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Abstract

The Clarke Pivotal Voting Mechanism (CPVM) elicits truthful revelation of utility functions by requiring any 'pivotal' voter to pay a monetary 'Clarke tax'. This neglects wealth effects and gives disproportionate power to rich voters. We propose to replace the 'Clarke tax' with a lottery, wherein the pivotal voter risks long-term exclusion from the CPVM (and any other formal political participation). The resulting voting mechanism is nonmanipulable, politically egalitarian, and implements something close to Relative Utilitarianism.

Let $\mathcal{A}$ be a set of social alternatives and let $\mathcal{I}$ be a set of individuals. For all $i \in \mathcal{I}$, let $u_i : \mathcal{A} \longrightarrow \mathbb{R}$ be a cardinal utility function describing i's preferences over $\mathcal{A}$. Classic utilitarianism prescribes the policy alternative in $\mathcal{A}$ which maximizes the social welfare function $\sum_{i \in \mathcal{I}} u_i$. By using cardinal utility functions rather than mere preference orderings as input, utilitarianism escapes the pessimistic conclusion of Arrow's Impossibility Theorem. Furthermore, it admits several appealing mathematical characterizations, due to Harsanyi (1953, 1955), d'Aspremont and Gevers (1977), Maskin (1978), Myerson (1981) and Ng (1975, 1985, 2000). However, like almost any other voting system or social welfare function, utilitarianism can be easily manipulated by voters who strategically misrepresent their utility functions. The Clarke (1971) *Pivotal Voting Mechanism* (CPVM)[1] is a 'truth-revealing' implementation of utilitarianism, through a hybrid between a referendum and an auction:

1. Each voter i declares a monetary *valuation* $v_i(a)$ for each alternative $a \in \mathcal{A}$. We regard $v_i(a)$ as a proxy for $u_i(a)$.

2. Given the set of valuations $\mathbf{v} := (v_i)_{i \in \mathcal{I}}$, society chooses the alternative $\alpha(\mathbf{v}) \in \mathcal{A}$ which maximizes the aggregate valuation $V(a) := \sum_{i \in \mathcal{I}} v_i(a)$.

[1] The CPVM is a special case of Groves' (1973) *demand-revealing mechanism*. It was also promoted by Tideman and Tullock (1976), and is extensively analyzed in the collection by Tideman (1977) and the monograph by Green and Laffont (1979). See also §8.2 of Moulin (1988), §23.C of Mas-Colell et al. (1995), §5 of Tideman (1997), and §8.1 of Mueller (2003).

3. If $\alpha(\mathbf{v}) = a$, then voter i is *pivotal* if there is some other $b \in \mathcal{A}$ with $V(a) - V(b) < v_i(a) - v_i(b)$; thus, if i had voted differently (i.e. if $v_i(b)$ had been bigger or $v_i(a)$ had been smaller), then the social choice would have been b instead of a. In this case, voter i must pay a *Clarke tax* $\tau_i(\mathbf{v}) := \sum_{j \neq i} [v_j(b) - v_j(a)]$.

It is easy to check that $\tau_i(\mathbf{v}) \leq v_i(a) - v_i(b)$; in other words, the Clarke tax never exceeds i's personal gain in obtaining a rather than b. Thus i should always be willing to pay the tax $\tau_i(\mathbf{v})$ in order to secure alternative a —assuming she expressed her preferences honestly. Indeed, suppose i has a *quasilinear* utility function

$$u_i(a, -t_i) \quad = \quad w_i(a) - t_i, \qquad \forall a \in \mathcal{A} \text{ and } \; t_i \in \mathbb{R}, \tag{1}$$

where $w_i : \mathcal{A} \longrightarrow \mathbb{R}$ is her utility function over the policy alternatives and t_i is the Clarke tax she must pay (thus, we could say that $w_i(a)$ is the monetary *worth* which voter i assigns to alternative $a \in \mathcal{A}$). Consider a collection $\mathbf{v}_{-i} := (v_j \; ; \; j \in \mathcal{I} \setminus \{i\})$ of valuations by all other voters. The valuation $v_i : \mathcal{A} \longrightarrow \mathbb{R}$ is i's *best response* to $\mathbf{v}_{-i}$ if, for any other possible valuation $v_i' : \mathcal{A} \longrightarrow \mathbb{R}$, we have $u_i\left[\alpha(v_i, \mathbf{v}_{-i}); -\tau_i(v_i, \mathbf{v}_{-i})\right] \geq u_i\left[\alpha(v_i', \mathbf{v}_{-i}); -\tau_i(v_i', \mathbf{v}_{-i})\right]$. We say that valuation v_i is a *dominant strategy* if it is a best response to every possible $\mathbf{v}_{-i}$.

Theorem 1 *Suppose voter i has quasilinear utility function* (1)*. Then i's dominant strategy is to set $v_i := w_i + c$, where c is any constant.*[2] □

If Theorem 1 holds for all $i \in \mathcal{I}$, then in the resulting dominant strategy equilibrium, the CPVM chooses the alternative which maximizes $\sum_{i \in \mathcal{I}} w_i$. If $\tau_i(\mathbf{v}) = 0$ for all $i \in \mathcal{I}$ (which will be true almost all the time, in large populations), then this outcome also maximizes $\sum_{i \in \mathcal{I}} u_i$; in this sense, the CPVM obtains the outcome prescribed by utilitarianism. The CPVM also satisfies other appealing axiomatic characterizations, due to Moulin (1986) and Sjostrom (1991). However, because it links voting to money, the CPVM has several major problems:

(a) Real people's utility functions are *not* 'quasilinear' as in eqn.(1). People are risk-averse, so utility is *concave* as a function of money. If we modify eqn. (1) to reflect this, then Theorem 1 is false (indeed, it is easy to construct nonpathological examples where a risk-averse voter does not have *any* dominant strategy in the CPVM).

(b) The CPVM gives more voting power to those who value money less. For example:

- Suppose Ivan and John have identical utility functions (both being concave functions of wealth), but Ivan is rich while John is poor. Then the CPVM gives more voting power to Ivan than to John, because Ivan assigns less marginal utility to each dollar than John.

[2] See Proposition 23.C.4 of Mas-Colell et al. (1995) or Lemma 8.1 of Moulin (1988).

- Suppose Ivan and John have equal wealth, but John assigns higher marginal utility to each dollar (e.g. John needs the money because he has expensive tastes, costly medical problems, and/or several children to feed, whereas Ivan is ascetic, healthy and childless. Then the CPVM again gives more voting power to Ivan than to John.

Good (1977) suggested the CPVM address this problem by rescaling each person's valuation according to her marginal utility for money. However, it is impossible to estimate these marginal utilities (and each person has considerable incentive to exaggerate her own). Tideman (1997) proposed that Clarke taxes be paid in *time* (spent, say, in community service) rather than money. However, Tideman's method is still inegalitarian: some people (e.g. retirees) have a lot of spare time, and hence presumably assign it a low marginal utility, whereas others (e.g. working parents or busy professionals) value their time very highly. Also, Good's and Tideman's variants still implicitly assume that utility is a linear function of money (or time), which is generally false.

(c) The quasilinear utility function (1) also assumes that person's preferences over the alternatives in $\mathcal{A}$ are totally *separable* from her wealth level. This is false. For example, rich people and poor people have very different preferences concerning redistributive taxation schemes and publicly funded goods.

(d) Any revenue collected by the Clarke tax must be removed from the economy (e.g. destroyed or donated to a faraway country), because otherwise voters who expect *not* to pay a Clarke tax have an incentive to distort their valuations so as to inflate the amount of revenue which is collected. Thus, the CPVM is never Pareto-efficient.

Solution: If CPVM is to work (i.e. if Theorem 1 is to be applicable), then Clarke taxes must be paid in some resource to which all voters assign *linear* utility. If the CPVM is to be politically egalitarian, then this resource must be equally valuable to all voters.

Recall the von Neumann-Morgenstern (vNM) theory of cardinal utility functions. Let $\mathcal{B}$ be some set of of alternatives, and define a *lottery* to be some probability distribution over $\mathcal{B}$. Assume that person i, when choosing amongst such lotteries, always makes choices which satisfy certain minimal axioms of consistency and rationality. Then there exists a utility function $u_i : \mathcal{B} \longrightarrow \mathbb{R}$ such that i will always choose the lottery which maximizes the expected value of u_i; see (Myerson, 1991, Theorem 1.1) or (Mas-Colell et al., 1995, Proposition 6.B.3).

By definition, the vNM utility of a lottery is a *linear* function of the probabilities of the alternatives. Thus, we can satisfy the quasilinearity assumption of Theorem 1 if the Clarke tax is paid by submitting to a *lottery* with some negative payoff as a prize. To be precise, suppose we require each voter to declare a valuation function $v_i : \mathcal{A} \longrightarrow [0, 1]$. It follows that the Clarke tax $t_i := \tau_i(\mathbf{v})$ will lie in the interval $[0, 1]$, so we can interpret it as a probability. Let $\mathcal{B} = \{b_0, b_1\}$, where b_1 represents some 'penalty' and b_0 represents the status quo. Consider the lottery $\mathcal{L}$ which assigns probability t_i to b_1 and $(1 - t_i)$ to

b_0. Then it is clear that the utility of $\mathcal{L}$ (for voter i) is exactly $-d_i t_i$, where $-d_i$ is the disutility of the penalty b_1 for voter i.

Suppose i's joint vNM utility function $u_i : \mathcal{A} \times \mathcal{B} \longrightarrow \mathbb{R}$ has the separable form

$$u_i(a,b) \quad = \quad \begin{cases} w_i(a) - d_i & \text{if} \quad b = b_1; \\ w_i(a) & \text{if} \quad b = b_0. \end{cases}$$

Then i's utility function over $\mathcal{A}$ and Clarke taxes has the desired 'quasilinear' form $u_i(a, t_i) = w_i(a) - d_i t_i$, so Theorem 1 holds: voter i's dominant strategy is to declare the valuation $v_i = w_i/d_i + c$, where c is any constant.

It remains to define the penalty b_1. If the GCPM is be politically egalitarian, then the disutility d_i of b_1 must be roughly the same for every voter, relative to the intensity of her political preferences. Clearly, a fine paid in money or time is not appropriate, because of problems (b), (c), and (d) above; the same objection applies to any other economic penalty. Even a penalty such as death, torture, imprisonment, or exile could yield very different disutilities for different voters (e.g. a suicidal or terminally ill person might not fear death), in addition to being barbaric and politically unacceptable.

It is a notorious philosophical conundrum to determine whether a penalty has the 'same' disutility for two people; indeed, it is not clear that this notion is even meaningful; see Sen (1970) or Roemer (1998) for discussions. Fortunately, we have a more modest goal: a penalty which has the same disutility for each voter, 'relative to the intensity of her political preferences'. One penalty which presumably has this property is exclusion from the political process itself. In other words, we propose that a pivotal voter who 'loses' the lottery $\mathcal{L}$ must pay the penalty by forfeiting her right to participate in the political process (e.g. vote) for some protracted period (e.g. N years, where $N \geq 5$).

The disutility generated for voter i by this protracted exclusion must be comparable in magnitude to the intensity of i's preferences over the policy alternatives in $\mathcal{A}$. In other words, if $-d_i$ is the disutility generated by an N year exclusion from the political process, then we must have

$$d_i \quad \geq \quad \max_{a \in \mathcal{A}} w_i(a) \; - \; \min_{a \in \mathcal{A}} w_i(a). \tag{2}$$

If not, then it is not possible for i to truthfully express her utility function using a valuation $v_i : \mathcal{A} \longrightarrow [0,1]$ of the form $v_i = w_i/d_i + c$. If 'political participation' means 'voting', then the magnitude of d_i reflects the utility of voting. But if the utility of voting is proportional to the chance of changing the outcome, then it is utterly infinitesimal in a large modern democracy, as noted by Downs (1957). Indeed, being 'pivotal' is such an astronomically improbable bonanza that it is easy to imagine a voter who would gladly forfeit her franchise for the rest of her natural life, as payment for being pivotal in just one referendum. In this case, the disutility of b_1 is not large enough to make the CPVM work properly.

To escape this Downsian conclusion, we must enrich the definition of 'political participation' to include other things besides voting in CPVM referenda. 'Informal' modes of political participation include political speech and political association. However, we cannot allow the penalty b_1 to curtail such informal political participation for two (ironically opposite) reasons: (1) Free speech and free association are fundamental human rights, and

it is unacceptable for the state to curtail them. (2) The 'Downsian voter' would probably assign political speech and association roughly the same utility as voting —i.e. almost none —because she would judge that these political acts have virtually no influence on public policy.

Instead, we must enrich the 'formal' aspect of political participation in some other way. There are at least two promising possibilities:

- Hylland and Zeckhauser (1979) have proposed another 'point-based' voting system which truthfully reveals each voter's preferences for budget allocations towards pre-existing government programs; see §8.3, p.170 of Mueller (2003) or §4 of Tideman (1997). Like the CPVM, the Hylland-Zeckhauser mechanism (HZM) makes it optimal for each voter to truthfully reveal her preferences, and thereby implements a utilitarian outcome. Like our proposal (and unlike the CPVM), the HZM does not involve real money, so it does not favour wealthy or ascetic voters.

- In addition to voting (in the CPVM or HZM), citizens can actively participate in policy creation and consensus formation through various forms of 'deliberative democracy' (DD) which have recently been proposed; see Bohman and Rehg (1997), Elster (1998), Fung (2003), or Fishkin and Laslett (2003).

In the HZM, every voter (not just the rare pivotal ones) exerts some influence (albeit minute) over budgetary allocations. In deliberative democratic fora, citizens have a sense of active engagement and empowerment within political institutions, rather than the alienation and futility of the Downsian voter. Hence, participation in either HZM or DD may confer much greater utility than mere voting; conversely, exclusion from both HZM and DD may confer sufficient disutility to make the penalty b_1 meaningful.

To translate Clarke taxes into probabilities, we have required each voter to declare a valuation function $v_i : \mathcal{A} \longrightarrow [0, 1]$. If the utility of participation in the CPVM, HZM, and/or DD is nontrivial, and the exclusion duration N is large enough, then the inequality (2) will hold. In this case, v_i will simply be the utility function u_i rescaled to range over $[0, 1]$. Loosely speaking, in the resulting voting mechanism, each person's worst possible political outcome (not just in $\mathcal{A}$, but over all political possibilities) is assigned a utility of zero, and each person's best possible political scenario obtains a utility of one. In other words, this mechanism implements a variant of utilitarianism called *Relative Utilitarianism*; see Cao (1982), Dhillon (1998), Karni (1998), Dhillon and Mertens (1999) and Segal (2000).

Conclusion: By replacing the monetary Clarke tax with a lottery, we have ensured the quasilinearity assumption necessary for applicability of Theorem 1. In this lottery, the pivotal voter risks long-term exclusion from formal political institutions, so the 'Clarke tax' has roughly the same disutility for each voter, relative to the intensity of her political preferences. This yields good approximation of Relative Utilitarianism. The resulting voting system simple, practical, nonmanipulable, politically egalitarian, and immune to Arrovian pathologies.

References

Bohman, J., Rehg, W. (Eds.), 1997. Deliberative Democracy : Essays on Reason and Politics. MIT Press.

Cao, X., 1982. Preference functions and bargaining solutions. In: Proceeedings of the 21st IEEE Conference on Decision and Control. Vol. 1. pp. 164–171.

Clarke, E., Fall 1971. Multipart pricing of public goods. Public Choice 11, 17–33.

d'Aspremont, C., Gevers, L., 1977. Equity and the informational basis of collective choice. Review of Economic Studies 44, 199–209.

Dhillon, A., 1998. Extended Pareto rules and relative utilitarianism. Soc. Choice Welf. 15 (4), 521–542.

Dhillon, A., Mertens, J.-F., 1999. Relative utilitarianism. Econometrica 67 (3), 471–498.

Downs, A., 1957. An Economic Theory of Democracy. Harper.

Elster, J. (Ed.), 1998. Deliberative Democracy. Cambridge UP.

Fishkin, J. S., Laslett, P. (Eds.), 2003. Debating Deliberative Democracy. Blackwell.

Fung, A. (Ed.), 2003. Real Utopias Project 04 Deepening Democracy. Verso.

Good, I., 1977. Justice in voting by demand-revalation. Public Choice 11 (2), 65–70.

Green, J. R., Laffont, J.-J., 1979. Incentives in Public Decision Making. Vol. 1 of Studies in Public Economics. North-Holland, Amsterdam.

Groves, T., 1973. Incentives in teams. Econometrica 41, 617–631.

Harsanyi, J., 1953. Cardinal utility in welfare economics and in the theory of risk-taking. Journal of Political Economy 61 (434-435).

Harsanyi, J., 1955. Cardinal welfare, individualstic ethics and interpersonal comparisons of utility. Journal of Political Economy 63, 309–321.

Hylland, A., Zeckhauser, R., August 1979. A mechanism for selecting public goods when preferences must be elicited. Tech. Rep. KSG Discussion Paper 70D, Harvard University.

Karni, E., 1998. Impartiality: definition and representation. Econometrica 66 (6), 1405–1415.

Mas-Colell, A., Whinston, M. D., Green, J. R., 1995. Microeconomic Theory. Oxford UP, Oxford, UK.

Maskin, E., 1978. A theorem on utilitarianism. Rev. Econom. Stud. 45 (1), 93–96.

Moulin, H., October 1986. Characterizations of the pivotal mechanism. Journal of Public Economics 31 (1), 53–78.

Moulin, H., 1988. Axioms of cooperative decision making. Cambridge University Press, Cambridge.

Mueller, D. C., 2003. Public Choice III. Cambridge UP, Cambridge.

Myerson, R. B., 1981. Utilitarianism, egalitarianism, and the timing effect in social choice problems. Econometrica 49 (4), 883–897.

Myerson, R. B., 1991. Game theory: Analysis of conflict. Harvard University Press, Cambridge, MA.

Ng, Y.-K., October 1975. Bentham or Bergson? Finite sensibility, utility functions, and social welfare functions. Review of Economic Studies 42, 545–569.

Ng, Y.-K., 1985. The utilitarian criterion, finite sensibility, and the weak majority preference principle. A response. Soc. Choice Welf. 2 (1), 37–38.

Ng, Y.-K., 2000. From separability to unweighted sum: a case for utilitarianism. Theory and Decision 49 (4), 299–312.

Roemer, J. E., 1998. Theories of Distributive Justice. Harvard UP, Cambridge, MA.

Segal, U., 2000. Let's agree that all dictatorships are equally bad. Journal of Political Economy 108 (3), 569–589.

Sen, A., 1970. Collective choice and social welfare. Holden-Day.

Sjostrom, T., July 1991. A new characterization of the Groves-Clarke mechanism. Economics Letters 36 (3), 263–267.

Tideman, T. N. (Ed.), Special Supplement to Spring 1977. Public Choice. Vol. 29(2). Center for the Study of Public Choice, Virginia Polytechnic Institute, Blacksburg, Virginia, (Special issue on the Groves-Clarke demand-revealing process).

Tideman, T. N., 1997. Voting and the revelation of preferences for public activities. In: Mueller, D. C. (Ed.), Perspectives on Public Choice: A Handbook. Cambridge UP, New York, Ch. 11, pp. 226–244.

Tideman, T. N., Tullock, G., December 1976. A new and superior method for making social choices. Journal of Political Economy 84 (6), 1145–1159.