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INTRODUCTION

A major implication of global climate change is that future generations will suffer severe damages while the current generation benefits. In this paper a model is developed to analyze the potential need for mitigating the adverse impacts of the greenhouse effect on efficiency grounds. The model characterises basic transfers, investigate the effect of greenhouse emissions, and analyze exogenous and endogenous uncertainty. The first (or current) generation faces the problem of dividing available resources amongst current consumption and transfers to future generations. A two-period model is presented in which the first generation may achieve beneficial transfers to the second by investment in capital (I), in technology (T), or by a direct bequest of final goods (B) and/or by leaving fossil fuel stocks undepleted.[1]

A commonly used device to focus attention on the intertemporal distribution problem is to assume that consumption is split equally among those of any given generation.[2] The assumption is made so as to avoid the intragenerational distribution and aggregation issues. Karl-Goran Maler has discussed the conditions under which the well-being of members of a generation can be aggregated and treated as a single unit.[3] A similar assumption to that of equity is to assume that each generation consists of homogeneous individuals who can be represented as a single agent, see Norgaard and Howarth.[4] The models presented in this paper follow this general approach and discuss the issue of transfers and compensation between generations.

A NEOCLASSICAL MODEL OF INTERGENERATIONAL TRANSFERS

Four variations of the model will be presented based upon the structure outlined above. In the first section the greenhouse effect is ignored and the rules for optimal resource allocation are derived given the four methods of making intergenerational transfers. Next the greenhouse effect is introduced as a certain loss of capital stock. This certainty is then relaxed so as to allow for two states of the world; one with greenhouse damages and one without. Finally, the model is adapted to allow for the potential to determine the probability of greenhouse damages, i.e., the different states of the world.

PRODUCTION FUNCTIONS. The basic neoclassical tool for the study of technology and of technical change is the production function. Structure is imposed upon the function f by the assumptions of continuity and differentiability in all variables. The marginal products of factors are assumed to be decreasing, i.e., if one factor is held constant and the other steadily increased, each extra unit of the latter gives a steadily decreasing increase in total product. In addition, constant returns to scale are assumed.

Three specific production functions are included in the models of this section, and are discussed below. These functions are related specifically to the first generation, the second generation without greenhouse damages, and the second generation with greenhouse gas damages, respectively. The production functions are of the general form:

$$Y^j = F(K^j, L^j, D^j) \quad j = 1,2 \tag{1}$$

where Y^j is aggregate output at time j; K^j, L^j, and D^j are total quantity of capital, labor supply (which would be affected by population changes) and fossil fuel depletion at time j, respectively, j=1,2. These functions are assumed neoclassical; thus they may be written in the intensive form:

$$y^j = f(k^j, d^j) \quad j = 1,2 \tag{2}$$

where $f_k > 0$, $f_{kk} < 0$, $f_d > 0$, $f_{dd} < 0$, and $f_{dk} = f_{kd}$.

The analysis of the model is conducted for two separate periods which run consecutively. By assumption, the members of society in the first period do not overlap with those of the second period. The production function in the first period is:

$$y^1 = f(k^1, \delta^1) A^1 \tag{3}$$

where k^1 and δ^1 are capital stock per worker which is fixed, or given, in the first period, and fossil fuel depletion per worker. The term A refers to Hicks neutral technical progress and is also given in the first period.

<u>TECHNICAL CHANGE</u>. Theories of technical change are concerned with explaining the rate and the direction of the change in technical knowledge. The term "direction" refers to the factor bias of the change, e.g., whether it saves on labor, capital, energy, or other inputs. Besides factor bias technical change may favor a specific sector (e.g., agriculture, mining, manufacturing, etc.), and can be in

the form of product innovation or process innovation.

Innovation can be defined as the production of new technical knowledge, while invention is the generation of some scientific idea, theory, or concept that may lead to an innovation when applied to a process of production. The rate of technical change is influenced both by the rate of scientific change and the rate of transformation of inventions into innovations. Both of these influences are in part shaped by economic processes. In the short to medium term the rate of invention may be taken as given so that the primary concern is then for the rate of transformation.[5]

The most general method of representing the effect of technological change in a model of economic growth involves writing the aggregate production function as:

$$Y = F(K,L,T) \tag{4}$$

According to Hicks, a technical invention is defined as neutral if the ratio of the marginal product of capital to that of labor remains unchanged at a constant capital-labor ratio.[6] Uzawa has noted that the technical invention represented by F(K,L,T) is Hicks neutral if and only if F(K,L) is decomposable:[7]

$$F(K,L,T) = A(T).F(K,L) \tag{5}$$

Further to this formulation, following Shell,[8] potential improvements can be made in the rate of technical change. That is, via investment, the rate of technical change can be altered through invention and innovation.

The rate of Hicks neutral technical progress can be influenced by the first generation so that technological progress is increased in the second period of the two period model. As shown in equation (6), the technological augmentation factor in the second period, A^2, is increased from A^1 by the proportion of successful innovations, rho, from the first period. At the same time the capital stock available to the second generation may be increased by investments, i, made by the first generation. There is no capital depreciation, nor any loss of technical knowledge. If these factors were included the level of growth in the economy would have to increase in order to prevent per capita declines. For example, capital augmentation via technology would need to be increased. If both capital and the non-renewable resource were considered to be depreciating over time, and there were no investment in technology (ie., t=0), the term A^1 would be less than one.

In the absence of any other considerations the second generation cannot have a smaller capital stock than the first and will have at least their technical knowledge. The first generation can increase production in the second period by investing in capital formation and/or research and development. Under these circumstances the production function in the second period is represented by equation (6).

$$y^2 = f^2(k^1 + i, s - \delta^1).(\rho t + A^1) \tag{6}$$

The amount of fossil fuel available to the second generation is reduced by the depletion carried out by the first generation. Thus, if no stock, s, is to be left at the end of the second period, depletion by the second generation, δ^2 will be $(s-\delta^1)$.

Hartwick has suggested reinvesting the rents from nonrenewable resource depletion in capital stock formation as adequate compensation to future generations for reducing the available stock of available natural resources.[9] Equation (6) shows how depletion of natural resources could be balanced by investments in "non-depletable" man-made capital. The two arguements of the production function are per capita capital stock and the non-renewable resource, fossil fuel. As the stock of fossil fuel is reduced production can be maintained by increases in the capital stock. Implicit in this argument is the stability of population, and the ability to convert fossil fuel into reproducible capital.

In the first model the aim is to include all the potential mechanisms identified above by which the first generation might make transfers to the future. The problem is to maximize utility subject to the production constraints, as shown in equation (7).

$$\begin{aligned} &Max\ U = U^1(y^1-i-t-b) + U^2(y^2+b) \\ &s.t.\quad y^1 = f^1(k^1,\delta^1).A^1 \\ &\qquad\ \ y^2 = f^2(k^1+i,s-\delta^1).(\rho t+A^1) \end{aligned} \tag{7}$$

This gives the Lagrangian shown in equation (8).

$$\begin{aligned} &\mathcal{L}(y^1,y^2,i,t,b,\delta^1,\alpha,\beta : k^1,s,\rho,A^1) \\ &= U^1(y^1-i-t-b) \\ &+ U^2(y^2+b) \\ &+ \alpha\,\{y^1-f^1(k^1,\delta^1).A^1\} \\ &+ \beta\,\{y^2-f^2(k^1+i,s-\delta^1).(\rho t+A^1)\} \end{aligned} \tag{8}$$

Solving for the first order conditions (FOC) gives the six partial differential equations shown as (9) to (14), plus the two constraints, not shown.

$$\frac{\partial\mathcal{L}}{\partial y^1} = \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial y^1} + \alpha = 0 \qquad \therefore\ \frac{dU^1}{dC^1} = -\alpha \tag{9}$$

The next three equations refer to the intergenerational transfer mechanism:

$$\frac{\partial \mathcal{L}}{\partial y^2} = \frac{dU^2}{dC^2}\frac{\partial C^2}{\partial y^2} + \beta = 0 \qquad \therefore \quad \frac{dU^2}{dC^2} = -\beta \tag{10}$$

investment in capital, investment in technology and direct bequests, in turn.

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial i} &= \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial i} - \beta \frac{\partial f^2}{\partial k^2}\frac{dk^2}{di}(\rho t + A^1) = 0 \\ \therefore \quad & -\frac{dU^1}{dC^1} \quad - \beta \frac{\partial f^2}{\partial k^2} \quad (\rho t + A^1) = 0 \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial t} &= \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial t} - \beta \frac{dA^2}{dt} f^2(k^1 + i, s - \delta^1) = 0 \\ \therefore \quad & -\frac{dU^1}{dC^1} \quad - \beta \;\; \rho \;\; f^2(k^1 + i, s - \delta^1) = 0 \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial b} &= \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial b} + \frac{dU^2}{dC^2}\frac{\partial C^2}{\partial b} = 0 \\ \therefore \quad & -\frac{dU^1}{dC^1} \quad + \frac{dU^2}{dC^2} \quad = 0 \end{aligned} \tag{13}$$

The final first order condition refers to the rate of fossil fuel depletion and will be required in order to derive the conditions for optimal non-renewable resource depletion over time.

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \delta^1} &= -\alpha \frac{\partial f^1}{\partial \delta^1} A^1 - \beta \frac{\partial f^2}{\partial \delta^2}\frac{\partial \delta^2}{\partial \delta^1}(\rho t + A^1) = 0 \\ \therefore \quad & -\alpha \frac{\partial f^1}{\partial \delta^1} A^1 + \beta \frac{\partial f^2}{\partial \delta^2} \quad (\rho t + A^1) = 0 \end{aligned} \tag{14}$$

However, before turning to the derivation of this condition some of the simpler requirements of the model can be obtained. These will be dealt with under subtitles relating to the intergenerational transfer mechanism from which they derive.

<u>BEQUESTS</u>. The FOC with respect to (w.r.t.) bequests gave equation (13) which shows that bequests will be distributed according to the marginal utilities they generate for the recipient generation. More specifically, equation (13) requires the equality of the marginal utility of consumption across time. That is, the marginal rate of substitution between consumption now and in the future is equal to one. This is a result of the fact that the possibility of discounting utility across generations has been excluded. These results are shown by equations (15a) and (15b):

$$\frac{dU^1}{dC^1} = \frac{dU^2}{dC^2} \qquad \therefore \frac{\frac{dU^1}{dC^1}}{\frac{dU^2}{dC^2}} = 1 \tag{15a}$$

$$\textit{That is, } \frac{MU^{C^1}}{MU^{C^2}} = 1 \qquad \textit{alternatively } MRS^{C1C2} = 1 \tag{15b}$$

<u>INVESTMENT IN TECHNOLOGY</u>. Next substitute equation (10) into equation (12), and write the second period production function in terms of second period capital and depletion, to get (16). Thus, investment in technology is dependent upon the relative marginal utilities of the two generations, the success rate of innovation and unaugmented second period output (ie., second period output in the absence of technological change).

$$\begin{gathered} MU^{C^1} = MU^{C^2} \rho f^2(k^2, \delta^2) \\ \therefore MRS^{C1C2} = \rho f^2(k^2, \delta^2) \end{gathered} \tag{16}$$

<u>CAPITAL INVESTMENT</u>. Next substituting equation (10) into equation (11) and manipulating gives equation (17). This requires the marginal rate of time

preference to be equated to capital productivity augmented by technological change. In the current model this is the equivalent of the Ramsey rule where the real return on capital equals the utility preference rate of consumption over time. The equation differs from the typical Ramsey rule in that the LHS (left hand-side) has no discount rate, which would appear as an additional term.[10] A further difference is the inclusion here of technological change, term A^2 on the RHS, allowing capital productivity to be augmented.

$$MU^{C^1} = MU^{C^2} \frac{\partial f^2}{\partial k^2}(\rho t + A^1)$$
$$\therefore MRS^{C1C2} = \frac{\partial f^2}{\partial k^2} A^2 \tag{17}$$

<u>FOSSIL FUEL DEPLETION</u>. The efficient depletion of a non-renewable resource such as fossil fuels is normally described by the Hotelling Rule discussed in more detail later. However, in this section only a partial step is taken in that direction because first we wish to summarise the optimal conditions of the model in terms of the marginal rates of transformation and substitution.

Substituting equations (9) and (10) into equation (14) and manipulating gives equation (18):

$$\frac{\partial f^1}{\partial \delta^1} A^1 MU^{C^1} = MU^{C^2} \frac{\partial f^2}{\partial \delta^2}(\rho t + A^1)$$
$$\therefore \frac{\partial f^1}{\partial \delta^1} A^1 MRS^{C1C2} = \frac{\partial f^2}{\partial \delta^2} A^2 \tag{18}$$

From equation (15b) we know that the marginal rate of substitution is equal to one. Thus, equation (18) can be written as equation (19).

$$\frac{\partial f^1}{\partial \delta^1} A^1 = \frac{\partial f^2}{\partial \delta^2} A^2 \tag{19}$$

Therefore, to the extent that productivity is greater in the second period due to technological change the rate of depletion will be altered. Depletion will be reduced in period one, so raising the real return to the resource, while it will be increased in period two where it is more productive. This is in contrast to the popular belief that because the future benefits from advances in technology they can be left with fewer natural resources.

<u>OPTIMALITY CONDITIONS</u>. The conditions so far derived for capital and fossil fuel depletion can be combined thus reducing the number of conditions to be met. Substituting equation (17), due to capital investment, in to equation (18), due to fossil fuel depletion, gives equation (20):

$$\frac{\partial f^1}{\partial \delta^1} A^1 \frac{\partial f^2}{\partial k^2} A^2 = \frac{\partial f^2}{\partial \delta^2} A^2 \tag{20}$$

Equation (20) can be rewritten as (21):

$$\frac{\partial f^1}{\partial \delta^1} A^1 = \frac{\frac{\partial f^2}{\partial \delta^2}}{\frac{\partial f^2}{\partial k^2}} = \frac{MPP^{\delta 2}}{MPP^{k2}} = MRT^{\delta 2k2} \tag{21}$$

There are now the following three conditions for optimality:

$$MRT^{\delta 2k2} = \frac{\partial f^1}{\partial \delta^1} A^1 \tag{22}$$

$$MRS^{C1C2} = 1 \tag{23}$$

$$MRS^{C1C2} = \rho f^2(k^2, \delta^2) \tag{24}$$

Technical change has altered the standard Neoclassical requirements for a socially optimal outcome. If there were no technical change in the model equation (12) would disappear as t=0. The terms A^1, A^2, would be equal to one and equation (19) would also lose these terms. Under these circumstances the FOC would give:

$$MRT^{\delta 2k2} = MRS^{C1C2} \tag{25}$$

HOTELLING RULE. Equation (19) shows how the augmentation of fossil fuel productivity can lead to a reduction in the rate of depletion of the fossil fuel reserve by the first generation. Once the real return on capital is included as was done in equation (20) the equivalent of the Hotelling Rule is apparent. Simply dividing (20) through by the real return on fossil fuel depletion in the first time period gives equation (26). That is, the marginal rate of transformation of fossil fuel over time is being equated to the return on capital.

$$\frac{\partial f^2}{\partial k^2} A^2 = \frac{\frac{\partial f^2}{\partial \delta^2} A^2}{\frac{\partial f^1}{\partial \delta^1} A^1} \tag{26}$$

As Hartwick states[11]:

> Efficiency of exhaustible resource extraction requires that the rate of return from a unit of reproducible capital equal the rate of return from owning a unit of deposits of the exhaustible resource. In price terms, this condition is characterized by the current capital gain on mineral deposits being equal to the interest rate or rate of return on reproducible capital. In our one-commodity world, this condition is satisfied by *the rate of change in the*

marginal product of the mineral being equal to the marginal product of reproducible capital. This is sometimes referred to as the Hotelling Rule. It characterizes the efficient exploitation of an exhaustible resources.

Equation (26) diverges from this rule in that no utility discounting is included in this model.

The greater the return on capital in the second period the greater amount of natural resources depleted today i.e., increased extraction today reduces the current rate of return and increases the rate of return in the future; increasing the RHS of (26). Thus, higher returns to capital via technical change increases fossil fuel depletion. Note, the rate of Hicks neutral technical change is predetermined (exogeneous) for the first generation while A^2 is endogeneous. If technology only affected returns to capital the rate of fossil fuel depletion could be altered by manipulating investment in technology. However, A^2 cancels out of (26) so that it has no impact on the depletion conditions. This is a result of Hicks neutral technical change so that any second period productivity increase in capital also occurs in fossil fuel.

As far as optimality conditions are concerned, equation (26) can be substituted for equation (22). Thus, rather than deriving the marginal rate of transformation an alternative is to derive the Hotelling conditions.

So far the components of the model presented are those which would exist without any consideration of the greenhouse effect. The inclusion of the greenhouse effect requires consideration of both the nature of expected damages and the probability of those damages. In this model we add the first of these considerations to model one; dealing with the second in the next two models.

Trace gases such as CO_2 and N_2O will increase in the atmosphere with fossil fuel combustion and can be assumed to be the main cause of global warming at present. This is a simplification of reality; abstracting from other non-fossil fuel gases such as CFCs and emissions from other sources such as deforestation. Climatic change is predicted to result in sea level rise and agricultural damages which can be summarised in economic terms as a loss of capital stock for future generations.[12]

This loss of capital will reduce the capital-labor ratio. The amount of capital lost is characterized here as a function of fossil fuel depletion g(δ). The capital-labor ratio will now be given by equation (27). Thus, the production function in the event that fossil fuel depletion causes the destruction of capital in the second period, due to climatic changes, becomes equation (28).

$$k^{2*} = [k^1 + i].[1 - g(\delta^1)] \tag{27}$$

$$y^{2*} = f^{2*}\{[k^1 + i].[1 - g(\delta^1)], s - \delta^1\}.(\rho t + A^1) \tag{28}$$

Modelling damages as capital stock reduction treats the potential effects of global warming as something less than a total catastrophe, even at its most extreme. If the function g were to equal one then the entire capital stock of the second generation would be lost. However, output need not fall to zero because there are still depletable resources remaining, i.e., no assumption has been made to the effect that capital is an essential input to production. In addition, the existence of direct bequests is assumed to be unaffected and therefore can be expected to play an important role in maintaining utility. This contrasts with Cropper's approach where an environmental catastrophe leads to the end of humanity.[13] Here g(.) can take values between zero and one and thus allow for a range of effects without necessitating a threshold assumption. Benefical affects of global warming could be shown by an enhancement of capital stock with g(.) being negative. (More appropriately this enhancement might be thought of in terms of natural capital e.g., faster tree growth due to CO_2 creating a larger stock of timber.)

Assuming the greenhouse effect is certain to happen, the maximization problem is now as shown in equation (29), with equation (28) as the second constraint:

$$\begin{aligned} &Max\ U = U^1(y^1 - i - t - b) + U^2(y^2 + b) \\ &s.t.\quad y^1 = f^1(k^1,\delta^1).A^1 \\ &\qquad y^{2^*} = f^{2^*}\{[k^1+i].[1-g(\delta^1)],\, s-\delta^1\}.(\rho t + A^1) \end{aligned} \tag{29}$$

The first order conditions are now more complex in several respects, but there are also some similarities with the first model. The differential w.r.t. y^1 is the same as equation (9), rewritten here as (30), and w.r.t. y^2 is the same as (10), except in the later case beta is replaced by a new Lagrange multiplier omega, rewritten as (31).

The partial differential w.r.t. bequests is also the same, giving equation (32), but consumption in the second period is now considered in light of damages due to the greenhouse effect, this is shown as C^{2*}.

$$MU^{C1} = -\alpha \tag{30}$$

$$MU^{C2} = -\Omega \tag{31}$$

$$MRS^{C1C2^*} = 1 \tag{32}$$

All the remaining FOC are substantially different and are reported in full. The three partial derivatives w.r.t. investment in capital, investment in technology, and depletion of fossil fuels are as follows.

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial i} &= \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial i} - \Omega\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{dk^{2^*}}{di}[1-g(\delta^1)](\rho t+A^1)=0 \\ \therefore \quad & -\frac{dU^1}{dC^1} \quad -\Omega\frac{\partial f^{2^*}}{\partial k^{2^*}} \quad [1-g(\delta^1)](\rho t+A^1)=0 \end{aligned} \tag{33}$$

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial t} &= \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial t} - \Omega\frac{dA^2}{dt}f^{2^*}([k^1+i][1-g(\delta^1)],s-\delta^1)=0 \\ \therefore \quad & -\frac{dU^1}{dC^1} \quad -\Omega \;\; \rho \;\; f^{2^*}([k^1+i][1-g(\delta^1)],s-\delta^1)=0 \end{aligned} \tag{34}$$

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial\delta^1} &= -\alpha\frac{\partial f^1}{\partial\delta^1}A^1 - \Omega\left[\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1}+\frac{\partial f^{2^*}}{\partial\delta^2}\frac{\partial\delta^2}{\partial\delta^1}\right](\rho t+A^1)=0 \\ &= -\alpha\frac{\partial f^1}{\partial\delta^1}A^1 - \Omega(\rho t+A^1)\left[\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1}-\frac{\partial f^{2^*}}{\partial\delta^2}\right]=0 \end{aligned} \tag{35}$$

INVESTMENT IN TECHNOLOGY. The marginal rate of substitution can be obtained by substituting equation (31) into equation (34) to give (36).

$$\begin{aligned} MU^{C1} &= MU^{C2^*} \rho f^{2^*}([k^1 + i][1 - g(\delta^1)], s - \delta^1) \\ MRS^{C1C2^*} &= \rho f^{2^*}([k^1 + i][1 - g(\delta^1)], s - \delta^1) \end{aligned} \tag{36}$$

This expression shows how damages to the capital stock now appears in the second period production function. The first term of the production function is the capital stock after damages have occured, k^{2^*}, and the second term is the depletion of fossil fuel in the second period, δ^2. Thus (36) can be rewritten as (37).

$$MRS^{C1C2^*} = \rho f^{2^*}(k^{2^*}, \delta^2) \tag{37}$$

CAPITAL INVESTMENT. Substitute equation (31) into (33) to get (38). This is equivalent to equation (17) of model one. The LHS shows the marginal rate of time preference, and the RHS shows how global warming reduces the productivity of capital. However, the presence of A^2 indicates that technology can increase productivity, but this will also augment damages as t is indirectly dependent upon δ^1.

$$\begin{aligned} MU^{C^1} &= MU^{C^{2^*}} \frac{\partial f^{2^*}}{\partial k^{2^*}} [1 - g(\delta^1)](\rho t + A^1) \\ \therefore\ MRS^{C1C2^*} &= \frac{\partial f^{2^*}}{\partial k^{2^*}} [1 - g(\delta^1)]\, A^2 \end{aligned} \tag{38}$$

The terms on the RHS can be broken down into their constituents in terms of investment in capital and the effect of technology. As shown in equation (39) the loss of capital productivity is ameliorated by the fraction of successful research and development, and the extent of first period technology.

$$MRS^{C1C2^*} = \frac{\partial f^{2^*}}{\partial k^{2^*}} \left(\rho t[1-g(\delta^1)] \quad + \quad A^1[1-g(\delta^1)] \right) \tag{39}$$

FOSSIL FUEL DEPLETION. Next substitute (30) and (31) into (35), divide through by the marginal utility of consumption in the second period, and rearranging terms gives (40). Note, A^2 has been substituted for rho times t plus A^1.

$$\frac{\partial f^1}{\partial \delta^1} A^1 MRS^{C1C2} = A^2 \left[\frac{\partial f^{2^*}}{\partial \delta^{2^*}} - \frac{\partial f^{2^*}}{\partial k^{2^*}} \frac{\partial k^{2^*}}{\partial g} \frac{dg}{d\delta^1} \right] \tag{40}$$

From equation (32) we know the marginal rate of substitution of consumption across time is equal to one. This allows (40) to be written as in (41).

$$\frac{\partial f^1}{\partial \delta^1} A^1 = A^2 \frac{\partial f^{2^*}}{\partial \delta^{2^*}} - A^2 \frac{\partial f^{2^*}}{\partial k^{2^*}} \frac{\partial k^{2^*}}{\partial g} \frac{dg}{d\delta^1} \tag{41}$$

The first term on the RHS and the term on the LHS together comprise the condition without any greenhouse effect, that is equation (19) in model one. The second term on the RHS is the loss of capital productivity due to the greenhouse effect. This term consists, in order, of the marginal physical product of capital as augmented by technical change, $f_k>0$; marginal loss of capital due to greenhouse gases, which is assumed negative (otherwise this would be a gain); and marginal damages, or additions to greenhouse gases, due to fossil fuel depletion, which is positive. This means that the second term will be positive. As a result the marginal physical product in the first time period must exceed that in the second time period ($MPP_{\delta 1}>MPP_{\delta 2}$). In order to achieve this fossil fuel depletion in the first period will need to be reduced while depletion in the second time period is increased.

OPTIMALITY CONDITIONS. In order to simplify the conditions and find the marginal rate of transformation equation (38) is substituted in to equation (40) giving equation (42).

$$\frac{\partial f^{1}}{\partial \delta^{1}}A^{1}\,\frac{\partial f^{2^*}}{\partial k^{2^*}}\,[1-g(\delta^{1})]\,A^{2}=A^{2}\left[\frac{\partial f^{2^*}}{\partial \delta^{2}}-\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^{1}}\right] \tag{42}$$

Dividing both sides of (42) by A^2 gives (43):

$$\frac{\partial f^{1}}{\partial \delta^{1}}A^{1}\,\frac{\partial f^{2^*}}{\partial k^{2^*}}\,[1-g(\delta^{1})] = \frac{\partial f^{2^*}}{\partial \delta^{2}}-\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^{1}} \tag{43}$$

Dividing both sides of (43) by the marginal physical product of capital gives (44).

$$\frac{\partial f^{1}}{\partial \delta^{1}}A^{1}\,[1-g(\delta^{1})] = MRT^{\delta^{2}k^{2^*}}-\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^{1}} \tag{44}$$

This can be rearranged to give (45):

$$MRT^{\delta^{2}k^{2^*}}=\frac{\partial f^{1}}{\partial \delta^{1}}A^{1}\,[1-g(\delta^{1})] + \frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^{1}} \tag{45}$$

Thus, the results of model two can be summarised by equations (32), (37) and (45).

<u>HOTELLING RULE</u>. Now, reconsider equation (43). This can be written in terms of the marginal productivity of fossil fuel depletion, as shown in equation (46).

$$\frac{\partial f^{2^*}}{\partial k^{2^*}}-\frac{\partial f^{2^*}}{\partial k^{2^*}}g(\delta^{1}) = \frac{\dfrac{\partial f^{2^*}}{\partial \delta^{2}}}{\dfrac{\partial f^{1}}{\partial \delta^{1}}A^{1}}-\frac{\dfrac{\partial f^{2^*}}{\partial k^{2^*}}\dfrac{\partial k^{2^*}}{\partial g}\dfrac{dg}{d\delta^{1}}}{\dfrac{\partial f^{1}}{\partial \delta^{1}}A^{1}} \tag{46}$$

On the LHS, the first term is the real return on capital after global warming, and the second term is the loss in that return. Thus, the LHS can be regarded as the net real return on capital after global warming. On the RHS, the first term is as without any greenhouse effect and with the first term from the LHS gives equation

(26) from model one. The second term gives the ratio of marginal capital productivity declines, due to fossil fuel depletion in the first period, to the marginal productivity of fossil fuel depletion in the first period. This term will be negative, ie., MPP_k declines as $MPP_{\delta 1}$ increases. Thus, the marginal physical product of capital in period two is reduced while fossil fuel depletion in period two becomes more attractive as damages are then avoided. The later effect is similar to an augmentation of the return on fossil fuel in period two, but is due to the negative effect of fossil fuel depletion in period one on capital productivity.

EXOGENEOUS UNCERTAINTY AND THE GREENHOUSE EFFECT

The second aspect of modelling the damages due to the greenhouse effect, as mentioned above, is the probability of their occurence. There are two main approaches to modelling this probability. First, the world could be viewed as being in one of two states of existence; either (i) suffering from the impacts of the greenhouse effect (model two), or (ii) finding no impacts from the greenhouse effect, a business as usual scenario (model one). If the greenhouse effect exists the damage function is relevant otherwise it is irrelevant. Second, a continuous series of world states could be defined. That is the damage function would have a variety of probabilities of occurrance. In this case a probability density function would need to be defined. This raises a new set of problems in that the appropriate probability density function is unknown. Therefore, the first approach is adopted below while the second approach is noted as an area for future research.

The next model follows an expected utility framework in which the sum of aggregate utility in the current period, U^1, plus the expected utility in future periods, $E(U^2)$, are maximized. The expected utility is given in equation (47). That is, the probability weighted sum of future utility if climate change does not occur and future utility if climate change does occur. The y terms are outputs, discussed above, and P is the probability of damages due to the greenhouse effect.

$$E(U^2) = [1-P]U^2(y^2) + [P]U^{2^*}(y^{2^*}) \tag{47}$$

The maximization problem is given in equation (48), and can be seen to combine the last two models:

$$E(Max\ U) = U^1(y^1 - i - t - b) + [1-P]\, U^2(y^2 + b) + [P]\, U^{2^*}(y^{2^*} + b)$$

$$\begin{aligned} s.t.\quad & y^1 = f^1(k^1, \delta^1).A^1 \\ & y^2 = f(k^1 + i, s - d^1).(\rho t + A^1) \\ & y^{2^*} = f^{2^*}\{[k^1 + i].[1 - g(\delta^1)], s - \delta^1\}.(\rho t + A^1) \end{aligned} \tag{48}$$

The Lagrange for the maximization of the objective function subject to the three production function constraints is:

$$\begin{aligned} & \mathcal{L}(y^1, y^2, y^{2^*}, i, t, b, \delta^1, \alpha, \beta, \Omega : k^1, s, \rho, A^1, P) \\ & = U^1(y^1 - i - t - b) \\ & + [1-P]\, U^2(y^2 + b) \\ & + [P]\, U^{2^*}(y^{2^*} + b) \\ & + \alpha\, \{y^1 - f^1(k^1, \delta^1).A^1\} \\ & + \beta\, \{y^2 - f^2(k^1 + i, s - \delta^1).(\rho t + A^1)\} \\ & + \Omega\, \{y^{2^*} - f^{2^*}([k^1 + i].[1 - g(\delta^1)], s - \delta^1).(\rho t + A^1)\} \end{aligned} \tag{49}$$

The first order conditions are derived in equations (50) through (52), but the three production constraints, as given above, are not shown. The first three equations give shadow price conditions.

$$\frac{\partial \mathcal{L}}{\partial y^1} = \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial y^1} + \alpha = 0 \qquad \therefore\ MU^{C^1} = -\alpha \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial y^2} = \frac{dU^2}{dC^2}\frac{\partial C^2}{\partial y^2}[1-P] + \beta = 0 \qquad \therefore\ MU^{C^2}[1-P] = -\beta \tag{51}$$

$$\frac{\partial \mathcal{L}}{\partial y^{2^*}} = \frac{dU^{2^*}}{dC^{2^*}}\frac{\partial C^{2^*}}{\partial y^{2^*}}[P] + \Omega = 0 \qquad \therefore\ MU^{C^{2^*}}[P] = -\Omega \tag{52}$$

The differentials w.r.t investment in capital, technology and bequests, the three intergenerational transfer mechanisms, follows next.

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial i} = {} & \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial i} - \beta\frac{\partial f^2}{\partial k^2}\frac{dk^2}{di}(\rho t + A^1) \\ & - \Omega\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{dk^{2^*}}{di}[1-g(\delta^1)](\rho t + A^1) = 0 \\ \therefore\ & -\frac{dU^1}{dC^1} - \beta\frac{\partial f^2}{\partial k^2}(\rho t + A^1) - \Omega\frac{\partial f^{2^*}}{\partial k^{2^*}}[1-g(\delta^1)](\rho t + A^1) = 0\end{aligned} \tag{53}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial t} = {} & \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial t} - \beta\frac{dA^2}{dt}f^2(k^1+i, s-\delta^1) \\ & - \Omega\frac{dA^2}{dt}f^{2^*}([k^1+i][1-g(\delta^1)], s-\delta^1) = 0 \\ \therefore\ & -\frac{dU^1}{dC^1} - \beta\,\rho f^2(k^1+i, s-\delta^1) \\ & - \Omega\,\rho f^{2^*}([k^1+i][1-g(\delta^1)], s-\delta^1) = 0\end{aligned} \tag{54}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial b} = {} & \frac{dU^1}{dC^1}\frac{\partial C^1}{\partial b} + \frac{dU^2}{dC^2}\frac{\partial C^2}{\partial b}[1-P] + \frac{dU^{2^*}}{dC^{2^*}}\frac{\partial C^{2^*}}{\partial b}[P] = 0 \\ \therefore\ & -\frac{dU^1}{dC^1} + \frac{dU^2}{dC^2}[1-P] + \frac{dU^{2^*}}{dC^{2^*}}[P] = 0\end{aligned} \tag{55}$$

The final FOC, w.r.t. fossil fuel depletion, is derived in equation (56).

$$\frac{\partial \mathcal{L}}{\partial \delta^1} = -\alpha \frac{df^1}{d\delta^1} A^1 - \beta \frac{\partial f^2}{\partial \delta^2} \frac{\partial \delta^2}{\partial \delta^1} (\rho t + A^1)$$
$$- \Omega \left[\frac{\partial f^{2^*}}{\partial k^{2^*}} \frac{\partial k^{2^*}}{\partial g} \frac{dg}{d\delta^1} + \frac{\partial f^{2^*}}{\partial \delta^2} \frac{\partial \delta^2}{\partial \delta^1} \right] (\rho t + A^1) = 0 \tag{56}$$
$$\therefore -\alpha A^1 \frac{\partial f^1}{\partial \delta^1} + \beta \frac{\partial f^2}{\partial \delta^2} A^2$$
$$- \Omega A^2 \left[\frac{\partial f^{2^*}}{\partial k^{2^*}} \frac{\partial k^{2^*}}{\partial g} \frac{dg}{d\delta^1} - \frac{\partial f^{2^*}}{\partial \delta^2} \right] = 0$$

<u>BEQUESTS</u>. Equation (55) can be written in terms of the marginal utility of consumption, which with some minor manipulation gives equation (57). This separates the marginal utility of consumption putting the first time period on the LHS and the second on the RHS.

$$MU^{C^1} = MU^{C^2} + P\,[MU^{C^{2^*}} - MU^{C^2}] \tag{57}$$

In the absence of the greenhouse effect P=0 and the second term on the RHS drops out. This gives equation (15) from model one. If the greenhouse effect is certain to occur P=1 and the marginal utility of consumtion in period one is equated with the marginal utility of consumption in period two after the greenhouse effect (MU^{C2^*}). Given a state of uncertainty over the occurence of the greenhouse effect the outcome depends upon the relative size of the marginal utility of consumption in period two across states of the world. If the marginal utility functions in the second period are the same across states of the world then:

$$\textit{If } MU^{C^2} = MU^{C^{2^*}}, \quad \textit{then } MRS^{C1C2} = 1. \tag{58}$$

However, if this were the case the greenhouse effect would have no impact on this condition regardless of the probability of its occurence. Thus, the need to increase bequests over the no greenhouse effect case (model one) depends upon the belief in how the marginal utility of consumption will be affected. The argument is that either (i) the marginal utility is increased due to losses caused by the greenhouse effect or (ii) the marginal utility of consumption is reduced due to being so much better-off. If the marginal utility of consumption in period two is increased, relative to that of period one, bequests will increase. This happens because $MU_c>0$ but $MU_c'<0$.

Equation (58) can be divided through by the marginal utility of consumption in period two to get equation (59) which is in terms of the marginal rate of substitution across time. This equation under normal neo-classical conditions would equal one plus the rate of time preference. In this model there is no discounting so that the marginal rate of substitution is expected to equal one, as in models one and two. However, due to the uncetain impact of global warming on future marginal utility there is an additional factor, the second term on the RHS of equation (59). This term acts as an effective discount factor if it is positive.

$$\frac{MU^{C^1}}{MU^{C^2}} = 1 + P\frac{[MU^{C^{2*}} - MU^{C^2}]}{MU^{C^2}} \tag{59}$$

In order for this "discount" factor to operate, ie., be positive, the marginal utility of consumption with the greenhouse effect need only be greater than the marginal utility of consumption in period two without the greenhouse effect. From the above discussion we may surmise this is the likely outcome from the greenhouse effect. Thus, the greenhouse effect could lead to discounting future consumption. The reasoning for this could be the risk associated with leaving consumption to the second time period. The effect of the "discount" factor is to reduce transfers to the second time period which might then reduce the size of the discount factor.

INVESTMENT IN TECHNOLOGY. Considering equation (54), the Lagrange multipliers can be replaced using equations (51) and (52). This gives equation (60) which is difficult to simplify using the same method as in previous models because of the presence of marginal utility of consumption for two states of the future world. Thus, if the division of the equation by the marginal utility of consumption is carried out a term for the marginal rate of substitution across future states remains in the second term on the RHS. This is shown in equation (61).

$$MU^{C^1} = MU^{C^2}[1-P]\,\rho f^2(k^2,\delta^2) + MU^{C^{2^*}}[P]\,\rho f^{2^*}(k^{2^*},\delta^2) \tag{60}$$

$$MRS^{C^1C^2} = [1-P]\,\rho f^2(k^2,\delta^2) + MRS^{C^{2^*}C^2}[P]\,\rho f^{2^*}(k^{2^*},\delta^2) \tag{61}$$

In order to seperate out the terms which are due to the uncertain future effects of global warming this can be rearraged to give (62). No greenhouse effect, P=0, gives the result from model one, equation (16), and a certain greenhouse effect, P=1, (after some manipulation) gives the result from model two, equation (37).

$$MRS^{C^1C^2} = \rho f^2(k^2,\delta^2) + P\,[MRS^{C^{2^*}C^2}\rho f^{2^*}(k^{2^*},\delta^2) - \rho f^2(k^2,\delta^2)] \tag{62}$$

The only way to simplify this further would be to assume that the marginal rates of substitution are equivalent under both states of the world in the second period. This also results in the LHS, the marginal rate of time preference, being equal to one due to equation (57).

CAPITAL INVESTMENT. Next substitute (51) and (52) into (53) to get

(63). This equation can be divided by the marginal utility of consumption in period two, with or without the greenhouse effect, to get an expression in terms of the marginal rate of substitution. Considering the former case gives equation (64).

$$MU^{C^1} = MU^{C^2}[1-P]\frac{\partial f^2}{\partial k^2}A^2 + MU^{C^{2*}}[P]\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)]\,A^2 \tag{63}$$

$$MRS^{C^1C^2} = [1-P]\frac{\partial f^2}{\partial k^2}A^2 + MRS^{C^{2*}C^2}[P]\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)]\,A^2 \tag{64}$$

Separating out those terms which are dependent upon the greenhouse effect and rearranging gives equation (65).

$$MRS^{C^1C^2} = \frac{\partial f^2}{\partial k^2}A^2 + P\left[MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)] \quad - \frac{\partial f^2}{\partial k^2}\right]A^2 \tag{65}$$

FOSSIL FUEL DEPLETION. Next substitute (50), (51) and (52) into (56).

$$MU^{C^1}A^1\frac{\partial f^1}{\partial \delta^1} = MU^{C^2}[1-P]\frac{\partial f^2}{\partial \delta^2}A^2 - MU^{C^{2*}}[P]A^2\left[\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1} - \frac{\partial f^{2*}}{\partial \delta^2}\right] \tag{66}$$

Dividing through by the marginal utility of consumption in the second period, for the state of the world without global warming, gives an equation including marginal rates of substitution. At the same time the second term has had its sign changed by altering the order of the terms in brakets.

$$MRS^{C^1C^2}A^1\frac{\partial f^1}{\partial \delta^1} = [1-P]\frac{\partial f^2}{\partial \delta^2}A^2 + MRS^{C^{2*}C^2}[P]\left[\frac{\partial f^{2*}}{\partial \delta^2} - \frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right]A^2 \tag{67}$$

Next the terms relating to the probability of the greenhouse effect are separated out to give equation (68).

$$MRS^{C^1C^2}A^1\frac{\partial f^1}{\partial \delta^1} = \frac{\partial f^2}{\partial \delta^2}A^2 + P\left[MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial \delta^2} - \frac{\partial f^2}{\partial \delta^2} - MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right]A^2 \quad (68)$$

<u>OPTIMALITY CONDITIONS</u>. Substituting equation (65) in to (68) gives equation (69).

$$\left(\frac{\partial f^2}{\partial k^2}A^2 + P\left[MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)] - \frac{\partial f^2}{\partial k^2}\right]A^2\right)A^1\frac{\partial f^1}{\partial \delta^1}$$
$$= \frac{\partial f^2}{\partial \delta^2}A^2 + P\left[MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial \delta^2} - \frac{\partial f^2}{\partial \delta^2} - MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right]A^2 \quad (69)$$

Dividing both sides of (69) by A^2 gives (70):

$$\left(\frac{\partial f^2}{\partial k^2} + P\left[MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)] - \frac{\partial f^2}{\partial k^2}\right]\right)A^1\frac{\partial f^1}{\partial \delta^1}$$
$$= \frac{\partial f^2}{\partial \delta^2} + P\left[MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial \delta^2} - \frac{\partial f^2}{\partial \delta^2} - MRS^{C^{2*}C^2}\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right] \quad (70)$$

If the marginal rate of substitution of consumption across states of the world in the second time period is equal to one (i.e., the marginal utility of consumption is equal with or without the greenhouse effect) then equation (70) can be simplified as in equation (71).

$$\left(\frac{\partial f^2}{\partial k^2} + P\left[\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)] - \frac{\partial f^2}{\partial k^2}\right]\right)A^1\frac{\partial f^1}{\partial \delta^1}$$
$$= \frac{\partial f^2}{\partial \delta^2} + P\left[\frac{\partial f^{2*}}{\partial \delta^2} - \frac{\partial f^2}{\partial \delta^2} - \frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right] \quad (71)$$

The second term on the RHS can be simplified if the marginal productivity of fossil fuel depletion is unchanged by the greenhouse effect. In this case the first two terms inside the brackets cancel each other out.

$$\left(\frac{\partial f^2}{\partial k^2} + P\frac{\partial f^{2^*}}{\partial k^{2^*}}[1-g(\delta^1)] \; - P\frac{\partial f^2}{\partial k^2}\right)A^1\frac{\partial f^1}{\partial \delta^1} = \frac{\partial f^2}{\partial \delta^2} - P\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1} \tag{72}$$

In order to get an expression in terms of the marginal rate of transformation of fossil fuel for capital, equation (72) is divided by the marginal productivity of capital in the second time period.

$$\left(1 - P + PMRT^{k^{2^*}k^2}[1-g(\delta^1)]\;\right)A^1\frac{\partial f^1}{\partial \delta^1} = MRT^{\delta^2 k^2} - PMRT^{k^{2^*}k^2}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1} \tag{73}$$

Rearranging terms to get the marginal rate of transformation of capital for fossil fuel on the LHS gives equation (74).

$$MRT^{\delta^2 k^2} = [1-P]A^1\frac{\partial f^1}{\partial \delta^1} + [P]MRT^{k^{2^*}k^2}\left(A^1\frac{\partial f^1}{\partial \delta^1}[1-g(\delta^1)] \; + \frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1}\right) \tag{74}$$

The FOC can therefore be summarised by equations (58), (62), and (74). Both (58) and (74) require that $MU^{C2}=MU^{C2^*}$. In addition, the derivation of (74) required that the marginal physical product of capital with and without the greenhouse effect is the same, i.e., $MPP^{k2^*}=MPP^{k2}$. Also, (74) assumes that the marginal rate of transformation between fossil fuel and capital is unchanged by the greenhouse effect. These assumptions reduce and define the type of uncertainty being considered.

HOTELLING RULE. In terms of the depletion of fossil fuel stocks the outcome is similarly dependent upon the extent to which uncertainty pervades the nature of greenhouse damages. If the marginal productivity of fossil fuel depletion is the same in the two states of the world, and likewise for capital productivity, several simplifications can be made. That is, from equation (70),

$$\frac{\partial f^2}{\partial k^2} + P\left[MRS^{c^{2*}c^2}\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)] - \frac{\partial f^2}{\partial k^2}\right] = \frac{\frac{\partial f^2}{\partial \delta^2}}{\frac{\partial f^1}{\partial \delta^1}A^1} + \frac{P}{\frac{\partial f^1}{\partial \delta^1}A^1}\left[MRS^{c^{2*}c^2}\frac{\partial f^{2*}}{\partial \delta^2} - \frac{\partial f^2}{\partial \delta^2} - MRS^{c^{2*}c^2}\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right] \tag{75}$$

If $MRS^{c^2c^{2*}} = 1$ *then,*

$$\frac{\partial f^2}{\partial k^2} + P\left[\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)] - \frac{\partial f^2}{\partial k^2}\right] = \frac{\frac{\partial f^2}{\partial \delta^2}}{\frac{\partial f^1}{\partial \delta^1}A^1} + \frac{P}{\frac{\partial f^1}{\partial \delta^1}A^1}\left[\frac{\partial f^{2*}}{\partial \delta^2} - \frac{\partial f^2}{\partial \delta^2} - \frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right] \tag{76}$$

If $\frac{\partial f^2}{\partial k^2} = \frac{\partial f^{2*}}{\partial k^{2*}} \wedge \frac{\partial f^2}{\partial \delta^2} = \frac{\partial f^{2*}}{\partial \delta^2}$ *then,*

$$\frac{\partial f^2}{\partial k^2} - P\frac{\partial f^{2*}}{\partial k^{2*}}g(\delta^1) = \frac{\frac{\partial f^2}{\partial \delta^2}}{\frac{\partial f^1}{\partial \delta^1}A^1} - P\frac{\left[\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{d}{d}\right.}{\frac{\partial f^1}{\partial \delta^1}A^1} \tag{77}$$

If the marginal rate of substitution of does not equal one the rule must make reference to marginal utility. That is, fossil fuel depletion cannot be carried out on grounds of productivity alone but must include utility weights.

This depletion is as in the second model except for the presence of the probability terms. The reductions in capital productivity are now weighted by the probability of their occurence. In this way the perspective of different optimal plans will differ depending upon the weights. Thus, international greenhouse gas control negotiations at Rio showed dramatically different approaches based upon the perception of the level of scientific uncertainty. In the next model the probability of the greenhouse effect is made into a choice variable, i.e., society is actually seen to chose the level of risk associated with global warming.

ENDOGENEOUS UNCERTAINTY AND THE GREENHOUSE EFFECT

In the last model the probability with which the greenhouse effect occurs was assumed to be beyond the control of either generation or the economic system. This assumption seems unrealistic when the influence of anthropogenic gas emissions on atmospheric chemistry is considered. That is, the more gas emitted the greater the probability that atmospheric reactions will result in climatic changes, and the damage function will become operative. In order to take this into account the probability of the greenhouse damages occuring will be made a function of fossil fuel depletion.

Thus, the following constraint is added,

$$P = h(\delta^1) \tag{78}$$

This is encorporated into the model and the respective shadow price is lambda. The maximization problem is given in equation (79), and the Lagrange for the maximization of the objective function subject to the three production function constraints and the new probability constraint is shown in (80).

$$E(Max\ U) = U^1(y^1 - i - t - b) + [1 - P]\,U^2(y^2 + b) + [P]\,U^{2^*}(y^{2^*} + b)$$

$$\begin{aligned} s.t.\quad & y^1 = f^1(k^1,\delta^1).A^1 \\ & y^2 = f(k^1+i, s-d^1).(\rho t + A^1) \\ & y^{2^*} = f^{2^*}\{[k^1+i].[1-g(\delta^1)], s-\delta^1\}.(\rho t + A^1) \\ & P = h(\delta^1) \end{aligned} \tag{79}$$

$$\begin{aligned} & \mathcal{L}(y^1, y^2, y^{2^*}, i, t, b, \delta^1, \alpha, \beta, \Omega, \lambda : k^1, s, \rho, A^1, P) \\ & = U^1(y^1 - i - t - b) \\ & + [1 - P]\,U^2(y^2 + b) \\ & + [P]\,U^{2^*}(y^{2^*} + b) \\ & + \alpha\,\{y^1 - f^1(k^1,\delta^1).A^1\} \\ & + \beta\,\{y^2 - f^2(k^1+i, s-\delta^1).(\rho t + A^1)\} \\ & + \Omega\{y^{2^*} - f^{2^*}([k^1+i][1-g(\delta^1)], s-\delta^1).(\rho t + A^1)\} \\ & + \lambda\,\{p - h(\delta^1)\} \end{aligned} \tag{80}$$

The result is two additional FOC and changes in the partial derivative of the Lagrangian w.r.t. fossil fuel depletion. All the other FOC remain the same as in the previous model. The two new FOC are:

$$\frac{\delta \mathcal{L}}{\delta \lambda} = P - h(\delta^1) = 0 \tag{81}$$

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta P} &= -U^2(y^2 + b) + U^{2^*}(y^{2^*} + b) + \lambda = 0 \\ & \therefore -\lambda = U^{2^*} - U^2 \end{aligned} \tag{82}$$

Equation (81) means that all P terms can be replaced by the function h, which is determined by the level of fossil fuel depletion by the first generation. Equation (82) describes the extent of uncertainty over future states of the world. For example, if utility is the same with or without the greenhouse effect lambda will equal zero. The risk of greenhouse damages is therefore summarised as the divergence of utility across states of the world.

Due to the presence of an additional term with respect to delta one (i.e., fossil fuel depletion in period one) the equation for the marginal rate of transformation will have an additional term included. Here the optimal depletion rule for fossil fuels is derived; being the alternative formulation of the marginal rate of transformation conditions.

FOSSIL FUEL DEPLETION AND HOTELLING RULE. The FOC w.r.t. fossil fuel depletion now has an additional term which includes lambda:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \delta^1} = -\alpha \frac{df^1}{d\delta^1} A^1 - \beta \frac{\partial f^2}{\partial \delta^2} \frac{\partial \delta^2}{\partial \delta^1} (\rho t + A^1) \\ - \Omega \left[\frac{\partial f^{2^*}}{\partial k^{2^*}} \frac{\partial k^{2^*}}{\partial g} \frac{dg}{d\delta^1} + \frac{\partial f^{2^*}}{\partial \delta^2} \frac{\partial \delta^2}{\partial \delta^1} \right] (\rho t + A^1) \\ - \lambda \frac{dh}{d\delta^1} = 0\end{aligned} \qquad (83)$$

In order to analyse the effect on fossil fuel depeletion further equations (50), (51), (52) and (81) can be substituted in to (83) to remove the lagrange multipliers. This gives equation (84).

$$MU^{C^1}\frac{df^1}{d\delta^1}A^1 + MU^{C^2}[1-P]\frac{\partial f^2}{\partial \delta^2}\frac{\partial \delta^2}{\partial \delta^1}(\rho t + A^1)$$
$$+ MU^{C^{2^*}}[P]\left[\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1} + \frac{\partial f^{2^*}}{\partial \delta^2}\frac{\partial \delta^2}{\partial \delta^1}\right](\rho t + A^1) \tag{84}$$
$$+(U^{2^*} - U^2)\frac{dh}{d\delta^1} = 0$$

Next replace the partial differential of delta two w.r.t delta one by its derivative, which is minus one. At the same time the augmentation terms have been simplified to give equation (85). Rearranging the terms, by taking all but the first term to the RHS, gives equation (86). Then reducing the number of negative signs by rearranging the last two terms on the RHS gives (87).

$$MU^{C^1}\frac{df^1}{d\delta^1}A^1 - MU^{C^2}[1-P]\frac{\partial f^2}{\partial \delta^2}A^2$$
$$+ MU^{C^{2^*}}[P]\left[\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1} - \frac{\partial f^{2^*}}{\partial \delta^2}\right]A^2 \tag{85}$$
$$+(U^{2^*} - U^2)\frac{dh}{d\delta^1} = 0$$

$$MU^{C^1}\frac{df^1}{d\delta^1}A^1 = MU^{C^2}[1-P]\frac{\partial f^2}{\partial \delta^2}A^2$$
$$- MU^{C^{2^*}}[P]\left[\frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1} - \frac{\partial f^{2^*}}{\partial \delta^2}\right]A^2 \tag{86}$$
$$-(U^{2^*} - U^2)\frac{dh}{d\delta^1}$$

$$MU^{C^1}\frac{df^1}{d\delta^1}A^1 = MU^{C^2}[1-P]\frac{\partial f^2}{\partial \delta^2}A^2 + MU^{C^{2^*}}[P]\left[\frac{\partial f^{2^*}}{\partial \delta^2} - \frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1}\right]A^2 + (U^2 - U^{2^*})\frac{dh}{d\delta^1} \tag{87}$$

In order to be able to substitute an expression containing the marginal product of capital, equation (87) needs to be expressed in terms of the marginal rate of substitution of consumption over time. Therefore, equation (87) is divided through by the marginal utility of consumption in the second period (for the state of the world without the greenhouse effect).

$$MRS^{C^1C^2}\frac{df^1}{d\delta^1}A^1 = [1-P]\frac{\partial f^2}{\partial \delta^2}A^2 + MRS^{C^{2^*}C^2}[P]\left[\frac{\partial f^{2^*}}{\partial \delta^2} - \frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1}\right]A^2 + \frac{(U^2 - U^{2^*})\frac{dh}{d\delta^1}}{MU^{C^2}} \tag{88}$$

Substituting for the marginal rate of substitution from equation (64).

$$\left[[1-P]\frac{\partial f^2}{\partial k^2} + [P]MRS^{C^{2^*}C^2}\frac{\partial f^{2^*}}{\partial k^{2^*}}[1-g(\delta^1)]\right]\frac{df^1}{d\delta^1}A^1A^2 = [1-P]\frac{\partial f^2}{\partial \delta^2}A^2 + MRS^{C^{2^*}C^2}[P]\left[\frac{\partial f^{2^*}}{\partial \delta^2} - \frac{\partial f^{2^*}}{\partial k^{2^*}}\frac{\partial k^{2^*}}{\partial g}\frac{dg}{d\delta^1}\right]A^2 + \frac{(U^2 - U^{2^*})\frac{dh}{d\delta^1}}{MU^{C^2}} \tag{89}$$

Dividing through by the term outside the brackets on the LHS, and rearranging so that the probability terms are separated out, gives equation (90).

$$\frac{\partial f^2}{\partial k^2} - P\left[\frac{\partial f^2}{\partial k^2} - MRS^{c^{2*}c^2}\frac{\partial f^{2*}}{\partial k^{2*}}[1-g(\delta^1)]\right]$$

$$= \frac{\dfrac{\partial f^2}{\partial \delta^2}}{\dfrac{\partial f^1}{\partial \delta^1}A^1} + \frac{P}{\dfrac{\partial f^1}{\partial \delta^1}A^1}\left[MRS^{c^{2*}c^2}\frac{\partial f^{2*}}{\partial \delta^2} - MRS^{c^{2*}c^2}\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1} - \frac{\partial f^2}{\partial \delta^2}\right] + \frac{(U^2-U^{2*})\dfrac{dh}{d\delta^1}}{MU^{c^2}\dfrac{\partial f^1}{\partial \delta^1}A^1A^2} \qquad (90)$$

This can be simplified by making the same assumptions as in model three.

$$\textit{If } MRS^{c^{2*}c^2}=1 \;\wedge\; \frac{\partial f^2}{\partial k^2}=\frac{\partial f^{2*}}{\partial k^{2*}} \;\wedge\; \frac{\partial f^2}{\partial \delta^2}=\frac{\partial f^{2*}}{\partial \delta^2} \quad \textit{then,}$$

$$\frac{\partial f^2}{\partial k^2} - P\frac{\partial f^{2*}}{\partial k^{2*}}g(\delta^1) = \frac{\dfrac{\partial f^2}{\partial \delta^2}}{\dfrac{\partial f^1}{\partial \delta^1}A^1} - \frac{P}{\dfrac{\partial f^1}{\partial \delta^1}A^1}\left(MRS^{c^{2*}c^2}\frac{\partial f^{2*}}{\partial k^{2*}}\frac{\partial k^{2*}}{\partial g}\frac{dg}{d\delta^1}\right) + \frac{(U^2-U^{2*})\dfrac{dh}{d\delta^1}}{MU^{c^2}\dfrac{\partial f^1}{\partial \delta^1}A^1A^2} \qquad (92)$$

The resulting optimal depletion conditions now contain utility functions regardless of the simplifying assumptions. Thus, efficiency can no longer be separated out from distribution.

CONCLUSIONS

The standard neoclassical model shows that augmentation of fossil fuel productivity reduces current depletion, so raising the real return to the resource, while increasing future depletion where the resource is more productive. This is counter balanced by the increase in productivity of capital which increases the rate of depletion. Thus, under Hicks neutral technical change these two effects cancel each other leaving the equivalent of the Hotelling rule unaffected.

Introducing certain greenhouse damages affects the marginal physical product of capital. This impact is ameliorated by the amount of successful innovation resulting from investment in research and development by the first generation and the technological augmentation factor for that generation. Fossil fuel depletion is postponed because of the requirement that the return to the resource in current period exceed that in the next period. In addition, the return on capital is directly reduced by the greenhouse effect which makes current depletion less desirable.

If the greenhouse effect is viewed as creating uncertainty over the future state of the world this can cause an effective discounting of the future. Even though the model excludes utility discounting we find a similar factor entering the

FOC via bequests. This factor is a positive discount rate if the marginal utility of the future generations under global warming exceeds the marginal utility of future generations when there is no global warming. Violation of the assumption that the marginal utility functions are the same in both states of the world in the second period would imply a change in the metric of measure. For example, future generations could benefit by being harmed because the value of the remaining capital assets and resources is different from that of the first generation. This is equivalent to the problem of income redistribution intragenerationally when the rich and poor have different marginal utility of money income functions (see Culyer[14]). In classical economics up until the time of Pigou the assumption of utilitarian economist was that equivalence was self evident (see Nath[15]). The uncertainty of future states of the world, and future utility functions, implies problems for intertemporal planning when future preferences are important. One possible solution is to appeal to a different ethical system to safeguard future generations, if their welfare is threatened.

The expression of uncertainty as an exogeneous factor makes the optimal depletion path uncertain and will then lead to differences of opinion depending on the risk assessed to exist. In the final model this probability is made endogeneous thus removing the risk factors from the depletion conditions. However, the result is that utility functions are present in what are normally defined as efficiency conditions. In contrast to the previous model simplifying assumptions concerning the marginal utility of future generations do not enable these utility terms to be removed. The conclusion here is that efficiency and equity are central to the depletion decision once endogeneous risk of future damages due to global warming are entered in to the inter-temporal model.

Table 4.1 Model Notation

Y^j	Aggregate output at time j, j = 1,2
L^j	Total labor supply at time j, j = 1,2
K^j	Total quantity of capital at time j, j = 1,2
S	Total stock of fossil fuels
D^j	Flow of fossil fuels, depletion, at time j, j = 1,2
U^j	Aggregate utility at time j, j = 1,2
I	Total investment in capital
B	Total bequests of final goods
T	Total investment in technology
C^j	consumption at time j, j=1,2
A^j	Technological augmentation at time j, j = 1,2
rho	Proportion of successful technological investment
P	Probability of disasters due to the greenhouse effect
k^j	Capital/labor ratio in the economy (K^j/L^j)
k^{j*}	Capital/labor ratio in the economy with the greenhouse effect (K^{*j}/L^j)
s	Fossil fuel stock per worker (S/L^j)
δ^j	Depletion of fossil fuel stock per worker (D^j/L^j)
y^j	Output/labor ratio (Y^j/L^j)
y^{j*}	Output/labor ratio with the greenhouse effect
i	Capital investment per worker (I/L^j)
b	Bequests per worker (B/L^j)
t	Technological investment per worker (T/L^j)

ENDNOTES

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