $n+1$ in the production stage is equal to the product's choke price. Under such a scenario, this firm will choose not to produce any positive level of output, and this means that the equilibrium in the production stage with firms $1, \ldots, n, n+1$ in the market is identical with the equilibrium with n firms in the market. Now let us lower the marginal cost of firm $n+1$ from the level of the product's choke price to c^0, the initial common marginal cost, while maintaining the marginal costs of firm 1 to firm n at c^0. During the process, the profit in the production stage of each of the firms $1, \ldots, n$ will be falling.[4] When the marginal cost of firm $n+1$ descends to c^0, each of the $n+1$ firms will produce the same level of output at the same marginal cost c^0, and earns the same level of profit in the production stage. Next, note that according to the hypothesis of the lemma, none of the firms finds it profitable to expend its own resource on R&D when the $n-1$ remaining firms choose not to incur any R&D cost on their own. Hence when one more firm enters the market and n of them choose not to expend any of their own resources on R&D, the remaining firm, which faces more competition when there are $n+1$ firms in the market than when there are n firms in the market, will not find it profitable, either, to expend its own resource on R&D. We have just demonstrated that under the equilibrium with $n+1$ firms in the market none of the firms chooses to expend its own resource on R&D.

[4] It is well known in Cournot oligopoly theory with linear demand curve and constant marginal costs that a fall in the marginal cost of a firm improves its profitability, but reduces the profit made by each of the other firms.

The argument just presented can be repeated *ad infinitum* to show that the entry process will continue indefinitely, and in the limit the equilibrium is the competitive equilibrium without innovation. ■

Lemma 2: *Suppose that for some* β*, there exists a positive integer* n *such that under the equilibrium with* β *as the degree of spillovers and with* n *as the number of firms in the market none of the firms chooses to expend any of its own resource on R&D. Then for any* β', $\beta < \beta' \leq 1$*, there exists a positive integer* $n' \leq n$ *such that under the equilibrium with* β' *as the degree of spillovers and with* n' *as the number of firms in the market none of the firms chooses to expend any of its own resource on R&D.*

PROOF: According to the hypotheses of Lemma 1, if β is the degree of spillovers then none of the n firms will expend a positive amount of its own resource on R&D when the remaining $n-1$ firms choose not to spend any positive amount of their own resources on R&D. This statement is still true when the degree of spillovers is $\beta' > \beta$, and when the number of firms in the market is still n. ■

Lemma 3: *If* β *is close to 1, then there exists a positive integer* n *such that under the equilibrium with* β *as the degree of spillovers and with* n *as the number of firms in the market none of the firms chooses to expend any of its own resource on R&D.*

PROOF: The proof of Lemma 3 requires some limiting arguments, and is given in Annex A.

When there are full spillovers ($\beta = 1$), the entire own R&D expenditure made by a firm flows freely to its rivals, and any rival firm – at no cost of its own – can obtain the same cost reduction obtained by the former firm. The full R&D spillovers destroy the incentive for any firm to spend its own resources on R&D. Indeed, if in equilibrium a firm chooses to incur a positive level of own R&D cost, then an infinite number of firms will enter the market and operate at the same marginal cost as the firm that spends a positive amount of its own resource on R&D. Under this scenario, there will be perfect competition in the post-innovation stage, with the ensuing consequence that all the firms in the market will earn zero profit (gross of own R&D cost) in the production stage. Thus, when there are full spillovers the equilibrium market structure is that of perfect competition without innovation. More generally, when the degree of spillovers is high, a firm that chooses to expend a positive amount of its own resource on R&D bears the entire burden of its own R&D cost, but most of the fruit of its R&D activities flow freely to its rivals, and this destroys the incentive for a firm to spend its own resource on R&D, as asserted by the following proposition.

Proposition 3: *If β is close to 1, then an infinite number of firms will enter the market, and a firm – once it has entered the market – will choose not to spend any of its own resources on R&D. That is, when the degree of spillovers is high, there will be no innovation, and the resulting equilibrium market structure is that of perfect competition without innovation.*

PROOF: To prove Proposition 3, invoke Lemma 3 and then Lemma 1. ■

Proposition 2 asserts the existence of a symmetric equilibrium with innovation when the degree of spillovers is low, while Proposition 3 asserts that perfect competition without innovation will prevail when the degree of spillovers is high. Proposition 4 asserts the existence of an asymmetric equilibrium – an equilibrium under which some firms choose to expend a positive amount of their own resources on R&D while other firms choose not to – when the degree of spillovers take on the intermediate values. Proposition 4 asserts that when the degree of spillovers is low, a firm that wishes to enter the market cannot rely on the spillovers to lower its own marginal cost and must bear most of the burden required to lower its own marginal cost. This factor reduces the incentive for entry, and the equilibrium is a symmetric equilibrium with innovation under which a small number of firms choose to enter the market. When the degree of spillovers has risen sufficiently, the spillovers allow more firms to enter the market, and some of the entrants can free ride on the R&D activities of other entrants: the equilibrium is then an asymmetric equilibrium. When the degree of spillovers reaches a high level, there is no incentive for any firm to expend its own resources on R&D, and the resulting equilibrium is the competitive equilibrium without innovation.

Proposition 4: *Suppose that pre-innovation common marginal cost is high. There exist two values of β, say $\underline{\beta}$and $\overline{\beta}$, with $0 < \underline{\beta} < \overline{\beta} < 1$, which have the following properties:*

(i) *For all* $0 < \beta < \underline{\beta}$, *the equilibrium with* β *as the degree of spillovers is a symmetric equilibrium with innovation.*

(ii) *For all* $\beta, \overline{\beta} < \beta \leq 1$, *the equilibrium with* β *as the degree of spillovers is the competitive equilibrium without innovation.*

(iii) *There exists a value of* $\beta \in \left(\underline{\beta}, \overline{\beta}\right)$, *such that the equilibrium with* β *as the degree of spillovers is an asymmetric equilibrium.*

PROOF: The proof of Proposition 4 is technical and is given in Annex B.

Lemma 4 deals with the effect of low degrees of spillovers on consumers' surplus, producers' surplus, and social welfare. In proving (iii) of Lemma 4, we assume that the R&D technology is given by (2).

Lemma 4: *When* β *rises in a small right neighborhood of 0, (i) consumers' surplus fall (ii) producers' surplus rises; and (iii) social welfare rises.*

PROOF: The proof of Lemma 4 involves the computations of various derivatives, and is quite technical. It is relegated to Annex C.

The following proposition describes how the degree of spillovers affects social welfare.

Proposition 5: *Suppose that the functional form of the R&D production function is given by (2). As the degree of spillovers rises from 0 to 1, social welfare first rises with* β, *reaches a maximum, and then declines to the lowest possible level, which is the social welfare associated with the competitive equilibrium.*

PROOF: Proposition 5 follows immediately from Proposition 2, Proposition 3, and Lemma 4. ■

The economic contents of Proposition 5 embody the tension between allocative efficiency and dynamic efficiency. When the degree of spillovers is low, a firm can appropriate most of the fruit of its own R&D, and this encourages innovation. This incentive is reduced when the degree of spillovers rises, and each firm spends less of its own resources on R&D. On the other hand, the rise in the degree of spillovers encourages entries. Social welfare rises with the degree of spillovers when it rises slightly above zero because the allocative efficiency effect dominates the dynamic efficiency effect. After the degree of spillovers has reached a sufficiently high level, the spillovers discourage firms from spending their own resources on R&D, with the ensuing consequence that the equilibrium is the competitive equilibrium without innovation: the rise in allocative efficiency generated by the high number of firms scarcely counteracts the dramatic loss in dynamic efficiency. For intermediate values for the degree of spillovers, the initial rise in social welfare is reversed when the gain in allocative efficiency cannot offset the loss in dynamic efficiency.

7 A numerical simulation

In the numerical simulation, the R&D technology is assumed to have the functional form represented by (2).To compute the equilibrium number of firms for the two-stage game, we proceed as follows. We begin with the monopoly case, and then successively raise the number of firms by 1 each time. At each step, use (29) to compute the effective R&D expenditure of the symmetric equilibrium with innovation for the current number of firms. If the symmetric equilibrium with innovation when there are n firms cannot deter entry, then raise the number of firms to $n+1$, and then try to compute the symmetric equilibrium with innovation when $n+1$ firms are in the market. If there is no symmetric equilibrium with innovation for the case of $n+1$ firms – either because there is no positive value of X that satisfies the first-order condition (24) or because there exists a positive value of X that satisfies the first-order condition (24), but using it will result in a negative profit net of R&D cost for each of the $n+1$ firms – then the equilibrium must be an asymmetric equilibrium under which some firms choose to incur a positive level of own R&D cost, while others choose not to do so.

At any step of the procedure just described, entry will occur if it is profitable for new firms to enter the market. The first time it is not profitable for a new firm to enter the market, we have found the equilibrium number of firms. The following table summarizes the results of the simulations[5] we carried out for various values of β.

[5] The results of the simulation are obtained with the help of a large number of *Mathematica* programs.

Table 1.
The equilibria for various values of β (Parameter values: $a = 3, b = 0.25, c^0 = 2.75, \gamma = 2.75$)

	$n[\beta]$	$n^+[\beta]$	$x^+[\beta]$	$c_i^1, i \leq n^+[\beta]$	$c_i^1, i > n^+[\beta]$	$p[\beta]$	$PS[\beta]$	$CS[\beta]$	$SW[\beta]$
$\beta = 0$	4	4	1.21	0.10	NA	0.68	0.54	10.78	11.32
$\beta = 0.1$	5	5	0.81	0.12	NA	0.60	0.57	11.49	12.06
$\beta = 0.2$	88	6	0.45	0.24	0.63	0.63	1.06	11.22	12.28
$\beta = 0.3$	∞	6	0.30	0.35	0.63	0.63	0.02	11.25	11.27
$\beta = 0.4$	∞	4	0.30	0.46	0.74	0.74	0.14	10.19	10.33
$\beta = 0.5$	∞	3	0.28	0.60	0.88	0.88	0.10	8.96	9.06
$\beta = 0.6$	∞	2	0.27	0.83	1.12	1.12	0.13	7.09	7.22
$\beta = 0.7$	∞	1	0.29	1.23	1.57	1.57	0.16	4.09	4.25
$\beta = 0.8$	∞	0	NA	NA	NA	2.75	0	0.125	0.125
$\beta = 0.9$	∞	0	NA	NA	NA	2.75	0	0.125	0.125
$\beta = 1$	∞	0	NA	NA	NA	2.75	0	0.125	0.125

The first row of the table gives the equilibrium for the case of no spillovers ($\beta = 0$). Under this equilibrium, 4 firms enter the market ($n[\beta] = 4$), and each of them ($n^+[\beta] = 4$) chooses to spend a positive amount ($x^+[\beta] = 1.21$) of its own resource on R&D. The equilibrium is thus a symmetric equilibrium with innovation. The post-innovation marginal cost is $c_i^1 = 0.10, i = 1, \ldots, n[\beta]$. The equilibrium product price is $p[\beta] = 0.68$. As for welfare, the producers' surplus (net of R&D cost) is $PS[\beta] = 0.54$ and the consumer surplus is $CS[\beta] = 10.78$, which sum up to a level of social welfare given by $SW[\beta] = 11.32$.

The second line of the table gives the equilibrium for the case $\beta = 0.1$. The equilibrium is a symmetric equilibrium with innovation under which 5 firms enter the market. The amount of own resource that a firm spends on R&D is 0.81, which yields an effective R&D of $0.81(1 + (5-1)(0.1)) = 0.81(1.4) = 1.134$, which is less than 1.21, the effective R&D expenditure of each firm under the symmetric equilibrium with innovation for $\beta = 0$. The post-innovation cost is $c_i^1 = 0.12, i = 1, \dots, n[\beta]$. The equilibrium product price is 0.60, which is lower than the value it takes under the symmetric equilibrium with innovation for the case of no spillovers. The lower product price implies a higher level of consumers' surplus. The producers' surplus is also higher. The end result is a higher level of social welfare. The spillovers make it possible for one more firm to enter the market. Although the post-innovation marginal cost is higher than when there are no spillovers, the gain in welfare due to a higher number of firms more than offset the impact of the higher post-innovation marginal cost relatively to the case of no spillovers.

The third line gives the equilibrium for the case $\beta = 0.2$. The equilibrium number of firms is now 88, with 6 firms choosing to spend a positive amount of their own resources ($x^+[\beta] = 0.45$) on R&D, while the remaining firms choose not to incur any own R&D cost. The equilibrium is asymmetric. The post-innovation marginal cost for the firms that spend a positive amount of their own resources on R&D is $c_i^1 = 0.24, i = 1, \dots, n^+[\beta]$. The post-innovation marginal cost for those who free ride on the R&D activities of other firms is $c_i^1 = 0.63, i = n^+[\beta] + 1, \dots, n[\beta]$. The equilibrium product price is 0.63, which is slightly higher than that under the equilibrium for the case $\beta = 0.1$, and this implies a slightly lower level of consumers' surplus ($11.22 < 11.49$). The much lower own R&D cost of each firm helps to raise the producers' surplus substantially ($1.06 > 0.57$). Social welfare is higher ($12.28 > 12.06$) than the level attained for the case $\beta = 0.1$.

The fourth line gives the equilibrium for the case $\beta = 0.3$. The equilibrium number of firms is now infinite, with 6 firms choosing to spend a positive amount of their own resources ($x^+[\beta] = 0.30$) on R&D, while the remaining firms choose not to incur any own R&D cost. The equilibrium is asymmetric. The post-innovation marginal cost for the firms that spend a positive amount of their own resources on R&D is $c_i^1 = 0.35, i = 1, \dots, n^+[\beta]$. The post-innovation marginal cost for those who free ride on the R&D activities of other firms is $c_i^1 = 0.63, i =$

$n^+[\beta]+1,\ldots,n[\beta]$. The equilibrium product price is 0.63. The producers' surplus is 0.02 and the consumers' surplus is 11.25, which sum up to a social welfare level of 11.27. Note that social welfare has dropped when β rises from 0.2 to 0.3. The fall in social welfare is due to the lower own R&D expenditures made by the firms that choose to do so and the competition from the fringe made possible by the higher degree of spillovers.

The fifth line gives the equilibrium for the case $\beta = 0.4$. The equilibrium number of firms is infinite, with 4 firms choosing to spend a positive amount of their own resources ($x^+[\beta] = 0.30$) on R&D, while the remaining firms choose not to incur any own R&D cost. The equilibrium is asymmetric. The post-innovation marginal cost for the firms that spend a positive amount of their own resources on R&D is $c_i^1 = 0.46, i = 1,\ldots,n^+[\beta]$. The post-innovation marginal cost for those who free ride on the R&D activities of other firms is $c_i^1 = 0.74, i \geq n^+[\beta]+1$. The equilibrium product price is 0.74. The producers' surplus is 0.14 and the consumers' surplus is 10.19, which sum up to a social welfare level of 10.33. Note that the producers' surplus rises when β rises from 0.3 to 0.4 because of the lower number of firms that incur a positive level of own R&D cost. Also, note that social welfare has dropped when β rises from 0.3 to 0.4. The fall in social welfare is due to the lower own R&D expenditures made by the firms that choose to do so and the competition from the fringe made possible by the higher degree of spillovers.

The sixth line gives the equilibrium for the case $\beta = 0.5$. An infinite number of firms enter the market, and among them 3 firms choose to spend a positive amount of their own resources on R&D. The post-innovation marginal cost of the 3 dominant firms is 0.60, while the post-innovation marginal cost of the firms of the fringe is 0.88. Relatively to the case $\beta = 0.5$, the producers' surplus (0.10) is higher; the consumers' surplus is lower; and social welfare is lower.

The seventh line gives the equilibrium for the case $\beta = 0.6$. An infinite number of firms enter the market, and among them 2 firms choose to spend a positive amount of their own resources on R&D. The post-innovation marginal cost of the 2 dominant firms is 0.83, while the post-innovation marginal cost of the firms of the fringe is 1.12. Relatively to the case $\beta = 0.6$, the producers' surplus (0.13) is higher; the consumers' surplus is lower; and social welfare (7.22) is lower.

The eighth line gives the equilibrium for the case $\beta = 0.7$. An infinite number of firms enter the market, and among them only one firm chooses to spend a positive amount of their own resources on R&D. The post-innovation marginal cost of the dominant firm is 1.23, while the post-innovation marginal cost of the firms of the fringe is 1.57. Relatively to the case $\beta = 0.7$, the producers' surplus (0.13) is higher; the consumers' surplus is lower; and social welfare (4.25) is lower.

The last three lines of the table give the equilibria for the cases $\beta = 0.8, 0.9,$ and 1.0, respectively. An infinite number of firms enter the market, and the resulting equilibrium market structure is perfect competition without innovation. Under perfect competition, the market price is equal to the initial common marginal cost; the producers' surplus is 0; and the consumers' surplus is 0.125. Thus, high values for the degree of spillovers yield the lowest level of social welfare possible.

The numerical simulation indicates that social welfare is at its highest level at $\beta = 0.2$. The market structure is that of an asymmetric equilibrium under which some firms expend a positive amount of their own resources on R&D while many firms free ride on the R&D activities of others to lower their own marginal cost.

8 Conclusion

In this essay we have formulated a model of the AJ type to analyze the impact of R&D spillovers on entry and the resulting equilibrium market structure. We find that the degree of spillovers plays a fundamental role on the number of firms entering the market, their R&D activities, and social welfare. Our analysis suggests that social welfare is maximized at some intermediate degree of spillovers. The policy implication of this result is that neither complete protection of intellectual property right nor lax enforcement of patent laws is socially optimal. Uncertainty and risk are important factors in R&D, but they have been ignored in the literature. These factors merit more attention. A more complete modeling of the innovation process should include an examination of the major drivers influencing the degree of spillovers: distance between the innovators, property rights, and the extent of telecommunication networks.

Annex A: The Proof of Lemma 3

We prove Lemma 3 by *reductio ad absurdum*. If Lemma 3 is not true, then we can find a sequence $\beta_k, k = 0,1, \ldots$, with $\beta_k < 1, \beta_k \uparrow 1$, such that for any k and any positive integer m, at least one firm will expend a positive amount of its own resource on R&D under the equilibrium with β_k as the degree of spillovers and m as the number of firms in the market. If the equilibrium number of firm when β_k is the degree of spillovers is finite, then a possible value for m is $m_k = n[\beta_k]$. When $n[\beta_k] = \infty$, a possible value for m is a positive integer m_k that satisfies the condition $m_k > \frac{1}{1-\beta_k}$.

According to (16), the following first-order condition characterizes the own R&D expenditure of a firm, say firm i, which expends a positive amount of its own resource on R&D under the equilibrium with β_k as the degree of spillovers and m_k as the number of firms in the market:

$$\text{(A.1)} \quad -1 + \frac{2}{b(m_k+1)^2} \begin{pmatrix} a - m_k\left(c^0 - f\left[x_i + \beta_k \sum_{j \neq i} x_j\right]\right) \\ + \sum_{j \neq i}\left(c^0 - f\left[x_j + \beta_k\left(x_i + \sum_{j' \neq i, j' \neq j} x_{j'}\right)\right]\right) \end{pmatrix} \times \begin{pmatrix} m_k f'\left[x_i + \beta_k \sum_{j \neq i} x_j\right] \\ -\beta_k \sum_{j \neq i} f'\left[x_j + \beta_k\left(x_i + \sum_{j' \neq i, j' \neq j} x_{j'}\right)\right] \end{pmatrix} = 0.$$

Now because $\beta_k \uparrow 1$ when $k \to \infty$, all of the firms, regardless of whether they choose to expend their own resources on R&D or choose to free ride on the R&D activities of others, will have the same post-innovation marginal cost in the limit. Hence the equilibrium number of firms, namely $n[\beta_k]$ will be indefinitely large when $k \to \infty$, and this implies $\ell im_{k \to \infty} m_k = \infty$. Furthermore, the total industry R&D expenditure must tend to 0 when $k \to \infty$. Indeed, if this is not the case, then the producers' surplus – the total industry profits in the production stage minus the total industry R&D expenditure – will be negative, and this cannot hold in equilibrium. Thus, in the limit, the first-order condition (A.1) becomes

$$\text{(A.2)} \quad -1 + \left(\ell im_{k \to \infty} \frac{2}{b(m_k]+1)^2}\right)(a - c^0) f'[0] = -1 < 0,$$

which is not consistent with (A.1) in the limit. ■

Annex B: The Proof of Proposition 4

Let $\underline{\beta}$ be the least upper bound of the degrees of spillovers β, such that for all $0 < \beta' < \beta$, the equilibrium associated with β' is a symmetric equilibrium with innovation. The existence of $\underline{\beta}$ follows directly from Proposition 2. Using Proposition 3, we can assert the existence of a value, say $\overline{\beta} < 1$, for the degree of spillovers, such that for all $\overline{\beta} < \beta \leq 1$, the equilibrium that prevails is perfect competition without innovation. To prove Proposition 4, we first establish a series of claims.

Claim 1: *We have* $\underline{\beta} < \overline{\beta}$.

PROOF: First, we claim that it is not possible to have $\overline{\beta} < \underline{\beta}$. Indeed, if this were the case, then for each value of $\beta \in \left(\overline{\beta} < \underline{\beta}\right)$, the equilibrium associated with β is both a symmetric equilibrium with innovation under which each firm earns net positive profit and the perfectly competitive equilibrium without innovation, and this is absurd. Thus, $\underline{\beta} \leq \overline{\beta}$. ■

Claim 2: $\underline{\beta} \neq \overline{\beta}$.

PROOF: The claim is proved by *reductio ad absurdum*. Suppose that $\underline{\beta} = \overline{\beta}$. Using the definition of $\underline{\beta}$, we can find a symmetric equilibrium with innovation when the degree of spillovers is $\underline{\beta} - \frac{1}{k}$ for large positive integers k. Such an equilibrium will converge to the equilibrium with $\underline{\beta}$ as the degree of spillovers when $k \rightarrow \infty$. Furthermore, because the first-order condition (18) must be satisfied by each of these equilibria, in the limit, the equilibrium number of firms under the equilibrium with $\underline{\beta}$ as the degree of spillovers must be finite, i.e., $n[\underline{\beta}] < \infty$.

Using the definition of $\overline{\beta}$, we can find a sequence of degrees of spillovers $(\beta_k)_{k=1}^{\infty}$, with $\beta_k \downarrow \overline{\beta}$, such that for each $k = 1,2, ...$, the equilibrium with β_k as the degree of spillovers is a competitive equilibrium without innovation. In the limit when $k \rightarrow \infty$, these equilibria converge to a competitive equilibrium without innovation. Thus, the equilibrium number of firms when $\overline{\beta}$ is the degree of spillovers is $n[\overline{\beta}] = \infty$. Thus, if $\underline{\beta} = \overline{\beta}$, then the equilibrium number of firms when the degree of spillovers is $\underline{\beta}$ will be both finite and infinite, and this is absurd. ■

Claim 3*: For some* $\epsilon > 0$ *sufficiently small, there exists an asymmetric equilibrium with* $\beta \in \left(\underline{\beta}, \underline{\beta} + \epsilon\right)$.

PROOF: First, note that there is no perfectly competitive equilibrium without innovation for each possible value for the degree of spillovers $\beta \in \left(\underline{\beta}, \underline{\beta} + \epsilon\right)$. Indeed, if this is not true, then we can find a sequence of degrees of spillovers decreasing to $\underline{\beta}$, such that the equilibrium associated with each of these degrees of spillovers is a competitive equilibrium without innovation, and this cannot be true by the argument used to establish Claim 2. Next, note that if there is no asymmetric equilibrium associated with some $\beta \in \left(\underline{\beta}, \underline{\beta} + \epsilon\right)$, then the equilibria associated with all $\beta \in \left(\underline{\beta}, \underline{\beta} + \epsilon\right)$ must be symmetric equilibria with innovation, and this contradicts the fact that $\underline{\beta}$ is the least upper bound of the values of β such that for all $\beta' < \beta$, the equilibrium with β' as the degree of spillovers is a symmetric equilibrium with innovation.

Together, the three claims constitute Proposition 4. ■

Annex C: The Proof of Lemma 4

To prove (i) of Lemma 4, first note that the consumers' surplus under a symmetric equilibrium with innovation is given by

(C.1) $\quad CS[n,\beta] = \frac{1}{2}(a - p[n,\beta])Q[n,\beta],$

where, according to (9),

(C.2) $\quad p[n[\beta],\beta] = \frac{a+n\left(c^0 - f[X[n[\beta],\beta]\right)}{n+1}$

is the equilibrium product price, and

(C.3) $\quad Q[n[\beta],\beta] = \frac{na-n(c^0-f[X[n[\beta],\beta]])}{(n[\beta]+1)b}$,

according to (8), is the equilibrium industry output.

Now the equilibrium number of firms when there are no spillovers is $n[0]$. As β rises slightly from 0, the equilibrium number of firms remains at the same level, i.e., $n[\beta] = n[0]$ when β is small. Next, note that for $n[\beta] = n[0]$, the first-order condition (18) shifts downward as β rises slightly from 0. Hence the equilibrium effective expenditure under the symmetric equilibrium with innovation falls – and this means that the equilibrium product price (C.2) rises – when β rises slightly from 0. The rise in the equilibrium product price implies a fall in the consumers' surplus, and (i) of Lemma 4 is proved.

To prove (ii) of Lemma 4, first, note that the producers' surplus under a symmetric equilibrium with innovation is given by

$$PS[n[\beta],\beta] = -\frac{n[\beta]X[n[\beta],\beta]}{1+(n[\beta]-1)\beta} + \frac{n[\beta]}{b(n[\beta]+1)^2}(a-c^0+f[X[n[\beta],\beta]])^2,$$

which assumes the following form when the degree of spillovers is low:

(C.4) $$PS[n[0],\beta] = -\frac{n[0]X[n[0],\beta]}{1+(n[0]-1)\beta} + \frac{n[0]}{b(n[0]+1)^2}(a-c^0+f[X[n[0],\beta]])^2.$$

Differentiating (C.4) with respect to β, we obtain

(C.5) $$\frac{1}{n[0]}\frac{\partial PS[n[0],\beta]}{\partial\beta} = -\frac{1}{1+(n[0]-1)\beta}\frac{\partial X[n[0],\beta]}{\partial\beta} + \frac{(n[0]-1)X[n[0],\beta])}{(1+(n[0]-1)\beta)^2} +$$

$$+\frac{2(a-c^0+f[X[n[0],\beta]])f'[X[n[0],\beta]]}{b(n[0]+1)^2}\frac{\partial X[n[0],\beta]}{\partial\beta}.$$

Multiply (C.5) with $(n[0]-(n[0]-1)\beta)$, we obtain

(C.6) $$\frac{(n[0]-(n[0]-1)\beta)}{n[0]}\frac{\partial PS[n[0],\beta]}{\partial\beta} = -\frac{(n[0]-(n[0]-1)\beta)}{1+(n[0]-1)\beta}\frac{\partial X[n[0],\beta]}{\partial\beta} + \frac{(n[0]-(n[0]-1)\beta)(n[0]-1)X[n[0],\beta])}{(1+(n[0]-1)\beta)^2} +$$

$$+\frac{2(n[0]-(n[0]-1)\beta)(a-c^0+f[X[n[0],\beta]])f'[X[n[0],\beta]]}{b(n+1)^2}\frac{\partial X[n[0],\beta]}{\partial\beta}$$

$$= -\frac{(n[0]-(n[0]-1)\beta)}{1+(n[0]-1)\beta}\frac{\partial X[n[0],\beta]}{\partial \beta} + \frac{(n[0]-(n[0]-1)\beta)(n[0]-1)X[n,\beta])}{(1+(n[0]-1)\beta)^2} + \frac{\partial X[n[0],\beta]}{\partial \beta}.$$

Note that the third line in (C.6) has been obtained by using the first-order condition (18), which asserts that

$$\frac{2(n[0]-(n[0]-1)\beta)\left(a-c^0+f\left[X[n[0],\beta]\right]\right)f'\left[X[n[0],\beta]\right]}{b(n[0]+1)^2} = 1.$$

Thus,

$$\text{(C.7)} \quad \frac{(n[0]-(n[0]-1)\beta)}{n[0]}\frac{\partial PS[n[0],\beta]}{\partial \beta} = \left(1 - \frac{(n[0]-(n[0]-1)\beta)}{1+(n[0]-1)\beta}\right)\frac{\partial X[n[0],\beta]}{\partial \beta} + \frac{(n[0]-(n[0]-1)\beta)(n[0]-1)X[n[0],\beta])}{(1+(n[0]-1)\beta)^2}$$

When $\beta \to 0$, (C.7) becomes

$$\text{(C.8)} \quad \frac{\partial PS[n[0],0]}{\partial \beta} = (1-n[0])\frac{\partial X[n[0],0]}{\partial \beta} + n[0](n[0]-1)X[n[0],0] > 0.$$

Note that the strict inequality (C.8) follows from the result $\frac{\partial X[n,\beta]}{\partial \beta} < 0$ and the fact that $n > 1$. That is, $\frac{\partial PS[n[\beta],\beta]}{\partial \beta} > 0$ when β is small. Hence the producer surplus rises with β when β is small, and (ii) of Lemma 4 is proved.

To prove (iii) of Lemma 4, first, note that social welfare is given by

$$\text{(C.9)} \quad SW[n,\beta] = CS[n,\beta] + PS[n,\beta].$$

For low degrees of spillovers, we have[6]

$$\text{(C.10)} \quad \frac{\partial SW[n[0],0]}{(n[0]-1)\partial \beta} = \frac{(n[0]-2)\left(a\sqrt{n[0]\gamma}+\sqrt{-2b(1+n[0])^2+a^2n[0]\gamma}\right)-4n[0]^2\sqrt{-2b(1+n[0])^2+a^2n[0]\gamma}\ \mathrm{Log}[Z[n[0],0]}{4n[0]\gamma\sqrt{-2b(1+n[0])^2+a^2n[0]\gamma}}.$$

[6] The symbolic calculations were carried out by *Mathematica*.

Because $n[\beta] \geq 2$, and because $Z[n, \beta]$, as given by (30), is less than 1, the right side of (C.10) is positive. Hence as β rises in a right neighbourhood of 0, social welfare also rises with β, and (iii) of Lemma 4 is proved. ■

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