MPRA
Munich Personal RePEc Archive

Performance of lag length selection criteria in three different situations

Asghar, Zahid and Abid, Irum

Quaid-i-Azam University, Islamabad

April 2007

Online at https://mpra.ub.uni-muenchen.de/40042/
MPRA Paper No. 40042, posted 13 Jul 2012 14:40 UTC

PERFORMANCE OF LAG LENGTH SELECTION CRITERIA IN THREE DIFFERENT SITUATIONS

Zahid Asghar[1], Irum Abid[2]

Abstract:

Determination of the lag length of an autoregressive process is one of the most difficult parts of ARIMA modeling. Various lag length selection criteria (Akaike Information Criterion, Schwarz Information Criterion, Hannan-Quinn Criterion, Final Prediction Error, Corrected version of AIC) have been proposed in the literature to overcome this difficulty. We have compared these criteria for lag length selection for three different cases that is under normal errors, under non-normal errors and under structural break by using Monte Carlo simulation. It has been found that SIC is the best for large samples and no criteria is not useful for selecting true lag length in presence of regime shifts or shocks to the system.

Keywords: Autoregressive, AIC, SIC, HQC, FPE, Monte Carlo

1.Author is Assistant Professor Quaid-i-Azam University Islamabad, Pakistan

2. Author is student at Quaid-i-Azam University Islamabad, Pakistan

Email:g.zahid@gmail.com

1.Introduction

The topic of order determination has attracted considerable attention in the literature of time series and in those areas of research which are closely related to time series analysis such as econometrics and statistics. It is rarely the case that the 'true' order of a process is known. One of the most difficult and delicate part of the time series analysis is the selection of the order of the process, based on a finite set of observations, since further analysis of that series is based on it. To overcome this difficulty several order selection criteria had been proposed in the literature but we don't have any criterion which could be considered as the best criterion in all situations. The current study is an effort to make comparison of some of the criteria most widely used in the research for order determination. In the present study, behavior of AIC, SIC, FPE, AICC and HQC have been studied under normal and non normal errors. Sometimes some external factors affect the structure or the generating process of the series and suddenly behavior of the series changes. Effect of such structural break on the behavior of lag length selection criteria have also been studied with three levels of structural breaks. Economists usually take the view that innovations with certain characteristics push the variables along the path which is led out for them. Occasionally, exogenous events which are not member of the usual class of innovations hit the economy and change some basic features like the mean or variance of the process (Muller). Structural break has important consequences. It can effect calibrations used in projections models, it can bias model estimation if not properly adjusted for, and it can effect the interpretation of the data (Alexandre, 2001)

Two of the important issues in constructing a model are: determining the model's lag length and checking the model's parameter stability. When there is no structural break the lag length of an AR process is estimated using any of the criteria discussed above. On the other hand when the lag length is known the parameter stability may be tested by employing various testing procedures (Yang, 2001).

In this research, through a simulation study, the performance of lag length selection criteria in the presence of a possible break in the mean is studied. This research focuses on the mean break mainly. This is because the break in the mean had severe impact on the forecast performance on the one hand and its simplicity helps to highlight it's interaction with the lag length selection on the other hand (Yang, 2001). It is observed that such structural break has very adverse effect on these selection criteria. We have excluded AIC, AICC for lag length selection under structural break because in both of these two cases true model should be known.

Although there are several studies on this issue but it is the first ever study in which lag length under structural break is considered. Liew and Khim (2004) have carried out this study for both normal and non-normal errors .They found that HQC is the best whereas our results show that SIC is the best for large samples. Moreover we have also included AICC which was not considered by Liew and Kim(2004). Difference may be due to the fact that we have restricted our AR coefficient between -0.5 and 0.5 in order to ensure that our process is stationary. Liew and Khim (2004) have selected coefficients between -0.8 and 0.8. Liew and Kim's model was AR(4) where as we have carried it for AR(5).

It is the first ever study in which performance of lag length selection criteria under structural break has been studied. In section 2 methodology and simulation procedure are discussed. In section 3 results and conclusions are made.

2.1 Methodology

Mathematically an AR(p) process of a series Y_t can be written as

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \dots\dots\dots + \alpha_p y_{t-p} + \varepsilon_t \qquad (1)$$

where $\alpha_1, \alpha_2, \alpha_3, \dots\dots, \alpha_p$ are autoregressive parameters and ε_t are normally distributed random error terms with a zero mean and a finite variance σ^2. To achieve our objective we have generated AR processes with p arbitrarily fixed at some value such that in last few values an intervention or structural break occurs. Then, by assuming that the true lag length is unknown, for each series lag length have been determined using different lag length selection criteria.

There are so many criteria used in the literature to determine the lag length of an AR process. Criteria that have been evaluated in this study are as follows:

1. Akaike's information criterion: $AIC_p = n\ln(\hat{\sigma}^2) + 2p$
2. Schwarz information criterion: $SIC_p = n\ln(\hat{\sigma}^2) + n^{-1} p \ln(n)$
3. Hannan-Quinn criterion: $HQC_p = n\ln(\hat{\sigma}^2) + 2n^{-1} p \ln(\ln(n))$
4. Final prediction error: $FPE_p = \ln(\hat{\sigma}^2)(n+p)(n-p)^{-1}$
5. Corrected version of AIC: $AIC_p = n\ln(\hat{\sigma}^2) + n\dfrac{1+p/n}{1-(p+2)/n}$

Where n is the sample size and $\hat{\sigma}^2 = (n-p-1)^{-1}\sum_{t=1}^{n}\varepsilon_t^2$, where ε_t are the model's residuals. Autoregressive parameters $\alpha_1, \alpha_2, \alpha_3, \dots\dots, \alpha_p$ with p = 5 have been generated independently from uniform distribution with values ranging from - 0.5 to 0.5 inclusively and values of parameters are taken in such a way that the sum of these simulated autoregressive parameters is less than unity in magnitude to avoid non-stationary AR process.

To achieve our objectives we compute the probability of correct estimation for each of these criteria. This probability could be any number between zero and 1. Possible results are as follows:

1. If this probability is 1 then it means that the criterion picks up the true lag length in all the cases and therefore is an excellent criterion.
2. If the probability is close to 1 or greater than 0.5 then it implies that the criterion manages to pick up the true lag length in most of the cases and hence is a good criterion.
3. If the probability is close to zero or less than 0.5 then it mean that the criterion fails to select the true lag length in most of the cases therefore is not a good criterion.
4. If this probability is zero it implies that criterion fails to pick up the true lag length in all the cases and hence is poor criteria.

A criterion under estimate the true lag length if it picks up a lag length which is lower than the true lag length and if it selects a lag length which is greater than the true lag length then it over estimates the lag length. Since we want to study the behavior of all these criteria, therefore, along with the cases of correct estimation we also observe the selected lag length of all these criteria in all the cases to compute the probability of under estimation and over estimation.

2.2 Simulation Procedure

Our simulation procedure consists of three major phases. At the first phase we generate a series from an AR process. At the second phase the autoregressive lag lengths of the simulated series have been selected. Third phase assesses the performance of the lag length selection criteria. Steps involved in the simulation procedure for each combination of sample size and AR lag length p are as follows:

1. Independently generate random numbers $\alpha_1, \alpha_2, ..., \alpha_p$ from uniform distribution in the interval (-0.5,0.5) such that

$$\alpha_1 + \alpha_2 + \alpha_3 + ... + \alpha_p = 1$$

   Where p = 5

2. Generate a series of random numbers of size 3n
   - From standard normal distribution to achieve our first objective
   - From standard normal distribution with a structural break in last (n/2) observations for our second objective.
   - From standard normal distribution with error term autoregressive in nature to achieve third objective

   And now denote it by ε_t.

3. Generate a series y_t of size 3n through the following AR process

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + ... + \alpha_p y_{t-p} + \varepsilon_t$$

   with $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_p$ obtained in step1. Initialize the starting value, $y_o = 0$.

4. Discard the first 2n observations to minimize the effect of the initial value
5. Now use each of the selection criteria to determine the autoregressive lag length for the last n observations generated in step3.
6. Repeat step 1 to step 6 B times where B is 100000 in this study.
7. Now compute the probabilities of
   - Correct estimate = (no. of times $\hat{p} = p$) / B

- Under estimate = (no. of times $\hat{p} < p$) / B
- Over estimate = (no. of times $\hat{p} > p$) / B

Repeat step1 to step 7 with p = 5

The error term is generated from normal distribution N(0,1) and for non normal errors we have adopted the following procedure Error term has been generated through

$$\varepsilon_t = z_t \sigma_t$$

where z_t is standard normal variable and

$$\sigma_t = \sqrt{\alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2}$$

Error term has been generated for $q = 2,3$. α_i's are random numbers generated through uniform distribution in the region $(0,1)$. The effect of ARCH errors is studied for different lag lengths and sample sizes.

Structural break in the second half of the values i.e. last (n / 2) values of error term are generated through N(μ , 1) with μ = 1, 2, 3. We have simulated data sets for various sample sizes, n: 30, 60, 120, 240, 480 and 960. To study the behavior of all these criteria probability of correct estimation, under estimation and over estimation has been computed for each case. . All these simulation experiments are carried out by using "R" and the program can be provided on request.

3. Results

According to our results all the criteria estimate the true lag length more than half of the times for all sample sizes and at all lag lengths. So long as the sample size is concerned, performance of all these criteria improves with an increase in the sample size. For n = 30, although AIC and FPE have the highest probability of correct estimation but all other criteria also perform very well. For sample size equal to 60, probability of correct estimation for HQC is highest but AICc and SIC also has probability of correct estimation close to that of HQC. For large sample size (120 or greater) performance of SIC is the best. This shows that AIC and FPE are efficient but not asymptotically consistent which matches with that of the results of Shibata (1976) where as SIC, AICc and HQC are asymptotically consistent criteria. Probability of under estimation is highest for SIC which is less than 0.35 for all sample sizes and AICc and FPE has least probability of under estimation which is less than 0.20 for all sample sizes. All the criteria has highest probability of under estimation for small sample i.e. 30 and as the sample size increases probability of under estimation decreases rapidly and becomes zero for samples equal to or greater than 240. As far as probability of over estimation is concerned it is low for all criteria and for all sample sizes which is less than 0.20. AIC and FPE has highest probability of over estimation which is between 0.175 to 0.195 and SIC has the least probability of over estimation. AICc and HQC lie between these two extremities in respect of probability of under estimation and over estimation.

These results are almost similar to a study carried out by Liew (2004) in which he compared five lag length selection criteria AIC, SIC, FPE, HQC and BIC with true lag length fixed at 4. According to his results for small sample size (60 or less) AIC and FPE has highest probability of correct estimation and for large sample (greater than 60) HQC has the best performance. In our case results for SIC are slightly better than HQC.

It is observed that such structural break has very adverse effect on these selection criteria. From our results it is clear that if there is very small change in the generating process then the results of small samples (less than or equal to 60) are effected more as compared to the results of large samples (greater than 60). HQC has the best performance for sample size equal to 120 and for sample size equal to or greater than 240 SIC has better performance than all other criteria. Now if we increase the change in the generating process then performance of FPE becomes poor no matter how big the sample size is, HQC also perform poorly for all sample sizes except for sample size equal to 960. Here also SIC performs better but for the sample size greater than 240. Now if we further increase the change in the generating process, the performance of all the criteria becomes very poor even for the sample size as big as 960. Even SIC performs poorly with highest probability of correct estimation of around 0.10 which is very low.

Table 1.1: Probabilities of Correct Estimation for AR(5)

Sample size	Simulation size	AIC	SIC	FPE	AICC	HQC
30	1000	0.533	0.471	**0.539**	0.515	0.524
	5000	0.5312	0.456	**0.5334**	0.5067	0.5158
	10000	0.5310	0.4554	**0.5323**	0.5021	0.5146
60	1000	0.769	0.740	0.771	0.779	**0.779**
	5000	0.7626	0.745	0.7638	0.7778	**0.7782**
	10000	0.7640	0.7405	0.7656	0.7760	**0.7766**
120	1000	0.896	**0.954**	0.896	0.935	0.946
	5000	0.8712	**0.9466**	0.8718	0.917	0.929
	10000	0.8900	**0.9449**	0.8910	0.9019	0.9120
240	1000	0.866	**0.983**	0.866	0.945	0.952
	5000	0.8510	**0.9770**	0.8510	0.9410	0.9510
	10000	0.8591	**0.9715**	0.8591	0.9394	0.9499
480	1000	0.886	**0.993**	0.886	0.950	0.959
	5000	0.8845	**0.9923**	0.8845	0.9467	0.9582
	10000	0.8857	**0.9922**	0.8857	0.9426	0.9540
960	1000	0.916	**0.995**	0.916	0.959	0.969
	5000	0.900	**0.9932**	0.900	0.9555	0.9676
	10000	0.9070	**0.9929**	0.9070	0.9503	0.9659

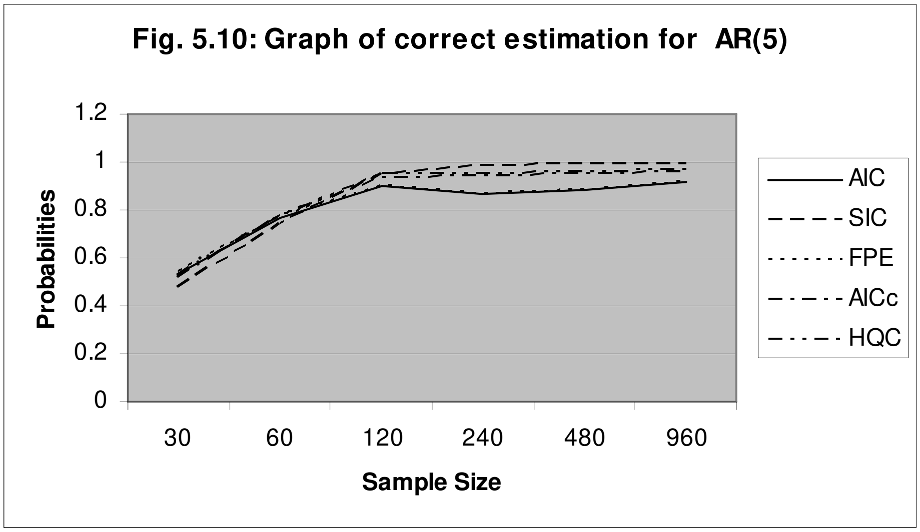
Table 1.2: Probabilities of Under Estimation for AR(5)

Sample size	Simulation size	AIC	SIC	FPE	AICC	HQC
30	1000	0.329	0.476	0.331	0.400	0.373
	5000	0.3308	0.488	0.3346	0.4146	0.381
	10000	0.3283	0.4902	0.3329	0.4202	0.3813
60	1000	0.100	0.222	0.100	0.185	0.147
	5000	0.097	0.221	0.0976	0.1690	0.1408
	10000	0.0990	0.2246	0.0999	0.1743	0.1446
120	1000	0.004	0.031	0.004	0.025	0.010

	5000	0.007	0.0334	0.007	0.0269	0.0142
	10000	0.003	0.0339	0.003	0.0298	0.0130
240	1000	0	0	0	0	0
	5000	0	0	0	0	0
	10000	0	0	0	0	0
480	1000	0	0	0	0	0
	5000	0	0	0	0	0
	10000	0	0	0	0	0
960	1000	0	0	0	0	0
	5000	0	0	0	0	0
	10000	0	0	0	0	0

Table 1.3: Probabilities of Over Estimation for AR(5)

Sample size	Simulation size	AIC	SIC	FPE	AICC	HQC
30	1000	0.138	0.053	0.130	0.085	0.103
	5000	0.138	0.056	0.132	0.0787	0.1032
	10000	0.1407	0.0544	0.1348	0.0777	0.1041
60	1000	0.131	0.038	0.129	0.036	0.075
	5000	0.1404	0.0332	0.1386	0.0532	0.081
	10000	0.1370	0.0349	0.1345	0.0497	0.0788
120	1000	0.100	0.015	0.100	0.04	0.044
	5000	0.1212	0.02	0.1206	0.0561	0.0568
	10000	0.107	0.0212	0.106	0.0683	0.075
240	1000	0.134	0.017	0.134	0.055	0.048
	5000	0.149	0.023	0.149	0.059	0.049
	10000	0.1409	0.0285	0.1409	0.0606	0.0501
480	1000	0.114	0.007	0.114	0.05	0.041
	5000	0.1155	0.0077	0.1155	0.0533	0.0418
	10000	0.1143	0.0078	0.1143	0.0574	0.046
960	1000	0.084	0.005	0.084	0.041	0.031
	5000	0.1	0.0068	0.1	0.0445	0.0324
	10000	0.093	0.0071	0.093	0.0497	0.0341



AR(5) Under Structural Breaks

Table 1.4: Probabilities of Correct Estimation AR(5)

Sample Size		SIC	FPE	HQC
30	μ = 0	0.471	**0.539**	0.524
	μ = 1	0.206	0.293	0.275
	μ = 2	0.014	0.031	0.022
	μ = 3	0.012	**0.019**	0.015
60	μ = 0	0.740	0.771	**0.779**
	μ = 1	0.445	0.477	0.493
	μ = 2	0.017	0.024	0.023
	μ = 3	0.011	0.006	0.008
120	μ = 0	**0.954**	0.896	0.946
	μ = 1	**0.798**	0.663	0.767
	μ = 2	**0.104**	0.035	0.068
	μ = 3	**0**	0	0
240	μ = 0	**0.983**	0.866	0.952
	μ = 1	**0.950**	0.763	0.884
	μ = 2	**0.414**	0.075	0.193
	μ = 3	**0**	0	0

480	μ = 0	**0.993**	0.886	0.959
	μ = 1	**0.980**	0.808	0.932
	μ = 2	**0.523**	0.132	0.301
	μ = 3	**0**	0	0
960	μ = 0	**0.995**	0.916	0.969
	μ = 1	**0.984**	0.854	0.952
	μ = 2	**0.651**	0.245	0.416
	μ = 3	**0**	0	0

AR(5) with ARCH Errors

Table 5.31: Results of AR(5) with ARCH(2) Errors

Sample Size	Probabilities	AIC	SIC	FPE	AICC	HQC
30	Correct	0.530	0.467	0.532	0.521	0.522
	Under	0.327	0.473	0.331	0.421	0.372
	Over	0.143	0.060	0.137	0.058	0.106
60	Correct	0.728	0.710	0.730	0.747	0.744
	Under	0.116	0.243	0.116	0.140	0.159
	Over	0.156	0.047	0.154	0.113	0.097
120	Correct	0.818	0.917	0.818	0.838	0.889
	Under	0.014	0.045	0.014	0.016	0.022
	Over	0.168	0.038	0.168	0.146	0.089
240	Correct	0.828	0.968	0.829	0.837	0.926
	Under	0.001	0.004	0.001	0.001	0.001
	Over	0.171	0.028	0.170	0.162	0.073
480	Correct	0.826	0.979	0.826	0.832	0.914
	Under	0	0	0	0	0
	Over	0.174	0.021	0.174	0.168	0.086
960	Correct	0.800	0.959	0.800	0.811	0.909
	Under	0	0	0	0	0
	Over	0.200	0.041	0.200	0.189	0.091

Table 5.32: Results of AR(5) with ARCH(3) Errors

Sample Size	Probabilities	AIC	SIC	FPE	AICC	HQC
30	Correct	0.502	0.436	0.511	0.466	0.498
	Under	0.332	0.492	0.337	0.461	0.382

	Over	0.166	0.072	0.152	0.073	0.120
60	Correct	0.696	0.688	0.696	0.718	0.706
	Under	0.130	0.253	0.132	0.158	0.181
	Over	0.174	0.059	0.172	0.124	0.113
120	Correct	0.752	0.856	0.752	0.773	0.815
	Under	0.028	0.065	0.028	0.030	0.050
	Over	0.220	0.079	0.220	0.197	0.135
240	Correct	0.694	0.892	0.694	0.708	0.823
	Under	0.002	0.014	0.002	0.002	0.004
	Over	0.304	0.094	0.304	0.290	0.173
480	Correct	0.655	0.901	0.655	0.663	0.792
	Under	0	0	0	0	0
	Over	0.345	0.099	0.345	0.337	0.208
960	Correct	0.613	0.868	0.613	0.635	0.759
	Under	0	0	0	0	0
	Over	0.387	0.132	0.387	0.365	0.241

References

1. Akaike, H. (1969). Fitting Autoregressive models for prediction. Ann. Inst. Statist. Math. 21, 243-247.
2. Akaike, H. (1970a). Statisticsl predictor identification. Ann. Inst. Statist. Math., 22, 203-217.
3. Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle. In 2nd international symposium on information theory, Ed. B.N. Petrov and F. Csaki, pp. 267-81. Budapest Akademia Kiado.
4. Akaike, H. (1974). A new look at the statistical model identification. IEE Trans. Auto. Control 19, 716-723.

5. Akaike, H. (1981). Likelihood of a model and information criteria. J. Econometrics, 16, 3-14.
6. Chow, G. C. (1981). A comparison of the information and the posterior probability criteria for model selection. J. Econometrics, 16, 21-33.
7. Hannan. E. J., Quinn. B. J. (1978). The determination of the lag length of an autoregression. Journal of Royal Statistical Society, **41**, 190-195.
8. Liew, Venus Khim-Sen (2004). Which Lag Length Selection Criteria Should We Employ? Economics Bulletin, **3(33)**, 1-9.
9. Quinn, B. G. (1980). Order determination for a multivariate autoregression. Journal of Royal Statistical Society B, **42(2)**, 182-185.
10. Schwarz, Gideon (1978). Estimating the dimension of a model. The Annals of Statistics, **6(2)**, 461-464.