Solving Two Sided Incomplete Information Games with Bayesian Iterative Conjectures Approach

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Abstract:

This paper proposes a way to solve two (and multiple) sided incomplete information games which generally generates a unique equilibrium. The approach uses iterative conjectures updated by game theoretic and Bayesian statistical decision theoretic reasoning. Players in the games form conjectures about what other players want to do, starting from first order uninformative conjectures and keep updating with games theoretic and Bayesian statistical decision theoretic reasoning until a convergence of conjectures is achieved. The resulting convergent conjectures and the equilibrium (which is named Bayesian equilibrium by iterative conjectures) they supported form the solution of the game. The paper gives two examples which show that the unique equilibrium generated by this approach is compellingly intuitive and insightful. The paper also solves an example of a three sided incomplete information simultaneous game.

Keywords: new equilibrium concept, two and multiple sided incomplete information, iterative conjectures, convergence, Bayesian decision theory, Schelling point

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1. INTRODUCTION

Solving a two sided incomplete information games using current prevailing games theory based upon Nash equilibrium would be a daunting task. There are typically too many equilibriums. Generally, the game has to be simplified to make analysis possible. Consequently, there are not many game theoretic research with two sided or multiple sided incomplete information.[1] However, real life situations involving strategic interactions abound with two sided or multiple sided incomplete information. Hence, there is a strong need for an approach that could solve such games more generally and generate lesser or even a unique equilibrium. It is to this task that this paper devotes itself to.

Given the definition of Nash equilibrium, the current prevailing games theory solves a game by asking which combinations of strategies constitute an Nash equilibrium. Implicit in such an approach is that agents know the strategies played by the others and they also know which equilibrium they are in. In contrast, the Bayesian iterative conjectures approach proposed by this paper solves a game by assuming that the agents have no idea about the actions or strategies adopted or to be adopted by other agents neither do they have any idea on which equilibrium they are in. Therefore, the Bayesian updating process starts with first order uninformative conjectures (or prior probability distribution functions) on the action or strategy of the other agents. The agents then keep updating their conjectures with game theoretic and Bayesian statistical decision theoretic reasoning until a convergence of conjectures is achieved.[2]

Given its ability to narrow down the number of equilibrium normally to one, the BEIC approach is useful for solving games with multiple side incomplete information, multiple heterogeneous players and multiple decision variables. This paper focuses on illustrating the use of BEIC on solving two sided incomplete information games, though the paper also contains an example of a three sided incomplete information game.

From the algorithmic point of view, the rationale to start with first order non informative conjecture is to let the game solves itself and selects its own equilibrium strategies and conjectures, rather than having the equilibrium and its strategies and conjectures being imposed or affected by the informative first order conjectures of the agents. Selten and Harsanyi (1988) propose a tracing procedure to select the most reasonable equilibrium among multiple Nash equilibriums. Their tracing procedure starts with first order non informative conjectures too and is quite similar to the approach of this paper. However, the approach of this paper does not start its tracing with only Nash equilibriums. It starts with all possible actions or strategies of the players. This is ensured through the enforced use of first order uninformative conjectures.

The BEIC approach differs from the current Nash Equilibrium based approach in several ways. First is that the BEIC approach achieves consistency with other major solution concepts while the equilibrium results of current Nash Equilibrium games theory sometimes contradict those derived through backward induction or iterative elimination of (weakly) dominated strategies. Second is that the current prevailing Nash Equilibrium games theory solves for equilibriums by constructing reaction functions and looks for their intersections. In contrast, the BEIC approach constructs reaction functions but uses first order uninformative conjectures and reaction functions to derive higher and higher orders of conjectures until a convergence of conjectures is achieved. Thirdly, Irationality in the processing of information and forming of predictions is the very foundation of the BEIC while the Nash Equilibrium based approach starts without defining rationality in the processing of information and forming of conjectures or prediction. The Nash equilibrium approach incorporates rationality

[1] For examples, see Aumann and Dreze (2008), Chatterjee and Samuelson (1987), Harsanyi and Selten (1988), Powell (1988) and Teng (2012).

[2] Refer to Schweizer (1989).

in the processing of information and forming of predictions in an ad hoc manner latter through Perfect Bayesian Equilibrium and its many refinements. Fourth is that the Nash equilibrium approach defines equilibrium in the strategic/actions space while the BEIC approach defines equilibrium in the subjective probability space with its use of convergence of conjectures. Needless to say, for conjectures to converge, they must also be consistent with the equilibrium they supported and so the BEIC's equilibrium in subjective probability space naturally incorporates equilibrium in strategic/action space as well. Fifth is that the BEIC is based on the Bayesian view of subjective probability which allows the tracing of updating of conjectures from first order uninformative conjectures to higher and higher order of conjectures and till convergence while the Nash equilibrium based approach largely sticks to the classical or frequentist view of probability (and in sequential games of incomplete information with pooling equilibriums, the use of off equilibrium beliefs is an exception that resort to subjective probability.)

Section 2 presents an example of two sided incomplete information simultaneous game. Section 3 solves an example of two sided incomplete information sequential game. Section 4 gives an example of a three sided incomplete information simultaneous game. Section 5 concludes the paper.

2. DOUBLE SIDED INCOMPLETE INFORMATION SIMULTANEOUS GAME

Example 1. Investment Entry Game.

Consider again the following investment entry game. Firm 1 is the incumbent. Firm 2 is the potential entrant. Both firm 1 and firm 2 have two types, high investment cost or low investment cost. The probability that firm 1 is of the high cost type is $\frac{1}{4}$ and the probability that firm 2 is the high cost time is $\frac{1}{10}$ and these probabilities are independent of each other.

When the high investment cost firm 1 faces the low investment cost firm 2 they have the following payoff matrix:

1\2	Enter (y)	Refrain (1-y)
Modern (w)	$0,-2$	$7,0$
Antique (1-w)	$4,2$	$6,0$

(There are three Nash equilibriums: (Antique, Enter), (Modern, Refrain), ($w=1$ for $y<1/5, w\in[0,1]$ for $y=1/5$ and $w=0$ for $y>1/5; y=1$ for $w<1/2$, $y\in[0,1]$ for $w=1/2$, $y=0$ for $w>1/2$).)

When the high cost firm 1 faces the high cost firm 2 they have the following payoff matrix:

1\2	Enter (z)	Refrain (1-z)
Modern (w)	$0,-5$	$7,0$
Antique (1-w)	$4,1$	$6,0$

(There are three Nash equilibriums: (Antique, Enter), (Modern, Refrain), ($w=0$ for $z>\frac{1}{5}, w\in[0,1]$ for $z=\frac{1}{5}$ and $w=1$ for $z<\frac{1}{5}; z=0$ for $w>\frac{1}{6}, z\in[0,1]$ for $w=\frac{1}{6}, z=1$ for $w<\frac{1}{6}$.)

If the low investment cost firm 1 encounters the low cost firm 2, then they have the following payoff matrix:

1\2	Enter (y)	Refrain (1-y)
Modern (x)	$3,-2$	$7,0$
Antique (1-x)	$4,2$	$6,0$

(There are three Nash equilibriums: (Antique, Enter), (Modern, Refrain), ($x=1$ for $y<1/2, x\in[0,1]$ for $y=1/2$ and $x=0$ for $y>1/2; y=1$ for $x<1/2$, $y\in[0,1]$ for $x=1/2$, $y=0$ for $x>1/2$).)

If the low investment cost firm 1 encounters the high investment cost firm 2, then they have the following payoff matrix:

1\2	Enter (z)	Refrain (1-z)
Modern (x)	$3,-5$	$7,0$
Antique (1-x)	$4,1$	$6,0$

(There are three Nash equilibriums: (Antique, Enter), (Modern, Refrain), ($x=1$ for $z<1/2, x\in[0,1]$ for $z=1/2$ and $x=0$ for $z>1/2; z=1$ for $x<1/6$, $z\in[0,1]$ for $x=1/6$, $z=0$ for $x>1/6$).)

The reaction functions of the respective types of firm 1 and firm 2 are:

I.

$w(y,z)=1$ if $\left(\frac{9}{10}\right)(7-7y)+\left(\frac{1}{10}\right)(7-7z)>\left(\frac{9}{10}\right)(6-2y)+\left(\frac{1}{10}\right)(6-2z)$

$w(y,z)\in[0,1]$ if $\left(\frac{9}{10}\right)(7-7y)+\left(\frac{1}{10}\right)(7-7z)=\left(\frac{9}{10}\right)(6-2y)+\left(\frac{1}{10}\right)(6-2z)$

$w(y,z)=0$ if $\left(\frac{9}{10}\right)(7-7y)+\left(\frac{1}{10}\right)(7-7z)<\left(\frac{9}{10}\right)(6-2y)+\left(\frac{1}{10}\right)(6-2z)$

or

$w(y,z)=1$ if $1>\left(\frac{9}{2}\right)y+\left(\frac{1}{2}\right)z$

$w(y,z)\in[0,1]$ if $1=\left(\frac{9}{2}\right)y+\left(\frac{1}{2}\right)z$

$w(y,z)=0$ if $1<\left(\frac{9}{2}\right)y+\left(\frac{1}{2}\right)z$

II.

$x(y,z)=1$ if $\left(\frac{9}{10}\right)(7-4y)+\left(\frac{1}{10}\right)(7-4z)>\left(\frac{9}{10}\right)(6-2y)+\left(\frac{1}{10}\right)(6-2z)$

$x(y,z)\in[0,1]$ if $\left(\frac{9}{10}\right)(7-4y)+\left(\frac{1}{10}\right)(7-4z)=\left(\frac{9}{10}\right)(6-2y)+\left(\frac{1}{10}\right)(6-2z)$

$x(y,z)=0$ if $\left(\frac{9}{10}\right)(7-4y)+\left(\frac{1}{10}\right)(7-4z)<\left(\frac{9}{10}\right)(6-2y)+\left(\frac{1}{10}\right)(6-2z)$

or

$x(y,z)=1$ if $1>\left(\frac{9}{5}\right)y+\left(\frac{1}{5}\right)z$

$x(y,z)\in[0,1]$ if $1=\left(\frac{9}{5}\right)y+\left(\frac{1}{5}\right)z$

$x(y,z)=0$ if $1<\left(\frac{9}{5}\right)y+\left(\frac{1}{5}\right)z$

III.

$y(w,x)=1$ if $\left(\frac{3}{4}\right)(2-4w)+\left(\frac{1}{4}\right)(2-4x)>0$

$y(w,x)\in[0,1]$ if $\left(\frac{3}{4}\right)(2-4w)+\left(\frac{1}{4}\right)(2-4x)=0$

$y(w,x)=0$ if $\left(\frac{3}{4}\right)(2-4w)+\left(\frac{1}{4}\right)(2-4x)<0$

or

$y(w,x)=1$ if $1>\left(\frac{3}{2}\right)w+\left(\frac{1}{2}\right)x$

$y(w,x)\in[0,1]$ if $1=\left(\frac{3}{2}\right)w+\left(\frac{1}{2}\right)x$

$y(w,x)=0$ if $1<\left(\frac{3}{2}\right)w+\left(\frac{1}{2}\right)x$

IV.

$z(w,x)=1$ if $\left(\frac{3}{4}\right)(1-6w)+\left(\frac{1}{4}\right)(1-6x)>0$

$z(w,x)\in[0,1]$ if $\left(\frac{3}{4}\right)(1-6w)+\left(\frac{1}{4}\right)(1-6x)=0$

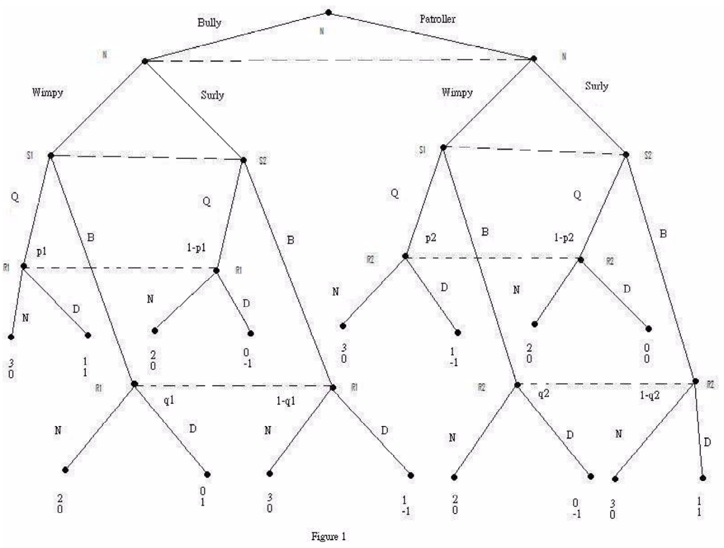

Figure 1:

$z(w,x) = 0$ if $\left(\frac{3}{4}\right)(1-6w) + \left(\frac{1}{4}\right)(1-6x) < 0$
or
$z(w,x) = 1$ if $1 > \left(\frac{9}{2}\right)w + \left(\frac{3}{2}\right)x$
$z(w,x) \in [0,1]$ if $1 = \left(\frac{9}{2}\right)w + \left(\frac{3}{2}\right)x$
$z(w,x) = 0$ if $1 < \left(\frac{9}{2}\right)w + \left(\frac{3}{2}\right)x$

The solution of the Bayesian iterative conjectures approach is presented below. In the table below, the first row identifies the respective probabilities (w, x, y and z) and the second row gives the first order uninformative conjectures (which are $w = 0.5, x = 0.5, y = 0.5$ and $z = 0.5$) and the later rows give second and higher order conjectures.

order\probabilities	w	x	y	z
1	0.5	0.5	0.5	0.5
2	0	0.5	0.5	0
3	0	1	1	1
4	0	0	1	1
5	0	0	1	1

So the Bayesian equilibrium by iterative conjectures is $w = 0, x = 0, y = 1, z = 1.3$. Two Sided Incomplete Information Sequential Game with Perfect Information

Example 2.

The above game is an extension of the famous beer-quiche game. The left hand side of the game is the original beer-quiche game. In the above game, there are two types of senders, wimpy and surly, and two types of receivers, bully and patrollers. The probability of wimpy is 0.1 and the probability of surly is 0.9. The bully type enjoys picking on the wimpy type. The patroller type, on the other hand, has the duty of challenging the surly type when the surly orders beer and only if the surly orders beer. However, if the patroller challenges the wimpy, the patroller is humiliated. The probability of bully is 0.1 and the probability of patroller is 0.9. The above game would have many equilibriums if solved by the perfect Bayesian equilibrium approach.

The Bayesian iterative conjectures approach solution is presented below:

Let the probability that the bully plays duels when Beer is observed be u, the probability that the bully plays duels when Quiche is observed be v, the probability that the patroller plays duels when Beer is observed be s and the probability that the patroller plays duels when Quiche is observed be t. Please note that when Quiche is observed, the patroller has the dominant strategy of choosing No Duel and therefore $t = 0$.

Let the probability that the surly plays Beer be x and the probability that the wimpy plays Beer be y.

Given u, v, s and t, the surly chooses beer if

$(0.1)[u + 3(1-u)] + (0.9)[s + 3(1-s)] > (0.1)\,2(1-v) + (0.9)\,2(1-t)$

$3 - (0.2)u - (1.8)s > 2 - (0.2)v - (1.8)t$

$1 > (0.2)u + (1.8)s - (0.2)v - (1.8)t$

$1 > (0.2)(u-v) + (1.8)(s-t)$

The above is the combination of

$[u + 3(1-u) - 2(1-v)] > 0$

and

$[s + 3(1-s) - 2(1-t)] > 0$

and weighted by 0.1 and 0.9.

Given u, v, s and t, the wimpy chooses beer if

$(0.1)[2(1-u)] + (0.9)[2(1-s)] > (0.1)[v + 3(1-v)] + (0.9)[t + 3(1-t)]$

$2 - (0.2)u - (1.8)s > 3 - (0.2)v - (1.8)t$

$(0.2)v + (1.8)t - (0.2)u - (1.8)s + > 1$

$(0.2)(v-u) + (1.8)(t-s) > 1$

The above is the combination of

$[2(1-u) - v - 3(1-v)] > 0$

$[2(1-s) - t - 3(1-t)] > 0$

and weighted by 0.1 and 0.9.

When observed Beer the bully plays

No Duel if $(-1)(x)\left(\frac{9}{10}\right) + (1)(y)\left(\frac{1}{10}\right) < 0$

is indifferent if $(-1)(x)\left(\frac{9}{10}\right) + (1)(y)\left(\frac{1}{10}\right) = 0$

Duel if $(-1)(x)\left(\frac{9}{10}\right) + (1)(y)\left(\frac{1}{10}\right) > 0$

When observed Quiche the bully plays

No Duel if $(-1)(y)\left(\frac{9}{10}\right) + (1)(x)\left(\frac{1}{10}\right) < 0$

is indifferent if $(-1)(y)\left(\frac{9}{10}\right) + (1)(x)\left(\frac{1}{10}\right) = 0$

Duel if $(-1)(y)\left(\frac{9}{10}\right) + (1)(x)\left(\frac{1}{10}\right) > 0$

When observed Beer the patroller plays

Duel if $(1)(x)\left(\frac{9}{10}\right) + (-1)(y)\left(\frac{1}{10}\right) > 0$

is indifferent if $(1)(x)\left(\frac{9}{10}\right) + (-1)(y)\left(\frac{1}{10}\right) = 0$

No Duel if $(1)(x)\left(\frac{9}{10}\right) + (-1)(y)\left(\frac{1}{10}\right) < 0$

Solving by the BEIC approach:

order\probability	u	v	s	t	x	y
1	0.5	0.5	0.5	0	0.5	0.5
2	0	0	1	0	1	0
3	0	1	1	0	0	0
4	0	0	1	0	0	0
5	0	0	0	0	0	0

The process therefore converges here with $u = 0, v = 0, s = 1, t = 0, x = 0$ and $y = 0$. Given the high probability of meeting the patroller, the surly chooses quiche. Consequently, the wimpy chooses quiche both for impersonating as the surly to avoid being challenge by the bully and for his intrinsic preference for quiche. The bully chooses not to duel when observed quiche since the probability of meeting the surly is high (0.9). Note that off-equilibrium beliefs as in perfect Bayesian equilibrium reasoning is not needed in the BEIC solution. Again, the resulting equilibrium seems intuitive and compelling enough.

4. Extending to Multiple (Three or More) Sided Incomplete Information Games

The method introduced in the previous sections could be extended into three or more sided incomplete information games. This section gives an example of a three sided incomplete information simultaneous game.

There are three firms simultaneously decide to enter and produce in a market or refrain from entering. Both firm 1, 2 and 3 has two types, type a or type b. Let the probability of 1a be 0.5 and the probability of 1b be 0.5, 2a be 0.5 and the probability of 2b be 0.5 and, 3a be 0.5 and the probability of 3b be 0.5. The payoff matrixes are as follow:

$1a\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$-1;-2;-20$	$(20);(0);(1)$	(3a Enter)
$Withhold$	$(0);(10);(1)$	$0;0,(5)$	

$1a\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$(20);(10);(0)$	$(100);0;0$	(3a Withhold)
$Withhold$	$0;(50);0$	$0;0;0$	

$1a\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$-1,-2,-40$	$(20),(0),(0.5)$	(3b Enter)
$Withhold$	$(0),(10),(0.5)$	$0,0,(2.5)$	

$1a\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$(20),(10),(0)$	$(100),0,0$	(3b Withhold)
$Withhold$	$0,(50),0$	$0,0,0$	

$1a\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$-1;-2;-20$	$(20);(0);(1)$	(3a Enter)
$Withhold$	$(0);(20);(1)$	$0;0,(5)$	

$1a\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$(20);(20);(0)$	$(100);0;0$	(3a Withhold)
$Withhold$	$0;(50);0$	$0;0;0$	

$1a\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$-1,-2,-40$	$(20),(0),(0.5)$	(3b Enter)
$Withhold$	$(0),(20),(0.5)$	$0,0,(2.5)$	

$1a\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$(20),(20),(0)$	$(100),0,0$	(3b Withhold)
$Withhold$	$0,(50),0$	$0,0,0$	

$1b\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$-1;-2;-20$	$(50);(0);(1)$	(3a Enter)
$Withhold$	$(0);(10);(1)$	$0;0,(5)$	

$1b\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$(50);(10);(0)$	$(100);0;0$	(3a Withhold)
$Withhold$	$0;(50);0$	$0;0;0$	

$1b\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$-1,-2,-40$	$(50),(0),(0.5)$	(3b Enter)
$Withhold$	$(0),(10),(0.5)$	$0,0,(2.5)$	

$1b\backslash 2a$	$Enter$	$Withhold$	
$Enter$	$(50),(10),(0)$	$(100),0,0$	(3b Withhold)
$Withhold$	$0,(50),0$	$0,0,0$	

$1b\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$-1;-2;-20$	$(50);(0);(1)$	(3a Enter)
$Withhold$	$(0);(20);(1)$	$0;0,(5)$	

$1b\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$(50);(20);(0)$	$(100);0;0$	(3a Withhold)
$Withhold$	$0;(50);0$	$0;0;0$	

$1b\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$-1,-2,-40$	$(50),(0),(0.5)$	(3b Enter)
$Withhold$	$(0),(20),(0.5)$	$0,0,(2.5)$	

$1b\backslash 2b$	$Enter$	$Withhold$	
$Enter$	$(50),(20),(0)$	$(100),0,0$	(3b Withhold)
$Withhold$	$0,(50),0$	$0,0,0$	

The Nash Equilibriums (pure strategy) are:
(Enter, Enter; Enter, Enter; Withhold, Withhold)
(Enter, Enter; Withhold, Withhold; Enter, Enter)
(Withhould, Withhold ; Enter, Enter; Enter, Enter)

In contrast, the BEIC approach gives a compelling unique equilibrium. Solving by the BEIC approach:

Let the probability that 1a enters be a and withholds be $1-a$.

Let the probability that 1b enters be b and withholds be $1-b$.

Let the probability that 2a enters be c and withholds be $1-c$.

Let the probability that 2b enters be d and withholds be $1-d$.

Let the probability that 3a enters be e and withholds be $1-e$.

Let the Probability that 3b enters be f and withholds be $1-f$.

The reaction functions are:

$a = 1$ if

$$[(-c + 20(1-c))e + (20c + 100(1-c))(1-e)](0.5) + [(-c + 20(1-c))f + (20c + 100(1-c))(1-f)](0.5) + [(-d + 20(1-d))e + (20d + 100(1-d))(1-e)](0.5) + [(-d + 20(1-d))f + (20d + 100(1-d))(1-f)](0.5) > 0$$

$b = 1$ if

$$[(-c + 50(1-c))e + (50c + 100(1-c))(1-e)](0.5) + [(-c + 50(1-c))f + (50c + 100(1-c))(1-f)](0.5) + [(-d + 50(1-d))e + (50d + 100(1-d))(1-e)](0.5) + [(-d + 50(1-d))f + (50d + 100(1-d))(1-f)](0.5) > 0$$

$c = 1$ if

$$[(-2a + 10(1-a))e + (10a + 50(1-a))(1-e)](0.5) + [(-2a + 10(1-a))f + (10a + 50(1-a))(1-f)](0.5) + [(-2b + 10(1-b))e + (10b + 50(1-b))(1-e)](0.5)$$

$+\left[\left(-2b+10\left(1-b\right)\right)f+\left(10b+50\left(1-b\right)\right)\left(1-f\right)\right]\left(0.5\right) > 0$

$d=1$ if

$\left[\left(-2a+20\left(1-a\right)\right)e+\left(20a+50\left(1-a\right)\right)\left(1-e\right)\right]\left(0.5\right)$
$+\left[\left(-2a+20\left(1-a\right)\right)f+\left(20a+50\left(1-a\right)\right)\left(1-f\right)\right]\left(0.5\right)$
$+\left[\left(-2b+20\left(1-b\right)\right)e+\left(20b+50\left(1-b\right)\right)\left(1-e\right)\right]\left(0.5\right)$
$+\left[\left(-2b+20\left(1-b\right)\right)f+\left(20b+50\left(1-b\right)\right)\left(1-f\right)\right]\left(0.5\right) > 0$

$e=1$ if

$\left[\left(-20c+\left(1-c\right)\right)a+\left(c+5\left(1-c\right)\right)\left(1-a\right)\right]\left(0.5\right)$
$+\left[\left(-20d+\left(1-d\right)\right)a+\left(d+5\left(1-d\right)\right)\left(1-a\right)\right]\left(0.5\right)$
$+\left[\left(-20c+\left(1-c\right)\right)b+\left(c+5\left(1-c\right)\right)\left(1-b\right)\right]\left(0.5\right)$
$+\left[\left(-20d+\left(1-d\right)\right)b+\left(d+5\left(1-d\right)\right)\left(1-b\right)\right]\left(0.5\right) > 0$

$f=1$ if

$\left[\left(-40c+\left(0.5\right)\left(1-c\right)\right)a+\left(\left(0.5\right)c+\left(2.5\right)\left(1-c\right)\right)\left(1-a\right)\right]\left(0.5\right)$
$+\left[\left(-40d+\left(0.5\right)\left(1-d\right)\right)a+\left(\left(0.5\right)d+\left(2.5\right)\left(1-d\right)\right)\left(1-a\right)\right]\left(0.5\right)$
$+\left[\left(-40c+\left(0.5\right)\left(1-c\right)\right)b+\left(\left(0.5\right)c+\left(2.5\right)\left(1-c\right)\right)\left(1-b\right)\right]\left(0.5\right)$
$+\left[\left(-40d+\left(0.5\right)\left(1-d\right)\right)b+\left(\left(0.5\right)d+\left(2.5\right)\left(1-d\right)\right)\left(1-b\right)\right]\left(0.5\right) > 0$

The process of conjectures is:

\	a	b	c	d	e	f
1	0.5	0.5	0.5	0.5	0.5	0.5
2	1	1	1	1	0	0
3	1	1	1	1	0	0

The unique BEIC is (Enter, Enter; Enter, Enter; Withhold, Withhold).

5. Conclusions.

At a more fundamental level, the approach expounded in this paper raised the question: Is current concept of rationality in games theory complete without defining rationality in the processing of information and forming of conjectures? The approach in this paper is an attempt to fill in this gap.

The approach traces how the conjectures of a rational player change and converge. A rational player here means that he is rational by both the definitions of rational in games theory and Bayesian statistical decision theory. The rational player forms his conjectures about how the game will be played by using all available information about the game and starting from first order uninformative conjectures and keeps updating with game theoretic and Bayesian statistical decision theoretic reasoning till a convergence in conjectures is achieved. He then acts rationally given his convergent conjectures.

As shown in the above examples, the Bayesian iterative conjectures approach allows two sided incomplete information games to be analyzed in a general way that avoids the generation of multiple equilibriums. The unique equilibrium generated in both examples are compellingly insightful and intuitive.

Much remains to be done. The next step is to use the approach to analyze two sided incomplete information games with continuous action space and multiple sided incomplete information games with discrete and continuous action space.

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