Malinvaud on Wicksell's legacy to capital theory: some critical remarks

Fratini, Saverio M.

20 June 2012

Malinvaud on Wicksell's Legacy to Capital Theory: Some Critical Remarks

Saverio M. Fratini*

Università degli Studi Roma Tre

Abstract

This critique of Malinvaud's article of 2003 on Wicksell's legacy to capital theory focuses in particular on three points raised there. The first regards the given amount of existing capital that appears in Wicksell's theory and its connection with his alleged "missing equation", the second the particular notion of the marginal product of capital adopted by Malinvaud and the meaning of its equality with the rate of interest, and the third the concept of the average period of production taken by Malinvaud from Hicks and its inverse relation to the rate of interest.

JEL Codes: B13, D24, D33.

1 Introduction

Because of the neo-Walrasian shift in general equilibrium theories of value and distribution, which can be symbolically associated with the publication of Hicks's *Value and Capital*,[1]

* I wish to express my gratitude toward the late Pierangelo Garegnani, with whom I had the opportunity to discuss an earlier version of this paper and from whom I learned most of what I know about capital theory. I am also grateful to E. Bellino, A. Dvoskin, P. Potestio and N. Salvadori for their comments and suggestions. The responsibility for all the remaining imperfections or errors is of course all mine.

Wicksell's contribution to economic theory has been forgotten by most of the modern mainstream economists. There is, for example, no reference to him in Debreu's *Theory of Value* (1959) or Arrow & Hahn's *General Competitive Analysis* (1971).

Malinvaud's article of 2003 therefore has the unquestionable merit of attempting to focus attention on Wicksell's theory, which is one of the most important and developed versions of the marginalist explanation of income distribution and value. His analysis of Wicksell's legacy is, however, misleading on at least the three points discussed in this paper.

The first refers to the given amount of capital in value terms that appears in Wicksell's theory. Following a certain interpretation to be found in the literature, Malinvaud presents this as connected with an alleged "missing equation" and it is argued on this view that any variable could be regarded as given in order to fill the vacant degree of freedom. We shall instead show in sec. 2 that Wicksell was obliged to take the value of the existing capital as a given magnitude in order to construct a marginalist theory of distribution.

The second, which will be discussed in sec. 3, concerns the "Wicksell effect" or lack of equality between the marginal product of value capital and the rate of interest. Malinvaud instead adopts a peculiar notion of the marginal product of capital and demonstrates an equality between it and the rate of interest. As we shall see, however, this result has neither the same origin nor the same function as the equalities to be found when factors of production are measured in "technical units" and not in value terms.

Finally, the third (sec. 4) refers to the concept of the average period of production taken by Malinvaud from Hicks and its inverse relation to the interest rate. A very simple example will be used to show that, contrary to Malinvaud's claims, the notion of the average period of production he uses is not "a measure of the degree of roundaboutness" of production, and its inverse relation to the rate of interest is therefore essentially irrelevant.

Even though many of the arguments used here are based on results already available in the literature, Malinvaud surprisingly appears to ignore them. In these cases, our original contribution will therefore be at least that of using those results in order to show the fallacy of Malinvaud's view of Wicksell's legacy. This concerns in particular the analysis presented in sec. 2, which is in fact largely based on Garegnani (1960 and 1990) and Kurz (2000), and part of the analysis in sec. 3, where we put forward our own generalization of arguments already developed in Pasinetti (1969) and Garegnani (1984).

2 The missing equation

2.1 Malinvaud's view of the "missing equation" and the given quantity of capital in Wicksell's theory

Malinvaud is one of a fairly large group of economists – including Hirshleifer (1967), Sandelin (1980), Neghishi (1982) and L. Samuelson (1982) – who claim that there is an equation missing in Wicksell's system of general equilibrium.[2] According to these authors, Wicksell attempted to focus attention on what Malinvaud (2003, p. 507) calls the 'production side' of capital theory – i.e. the mechanism of choice of the optimal technique and the resulting demand for labour, land and capital – and thus ended up neglecting the 'consumption side', understood as the choice between present and future consumption, which neoclassical theory sees as the basis of decisions about saving.

This neglect of the consumption side is then viewed as the cause of the under-determinacy of Wicksell's general equilibrium system. As Hirshleifer put it, Wicksell's formalisation is 'an incomplete theoretical structure' and 'to complete the Bohm-Wicksell formal system and actually determine the rate of interest, at least one other equation is needed – specifically a time-preference relation' (Hirshleifer, 1967, pp. 191 and 197).

In other words, to be more precise, since Wicksell's equilibrium refers to a situation in which distribution variables and relative commodity prices do not vary period by period, what

Hirshlefer expects to find is a condition of zero net savings (per unit of labour). Since income distribution depends in neoclassical theory on the relative scarcity of factors of production (among other things), the distribution variables – and relative prices – can only be stationary if there is no net capital accumulation (per unit of labour). The amount of capital in use must therefore adjust so as to generate the level of the interest rate that – given the intertemporal preferences of consumers – leads to zero net savings (per unit of labour). As there is, however, no such zero net savings condition in Wicksell's analysis, an equation appears to be missing. 'The missing equation is, of course, that which follows from the behaviour of the consumption sector.' (Malinvaud, 2003, p. 510).

Moreover, according to this particular interpretation of Wicksell's theory, the principal consequence of the 'missing equation' – with one degree of freedom left open – is, as Sandelin writes, 'that one central magnitude has to be determined exogenously' and therefore '[a]fter some vacillation Wicksell chooses the value of capital as an exogenous variable of his system' (Sandelin, 1980, p. 29). But since, on this view, any variable or even any proper combination of variables could be taken exogenously in order to close the system, the decision to take the value of the existing capital as a given magnitude is 'an unfortunate feature' of Wicksell's theory (cf. Malinvaud, 2003, p. 510).

Malinvaud's suggestion, in particular, is that the degree of freedom left open by Wicksell can be filled by regarding the interest rate as exogenous (cf. Malinvaud, 2003, p. 513). He then goes on to examine what he calls 'the production side' of capital theory – the choice of techniques and hence of the employment of capital per unit of labour – by taking the rate of interest as the independent variable. This is obviously not very satisfactory, since one of the principal goals – if not indeed the most important goal – of Wicksell's analysis, both in *Value, Capital and Rent* ([1893] 1970) and in the first book of the *Lectures* ([1901] 1967), was to explain the determinants of the (real) rate of interest.

2.2 The quantity of capital and the marginalist theory of value and distribution

The reconstruction of Wicksell's analysis emerging from the 'missing equation' debate is misleading in our view because Malinvaud and the other authors mentioned above do not appear to attach much importance to what can instead be regarded as its fundamental characteristic, namely the fact of being a 'marginalist' theory of distribution.

In marginalist theories, as is known, the price system – which includes distribution variables, understood here as the prices of production factors – was expected to adjust to the point where excess demand is zero in every market. And the principles of decreasing marginal utility and productivity were used in order to construct excess demands for commodities and factors sensitive to changes in the price system.

Focusing attention in particular on the idea of decreasing marginal productivity, we can start by noting that, as Wicksell himself stressed (cf. Wicksell, 1967, pp. 116, 17), this derived from a generalization of the Ricardian theory of intensive rent.[3] While Ricardo made use of this principle in order to determine rents for a given wage rate, the marginalist economists tried to use it in order to determine all of the distributive variables. For such a generalisation to hold, however, capital must be regarded as capable of changing its physical form while remaining fixed in terms of quantity.

Let us consider the marginal product of labour, for example. According to a standard definition, this is the increase in output obtained from a given capital stock when an additional worker is employed. It is quite clear here that the given capital stock cannot be regarded as a vector of physical quantities of capital goods, otherwise no change would be possible in the technique used and the output obtained. Unlike labour and land, most capital goods are in fact highly specialised inputs invented and produced in order to perform a specific task in a specific way, and the switch to a more labour-intensive technique therefore entails a change in the kind of capital goods employed. If there is no change in the latter, there can be no change in the technique in use. Therefore, in order to have the marginal product of labour, capital must be conceived as a given magnitude that can take different forms.[4]

Accordingly, Jevons, Böhm-Bawerk, J.B. Clark and many other economists attempted to construct a marginalist theory of distribution by adopting a conception of capital based on the average period of production. This seemed to allow the possibility of an adjustment in the physical composition of the capital in use on the one hand and a measurement of the amount of capital independently of prices and income distribution on the other. If this idea of capital had not run up against the problems outlined below (sec. 4), the marginalist theory would be able to determine the interest rate capable of bringing the average period of production of the technique in use into line with the amount of existing capital, namely the wage fund available to pay the workers during the period of production.[5] And this was exactly what Wicksell, following Böhm-Bawerk, intended to do in *Value, Capital and Rent*. Even in the *Lecture*, despite his awareness of the difficulties of using the average period in general, he was interested in answering 'the question why a given amount of existing social capital gives rise to a certain rate of interest, neither higher nor lower' (Wicksell, [1901] 1967, p. 171).

The supply of capital found in Wicksell's theory, as well as many other marginalist theories, is therefore not the particular amount of capital that – given also the other data – generates the rate of interest that makes the net savings (per unit of labour) zero, as it would be if the missing equation were added. It is instead the 'existing social capital'. And since the total amount of existing capital in actual economies does not usually vary very much year by year, even without any zero net saving condition being imposed, treating it as an unchanging amount was considered a good approximation and a useful simplification.[6]

In Wicksell's theory, as a result, the zero net savings condition is not so much missing, but unwanted. Wicksell deliberately[7] omitted any such condition because he did not intend to confine his theory to the study of a hypothetical economic system in which the inducement to net capital accumulation is zero. He was instead interested in studying cases as close as possible to real life, and in real economies there is capital accumulation, even if the resulting changes in capital stock are usually very gradual.

To conclude, what is missing in Wicksell's theory, and in marginalist theories in general, is not a zero net saving condition but rather and most importantly a notion of capital capable of making it work consistently. In actual fact, when Wicksell became aware of the restrictions entailed by the conception of capital in terms of average period of production (cf. sec. 4), he had no way to express the amount of capital other than in value terms. And this opened up two different kinds of problem.

The first, as discussed and (partially) solved by Wicksell himself, stemmed from the fact that while it is necessary, in order to have a marginal product, to measure every factor of production in its 'technical unit', value is not a technical unit of measurement of capital because, for example, '[t]he productive contribution of a piece of technical capital, such as a steam engine, is determined not by its cost but by the horse-power which develops' (Wicksell, [1901] 1967, p. 149). As is known, the solution he proposed consisted in regarding the employment of capital as the employment of saved labour and saved land (cf. Wicksell, [1901] 1967, pp. 148 and ff). While there is no need to pursue this point any further here, it can be observed for future reference that the dual conception of capital – value and technical units – is the cause of the Wicksell effect considered in sec. 3.

The second problem, which lies at the root of the search for a supposedly missing equation, concerns the fact that Wicksell included the value of the existing capital stock among the data of his theory. An amount of value thus appears in Wicksell's theory among the determinants of value and distribution and this constitutes a fatal inconsistency.[8]

It should be clear that he had no alternative, as Garegnani (1960) argued more than fifty years ago. In the first place, the principle of marginal productivity he intended to use entailed a view of capital as a single magnitude capable of taking different forms. Second, like all the marginalist economists of his time, Wicksell was interested in the determination of the rate of interest associated with the existing capital stock. Third and last, once the impossibility of using the average period of production had been established, Wicksell was left with no way other than value to express aggregate capital. It was these three points together and not any supposedly missing equation that forced Wicksell to consider capital stock in value terms among the data of his theory.

3 The marginal product of capital

3.1 Analysis of Åkerman's problem

As pointed out above, the abandonment of the average period of production left Wicksell with a dual conception of capital, aggregate capital being expressed in value terms and technical capital conceived as a series of quantities of saved labour and saved land. This duality lies at the root of what Uhr (1951) called the "Wicksell effect", i.e. the non-equality of the rate of interest with the marginal product of (aggregate) capital.

The point is quite simple. Capital is an amount of value and its marginal product cannot therefore be something technical because value is not a "technical unit" of measurement for capital.[9] As a result, the choice of the optimal – i.e. profit-maximising – technique does not involve the satisfaction of a condition of equality between the marginal product of value capital and the rate of interest, which is the cost of its employment. In actual fact, as Wicksell was the first to observe (cf. Wicksell, [1901] 1967, p. 180), the two generally differ.

In this connection, Malinvaud (2003, p. 523 n.7) quotes Swan's description of the Wicksell effect as "[n]othing but an inventory revaluation" (Swan 1956, p. 355). There would thus appear to be some possibility of restoring the equality between the marginal product of capital and the interest rate by preventing such a revaluation. The idea is ascribed to Hicks, who is credited by Malinvaud (2003, pp. 509, 517 and 519) with putting forward the right methodology for "comparative assessments" concerning capital theory in *Value and Capital* (1946).

On the assumption of a variation in the interest rate and hence a change in the technique in use, the methodology consists in aggregating the capital employed (for a given amount of labour) with the different techniques by means of the same system of prices, namely the initial one, which is also the equilibrium one. When the variation of capital is determined in this way, the ratio of the change in the net product obtained to the change in capital employed (with fixed employment of labour) will ultimately prove equal to the rate of interest. This result has been known for a long time and its meaning has already been the object of discussion. (See in particular Swan 1956, Bhadurj 1966, Pasinetti 1969 and Garegnani 1984.)

Since Malinvaud illustrates this point with reference to the model Wicksell constructed for his analysis of Åkerman's problem,[10] we will introduce this model briefly here and return to Malinvaud's equality of the marginal product of capital and the interest rate in sec. 3.2.

Wicksell's analysis of Åkerman's problem refers to an economy with just one consumption good produced by means of labour and capital goods (axes). In particular, with L_2 units of labour and J axes, it is possible to obtain a quantity y_2 of the consumption good, with:

$$y_2 = cL_2{}^{\alpha} \cdot J^{1-\alpha}. \tag{1}$$

Axes are produced by means of labour alone. The length of the utilisation of an axe – θ in Malinvaud's notation – depends on the amount of labour z employed in its production, with:

$$z = k\theta^{\omega}. \tag{2}$$

If ν and w are respectively the price of the annual services of an axe and the wage rate, profit maximisation in the consumption good sector requires:

$$w = \frac{\alpha y_2}{L_2} \tag{3}$$

and

$$\nu = \frac{(1-\alpha)\cdot y_2}{J}. \tag{4}$$

In equilibrium, the cost of production of an axe – i.e. w·z – must equal the present value of the services that it will provide. Therefore:

$$w\cdot z = \nu\int_0^\theta e^{-\rho\cdot\tau}d\tau = \nu\frac{(1-e^{-\rho\cdot\theta})}{\rho}. \tag{5}$$

Moreover, taking equation (5) also in consideration, we find that the lifetime θ maximising profit in the axe sector – i.e. the difference between $\nu(1-e^{-\rho\cdot\theta})/\rho$ and $w\cdot k\theta^{\omega}$ – must satisfy the condition:

$$e^{\rho\cdot\theta} = 1+\frac{\rho\cdot\theta}{\omega}. \tag{6}$$

Using the same principle adopted for the RHS of equation (5), we find that the price of an axe with a residual life t is $q(t) = \nu(1-e^{-\rho\cdot t})/\rho$. Therefore, on the assumption that the axes employed in the economy are uniformly distributed for residual life (or that there are J/θ axes for every residual life t, with $0 < t \leq \theta$, which is the same thing), the value of the total employment of capital in the economy is:

$$V = \frac{J}{\theta}\int_0^\theta q(t)dt = \frac{J\cdot\nu}{\theta\cdot\rho}\int_0^\theta(1-e^{-\rho\cdot t})dt = \frac{J\cdot\nu}{\theta\cdot\rho^2}(\rho\cdot\theta-1+e^{-\rho\cdot\theta}). \tag{7}$$

Finally, since J/θ new axes must be produced in every period (or moment) in order to keep the physical capital of the economy stationary, zJ/θ units of labour must be devoted to this function. Letting L be the given supply of labour, we therefore have:

$$L = z\cdot J/\theta + L_2. \tag{8}$$

We thus have eight equilibrium conditions with nine unknown variables: z, ν, w, ρ, L_2, J, y_2, V and θ. As a result, Wicksell regarded the lifetime of the axes θ as an independent variable.[11] In this way, by solving the system, he arrived at the conclusion[12] that the lifetime of the axes θ is related inversely to the rate of interest ρ, and directly to the value of the capital V and to the quantity of consumption good produced y_2. In particular, with λ_1, λ_2, λ_3 and λ_4 as four positive coefficients, the solution worked out by Wicksell ([1901] 1967, p. 289) is:

$$\rho = \lambda_1 \theta \tag{9}$$

$$V = \lambda_2 \theta^{1+(1-\alpha)(1-\omega)} \tag{10}$$

$$y_2 = \lambda_3 \theta^{(1-\alpha)(1-\omega)} \tag{11}$$

$$\nu = \lambda_4 \theta^{-\alpha(1-\omega)}. \tag{12}$$

Equations (9)–(11) can easily be used to verify the Wicksell effect. In particular, we can differentiate y_2 and V with respect to θ, determine the ratio of the two results, and then – because of equation (9) – substitute λ_1/ρ for θ. This gives us:

$$\frac{dy_2}{dV} = \frac{\lambda_3}{\lambda_1 \lambda_2} \cdot \frac{(1-\alpha)(1-\omega)}{1+(1-\alpha)(1-\omega)} \cdot \rho \ . \tag{13}$$

Therefore, since $[\lambda_3(1-\alpha)(1-\omega)]/\{\lambda_1\lambda_2[1+(1-\alpha)(1-\omega)]\} \neq 1$ in general, the result is $dy_2/dV \neq \rho$, as Wicksell himself remarked ([1901] 1967, p. 292).

3.2 Malinvaud's equality between the marginal product of capital and the rate of interest

Malinvaud's approach in dealing with the model just outlined is different from Wicksell's. In the first place, he closes it by taking the interest rate ρ – rather than the lifetime θ of a new axe –

as an independent variable (Malinvaud, 2003, p. 513). Second, he seeks to obtain an equality between the marginal product of capital and the rate of interest by means of Hicks's above-mentioned "methodology".

Malinvaud introduces his argument as follows:

> [r]easoning as Hicks [...] Wicksell would have argued (i) that the volume K of capital must be defined by aggregating the numbers J/θ of the machines of various ages $a = \theta - \tau$ using proper weights, namely the respective prices $q(\theta,a)$; (ii) that for comparative assessments in the neighbourhood of a given stationary state, where by definition $K = V$, the physical marginal productivity of capital should not be defined with respect to infinitesimal changes in V, from one stationary state to another neighbouring stationary state, but to changes $\hat{d}K$ in K as computed with unchanged weigh. [Malinvaud, 2003, p. 519]

In greater detail, Malinvaud's idea is the following. There are different kinds of capital goods – axes of different ages – in Wicksell's model. These different capital goods are aggregated into a value capital V by means of their prices $q(t)$ for every length of residual life t. The value capital V therefore depends: (i) on the number J/θ of axes of different ages (there is in fact the same number of axes for every age), (ii) on the lifetime θ of a new axe, and (iii) on the prices $q(t)$. If J/θ and θ alone are allowed to change while the prices $q(t)$ are kept at their equilibrium level, we obtain the incomplete variation $\hat{d}K$ that Malinvaud intends to use for the calculation of his marginal productivity of capital.

Let us follow Malinvaud and take equation (7) as our starting point. By differentiating the RHS of the equation with respect to J/θ and θ but taking ρ and ν as fixed, we obtain:

$$\hat{d}K = \frac{\nu}{\rho^2}(\rho \cdot \theta - 1 + e^{-\rho\cdot\theta}) \cdot d(J/\theta) + \frac{J \cdot \nu}{\theta \cdot \rho}(1 - e^{-\rho\cdot\theta}) \cdot d\theta . \tag{14}$$

Because of equations (4), (11) and (12), the variation d(J/θ) is:

$$d(J/\theta) = -\omega \frac{J}{\theta^2} d\theta \tag{15}$$

Therefore, by substituting equation (15) into equation (14) and using equation (6), $\hat{d}K$ becomes:

$$\hat{d}K = (1-\omega)\frac{J \cdot \nu}{\rho \cdot \theta} d\theta \ . \tag{16}$$

Moreover, since from equation (11) we have:

$$dy_2 = (1-\omega)(1-\alpha)\frac{y_2}{\theta} d\theta \tag{17}$$

we can now calculate Malinvaud's marginal productivity of capital:

$$\frac{dy_2}{\hat{d}K} = \frac{(1-\alpha) \cdot y_2}{J \cdot \nu} \rho = \rho \ . \tag{18}$$

Here we have the equality between the marginal product of capital – as "properly defined" – and the interest rate. Malinvaud comments on this result with a certain satisfaction:

> [f]ew readers today will be surprised to find, in a discussion of capital theory under perfect competition, the assertion that the rate of interest is equal to the marginal productivity of capital when this marginal productivity is properly defined for comparative analysis. Some may perhaps be puzzled by the idea that the assertion follows from Wicksell's model. [Malinvaud, 2003, p. 521]

Being an already known result, as stated above, this equality is in fact not very surprising. Equally well-known is the fact that actually has very little bearing on the marginalist theory of distribution, as shown below in sec. 3.3.

3.3 An illusory equality

Let us begin our discussion of Malinvaud's result by pointing out that the equilibrium system formed by equations (1)–(8) includes two conditions of equality between the marginal product of an input and its price, namely equations (3) and (4). They are, as noted above, the first-order conditions of the profit-maximisation problem regarding the consumption good sector. Therefore, as usual, these conditions make it possible to determine for every level of w and ν, the corresponding demand for labour and axes in the production of the consumption good.

The equality that Malinvaud finds between his "properly defined" marginal product of capital and the interest rate has neither the same origin nor the same function as conditions (3) and (4). It is not in fact the first-order condition of a profit-maximisation problem and does not make it possible to determine the demand for capital in value terms associated with a certain rate of interest,[13] since this is already determined by solving the system (1)–(8) and is the amount resulting from equations (9) and (10).[14] Having established what Malinvaud's equality is not, we can now go on to see what it is.

In order to show the real meaning of Malinvaud's equality, we shall refer to an economy in which a single consumption good is produced by means of labour and n different kinds of capital goods. Assuming constant returns to scale, we denote by $y^{(i)}$ and $k^{(i)} = [k_1^{(i)}, k_2^{(i)}, ..., k_n^{(i)}]$ respectively the net output obtained and the vector of capital goods employed with technique (i), both per unit of labour. Given a vector $p = [p_1, p_2, ..., p_n]$ of prices expressed in terms of the consumption good, an interest rate ρ and a wage rate w, the (extra) profits per unit of labour entailed by the use of technique (i) are:

$$\pi^{(i)}(p,\rho,w) = y^{(i)} - \rho\sum_{\ell=1}^{n} k_\ell^{(i)} \cdot p_\ell - w. \tag{19}$$

Following the argument in Pasinetti (1969), if another technique (j) entails the same amount of profits per unit of labour as technique (i) for the same system of prices and distribution variables, this means that:

$$y^{(j)} - y^{(i)} = \rho\sum_{\ell=1}^{n} (k_\ell^{(j)} - k_\ell^{(i)}) \cdot p_\ell. \tag{20}$$

The meaning of equation (20) is quite evident. If the (extra) profits are the same, and the wage rate is the same, then the difference in the net product (per unit of labour) with the two techniques must correspond to the difference in the interest paid (per unit of labour).

We can now generalize the result of equation (20) as follow. When the system of prices and distribution is the equilibrium one, and y* and k* = [k_1*, k_2*, ..., k_n*] are the net output and the vector of capital employment per unit of labour with the optimal technique, then:

$$y^* - \rho\sum_{\ell=1}^{n} k_\ell^* \cdot p_\ell - w = 0 \tag{21}$$

and

$$(y^* + dy) - \rho\sum_{\ell=1}^{n} (k_\ell^* + dk_\ell) \cdot p_\ell - w = 0 \tag{22}$$

for every infinitesimal variations dy, dk_1, ..., dk_n.

Therefore, as before, equations (21) and (22) imply:

$$dy = \rho \sum_{\ell=1}^{n} dk_{\ell} \cdot p_{\ell} \quad \text{or} \quad \frac{dy}{\sum_{\ell=1}^{n} dk_{\ell} \cdot p_{\ell}} = \rho \tag{23}$$

and equation (23) is of exactly the same kind – and therefore has exactly the same meaning – as the equality that Malinvaud finds between his "properly defined" marginal product of capital and the rate of interest for the model constructed by Wicksell in order to study Åkerman's problem.

We can conclude our discussion of the result presented by Malinvaud in his paper with a couple of remarks. First, the equality in equation (23) – and in equation (18) – derives from the fact that: (a) the prices used to aggregate the quantities of different kinds capital goods (or axes with different residual life-periods) and the interest rate ρ are at their equilibrium levels;[15] (b) the variations in the level of net output and in the quantities employed of the different kinds of capital goods occur with respect to the technique that is optimal in equilibrium. Equations (21) and (22) – on which equality (23) is based – are in fact the consequence of conditions (a) and (b) (on this point see also Garegnani, 1984, pp. 146 and 156).

Second, far from disproving the validity of the Wicksell effect, equality (23) is another way to prove it. As is known, the change in value of capital (per unit of labour) in use can be broken down into a real effect and a price effect:

$$dv = \underbrace{\sum_{\ell=1}^{n} dk_{\ell} \cdot p_{\ell}}_{\text{real effect}} + \underbrace{\sum_{\ell=1}^{n} k_{\ell} \cdot dp_{\ell}}_{\text{price effect}} \tag{24}$$

and since, because of equation (23), the real effect is $\Sigma dk_{\ell} \cdot p_{\ell} = dy/\rho$, we have:

$$dv = \frac{dy}{\rho} + \sum_{\ell=1}^{n} k_{\ell} \cdot dp_{\ell} \tag{25}$$

which implies in general $dy/dv \neq \rho$.

4 The average period of production

4.1 The average period of production: the traditional formula

We shall begin our discussion of the average period of production by elucidating the role it played – as mentioned above (sec. 2) – in the earlier versions of marginalist theory. For this purpose, we shall consider the "Austrian model", where the sole consumption good is obtained by the employment of labour during the T periods of time preceding the moment of output. Let u_t be the share of labour employed t periods before output is obtained, with t = 1, 2, ... T, so that $\sum_{t=1}^{T} u_t = 1$. According to the traditional marginalist theory, the average period of production can then be defined by the following formula:

$$\overline{\theta} = \sum_{t=1}^{T} t \cdot u_t \ . \tag{26}$$

The amount of (net) output obtained per unit of labour is then assumed to be a function of this average period of production $y = f(\overline{\theta})$, with $f'(\overline{\theta}) > 0$ and $f''(\overline{\theta}) < 0$.

Moreover, if simple interest is assumed at a rate r and w is used to denote the wage rate paid at the beginning of each period in terms of the consumption good, the cost of production per unit of labour is:

$$c = \sum_{t=1}^{T} w \cdot u_t \cdot (1 + t \cdot r) = w \cdot (1 + \overline{\theta} r). \tag{27}$$

For given levels of the wage rate and the interest rate, the optimal average period of production can therefore be found by solving the following first-order condition:

$$f'(\bar{\theta}) - w \cdot r = 0. \tag{28}$$

At the same time, since extra-profits must vanish under the hypothesis of free competition, we have:

$$f(\bar{\theta}) - w \cdot (1 + \bar{\theta} \cdot r) = 0. \tag{29}$$

Equations (28) and (29) make it possible to associate each possible interest rate r with a wage rate w and an average period of production $\bar{\theta}$. In particular, we obtain:

$$\frac{f'(\bar{\theta})}{f(\bar{\theta}) - f'(\bar{\theta}) \cdot \bar{\theta}} = r\ . \tag{30}$$

And therefore, because of $f''(\bar{\theta}) < 0$, a decrease in the interest rate involves a longer average period of production.[16] In Samuelson's words, this is 'the simple tale told by Jevons, Böhm-Bawerk, Wicksell, and other neoclassical writers', according to which, 'as the interest rate falls in consequence of abstention from present consumption in favour of future, technology must become in some sense more "roundabout", more "mechanized", and more "productive"' (cf. Samuelson, 1966, p. 568).

As demonstrated, however, this "simple tale" is not universally valid. To be precise, in the form presented here it is clearly based on extremely strong assumptions, such as the application of the simple interest formula and the presence of a single primary factor (labour).[17] It is precisely because of the strong assumptions required that Wicksell, as already stated in sec. 2, abandoned this conception of capital in his *Lectures* after initially adopting it in *Value, Capital and Rent*.

4.2. The inverse relationship between the Hicks-Malinvaud average period of production and the interest rate

In his discussion of Wicksell's legacy, Malinvaud takes up the idea of the average period of production introduced by Hicks in *Value and Capital* (1946) and *Capital and Time* (1973). Hicks's idea is simple: it involves using shares of cost rather than shares of labour as weights in determining the average period of production.[18]

If compound interest is assumed at a rate ρ, the cost of production per unit of labour is not as in equation (27) but rather:

$$c = \sum_{t=1}^{T} w \cdot u_t \cdot (1+\rho)^t \tag{31}$$

where $w \cdot u_t \cdot (1+\rho)^t$ is the part of the cost that can be ascribed to the employment of labour t periods before the output. By using the ratio of this part of the cost to the total – i.e. $u_t \cdot (1+\rho)^t \big/ \sum u_t \cdot (1+\rho)^t$ – as the weight for t in the formula for the average period of production, we therefore obtain (cf. Malinvaud 2003, p. 516):

$$\bar{\theta} = \sum_{t=1}^{T} t \cdot \frac{u_t \cdot (1+\rho)^t}{\sum_{t=1}^{T} u_t \cdot (1+\rho)^t} = \frac{\sum_{t=1}^{T} t \cdot u_t \cdot (1+\rho)^t}{\sum_{t=1}^{T} u_t \cdot (1+\rho)^t} \,. \tag{32}$$

Now, since the weights are shares of labour in the traditional average-period formula (equation (26)) but shares of costs in the Hicks-Malinvaud version (equation (31)), while the former is completely independent of prices and distribution variables, the average period associated with a technique depends in the latter on the rate of interest. This appears to be the main concern of Hicks and Malinvaud.

In particular, there are two different effects of a change in the rate of interest on the average period of production. As the rate of interest performs two functions, a) entering into the determination of the average period associates with the techniques and b) making it possible to establish which technique is optimal and hence in use, any change in it affects the average period of production by two ways, involving both a change of the average periods associated with the various techniques and a change of the technique in use.

Malinvaud suggests that this problem can be avoided by means of the "methodology" used in sec. 3.2 to determine the "properly defined" marginal product of capital. In this way, he seeks to focus attention on the second effect alone, separating it from the first, and suggests that the average period associated with each technique should be kept the same in examining variations in the average period of the technique due to change in the interest rate. More precisely, with reference to equation (31), in Malinvaud's analysis the change in the interest rate affects the technique in use, and therefore the labour shares u_t, but is not allowed to affect the interest factors $(1+\rho)^t$, for every $t = 1, 2, ..., T$.[19]

Therefore, assuming a change in the interest rate, if we focus attention on the change in labour terms u_t, due to the change in the technique in use, while keeping the interest factor $(1+\rho)$ – hereafter R – constant, we obtain the change in the average period $\hat{d}\overline{\theta}$ that Malinvaud considers "relevant for comparative analysis" (2003, p. 517):

$$\hat{d}\overline{\theta} = \frac{\sum_{t=1}^{T}(t-\overline{\theta})\cdot R^t \cdot du_t}{\sum_{t=1}^{T} u_t \cdot R^t} \, . \tag{33}$$

After a long series of mathematical operations not shown here,[20] Malinvaud arrives at the conclusion that the change in the average period $\hat{d}\overline{\theta}$ must always be opposite in sign to the change in the rate of interest. In particular, he writes (2003, p. 518):

> [a] decrease in the real interest rate ρ [...] is associated with a lengthening of the average period of production, given what we mean by such lengthening

and comments:

> [i]t is interesting to know that the average period of production, *a measure of the degree of roundaboutness*, contra-varies with the interest rate. [Emphasis added.]

Following Hicks's 'methodology', Malinvaud thus seems to have arrived back at the simple tale of the old neoclassical writers but within a far more general framework. His result is, however, not exactly the same as the traditional one and, as will be shown below, the Hicks-Malinvaud average period is in fact far from being 'a measure of the degree of roundaboutness' of production.

4.3 An example with two techniques

As shown above, the Hicks-Malinvaud average period of production associated with a given technique is generally a function of the rate of interest and can therefore change with no change in the technique in use. This fact and its possible implications are viewed by Hicks and Malinvaud as the main problem connected with the use of their idea of the average period. We shall see in this section, however, that the problem is much more serious and concerns the ranking of techniques on the basis of Hicks-Malinvaud average period, which can change with the interest rate, as we shall see.

Let us consider an example in which there are only two possible techniques, (α) and (β), and denote by $y^{(i)}$ and $u_t^{(i)}$ respectively, with i = α, β, the net product per unit of labour and the share of labour required t periods before the final output is obtained with the two techniques.

For each technique, the maximum wage rate that can be paid is a function of the interest factor R:

$$w^{(i)}(R) = \frac{y^{(i)}}{\sum_{t=1}^{T} u_t^{(i)} \cdot R^t}, \text{ with } i = \alpha, \beta. \tag{34}$$

By differentiating the wage rate $w^{(i)}(R)$, we obtain:

$$\frac{dw^{(i)}(R)}{dR} = -\frac{y^{(i)}}{R} \cdot \frac{\sum_{t=1}^{T} t \cdot u_t^{(i)} \cdot R^t}{\left[\sum_{t=1}^{T} u_t^{(i)} \cdot R^t\right]^2} = -\frac{w^{(i)}(R)}{R} \cdot \frac{\sum_{t=1}^{T} t \cdot u_t^{(i)} \cdot R^t}{\sum_{t=1}^{T} u_t^{(i)} \cdot R^t} \tag{35}$$

and since, according to the Hicks-Malinvaud conception, the average period of production associated with technique (i), with $i = \alpha, \beta$, is:

$$\overline{\theta}^{(i)}(R) = \frac{\sum_{t=1}^{T} t \cdot u_t^{(i)} \cdot R^t}{\sum_{t=1}^{T} u_t^{(i)} \cdot R^t}, \tag{36}$$

equation (35) implies:

$$\overline{\theta}^{(i)}(R) = -\frac{dw^{(i)}}{dR} \cdot \frac{R}{w^{(i)}(R)}. \tag{37}$$

Equation (37) is very important in our argument. It clearly states that the average period of production associated with technique (i), with $i = \alpha, \beta$, is equal to the elasticity of the wage rate $w^{(i)}$ with respect to the interest factor R, with the sign changed. Given a certain interest factor, the technique with the most elastic wage-interest curve is the one with the highest average period of production.

In order to show the consequences of the above result, let R' be a switch point or, in other words, an interest factor such that $w^{(\alpha)}(R') = w^{(\beta)}(R')$. Because of equation (37), $\overline{\theta}^{(\alpha)}(R') > \overline{\theta}^{(\beta)}(R')$ if and only if $\left|dw^{(\alpha)}/dR\right| > \left|dw^{(\beta)}/dR\right|$ in R', which means that the technique with the steepest wage-interest curve has the highest average period of production at a switch point.

If we assume, however, the existence of another interest factor R'', with $R'' > R'$, such that $w^{(\alpha)}(R'') = w^{(\beta)}(R'')$, the ranking of techniques based on the period of production calculated at R'' must be opposite to the one calculated at R', i.e. $\overline{\theta}^{(\alpha)}(R'') < \overline{\theta}^{(\beta)}(R'')$. This result follows very simply from the observation that if the wage-interest curve $w^{(\alpha)}(R)$ is steeper than $w^{(\beta)}(R)$ at the switch point R', then it must be less steep than $w^{(\beta)}(R)$ at the subsequent switch point, as shown in fig. 1. Equation (37) therefore implies that $\overline{\theta}^{(\alpha)} > \overline{\theta}^{(\beta)}$ at R' and $\overline{\theta}^{(\alpha)} > \overline{\theta}^{(\beta)}$ at R''.

[fig. 1 about here]

Moreover, when R moves in the vicinity of a switch point, the technique in use (the technique that makes it possible to pay the highest wage) for interest factors lower than the switch level is the one with the steepest wage-interest curve. The technique with the flattest wage-interest curve therefore comes into use for interest rates higher than the switch level.[22] This is Malinvaud's result, according to which an increase in the rate of interest is associated with the use of a technique with a shorter average period. And this is true at both switch points, since the technique with the highest average period at the interest factor R' – i.e. technique α – is the one with the lowest average period at the interest factor R''. Therefore, despite the reswitching of techniques, thanks to the Hicks-Malinvaud definition, a technique with a lower average period of production is adopted at both switch points as the rate of interest increases.

This result appears, however, to have little or no significance. Contrary to what Malinvaud claims, the Hicks-Malinvaud average period does not express the "degree of roundaboutness" or "capital deepening" of the production techniques. This is clearly proved by our simple example, where the technique with the longest average period at the first switch point becomes the one with the shortest average period at the second.

5 Conclusions

To summarise, the analysis put forward in this paper concerns three claims that Malinvaud makes in his article of 2003: 1) 'the average period of production, a measure of the degree of roundaboutness, contra-varies with the interest rate' (p. 518); 2) 'the rate of interest is equal to the marginal productivity of capital when this marginal productivity is properly defined for comparative analysis' (p. 521); 3) taking the value of the existing capital as a given magnitude is 'an unfortunate feature' of Wicksell's theory, since any variable could be taken exogenously in order to close the degree of freedom left open because of the "missing equation" (cf. Malinvaud, 2003, p. 510).

The validity of these three claims would mean complete rehabilitation of the marginalist theory of capital and hence of distribution and value. As we have shown, this is not the case.

In particular, as seen in sec. 2, there is no missing equation in Wicksell's theory and the given amount of capital found there – like the given amounts of labour and land – is a characteristic feature of the marginalist explanation of distribution. The problem is rather the lack of a consistent way – due to the particular structure of the theory – to express the quantity of this given capital.

With respect to Malinvaud's equality of the interest rate and the "properly defined" marginal product of capital, as shown in sec. 3 and contrary to the case in which the marginal product is related to factors of production measured in "technical units", this is not a first-order condition of the (extra) profit-maximisation problem and therefore does not perform the function of determining the optimal employment of capital. It is instead a well-known equality found, when Wicksell price effects are ignored, for variations in the technique in use around an equilibrium position.

Finally, by means of a very simple example, it is proved in sec 4 that the Hicks-Malinvaud average period of production does not express the 'degree of roundaboutness' of the techniques, since the ranking of techniques in terms of it can change when the rate of interest varies and it is therefore impossible to say, independently of the rate of interest, which technique is more capital-intensive.

Notes

1. On this point, see Garegnani (1976 and 2011).
2. See Kurz (2000) for a critical survey of the views of these authors.
3. Ricardo's theory of intensive rent is grounded on the possibility of applying successive doses of labour on a fixed area of land and thus giving rise to successive but always smaller increments in the amount of produce obtained.

   Although Ricardo referred to the application of successive doses of capital on a given area of land, capital is assumed in his analysis to consist (essentially) of wages paid at the beginning of the process and the wage rate is taken as a given. Each dose of capital therefore corresponds to a dose of labour (see also Wicksell, [1901] 1967, p. 117).
4. On the conception of capital and marginal productivity, see also Trabucchi (2011).
5. See Garegnani (1960, pp. 123–34 and 147–55) for a detailed analysis of this argument.
6. As Knight wrote:

   [w]e assume that the fundamental conditions of economic life in the aggregate, on both supply and demand sides of the relation, remain unchanged. These fundamental conditions include (a) the total supply of productive resources ("land, labor, and capital"); (b) the "state of the arts" or

knowledge of productive methods and processes; and (c) the "psychology", tastes and habits of the people. Significant changes in these things are generally progressive in character, in contrast to the readjustments to accidental fluctuations [Knight, 1921, p. 311].

Thus, as Knight stated very clearly, it is assumed that the total supply of capital, together with all the other data, remains unchanged because significant changes in it are generally gradual or "progressive in character", which means that they will become relevant only in the long run, while the adjustment to an equilibrium position is instead assumed to be very rapid.

It is after all common practice in every science to examine quick dynamical processes by assuming the invariance of magnitudes that vary extremely slowly with respect to the others, and the conception of equilibrium that thus emerges has been called a "quasi-stationary state" precisely in order to stress that what is involved in these cases is an approximation.

7. Wicksell (1967: p. 171) criticised Walras for including the zero net saving condition in his equilibrium system.
8. Negishi (1982: p. 192) describes the given value of capital in Wicksell's theory as 'quite unsatisfactory' and Malinvaud (2003: p. 510) as 'an unfortunate feature'. They tend therefore to underestimate the relevance of the problem, even though it is almost clear in every paper of the 'missing equation' debate that the search for an equation to add to Wicksell's system is aimed at circumventing this difficulty. See also Potestio (1999) and Kurz & Salvadori (2001).
9. As Wicksell himself argued, the employment of capital in value terms cannot appear in a production function, since the link between it and the amount of product obtained is not technical.
10. The problem Åkerman highlighted consists essentially in verifying the validity of the fundamental relations of marginal productivity in the case with fixed or durable capital goods in which a further variable must be taken into account, namely the lifetime of capital goods employed.

11. See Garegnani (1960, p. 143 n. 51) for a critical discussion of this procedure.

12. Wicksell's aim in constructing this model was to show that an increase in the lifetime of durable capital goods does not necessarily imply – as Åkerman instead seemed to believe – a decrease in the amount of labour employed in order to keep the physical capital available unchanged. As he explicitly admitted, all the characteristics of his model were indeed selected specifically for this purpose.

13. There is a way in which the equality between the interest rate and a certain conception of the marginal product of value capital can be viewed as a first-order condition of a maximization problem also in the case with heterogeneous capital goods. This is the case considered in Salvadori (1996).

    The procedure can be briefly summarized as follows. Given a rate of interest ρ, it is possible to determine the price vector associated with the use of a certain technique θ and then the value of the capital per worker v^θ in cases where this technique is in use. If y^θ is the net product per worker with technique θ, we have a pair (y^θ, v^θ) associated to this technique. Moreover, $w^\theta = y^\theta - \rho\cdot v^\theta$ is the wage rate that can be paid when technique θ is in use. If there are enough techniques to express y as a continuous function of v: $y = \phi_\rho(v)$, then, since the optimal technique for a given interest rate is the one making it possible to pay the highest wage rate, it is the technique that maximizes the difference $\phi_\rho(v) - \rho v$. Therefore, if $\phi_\rho(v)$ is a differentiable function, $\phi_\rho'(v) = \rho$ is the first-order condition of the wage rate maximization problem.

    It should be noted, however, that while in the present argument we have aggregated the capital goods of each technique using the prices associated with the use of that technique, in Malinvaud's case the value of capital employed with each technique is determined by means of the same vector of prices, namely the one associated with the use of the optimal technique for the given ρ. As a result, in Malinvaud's analysis, the difference between net product per worker and interest on value capital per worker with technique θ is not, in general, the wage rate that the use of technique θ makes it possible to pay. The difference has this meaning in fact only for the optimal technique. Malinvaud's equality cannot therefore be interpreted as the first-order condition of the wage rate maximization problem.

It can also be observed that the above equality $\phi_\rho'(v) = \rho$ does not imply a decreasing demand for capital in value terms, since the function $\phi_\rho(v)$, and then $\phi_\rho'(v)$, changes when ρ varies. (Readers are referred to Salvadori, 1996 and Kurz & Salvadori 2010 for further details.)

14. In particular, using equations (9) and (10), we have: $V = \lambda_2 \left(\frac{\rho}{\lambda_1} \right)^{1+(1-\alpha)(1-\omega)}$.

15. Malinvaud takes ρ as an exogenous variable, even though this level of the interest rate is assumed to be the one that brings the production (demand) side of capital theory into equilibrium with its consumption (supply) side (cf. Malinvaud, 2003, pp. 510 and 513).

16. If we assume $g(\bar{\theta}) = \frac{f'(\bar{\theta})}{f(\bar{\theta}) - f'(\bar{\theta}) \cdot \bar{\theta}}$, it follows that $g'(\bar{\theta}) = \frac{f'(\bar{\theta}) \cdot f''(\bar{\theta})}{[f(\bar{\theta}) - f'(\bar{\theta}) \cdot \bar{\theta}]^2}$ and $f''(\bar{\theta}) < 0$ implies $g'(\bar{\theta}) < 0$. From equation (30) – i.e. $g(\bar{\theta}) = r$ – the average period $\bar{\theta}$ and the interest rate r must therefore vary in opposite directions.

17. For a discussion of the traditional average period of production, see also Garegnani (1960, pp. 123–36) and Petri (2004, pp. 99–117).

18. See Fratini (2012) for an in-depth discussion of the Hicks-Malivaud average period of production.

19. In the words of Hicks, as quoted by Malinvaud:

[i]f the average period changes, without the rate of interest having changed, it must indicate a change in the stream [of inputs]; but if it changes, when the rate of interest changes, this need not indicate any change in the stream at all. Consequently, even when we are considering the effect of changes in the rate of interest on the production plan, we must not allow the rate of

interest which we use in the calculation of the average period to be changed. [Hicks, 1946, p. 220].

20. See Malinvaud (2003, pp. 517, 18) or Fratini (2012, appendix) for the mathematical steps leading to Malinvaud's conclusion.

22. Hicks writes as follows in *Capital and Growth* (1965):

> [w]hen there is a rise in the rate of real wages (or a fall in the rate of profit) there will always be a tendency to shift to a technique with a wages curve which (in the way we have drawn our diagrams) is, at that, level of wages, a curve with a slope that is less. That is to say, the new wage curve must be one on which, at that level, profits are less affected by a given rise in wages. In that sense, and in that sense only, the new technique must be one with a lower labour-intensity. And since the whole thing can be put the other way, it is also a technique in which wages are more affected by a given rise in profits. In that sense, and only in that sense, we can safely say that the new technique is one of greater capital intensity. [Hicks, 1965, pp. 166, 7]

The point is also considered in *Capital and Time* (1973, p. 45).

References

Arrow, K.J. & Hahn, F.H. (1971) *General Competitive Analysis* (San Francisco: Holden-Day).

Bhaduri, A. (1966) 'The Concept of the Marginal Productivity of Capital and the Wicksell Effect', *Oxford Economic Papers*, XVIII, 284–8.

Burmeister, E. (1980) *Capital Theory and Dynamics* (Cambridge: Cambridge University Press).

Burmeister, E. (2006): ‘A Retrospective View of *Hicks’s Capital and Time: A Neo-Austrian Theory*’, in H. Hagemann and R. Scazzieri (eds) *Capital, Time and Transitional Dynamics* (London: Routledge).

Debreu, G. (1959) *Theory of Value* (New York: Wiley).

Fellner, W. & Ellis, H.S. (1940) ‘Hicks and the Time-Period Controversy’, *The Journal of Political Economy*, XLVIII(4), 563–78.

Fratini, S.M. (2010) ‘Reswitching and Decreasing Demand for Capital’, *Metroeconomica*, LXI(4), 676–82.

Fratini, S.M. (2012): ‘The Hicks-Malinvaud average period of production and ‘marginal productivity’: a critical assessment’, *European Journal of the History of Economic Thought*, forthcoming.

Garegnani, P. (1960) *Il Capitale nelle Teorie della Distribuzione* (Milan: Giuffré)

Garegnani, P. (1970) ‘Heterogeneous Capital, the Production Function and the Theory of Distribution’, *The Review of Economic Studies*, XXXVII, 407–36.

Garegnani, P. (1976) ‘On a change in the notion of equilibrium in recent work on value and distribution’, in M. Brown, K. Sato & P. Zaremka (eds) *Essays in Modern Capital Theory* (Amsterdam: North-Holland).

Garegnani, P. (1984) ‘On some illusory instances of «marginal products»’, *Metroeconomica*, XXXVI(2–3), 143–60.

Garegnani, P. (2011) ‘Capital in the Neoclassical Theory. Some Notes’, *Nómadas. Revista Crítica de Ciencias Sociales y Jurídicas*, MA/2011.1.

Hicks, J.R. (1946) *Value and Capital*, 2nd ed. [1st ed. 1939] (Oxford: Clarendon Press).

Hicks, J.R. (1965) *Capital and Growth* (Oxford: Clarendon Press).

Hicks, J.R. (1973) *Capital and Time: a Neo-Austrian Theory* (Oxford: Clarendon Press).

Hirshleifer, J. (1967) ‘A Note on Böhm-Bawerk/Wicksell Theory of Interest’, *The Review of Economic Studies*, XXXIV(2), 191–9.

Kurz, H.D. (2000) 'Wicksell and the Problem of the "Missing" Equation', *History of Political Economy*, XXXII(4), 765–88.

Kurz, H.D. & Salvadori, N. (1995) *Theory of production* (Cambridge: Cambridge University Press).

Kurz, H.D. & Salvadori, N. (2001) 'The aggregate neoclassical theory of distribution and the concept of a given value of capital: a reply', *Structural Change and Economic Dynamics*, XII, 479–85

Kurz, H.D. & Salvadori, N. (2010) 'Spurious 'margins' versus the genuine article', in A. Birolo, D.K. Foley, H.D. Kurz, B. Schefold, and I. Steedman (eds) *Production, Distribution and Trade: Alternative Perspectivde. Essays in honour of Sergio Parrinello* (London: Routledge).

Malinvaud, E. (2003) 'The Legacy of Knut Wicksell to Capital Theory', *Scandinavian Journal of Economics*, CV(4), 507–25.

Negishi, T. (1982) 'Wicksell's Missing Equation and Bohm-Bawerk's Three Causes of Interest in a Stationary State', *Zeitschrift für Nationalokonomie*, XLII, 161–74.

Pasinetti, L.L. (1969) 'Switches of technique and the 'rate of return' in capital theory', *The Economic Journal*, LXXIX, 508–31.

Petri, F. (2004) *General Equilibrium, Capital and Macroeconomics* (Cheltenham: Edward Elgar Publishing).

Potestio, P. (1999) 'The Aggregate Neoclassical Theory of Distribution and the Concept of a Given Value of Capital: Toward a More General Critique', *Structural Change and Economic Dynamics*, X, 381–94.

Salvadori, N. (1996) ''Productivity curves' in the Accumulation of Capital', in M.C. Marcuzzo, L.L. Pasinetti & A. Roncaglia (eds) *The Economics of Joan Robinson* (London and New York: Routledge). Reprinted in H. Kurz & N. Salvadori (1998) *Understanding 'Classical' Economics. Studies in Long-Period Theory* (London and New York: Routledge).

Samuelson. L. (1982) ‘On Wicksell's Missing Equation’, *History of Political Economy*, XIV, 301–7.

Samuelson, P.A. (1966): ‘A Summing Up’, *The Quarterly Journal of Economics*, LXXX(4), 568–83.

Sandelin. B. (1980) ‘Wicksell's Missing Equation, the Production Function, and the Wicksell Effect’, *History of Political Economy*, XII, 29–40.

Sargan, J.D. (1955) ‘The Period of Production’, *Econometrica*, XXIII(2), 151–65.

Swan, T.W. (1956) ‘Economic Growth and Capital Accumulation’, *The Economic Record*, XXXII, pp. 334–61.

Trabucchi, P. (2011) ‘Capital as a Single Magnitude and the Orthodox Theory of Distribution in Some Writings of the Early 1930s’, *Review of Political Economy*, XXIII(2), 169–88.

Uhr, C.G. (1951) ‘Knut Wicksell—A Centennial Evaluation’, *American Economic Review*, XLI(5), 861-78.

Wicksell, K. ([1893] 1970) *Value, Capital and Rent* (New York: Augustus M. Kelley Publishers).

Wicksell, K. ([1901] 1967) *Lectures on Political Economy, vol. I*, (New York: Augustus M. Kelley Publishers).

Fig. 1