# Aggregate efficiency and interregional equity: a contradiction?

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15 June 2012

# Aggregate Efficiency and Interregional Equity: A Contradiction?

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**Abstract**

This paper attempts to rekindle interest on regional allocation of investment and to show that a trade-off between aggregate efficiency and interregional equity is implied. Modifying, however, the objective function it is established that this trade-off can be avoided.

JEL Classification: R10

## 1. Introduction

A major concern for policy-makers is how to allocate resources across space (regional investment) in order to achieve aggregate efficiency (maximum output) without increasing regional inequalities (interregional equity). The purpose of this paper is to contribute in that direction using Optimal Control Theory (hereafter OCT) and amending the framework developed by Intriligator (1964). The rest of the paper is laid out as follows. Section 2 summarises what we can learn from the model of regional allocation of investment. Section 3 then outlines an alternative framework, modifying the objective function by attaching a 'weight' in each region. Section 4 summarises the arguments and considers the lessons for policy making.

## 2. Regional Allocation of Investment

A starting point is provided by Intriligator (1964), who building upon the work of Rahman (1963), showed that OCT can be applied in order to maximise national output ($Y_N$) in a 'two-region' economy at some terminal time ($T$). Given a

* The findings, interpretations and conclusions are entirely those of the authors and, do not necessarily represent the official position, policies or views of the Ministry of Rural Development & Foods and/or the Greek Government.

production function $Y_i = v_i K_i$, $i = 1,2$, fixed capital coefficient and assuming that a constant proportion of output is saved ($S_i = s_i Y_i$), capital accumulation evolves as $\dot{K}_1 + \dot{K}_2 = \gamma_1 K_1 + \gamma_2 K_2$ [1]. Intriligator (1964) assumes that once capital is placed in one region, it cannot be shifted into the other region. Total savings are pooled in a central agency and allocated to each region, given an 'allocation parameter' ($\delta$). The objective function is, therefore, $\underset{\delta}{Max} Y_N(T)$, subject to the constraints, given by equations (1), (2) and (2) below.

$$\dot{K}_1 = \delta(\gamma_1 K_1 + \gamma_2 K_2) \qquad (1)$$

$$\dot{K}_2 = (1-\delta)(\gamma_1 K_1 + \gamma_2 K_2) \qquad (2)$$

$$0 \leq \delta \leq 1 \qquad (3)$$

Assuming constant returns, at any point in time either $\delta^*(t) = 0$ or $\delta^*(t) = 1$, The problem can be solved by identifying the value of $\delta(t)$ that maximises the Hamiltonian function, $H = [\delta(p_1 - p_2) + p_2](\gamma_1 K_1 + \gamma_2 K_2)$, where $p_1$ and $p_2$ are the auxiliary variables, frequently referred to as the implicit price of capital. The optimal path of $\delta$ depends on the sign of the difference $p_1(t) - p_2(t)$. Put simply, the optimal solution suggests that the funds should be invested in the region where the shadow price of capital is higher. Thus, if $p_1(t) - p_2(t) > 0$, then $\delta^* = 1$ while if $p_1(t) - p_2(t) < 0$, then $\delta^* = 0$. The Hamiltonian system must satisfy the conditions $\dot{p}_1 = -\frac{\partial H}{\partial K_1} = -\gamma_1 \max\{p_1, p_2\}$ and $\dot{p}_2 = -\frac{\partial H}{\partial K_2} = -\gamma_2 \max\{p_2, p_1\}$, implying that $\dot{p}_1 = -[\delta(p_1 - p_2) + p_2]\gamma_1$ and $\dot{p}_2 = -[\delta(p_1 - p_2) + p_2]\gamma_2$. Therefore,

$$p_1(t) - p_2(t) = p_2(t)\left(\frac{\gamma_1 - \gamma_2}{\gamma_2}\right) \qquad (4)$$

The auxiliary variables must satisfy the terminal conditions $p_1(T) = \frac{\partial Y_N(T)}{\partial K_1(T)} = v_1$ and $p_2(T) = \frac{\partial Y_N(T)}{\partial K_2(T)} = v_2$. Given that $\frac{p_1(T)}{p_2(T)} = \frac{v_1}{v_2}$, then

$$p_1(T) - p_2(T) = p_2(T)\left(\frac{v_1 - v_2}{v_2}\right) \qquad (5)$$

[1] The term $\gamma_i = s_i v_i$ can be interpreted as the autonomous growth rate of each region.

For a given planning period, $[0\ldots T]$, $\delta^*(t)=1$ at $0\le t<T$ if $\gamma_1>\gamma_2$ or $\delta^*(t)=0$ if $\gamma_1<\gamma_2$. Conversely, at $t=T$ invest only in the region with the highest output/capital ratio; that is to say if $v_1>v_2$, then $\delta^*(t)=1$ while if $v_1<v_2$, then $\delta^*(t)=0$. Following Takayama (1967), setting $\delta=0$ yields $\dot{p}_2=-p_2\gamma_2$, which is a first order differential equation with the solution $p_2(t)=v_2e^{\gamma_2(T-t)}$. Solving for $t$, the switching time can be estimated:

$$t^*=T-\frac{1}{\gamma_2}\log\left(\frac{s_1-s_2}{\gamma_1-\gamma_2}v_1\right) \tag{6}$$

In the model so far, the possibility of uneven distribution of regional incomes is not considered, at least explicitly. Nevertheless, a number of interesting implications can be derived. In order to have a concrete vocabulary, define the initial income gap between region 1 and 2 as $G(0)=Y_1(0)-Y_2(0)$. Assuming that $G(0)>0$, $\gamma_1>\gamma_2$ and $v_1>v_2$, then the optimal solution $\delta^*(t)=1$, $\forall\, t\in[0\ldots T]$ yields maximum output but at the expanse of closing the income 'gap' between region 1 and 2, i.e. a trade-off between aggregate efficiency and regional equity. When $\gamma_1>\gamma_2$ and $v_1<v_2$ a switch in the allocation parameter is necessary if total output is to be maximised. This will also reduce regional inequalities at $t=T$ [2]. A steady elimination of the income gap between the two regions while a constant increase in total output is feasible if $\gamma_1<\gamma_2$ and $v_1<v_2$. Given that $v_1<v_2$ and $\gamma_1<\gamma_2$, the solution $\delta^*(t)=0$, $\forall\, t\in[0\ldots T]$, enables the planner to overcome the trade-off between efficiency and equity[3].

Nevertheless, it is possible to extend the argument by attaching a 'weight' in each region. While Takayama (1967) acknowledges that this possibility, nevertheless, to the best of our knowledge, this remained a rather unexplored area and is examined in the next section.

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[2] If $\gamma_1<\gamma_2$ and $v_1>v_2$, then $\delta^*(t)=0$ at $0\le t<T$ and $\delta^*(t)=1$ at $t=T$. Regional disparities can be reduced at $0\le t<T$ and increase again at the end of the period.

[3] Similarly, the trade-off is absent when $\delta^*(t)=1$, $\forall\, t\in[0\ldots T]$ if $G(0)<0$.

**3. Regional Allocation of Investment: An Alternative Perspective**

According to Intriligator (1964), the investment decision is determined exclusively by different growth potentials in each region. It is possible, however, to attach a different 'weight' to each region[4], reflecting political or social reasons. National output, therefore, can be expressed as: $Y_N(T) = \omega_i Y_i(T)$, where $\omega_i > 0$ is the weight attached to each region[5]. Maintaining OCT as the basic vehicle of analysis, the problem is defined as

$$\underset{\delta}{Max} Y_N(T) = \omega_1 v_1 K_1(T) + \omega_2 v_2 K_2(T), \text{ with } \omega_1 - \omega_2 \neq 0 \tag{7}$$

subject to the constraints defined by equations (1), (2) and (3).

It follows that $p_1(t) - p_2(t) = p_2(t)\left(\dfrac{\gamma_1 - \gamma_2}{\gamma_2}\right)$ while the terminal conditions $p_1(T) = \omega_1 v_1$ and $p_2(T) = \omega_2 v_2$, imply that

$$p_1(T) - p_2(T) = p_2(T)\left(\frac{\omega_1 v_1 - \omega_2 v_2}{\omega_2 v_2}\right) \tag{8}$$

Before the end of the planning period ($0 \leq t < T$) the optimal solution is $\delta^*(t) = 1$ if $\gamma_1 > \gamma_2$ or $\delta^*(t) = 0$ if $\gamma_1 < \gamma_2$. At the end of the planning period ($T$) if $\omega_1 v_1 > \omega_2 v_2$, then $\delta^*(t) = 1$ while if $\omega_1 v_1 < \omega_2 v_2$, then $\delta^*(t) = 0$. The switching time is given by the following expression:

$$t^{\omega} = T - \frac{1}{\gamma_2}\log\left(\frac{(\omega_2 s_1 - \omega_1 s_2)v_1}{(\gamma_1 - \gamma_2)\omega_2}\right) \tag{9}$$

Essentially, the objective function in equation (7) encapsulates two components, aggregate efficiency and interregional equity. When $G(0) > 0$ and $\omega_1 > \omega_2$, the efficiency component dominates. By analogy, if $G(0) > 0$ and $\omega_1 < \omega_2$, then the equity element is of primary concern[6]. Nevertheless, the aim $G(T) = 0$ might not be feasible. Hence, it would be more reasonable to define an aim $G(T) \to 0$. The

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[4] The notion of 'region' can be extended to include groups of geographical areas. Just as an example consider 'agricultural' and 'industrial' regions or 'northern' and 'southern' regions.

[5] From a technical standpoint attaching weights ($\sum_{i=1}^{n} \omega_i = 1$) does not alter the structure of the production functions.

[6] It is possible to consider several planning periods, e.g. a period with an exclusive domination of efficiency and regional equity appearing in the subsequent period.

investment sequence when $\omega_1 > \omega_2$ and $\omega_1 < \omega_2$ are set out in Table 1 and 2, respectively.

Table 1: Regional Allocation of Investment when $\omega_1 > \omega_2$

| | $\gamma_1 > \gamma_2$ | | $\gamma_1 < \gamma_2$ | |
|---|---|---|---|---|
| | $v_1 > v_2$ | $v_1 < v_2$ | $v_1 > v_2$ | $v_1 < v_2$ |
| $\gamma_1 - \gamma_2$ | $\gamma_1 - \gamma_2 > 0$ | $\gamma_1 - \gamma_2 > 0$ | $\gamma_1 - \gamma_2 < 0$ | $\gamma_1 - \gamma_2 < 0$ |
| $p_1(T) - p_2(T)$ | $\omega_1 v_1 - \omega_2 v_2 > 0$ | $\omega_1 v_1 - \omega_2 v_2 > 0$ | $\omega_1 v_1 - \omega_2 v_2 > 0$ | $\omega_1 v_1 - \omega_2 v_2 > 0$ |
| $\delta^*(t)$ at $0 \le t < T$ | **1** | **1** | **0** | **0** |
| $\delta^*(t)$ at $t = T$ | **1** | **1** | **1** | **1** |

Table 2: Regional Allocation of Investment when $\omega_1 < \omega_2$

| | $\gamma_1 > \gamma_2$ | | $\gamma_1 < \gamma_2$ | |
|---|---|---|---|---|
| | $v_1 > v_2$ | $v_1 < v_2$ | $v_1 > v_2$ | $v_1 < v_2$ |
| $\gamma_1 - \gamma_2$ | $\gamma_1 - \gamma_2 > 0$ | $\gamma_1 - \gamma_2 > 0$ | $\gamma_1 - \gamma_2 < 0$ | $\gamma_1 - \gamma_2 < 0$ |
| $p_1(T) - p_2(T)$ | $\omega_1 v_1 - \omega_2 v_2 < 0$ | $\omega_1 v_1 - \omega_2 v_2 < 0$ | $\omega_1 v_1 - \omega_2 v_2 < 0$ | $\omega_1 v_1 - \omega_2 v_2 < 0$ |
| $\delta^*(t)$ at $0 \le t < T$ | **1** | **1** | **0** | **0** |
| $\delta^*(t)$ at $t = T$ | **0** | **0** | **0** | **0** |

Table 1 implies that if $G(0) > 0$ and $\omega_1 > \omega_2$, then the optimal policy sustains the initial gap in regional incomes, provided that $\gamma_1 > \gamma_2$. A change in the allocation parameter triggers a reduction in regional inequalities if $\gamma_1 < \gamma_2$. Assume, however, that $G(0) > 0$ and that the planner has an explicit interest in reducing the income gap between the two regions. This assumption can be expressed as $\omega_1 < \omega_2$ [7]. According to the conditions set out in Table 2, $\delta^*(t) = 1$ at $0 \le t < T$ if $\gamma_1 > \gamma_2$. Given that $\omega_1 < \omega_2$, then $\delta^*(t) = 0$ at $t = T$, *irrespective* of the difference in the capital coefficients. In this case, although initial regional disparities increase during the period $0 \le t < T$, a reduction in the income gap between the two regions is possible due to a switch in $\delta$ at $t = T$. While Intriligator (1964) implies a trade-off when $\gamma_1 > \gamma_2$ and $v_1 > v_2$, this can be avoided by imposing $\omega_1 < \omega_2$. Contrary to the possibility of perpetuating regional inequalities, implied by Intriligator (1964) when $\delta^*(t) = 1, \forall t \in [0 \ldots T]$. It is conceivable that regional equity dominates when $\gamma_1 < \gamma_2$ and $\omega_1 < \omega_2$. The allocation parameter remains unchanged and a steady elimination

[7] A similar analysis can be conducted if $G(0) < 0$ and $\omega_1 > \omega_2$.

of the income gap is observed. The optimal solution implied by the conditions $\gamma_1 < \gamma_2$ and $\omega_1 < \omega_2$ is $\delta^*(t) = 0$, $\forall t \in [0 \ldots T]$. Thus, *irrespective* of the productivity differences between regions, a simultaneous reduction of regional inequalities and maximisation of aggregate output is possible. In terms of the analysis by Intriligator (1964) this is feasible only if $v_1 < v_2$ and $\gamma_1 < \gamma_2$.

Table 3 and 4 compare the switching times implied by equation (6) and (9) when $\omega_1 > \omega_2$ and $\omega_1 < \omega_2$, respectively.

Table 3: Switching times when $\omega_1 > \omega_2$

| | |
|---|---|
| $t^* > t^\omega$ | $v_1 > v_2$, $s_1 < s_2$, $\gamma_1 < \gamma_2$ |
| $t^* < t^\omega$ | $v_1 < v_2$, $s_1 > s_2$, $\gamma_1 > \gamma_2$ |
| $t^* > t^\omega$ | $v_1 < v_2$, $s_1 < s_2$, $\gamma_1 < \gamma_2$ |

Table 4: Switching times when $\omega_1 < \omega_2$

| | |
|---|---|
| $t^* < t^\omega$ | $v_1 > v_2$, $s_1 < s_2$, $\gamma_1 < \gamma_2$ |
| $t^* > t^\omega$ | $v_1 < v_2$, $s_1 > s_2$, $\gamma_1 > \gamma_2$ |
| $t^* < t^\omega$ | $v_1 < v_2$, $s_1 < s_2$, $\gamma_1 < \gamma_2$ |

## 4. Conclusion

An aggregate efficiency and regional equity component is normally involved in the design of regional policies. Nevertheless, these might contradict each other, given that maximising aggregate efficiency may increase regional income differentials; a topic that appears to be attracting increasing attention and interest amongst policy-making bodies. This paper has shown that this contradiction can be avoided by a simple modification of the model, developed by Intriligator (1964). Introducing, a weight to the income of each region, provides a range of policies which result to a simultaneous reduction of regional inequalities and maximisation of total output, allowing for a more efficient allocation of scarce resources. Nevertheless, this depends on the time horizon and the chosen spatial units. The variation discussed in this paper is versatile and flexible enough to be applied in various contexts and provides a range of choices to policy-makers in designing regional (or even sectoral) policies. Yet, economic knowledge cannot be gleaned from theory alone. For theoretical innovation to convince they need to be evaluated through observed facts. This clearly implies the need for more detailed and focused analysis and research before the trade-off can be discussed with confidence. What then is the purpose of such analysis? Perhaps the main purpose should be to provoke interest and further discussion in the possibility of ‘bridging’ the gap between aggregate efficiency and interregional equity.

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