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The paper proposes a theory of innovation and market structure. The model incorporates n firms with horizontal spillovers all interacting within a hypothetical industry. In a two-stage sequential game framework, four types of cooperation are studied: full non-cooperation; cooperation in both stages; cooperation only in the R&D stage; and simultaneous cooperation and non-cooperation in the R&D stage. It is shown that the effect of competition on total innovation investment varies among all four cases and mostly depends on the level of spillover effects as well as the level of coordination among competing firms.

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1. Introduction

The incentive to invest in R&D activities is the expectation of realizing an increase in profits as a result of the innovation activity. The change in the level of profit varies depending on the market structure and environment surrounding the firm. For example, incentive is argued to be high under a highly-concentrated product market where monopoly profit can guarantee R&D financing, and when strong innovation protection is in place (Schumpeter-1942). For process innovation, the incentive may be high or low depending on the existence of exclusive rights to the outcome of innovation for the innovators (Arrow-1962). Incentive is also high when innovation increases the monopolistic power of a firm by differentiating its products or reducing its costs significantly. It may also deter entry, and therefore preserve the firm's monopolistic power. The incentive to innovate, however, is negatively affected by a high pre-innovation profit which usually results from a highly concentrated market structure.

Many scholars have examined the incentive to invest in R&D activities by considering market structure, protection of innovation, spillover effects and extent of the innovation activity, *inter alia*. The discussion goes back to 1942 when Schumpeter analyzed the relationship for the first time. In his work, *Capitalism, Socialism, and Democracy* (1942) he concluded that ex-ante monopoly would promote innovation by facilitating both spending (in the form of investment) and benefiting from the R&D activity. The Schumpeterian view of the relationship between innovation investment and competition is later expanded upon by Kamien and Schwartz (1982), Romer (1990), and Grossman and Helpman (1991).

This view, however, was seriously challenged by many in the years which followed. Baldwin and Scott's (1987) empirical work suggests that there is no significantly valid relationship between market power and innovation investment after accounting for industry and firm-specific differences. Levin et al. (1985), Scott (1993) and Weinberg's (1992) works tend to reject Schumpeter's hypothesis by controlling for industry differences in technological opportunities, and the ability of firms to capture the full value of their innovation activities.

Porter's (1990) results rely on a cross-sectional statistical analysis to confirm that innovation investment is higher in competitive markets. His analysis, however, fails to control for other factors influencing innovation activity. MacDonald (1994) uses labour productivity as a proxy for technical changes and shows that highly concentrated markets have benefited from technical progress as a result of a sudden increase in competition.

Bresnahan (1985) examines the Xerox Cooperation case and concludes that the Federal Trade Commission consent decree, which required Xerox to allow competition in the plain paper copier industry, increased innovative activity by both Xerox and new entrants. Mentioning the difficulty of excluding the effect of variations in technological changes, he focused on the direction of the innovation and showed that new entrants tended to enter segments where products were not close substitutes for their existing products, and in which higher innovative activities would have no negative effect on their own profit margins but would destroy the others' (this is congruent with the Arrow replacement effect).

Koller (1995) uses data for four-digit SIC manufacturing industries and a two-equation model to confirm that market concentration negatively affects innovation output. He also studies differences between large- and small-firm innovation activities to state that innovation output for large firms significantly affects market structure, while it has no explanatory power for small firms.

Dasgupta *et al* (1982) take the case of natural resources and distinguish between invention and innovation timing. They conclude that resource sector monopolies invest less than the social optimum in R&D. They also argue that competition over a small stock of natural resources causes under-investment on innovation compared to the socially optimum amount, but for a sufficiently large stock and under some conditions, competition brings about exceptionally rapid technical advancements.

Blundell *et al.* (1999) study the relationship between the size of firm and innovation activity at a firm level as well as industry-wide and conclude that bigger firms tend to spend more on innovation, but industry-wide innovation is discouraged by market concentration. Cohn and Klepper (1996) also relate intensity in innovation activity to a firm's size rather than market structure, and argue that the former explains most of the variance in process innovation intensity. The relationship is shown to be weaker with high-growth industries and/or when patenting and protection of innovation is in place.
Carlin et al (2004) take an international step in doing analysis and use data from a survey of nearly 4000 firms in 24 transition countries to show that mild competition improves

innovation both directly and indirectly. Their argument concludes that performance is better under conditions of mild competition than when competition is dense.

Aghion et al (2005) use panel data to investigate the relationship between product market competition and innovation. They demonstrate that the relationship between these two elements is bell shaped. Using the model, they make two additional predictions – the average technological distance between leaders and followers increases with competition, and the bell is steeper when industries are in closer competition.

Arrow (1962), Wang and Yang (2002), Loury (1979), Lee and Wilde (1980), and Dasgupta and Stiglitz (1980) give a more theoretical concept to the idea. Arrow (1962) distinguishes between process and product innovation and shows that in the presence of permanent innovation intellectual rights, investment in product innovation is encouraged under a complete monopolistic regime in which innovation and production are both protected and investment in process innovation is discouraged under a complete monopolistic regime. He altered his results under a drastic innovation outcome when innovation makes a revolution in the existing product or process.

Wang and Yang (2002) examine R&D cooperation and competition in a vertically integrated market and in the presence of spillover effects. Their results show that in a vertical market structure with an intermediate and a final good producer, R&D outcomes and production levels follow a descending order - social optimum; vertical integration;

cooperative R&D; and non-cooperative R&D. For the consumer price the order is ascending.

Emphasizing on the importance of fixed costs, Loury (1979) concludes that firm level investment on innovation declines with competition in the research sector, while Lee and Wilde (1980) use the same model to show that when emphasis is on variable costs, an increase in competition in the research sector results in an increase in a firm's equilibrium level of innovation investment.

Dasgupta and Stiglitz (1980) show that under a fairly competitive structure, competition in the research sector is negatively correlated with industry-wide and firm-level innovation investment.

Using a linear model, Dubey and Wu (2001) analyze firms engaging in A la Cournot competition over a common product, undertaking innovation, and conclude that innovation is maximized in an intermediate market size.

Boone (2000 and 2001) provides more general results by adopting an axiomatic approach to defining the intensity of competition in the product market, and concludes that the relationship between market structure and innovation incentives is non-monotonic.

Symeonidis (2001) focuses on R&D-intensive industries, examining the effects of price competition on innovation and market structure.

This paper presents an analysis of the relationships between competition and total innovation under different coordination scenarios and in the presence of appropriability. Appropriability, in the form of horizontal spillover, explains how a technological environment affects the relationship between competition and total innovation. Coordination scenarios are visualized as cooperation and non-cooperation, with and without information sharing and in a two-stage game framework. The major contribution of the current work to the literature is to discuss the topic in a formal theoretical framework considering the spillover effects. It differs from the existing literature by providing a common framework, in the presence of spillover effects, under which different coordination scenarios are introduced. The findings suggest that the relationship between competition and innovation outcomes is highly dependent on the level of spillover effects as well as coordination among competing firms. The results classify the conditions under which the effect is described and provide explanations to help to understand the reasoning behind the reaction of innovation activity and outcome to the market structure. In this sense, the findings obtained under the third scenario are in line with the Schumpeterian hypothesis, but also explain the direction of the changes to clarify the findings of MacDonald (1994) and Arrow (1962) for process innovation, *inter alia.* The model results are also in line with Boone (2000 and 2001) in expressing a non-monotonic relationship between competition and innovation outcome.

One other major finding of the paper is that the innovation output is larger under partial collaboration (the second scenario) than full cooperation (the third scenario). It will be argued that total output under the partially collaborative regime is higher than that under full cooperation, and therefore total innovation output would be higher under the former than under the latter.

It is important to note that the results, in contrast to most of the empirical literature, relate the innovation output, and not innovation spending, to competition. This, however, would be used more as an explanatory factor since the linear innovation cost function introduced throughout the paper well relates the two.

The paper is organized into six sections as follows: the second section studies the case of non-cooperation in both stages of the game; the third concentrates on the case when firms act cooperatively in the R&D stage but non-cooperatively in the production stage; the fourth section analyzes the case of cooperation in both stages; the fifth studies simultaneous cooperation and non-cooperation in the innovation stage; and section six is the conclusion.

2. *Non-cooperation in both stages*

Let us assume that there are n firms in the market each producing q_i and facing an inverse demand function $P_i = f(Q)$ in which P is the price each firm faces and Q is the total quantity demanded in the market. $P(.)$ is assumed to be a differentiable function with

$P'(Q) < 0$ at all $Q \geq 0$. Each firm undertakes innovation activity which costs it $\gamma(x_i)$ and has a cost reduction benefit for it and its competitors depending on the level of spillover. $\gamma(x_i)$ is an increasing convex function of *x.* It is increasing in *x* because we assume firms produce in an optimal manner and then face an increasing cost function. The R&D cost function is a convex function of *x* if our production set is concave, closed, and satisfies the free disposal property. Having fulfilled all these assumptions, we can insure that we have a concave profit function and feasible production activity.

The output level q_i is producible at a cost $C(q_i, x_i, x_j)$ where $C(.)$ is a differentiable increasing convex function of q_i, and a differentiable decreasing concave function of x_i and x_j.

Each firm has a two-period decision platform. In the first period it decides about the level of its R&D investment, and in the second stage it will finalize its production decision based on the information obtained in the first stage. Using a backward induction approach, the discussion begins from the second stage. In this stage, as previously stated, the firm decides about its production level:

$$\underset{q_i}{Max} \quad \pi_i = f(Q) q_i - C(q_i, x_i, x_j) - \gamma(x_i). \qquad (1)$$

The optimal quantity of the firm must satisfy the following first order condition (knowing that $q_i > 0$):

$$f'(Q)q_i + f(Q) - C'(q_i, x_i, x_j) \leq 0 \quad \textit{with equality if } q_i > 0. \quad (2)$$

This is the well known condition of a profit maximizing. The first two terms in equation (2) refer to the marginal revenue from a differential increase in q_i. These two terms are equal to the derivative of the total revenue for the firm. The third term is the corresponding marginal cost. As we know that $f(0) > C'(0)$ then condition 2 could be satisfied only at $q_i > 0$. Consequently, marginal cost has to be equal to marginal revenue at the firm's optimal output level:

$$f'(Q)q_i + f(Q) - C'(q_i, x_i, x_j) = 0. \quad (3)$$

Solving the *n* first order conditions obtained from the maximization of (1) for *i=1,…,n* simultaneously yields $q_i^*(x_i, x_j)$ and $Q^*(x_i, x_j)$. At the first stage, the firm decides about its innovation investment:

$$\underset{x_i}{Max} \quad \pi_i = f\left(Q^*(x_i, x_j)\right)q_i^*(x_i, x_j) - C\left(q_i^*(x_i, x_j), x_i, x_j\right) - \gamma(x_i). \quad (4)$$

The corner solution to this maximization problem is:

$$Q^{*\prime}(x_i,x_j)\,f'\big(Q^*(x_i,x_j)\big)\,q_i^*(x_i,x_j)+f\big(Q^*(x_i,x_j)\big)\,q_i^{*\prime}(x_i,x_j)-$$
$$\big(1+q_i^{*\prime}(x_i,x_j)\big)\,C'\big(q_i^*(x_i,x_j),x_i,x_j\big)-\gamma'(x_i)=0. \qquad (5)$$

Equation (5) explains that at optimum, the total marginal benefits of having one more unit of R&D activity outcome is equal to the total marginal costs of producing that extra unit. Imposing ex-post symmetry, one can calculate $x^*(n)$ as the notion of optimum innovation investment at the firm level and $X(n)=nx(n)$ as the total innovation outcome. Under the non-cooperative scenario, the firm level innovation outcome, the marginal effect of competition on innovation and number of competing firms determines the final effect of competition on innovation. This effect consists of the following components:

$$\frac{\partial X}{\partial n}=\frac{\partial\big(nx(n)\big)}{\partial n}=x(n)+nx'(n). \qquad (6)$$

The first expression on the right side of (6), which indicates the firm level innovation outcome, is always positive. The second term, which specifies the marginal effect of an increase in competition on the firm level innovation outcome, is always negative. This is due to the fact that an increase in the number of competing firms reduces the market share of the firm and its total benefits of innovation (since innovation is implemented on lower production) which in turn discourages innovation at the firm level. Using explicit functional forms for the demand and the cost functions it is easy to show that in absence of spillover effects, the sum of negative marginal effects is smaller than the increase in the innovation outcome of the new entrant and therefore the final effect of competition on innovation outcome would be positive.

A similar discussion applies for the perfect spillover scenario but this time the total decrease in the firm level innovation would be more than the increase in the innovation outcome of the new entrant since firms have also a free riding opportunity. The new entrant influences the total innovation in status quo by reducing each firm's production and then by providing each firm with some free innovation outcome. The first effect, as discussed before, influences the firm's innovation level negatively and the second, by creating a free-riding opportunity, intensifies this negative effect. The bottom line is that the sum of the negative marginal effects is greater than the increase in the innovation outcome of the new entrant and therefore the final effect would be negative.

Proposition 1*: In the event of non-cooperation in both stages, an increase in the number of firms - increase in competition - may increase or decrease total innovation depending on the level of spillover, e.g. in the presence of perfect spillover, increase in competition affects innovation negatively and in the absence of spillover, competition has a positive effect on innovation.*

An example:

We assume there are n firms in the market each producing q_i. We also assume that all firms are facing a linear demand function $P = a - bQ$ in which P is the price each firm faces and Q is the total quantity demanded in the market. Each firm undertakes an innovation activity which costs it $\gamma(x_i) = \gamma x_i^2$ which has a cost reduction benefit of x_i for it and βx_i for its rivals (competitors). Each firm has a two-period decision making platform. In the first period it decides about the level of its R&D investment, and in the second stage it will finalize its production decision based on the information obtained in the first stage. Using a backward induction approach, the discussion begins from the second stage. In this stage, as previously stated, the firm decides about its production level, then:

$$\underset{q_i}{Max} \quad \pi_i = (a-bQ)q_i - \left(A - x_i - \sum_{j\neq i}\beta x_j\right)q_i - \gamma x_i^2, \qquad (7)$$

in which the firm *i*'s unit cost is:

$$c_i = A - x_i - \sum_{j\neq i}\beta x_j. \qquad (8)$$

Where A is the autonomous unit cost, x_i is the innovation outcome originated by the firm *i*'s R&D activity and $\sum_{j\neq i}\beta x_j$ is the innovation spillover generated by the other firms' R&D activities.

Solving all *n* first order conditions, it is straightforward to show that the pure strategy Nash equilibrium outcome of firm *i* is:

$$q_i = \frac{a - A + (1+n)x_i + (-1+\beta(1-n))\sum_{i=1}^{n} x_i + (1+n)\beta\sum_{j\neq i} x_j}{b(1+n)}. \qquad (9)$$

Substituting q_i back into the profit function will result in:

$$\pi_i = -\gamma x_i^2 - \frac{\left(A - x_i - \beta\sum_{j\neq i} x_j\right)\left(a - A + (1+n)x_i + (-1+\beta(1-n))\sum_{i=1}^{n} x_i + (1+n)\beta\sum_{j\neq i} x_j\right)}{b(1+n)} + \frac{\left(a + An + (-1+(\beta(1-n)))\sum_{i=1}^{n} x_i\right)\left(a - A + (1+n)x_i + (-1+\beta(1-n))\sum_{i=1}^{n} x_i + (1+n)\beta\sum_{j\neq i} x_j\right)}{b(1+n)^2}. \quad (10)$$

Now at the first stage, the decision making firm *i* maximizes its profit with respect to x_i, having knowledge of the optimal value of production from the previous stage. The first order condition is:

$$\frac{1}{b(1+n)^2}\left(-\left((\beta+n(1-\beta))^2 - b(1+n)^2\gamma\right)x_i - (\beta+n(1-\beta))\left(a - A + (1+n)\beta\sum_{j\neq i} x_j\right)\right) = 0. \quad (11)$$

This provides us with the notion of optimum innovation investment for firm *i*. Since we are interested in the total innovation investment in this hypothetical economy and appeal to symmetry, we will have:

$$X = nx = -\frac{n\left((a-A)\left(n(-1+\beta)-\beta\right)\right)}{\left((-1+\beta)\beta + b\gamma + n^2\left((-1+\beta)\beta + b\gamma\right) + n\left(-1-2(-1+\beta)\beta + 2b\gamma\right)\right)}. \quad (12)$$

To find out what the marginal effect of an increase in the number of firms on the total innovation outcome is:

$$\frac{\partial X}{\partial n}=\frac{\left((a-A)\left((-1+\beta)(n+\beta-n\beta)^{2}-b(1+n)\left(-\beta+n(-2+3\beta)\right)\gamma\right)\right)}{\left((-1+\beta)\beta+b\gamma+n^{2}\left((-1+\beta)\beta+b\gamma\right)+n\left(-1-2(-1+\beta)\beta+2b\gamma\right)\right)^{2}}. \qquad (13)$$

The effect of competition on innovation could be negative or positive depending on the level of spillover. If we assume the extreme case of full information sharing ($\beta=1$) we will have:

$$\frac{\partial X}{\partial n}=-\frac{b(a-A)(-1+n)(1+n)\gamma}{\left(b\gamma+bn^{2}\gamma+n(-1+2b\gamma)\right)^{2}}. \qquad (14)$$

This expression is always negative, meaning that in the presence of perfect spillover, increase in competition affects innovation negatively. Under a fully non-cooperative market structure, an increase in competition triggers two effects – the first is an increase in total innovation resulting from an increase in the number of incumbents in the market, and the second is a decrease in the firm's level innovation activity resulting from having full access to other firms' innovation outcome (free riding of other firms' innovation efforts) and reducing the extent of redundant innovation efforts. The second one overcomes the first and therefore total innovation is reduced by an increase in competition. Under the absence of spillover, however, we will have:

$$\frac{\partial X}{\partial n}=\frac{(a-A)\left(-n^{2}+2bn(1+n)\gamma\right)}{\left(b\gamma+bn^{2}\gamma+n(-1+2b\gamma)\right)^{2}}. \qquad (15)$$

This always has a positive sign, indicating that in the absence of spillover competition has a positive effect on innovation. Here, under the absence of the second effect, competition increases total innovation.

3. Cooperation in R&D

This refers to a case similar to the previous one, but this time the innovation decision is made cooperatively. In this case, under the same assumptions, the second stage stays non-cooperative but in the first stage firms maximize joint profits:

$$\begin{aligned} &\underset{x_i}{Max} \quad \sum \pi_i = \pi_1 + \pi_2 + \ldots + \pi_n = \\ &\sum_{i=1}^{n} \Big[f\left(Q^*\left(x_i, x_j\right)\right) q_i^*\left(x_i, x_j\right) - C\left(q_i^*\left(x_i, x_j\right), x_i, x_j\right) - \gamma\left(x_i\right) \Big], \end{aligned} \tag{16}$$

And the first order condition of the firm *i* is:

$$\begin{aligned} &\sum_{i=1}^{n} Q^{*\prime}\left(x_i, x_j\right) f'\left(Q^*\left(x_i, x_j\right)\right) q_i^*\left(x_i, x_j\right) + \sum_{i=1}^{n} f\left(Q^*\left(x_i, x_j\right)\right) q_i^{*\prime}\left(x_i, x_j\right) - \\ &\sum_{i=1}^{n} C'\left(q_i^*\left(x_i, x_j\right), x_i, x_j\right)\left(1 + q_i^{*\prime}\left(x_i, x_j\right)\right) - \sum_{i=1}^{n} \gamma'\left(x_i\right) = 0. \end{aligned} \tag{17}$$

Equation (5) explains that at optimum, total marginal benefits of having one more unit of R&D activity outcome is equal to the total marginal costs of producing that extra unit. Imposing ex-post symmetry, one can calculate $x^*(n)$ as the notion of optimum innovation investment at the firm level, and $X(n) = nx(n)$ as the total innovation outcome. Under the cooperation in innovation scenario, the firm level innovation outcome – the marginal effect of competition on innovation and number of competing firms – determines the final effect of competition on innovation.

Using explicit functional forms for the demand and the cost functions it is easy to show that, in the absence of spillover effects, the sum of the negative marginal effects is larger than the innovation outcome of the new entrant and therefore the final effect of an increase in number of firms on the innovation outcome would be negative. A similar discussion applies for the perfect spillover scenario but this time the sum of decrease in the firm-level innovations would be less than the increase in the innovation outcome of the new entrant.

Proposition 2: *In the event of cooperation in R&D, the effect of competition on total innovation outcome varies with the level of spillover. Under this scenario, the final effect of increase in the number of firms on the total innovation outcome would be negative in the absence of spillover effects, and positive in the presence of perfect spillover.*

An example:

As already explained, in this case the second stage stays non-cooperative but in the first stage, firms maximize the joint profits. Using the same set of assumptions we have:

$$\underset{x_i}{Max} \quad \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} \{ -\gamma x_i^2 - \frac{\left(A - x_i - \beta \sum_{j \neq i} x_j\right)\left(a - A + (1+n) x_i + (-1 + \beta(1-n)) \sum_{i=1}^{n} x_i + (1+n)\beta \sum_{j \neq i} x_j\right)}{b(1+n)} + \frac{\left(a + An + (-1 + (\beta(1-n))) \sum_{i=1}^{n} x_i\right)\left(a - A + (1+n) x_i + (-1 + \beta(1-n)) \sum_{i=1}^{n} x_i + (1+n)\beta \sum_{j \neq i} x_j\right)}{b(1+n)^2} \}. \qquad (18)$$

The first-order condition, after imposing ex-pose symmetry on the innovation, would be:

$$\frac{1}{b(1+n)^2}\left(2\left(1+(-1+n)\beta\right)\left(a-A+x+(-1+n)\beta x\right)\right)-2\gamma x=0, \qquad (19)$$

And the innovation outcome at the firm level is:

$$x=-\frac{(a-A)\left(1+(-1+n)\beta\right)}{\left(\left(1+(-1+n)\beta\right)^2-b(1+n)^2\gamma\right)}. \qquad (20)$$

Summing over the *x* results:

$$X=nx=-\frac{n(a-A)\left(1+(-1+n)\beta\right)}{\left(\left(1+(-1+n)\beta\right)^2-b(1+n)^2\gamma\right)}, \qquad (21)$$

And differentiating with respect to *n* results:

$$\frac{\partial X}{\partial n}=\frac{\left((a-A)\left((-1+\beta)\left(1+(-1+n)\beta\right)^2+b(1+n)\left(1-\beta+n(-1+3\beta)\right)\gamma\right)\right)}{\left(\left(1+(-1+n)\beta\right)^4-2b\left(1+n+\left(-1+n^2\right)\beta\right)^2\gamma+b^2(1+n)^4\gamma^2\right)}. \qquad (22)$$

As seen, the effect of the increase in the number of firms on innovation depends on the level of spillover. In fact in the absence of spillover ($\beta=0$) we have:

$$\frac{\partial X}{\partial n} = -\frac{(a-A)\left(1+b\left(-1+n^2\right)\gamma\right)}{\left(-1+b(1+n)^2\gamma\right)^2}. \qquad (23)$$

This expression is always negative, expressing a negative relationship between the number of firms and the total innovation outcome in the absence of spillover effects. Here firms undertake innovation activities individually and then they collectively decide about the production level. The imbedded assumption is that innovation is a cost-reducing and not a quality-improving activity, thus it does not increase the market demand for the product. As a result, increase in number of firms which cooperatively produce and share the market among them reduces firm-level profit and therefore discourages innovation investment funded by the firm's cash flow.

Assuming the other extreme case of perfect spillover $(\beta = 1)$ we have:

$$\frac{\partial X}{\partial n} = \frac{2bn(a-A)(1+n)\gamma}{\left(n^2 - b(1+n)^2\gamma\right)^2}. \qquad (24)$$

This is, again, an always-positive expression, meaning that in the presence of full innovation information sharing, the increase in the number of firms increases the total innovation outcome. Here firms innovate individually but bring the outcome to a single table to make a collaborative decision on the total production. An increase in the number of active firms in the market decreases each firm's market share but at the same time increases the total innovation outcome accessible to each firm, reducing the firm's marginal cost of production. Under the mixed market structure and full information sharing, the latter overcomes the former and the total innovation outcome increases.

4. *Cooperation in both stages*

In this case, firms behave cooperatively in both stages of their operation – innovation at the first stage and production at the second. Having the same set of assumptions, the objective in the first stage would be the level of production which maximizes the total profit, meaning:

$$\underset{Q}{Max} \quad \pi = f(Q)Q - C(Q, x_i, x_j) - \sum_{i=1}^{n} \gamma(x_i), \qquad (25)$$

And the first order condition is:

$$f'(Q)Q + f(Q) - C'(Q, x_i, x_j) \leq o \quad \textit{with equality if } Q > 0. \qquad (26)$$

This is the well-known condition of a profit maximizing monopolist. The first two terms in the equation (26) refer to the marginal revenue from a differential increase in *Q*. These two terms are equal to the derivative of the total revenue of the firm. The third term in this equation is the corresponding marginal cost. We know that if $f(0) > C'(0)$ then conditions (26) could be satisfied only at $Q > 0$. Consequently marginal cost has to be equal to marginal revenue at the monopolist's optimal output levels:

$$f'(Q)Q + f(Q) - C'\left(Q, x_i, x_j\right) = 0. \qquad (27)$$

For $x_1 = x_2 = ... = x_i = ... = x_n = x$ the symmetric solution leads to $Q^*\left(x_i, x_j\right)$. Now, at the first stage, the firm decides about its innovation investment:

$$\underset{x}{Max} \quad \pi = f\left(Q^*(x)\right)Q^*(x) - C\left(Q^*(x), x\right) - n\gamma(x). \qquad (28)$$

The interior solution to this maximization problem is:

$$Q^{*\prime}(x) f'\left(Q^*(x)\right)Q^*(x) + f\left(Q^*(x)\right)Q^{*\prime}(x) - \left(1 + Q^{*\prime}(x)\right)C'\left(Q^*(x), x\right) - n\gamma'(x) = 0. \qquad (29)$$

Equation (4) explains that at optimum the total marginal benefits of having one more unit of R&D activity outcome is equal to the total marginal costs of producing that extra unit. Imposing ex-pose symmetry, one can calculate $x^*(n)$ as the notion of optimum innovation investment at the firm level and $X(n) = nx(n)$ as the total innovation outcome. Under the fully cooperative scenario, as before, the firm level innovation outcome, the marginal effect of competition on innovation and number of competing firms, determines the final effect of competition on innovation.

Using explicit functional forms for the demand and the cost functions, one can easily show that in absence of spillover effects the sum of the negative marginal effects is larger than the innovation outcome of the new entrant, and therefore the final effect of increase in the number of firms on the total innovation outcome would be negative. A similar discussion applies for the perfect spillover scenario but this time the sum of decrease in the firm-level innovations would be less than the innovation outcome of the new entrant.

It is interesting to note that under the fully cooperative scenario the total innovation outcome is less than that of the collaboration at R&D case and this is due to the fact that the total output under the partially collaborative regime is higher compared to that of the fully cooperative one.

Proposition 3: *In the event of cooperation in both stages, the effect of competition on total innovation outcome varies with the level of spillover, i.e. the effect of increase in number of firms on the total innovation outcome would be negative in the absence of spillover effects, and positive in the presence of perfect spillover.*

An example:

As already explained, in this case firms behave cooperatively in both stages of their operation. Having the same set of assumptions:

$$\underset{Q=\sum_i q_i}{Max} \quad \pi = \sum_i \left(\left(a - bQ\right) q_i - \left(A - x_i - \beta \sum_{j \neq i} x_j \right) q_i - \gamma x_i^2 \right), \qquad (30)$$

And the first order condition, under the symmetry assumption, will be:

$$\frac{\partial \pi}{\partial Q} = 0 \Rightarrow Q = \frac{a - A + \left(\left(n-1\right)\beta + 1 \right) x}{2b}. \qquad (31)$$

Substituting the results into the profit function will result in:

$$\pi = \frac{1}{b}\left[\frac{(a-A)+\left(\left((n-1)\beta+1\right)x\right)}{2}\right]^2 - n\gamma x^2. \qquad (32)$$

Now at the preceding stage the innovation is the objective of the maximization process:

$$x = -\frac{(a-A)\left((n-1)\beta+1\right)}{\left((n-1)\beta+1\right)^2 - 4bn\gamma}, \qquad (33)$$

And then for all firms:

$$X = nx = -\frac{(a-A)\left(\left(n^2-n\right)\beta+n\right)}{\left((n-1)\beta+1\right)^2 - 4bn\gamma}. \qquad (34)$$

An increase in the number of firms affects the total innovation outcome as follows:

$$\frac{\partial X}{\partial n} = \frac{(a-A)\left((\beta-1)\left((n-1)\beta+1\right)\beta^2 + 4bn^2\beta\gamma\right)}{\left(\left((n-1)\beta+1\right)^2 - 4bn\gamma\right)^2}. \qquad (35)$$

As seen, the effect of competition on innovation depends on the level of spillover. In fact if we assume $\beta = 0$ we have:

$$\frac{\partial X}{\partial n} = -\frac{a-A}{\left(1-4bn\gamma\right)^2}. \qquad (36)$$

which is an always negative expression. Assuming $\beta = 1$, we have:

$$\frac{\partial X}{\partial n} = \frac{4\left(a-A\right)bn^2\gamma}{\left(n^2-4bn\gamma\right)^2} \qquad (37)$$

Equation (37) is an always positive expression expressing a positive effect of increase in number of firms on total innovation. The intuition is similar to the previous case except that the innovation decision is being made cooperatively under this scenario.

5. Simultaneous cooperation and non-cooperation in the innovation

In this case we assume that firms act non-cooperatively in the production and innovation stages while, simultaneously, a sub-group of firms cooperates in the innovation stage.

Under this scenario the first stage results stay the same as those of the fully non-cooperative case. At the second stage, however, we assume that m firms cooperate in their innovation and the rest $n-m=r$ of the firms keep their non-cooperative status. This concludes that *r+1* firms interact non-cooperatively in the market. Relying on the same assumptions, the second stage profit function for the non-cooperative and cooperative firms would be:

$$\underset{x_r}{Max} \quad \pi_r = f\left(Q^*\left(x_r, x_m\right)\right)q_r^*\left(x_r, x_m\right) - C\left(q_r^*\left(x_r, x_m\right), x_r, x_m\right) - \gamma\left(x_r\right), \qquad (38)$$

And:

$$\underset{x_m}{Max} \sum_{m=r+1}^{n} \pi_m = \sum_{m=r+1}^{n} \Big(f\big(Q^*(x_r,x_m)\big) q_m^*(x_r,x_m) - C\big(q_m^*(x_r,x_m),x_r,x_m\big) - \gamma(x_m) \Big). \tag{39}$$

Where $r = 1,...,r$ and $m = r+1,...,n$. The first order condition for the first r firms is:

$$\begin{aligned} &Q^{*\prime}(x_r,x_m) f'\big(Q^*(x_r,x_m)\big) q_r^*(x_r,x_m) + f\big(Q^*(x_r,x_m)\big) q_r^{*\prime}(x_r,x_m) - \\ &\big(1+q_r^{*\prime}(x_r,x_m)\big) C'\big(q_r^*(x_r,x_m),x_r,x_m\big) - \gamma'(x_r) = 0. \end{aligned} \tag{40}$$

And for the remaining m firms:

$$\begin{aligned} &\sum_{m=r+1}^{n} Q^{*\prime}(x_r,x_m) f'\big(Q^*(x_r,x_m)\big) q_m^*(x_r,x_m) + \sum_{m=r+1}^{n} f\big(Q^*(x_r,x_m)\big) q_m^{*\prime}(x_r,x_m) - \\ &\sum_{m=r+1}^{n} C'\big(q_m^*(x_r,x_m),x_r,x_m\big)\big(1+q_m^{*\prime}(x_r,x_m)\big) - \sum_{m=r+1}^{n} \gamma'(x_m) = 0. \end{aligned} \tag{41}$$

Imposing ex-pose symmetry to (40) and (41), the optimal firm level innovations; $x_r^*(r,m)$ and $x_m^*(r,m)$; and the total innovation outcome; $X(r,m) = r x_r^*(r,m) + m x_m^*(r,m)$ could be calculated.

Under the simultaneous cooperation and non-cooperation in the innovation scenario, the final effect of an increase in competition among non-cooperative (cooperative) firms on total innovation is determined by the followings:

1) The firm-level innovation outcome of non-cooperative (cooperative) firms.
2) The marginal effect of an increase in competition among the non-cooperative (cooperative) firms on the innovation level of non-cooperative firms.
3) The marginal effect of an increase in competition among non-cooperative (cooperative) firms on the innovation level of cooperative firms.
4) The number of non-cooperative and cooperative firms.

This effect consists of the following components:

$$\frac{\partial X(r,m)}{\partial r}=\frac{\partial\left(rx_r^*(r,m)+mx_m^*(r,m)\right)}{\partial r}=x_r^*(r,m)+rx_r^{*\prime}(r,m)+mx_m^{*\prime}(r,m). \qquad (42)$$

As explained before, the first term on the right side of the equation (42) is always positive, while the second and third terms are always negative. Using explicit functional forms for demand and cost functions, it is easy to show that in the absence of spillover effects, the final effect of an increase in competition among non-cooperative (cooperative) firms on the total innovation outcome would be positive (negative) because the sum of negative marginal effects is smaller (larger) than the innovation outcome of new entrant.

A similar discussion applies for the perfect spillover scenario but this time the total decrease in the firm-level innovations would be greater (smaller) than the innovation outcome of the new entrant and therefore the final effect would be negative (positive).

Proposition 4: *In the event of simultaneous cooperation and non-cooperation in R&D, the effect of an increase in the number of firms on the total innovation outcome varies with the level of spillover. More specifically, when the increase in competition is among the non-cooperative firms the final effect is similar to that of the first case, and when the increase is among the cooperative firms the final effect is more in line with that of the second and third cases.*

An Example:

Relying on similar assumptions, and assuming that m firms cooperate in their innovation and the rest of the $n - m = r$ firms maintain their non-cooperative status, the first order conditions of non-cooperative and cooperative firms are respectively as follows:

$$\frac{1}{b(r+2)^2}\left(\left(-(-1+r(\beta-1))^2+b(r+2)^2\gamma\right)x_r+(1+r-r\beta)\left(a-A+\beta(2+r)\left((r-1)x_r+mx_m\right)\right)\right)=0, \tag{43}$$

$$\begin{aligned}&\frac{2}{b(r+2)^2}\{(a-A)\left(2-m+r-(2-2m+r)\beta\right)-r(2\beta-1)\left(m+(2+r)(\beta-1)-2m\beta\right)x_r\\&\qquad+\left(\left(m+(2+r)(\beta-1)-2m\beta\right)^2-b(r+2)^2\gamma\right)x_m\}=0.\end{aligned} \tag{44}$$

Solving these two equations simultaneously for x_r and x_m we can calculate *X*. Differentiating *X* with respect to *r* will provide us with the effect of an increase in the number of non-cooperative firms on total innovation.

The effect of an increase in competition among non-cooperative firms on total innovation could be negative or positive depending on the level of spillover. If we assume the extreme case of perfect information sharing ($\beta = 1$) we will have:

$$\frac{\partial X}{\partial r} = \frac{b\gamma(a-A)(r+2)\{m^2\left(r\left(r(3r+14)+9\right)-12\right)+b(r+2)^2\left(-m^2(r-2)(2r+5)+(r+2)\left(r^2+3\right)\right)\gamma+b^2(r+2)^4\left(r-2m^2-2\right)\gamma^2\}}{\left(m^2\left(r+3-b\gamma(r+2)^2\right)+b\gamma(r+2)^2\left(r^2+r-3+b\gamma(r+2)^2\right)\right)^2}. \quad (45)$$

This expression is always negative meaning that in the presence of perfect spillover increase in competition affects innovation negatively. The absence of spillover, however, results in:

$$\begin{aligned}\frac{\partial X}{\partial r} = &\{(1+r)^2(r-m+2)^2\left(-m+(r+2)^2\right)- \\ &b(r+2)\left(m^3r^2(r+2)+(r+2)^3\left(r(5r+12)+6\right)+m(r+2)\left(r\left(r\left(r(r-4)-33\right)-52\right)-21\right)+m^2\left(r\left(19-2r\left(r(r+3)-2\right)\right)+8\right)\right)\gamma \\ &+b^2(r+2)^3\left(m^3(r+2)+2mr\left(r^2+r-2\right)-m^2\left(r(4r+11)+10\right)+(r+2)\left(r(7r+18)+9\right)\right)\gamma^2 \\ &-b^3(r+2)^5\left(3r+2+m\left(r-2m+2\right)\right)\gamma^3\}\times\frac{(a-A)}{\left(\left((r+1)^2-b\gamma(r+2)^2\right)^2\left((r-m+2)^2-b(r+2)^2\gamma\right)^2\right)}.\end{aligned} \quad (46)$$

This always has a positive sign, indicating that in the absence of spillover, competition has a positive effect on innovation. The intuition would be similar to the first case.

Differentiating X with respect to m provides us to solve for the effect of an increase in the number of cooperative firms on total innovation.

The effect of an increase in competition among cooperative firms on total innovation could be negative or positive depending on the level of spillover. Under the perfect spillover condition ($\beta = 1$) we will have:

$$\frac{\partial X}{\partial m} = \frac{2b\gamma m(a-A)(r+2)^2\left(b(r+2)^2\gamma - r - 3\right)\left(r^2 + b(r+2)^2\gamma - 3\right)}{\left(m^2\left(r - b(r+2)^2\gamma\right) + b\gamma(r+2)^2\left(r^2 + r + b\gamma(r+2)^2 - 3\right) + 3\right)^2}. \qquad (47)$$

This expression is always positive meaning that in the presence of perfect spillover, an increase in the number of firms affects innovation positively. Absence of spillover, on the other hand results:

$$\frac{\partial X}{\partial m} = \frac{(a-A)(r+2)\left(b(r+2)^2\gamma - r - 1\right)\left(b(r+2)(r-2m+2)\gamma - (r-m+2)^2\right)}{\left((r-m+2)^2 - b(r+2)^2\gamma\right)^2\left(b\gamma(r+2)^2 - (r+1)^2\right)}. \qquad (48)$$

This always has a negative sign indicating that, in the absence of spillover, competition has a negative effect on innovation. The intuition is similar to the second case.

6. *Conclusion*

Using a two-stage game model, this paper examined the effect of competition on innovation under different coordination regimes. In general, the findings suggested that the relationship is highly dependent on the level of spillover effects as well as the level of coordination among competing firms. When firms interact non-cooperatively in both stages, the effect of an increase in competition on total innovation outcome depends on the level of spillover (i.e. in the presence of perfect spillover, increase in competition affects innovation negatively and in the absence of spillover, competition has a positive effect on innovation.) The relationship, under both R&D cooperation and complete cooperation regimes, again depends on the level of spillover, but the effect is in an opposite direction to the fully non-cooperative case.

Simultaneous cooperation and non-cooperation at the R&D level was the fourth scenario investigated in the paper. It was shown that when the increase in competition is among the non-cooperative firms the final effect is similar to that of the first case, and when the increase is among the cooperative firms, the final effect is more in line with that of the second and third cases.

As seen throughout the paper, the major contribution of the current work was to provide a common framework to compare the mixed results provided by the literature. The topic was discussed in a formal theoretical framework where the importance of spillover effects and coordination among firms were recognized. The results concur largely with those of Boone (2000 and 2001) in finding that there is a non-monotone relationship between market structure and innovation incentives, but go further to specify the criteria under which the differences are distinguished.

As the other major finding of the paper, the innovation output was shown to be larger under the partial collaboration (second scenario) than the full cooperation (third scenario). The argument was supported by the fact that the total output under the partially collaborative regime was higher than that of the fully cooperative one, and therefore total innovation output would be higher under the former than the latter.

As discussed in the literature, communication network, distance between the innovators, and intellectual property right protections, such as patent and secrecy, are considered to be major drivers of innovation spillover. As expected, the spillover rate is positively related to the telecommunication network, and negatively related to the distance between innovating firms (or geographic size of the market) and protection of the property rights. Incorporating these variables to the discussion provides the following interesting results:

- Non-cooperation scenario: Telecommunication networks negatively affect the relationship between competition and innovation outcomes while distance and protection of property rights positively affect the relationship under this scenario.
- Cooperation in the innovation stage: Under this scenario telecommunications networks play a positive role while distance between firms and protection of property rights has a negative effect.
- Cooperation in both stages: exhibits a similar order of effects to the previous case.
- Simultaneous cooperation and non-cooperation in place: in the event of an increase in the number of non-cooperative firms, the findings would be in line with the first case. The findings, however, would be similar to the second (and the third) cases if the increase happened among the cooperative firms.

The purpose of this study is to create a primary baseline for future works. The sequential game approach could well be replaced by a simultaneous game framework where firms simultaneously decide about their production and R&D investments. The discussion of the protection of intellectual property rights with certain significant effects on the relationship deserves more attention in any future work. Uncertainty and risk have been ignored in the literature and therefore merit more attention. And, finally, the symmetric discussion can be well replaced by an asymmetric framework.

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