Backward recalculation of seasonal series affected by economic crisis: a Model-Based-Link method for the case of Turkish GDP

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Abstract

When attempting to deal with the recalculation process, it is hard to answer the question "Does the recalculated series include economic events and seasonal behaviours in the past?". This paper discusses some alternative backward recalculation methods and presents the applications and their results relative to the Turkish Gross Domestic Product (GDP) series. Using comparative analysis, it is shown that ordinary ARIMA forecasts and signal extraction methods are not successful at taking into account past events in the backward recalculated series. A new innovative method, named Model-based-link, is then proposed and suggested by the authors in order to be able to take past economic events and seasonal patterns into account when the series is to be backward recalculated. A first application of this new method is run on the quarterly series of the Turkish GDP. In addition, it is shown that the Model-based-link method can be extended to data sets of different frequencies (i.e. annual data). Consequently, it can be claimed that a comparable recalculated quarterly and annual Turkish GDP series for forthcoming data is obtained.

The paper is structured as following: section 1 introduces the reader to the state of the art in the current literature; section 2 defines the information set to be backward recalculated and presents some statistics on the data while section 3 presents the main methodological statistical aspects of classical methods compared to the methodological scheme of the Model-based-link that can be used for the recalculation process. Section 4 presents results of the methods mentioned in the previous section and section 5 discusses the extension of the Model-based-link method to monthly data and includes an application for annual data; section 6 concludes. Finally, section 7 presents topics for discussion and challenges for continuation of the analysis.

1. Introduction

Statistical time series are comparable as long as they are produced according to a commonly harmonised set of definitions and concepts. In the last decades, the definitions and concepts have undergone various updating processes in order to maintain compatibility with phenomena being represented by data (Mazzi and Savio, 2005). On the other hand, the updates and changes have also risked reducing the comparability of the data with respect to their previous versions. In order to minimize the trade-off costs, it might be suggested that one appropriate method would be to backward recalculate the time series according to the new definitions. As mentioned in Mazzi et al. (2010), the backward recalculations required for users of official statistics often require a long time-series for their analysis, forecasting exercises, modelling activities and policy oriented simulations.

In the recalculation process, it is important to take in account of past economic events and seasonality. The first consideration is past economic events. For example, there may have been previous economic crises or fluctuations. If backward recalculatation of the series disregards past economic events, the result of the process risks to lose significance in terms of economic reliability. On the other hand, this situation also may cause the recent crisis or depression period in the economic time series to be wrongly identified. During periods of economic depression, the identification of outliers may be influenced from past outliers or economic events. These observations should be truly identified and treated as outliers if there is to be as small a revision as possible in the seasonal adjustment process. The second consideration is seasonality. If additional information is used in the backward recalculation process (for example, a previous version of the series to be recalculated), and if there are differences between seasonal components of the series and other series, these differences should be taken in account in the process.

Several papers can be given as references in relation to the problem of backward recalculation. Capolin and Sartore (2006) discussed the use of aggregation-disaggregation methods in the backward recalculation process. On the other hand, Di Fonzo (2003) tried to solve this problem with constrained retropolation which consists of benchmarking the series using numerous additional time series. There is no consensus on the solution for the backward recalculation problem.

GDP is one of the most widely used data sets in economic analysis. The length of time series that the GDP figure monitors is also crucial to the analysis. It is clear that the higher the number of observations of GDP data available, the more consistent and robust are the estimations of the relationship between the past and the present.

The Turkish Statistical Institute has produced quarterly GDP estimates according to the System of National Accounts (SNA-68) in current and in constant 1987 prices between the periods 1987-2007. In accordance with emerging conditions in economic and social areas, the new GDP series at current and constant 1998 prices between the periods 1998 and up to the present have been published by changing calculations relating to the scope or method according to European System of Accounts (ESA-95) from the year 2008[§] onwards.

There are two main differences between GDP in constant 1987 prices calculated with SNA-68 (hereafter SNA-GDP) and GDP in constant 1998 prices calculated using ESA-95 (hereafter ESA-GDP):

- SNA-GDP covers the time period from 1st quarter of 1987 to 3rd quarter of 2007, while ESA-GDP covers the time from 1st quarter of 1998 to the present date (last data for 3rd quarter of 2009). Therefore, there is a gap between the periods of 1987 to 1998 for the ESA-95 series.
- The ESA-GDP series is calculated and disseminated in units of new, revalued Turkish Lira which started to be used after 2009 while SNA-GDP used to be estimated in old Turkish Lira. So in terms of unit values, there is a huge difference in the levels between the two series if comparing the equavalence of "1 Turkish Lira = 1.000.000 Old Turkish Lira".

The aim of this paper is to propose a model-based method which takes account of past economic crisis, to backward recalculate the series including changed seasonality, definitions and concepts, and also to obtain a useful extended Turkish ESA-GDP series from the period 1st quarter of 1987 up to the present.

2. Data

The SNA-GDP series covers the time span between Q1-1987 to Q3-2007 while ESA-GDP covers Q1-1998 to Q3-2009, as shown in Figure 1 (SNA-GDP in left axis and ESA-GDP in right axis). Clearly, the difference between the calculation units of the two series has caused different trend levels. The seasonal patterns however are similar and coincide. Both series are at a maximum at the third quarter and a minimum at the fourth quarter in a year. However, the third quarters of SNA-GDP are higher than ESA-GDPs'. Moreover, the fourth quarters of SNA-GDP are lower than ESA-GDPs'. The difference between the sizes of seasonality can be easily seen using a simple variance test done on the seasonal components of the two series for a common period which is between 1998-Q1 and 2007-Q3. According to the results of the variance test shown in Table 1, the difference between the sizes of seasonal components is statistically significant at the 1% level.

Table 1. Result of variance test for seasonal factors for common period

	SNA-GDP	ESA-GDP
Mean	97.6	99.0
Variance	228.4	84.3
Observations	39	39
Degree of freedom	38	38
F-test value	2.709	
P-Value	0.001	
Critical Value (0.01)	2.157	

Because of these differences between the two series, it is difficult to compare the values between two specific dates i.e. 1989-Q1 and 2008-Q1. To solve this comparability problem, the various methods available to backward recalculate time series are given in the following part of this paper.

[§]See the following link for detailed information and the difference between the two series, http://www.turkstat.gov.tr/PreHaberBultenleri.do?id=3912&tb_id=15 .

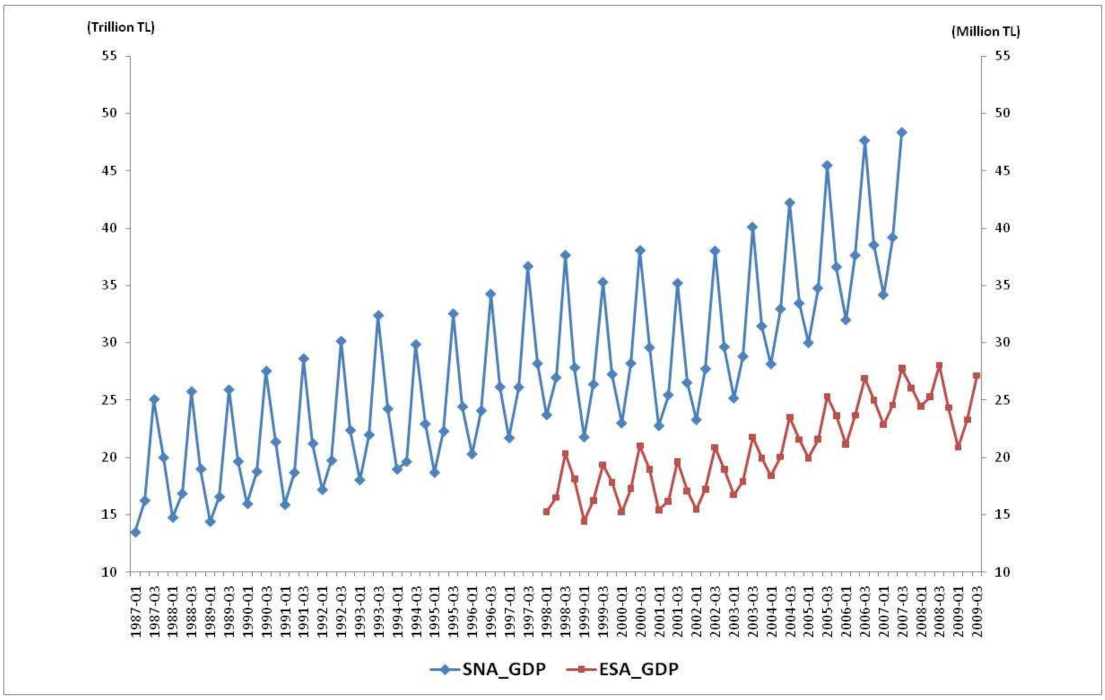

Figure 1. GDP series at 1987 constant prices (SNA-GDP) and at 1998 constant prices (ESA GDP)

3. Classic recalculation methods vs. Model-based-link method

Prior to the implementation of **classical recalculation methods** (the first and second approaches in this paper), as discussed in Caporin and Sartore (2006), the analysis should first assess the time series temporal reversion. Each property must be carefully considered. A time reversibility test exists, (Ramsey and Rothman (1996)), but it requires symmetry of the series (i.e. no trends, no asymmetric seasonal components and no asymmetric cycles). According to some tests conducted by Caporin (2006) on some real data sets, these hypotheses are found to often be rejected due to the presence, in almost all economic time series, of permanent trend components and are influenced by the business cycle that can be asymmetric (Kim, 2005).

The seasonal ARIMA (SARIMA) model (Box and Jenkins, 1970) can be considered **as the first approach** to backward recalculate a time series. The general seasonal multiplicative ARIMA model is,

$$\phi(B)\Phi(B_S)\nabla^d\nabla_S^D z_t = c + \theta(B)\Theta(B_S)a_t \qquad (3.1)$$

where "c" is a constant, s, is the number of periods in a year, d=0,1,2, D = 0,1, ∇ =1-B is a regular difference, ∇_S =1-B_S is seasonal difference, and B is the backshift operator, $Bz_t=z_{t-1}$. $\phi(B)$ and $\theta(B)$ are regular auto-regressive and moving average polynomials, respectively. $\Phi(B_S)$ and $\Theta(B_S)$ are seasonal auto-regressive and moving average polynomials. In this study, the TRAMO approach suggested by Gomez and Maravall (2000a) is used to identify "d", "D" and degrees of $\phi(B)$, $\theta(B)$, $\Phi(B_S)$, and $\Theta(B_S)$ and also estimate these parameters. TRAMO is a program[**] for the estimation and forecasting of regression models with errors that follow in general nonstationary ARIMA processes, when there may be missing observations in the series, as well as contamination by outliers and other special (deterministic) effects. Therefore, we could also take in account the outliers and calendar effects in the identification process. Parameters of the ARIMA model are estimated by the Exact Maximum Likelihood method to satisfy the conditions on the auto-regressive and moving average parameters. These conditions are stationarity and invertibility which are clearly defined in Box and Jenkins (1970).

[**] **T**ime Series **R**egression with **A**RIMA Noise, **M**issing values and **O**utliers. To apply TRAMO, TSW (**T**ramo&**S**eats for Windows) software is used (Gómez and. Maravall, 1996, version: 19 March 2009).

Once parameters are estimated, the forecast and standard errors of these forecasts can be calculated. As mentioned before, our ESA-GDP series is between 1998Q1 and 2009Q3. Therefore, we use inverted series to obtain extended series (backward recalculated to 1987Q1) . When we obtain the ARIMA forecast for 2009Q3 to 2020Q4, we also obtain values backward recalculated from 1987Q1 and 1997Q4.

The second approach, forecasting based on signal extraction of the ARIMA model suggested by Burman (1980), can also be considered to backward recalculate time series. In this approach, once the ARIMA model is identified for a series, sub-ARIMA models are constructed for each component, i.e. trend-cycle, seasonal and irregular. In this study, the SEATS (**S**ignal **E**xtraction in **A**RIMA **T**ime **S**eries) approach (Gomez and Maravall, 2000) is used to identify and estimate sub-component models. For simplification, one may consider additive decomposition of the series z_t,

$$z_t = p_t + s_t + u_t \,, \qquad (3.2)$$

where p denotes trend-cycle components, s, seasonal components, and u, irregular components. Once it is assumed that Z has an ARIMA model (3.1) that satisfies the conditions and that each component has an ARIMA model (3.3, 3.4 and 3.5) that satisfies the canonical conditions (Burman, 1980; Hillmer and Tiao, 1982; Bell and Hillmer, 1984) this means that the variance of irregulars should be maximised at the end of the decomposition.

Trend: $\phi_p(B)\nabla^d p_t = \theta_p(B)a_{p,t}$, $\quad a_{p,t} \approx \text{n.i.i.d}\left(0,\sigma_p^2\right)$ (3.3)

Seasonal: $\phi_s(B)Ss_t = \theta_s(B)a_{s,t}$, $\quad a_{s,t} \approx \text{n.i.i.d}\left(0,\sigma_s^2\right)$, $\quad S = 1 + B + \ldots + B^{s\ 1}$, (3.4)

Irregular: $u_t \approx$ white noise (3.5)

In the equilibria 3.3 and 3.4, a_{pt} and a_{st} the innovation term of the trend and seasonal components is shown. And, 3.5 shows that the irregular component is white-noise. Gomez and Maravall (2000b) show an estimations of these sub-component models in detail. Then, forecasts of original series are calculated by summing forecasts of the trend-cycle and seasonal component since the irregular component is white noise. As before, we use inverted series to obtain backward recalculated values from 1987Q1 and 1997Q4.

The SARIMA and signal extraction approaches (namely classic methods) can be implied easily and use forecasts (actually, backcasts) to recalculate series to the past. However, they have serious disadvantages as well as advantages. Firstly, there is no possibility to take account of past events (in general terms outliers) since forecasts of series are calculated based on past characteristics of the series and cannot include information about the future. Second, classic methods don't use additional information (extra time series) which carries information about the past of the series. What seems an advantage in terms of process implementation, actually is a disadvantage in terms of process reliability; if past information about the series is used, outliers and trend/seasonal movements can easily be seen.

The third approach we suggest as an alternative in this paper, is the Model-based-link method based on the ARIMA model which can be used for backward recalculation of time series whose definitions and concepts are changed by taking account of past economic events or outliers. In this method, we use additional information about the past for backward recalculating the series. This information is a time series which is the old version of the series to be recalculated. In other words, we link two series in a way to provide a compatible trend and seasonal component within two series. In this way, information about past crises or fluctuations is protected and transferred to the new recalculated series.

In general, three different types of outliers can be captured in a time series. These outlier types can be differentiated by the way the time series moves back to its original level. In the case of an *additive outlier* the time series moves immediately back to its normal level. A *transitory change* is an outlier where the level of the time series changes abruptly but moves gradually back to its original level over the next few reference periods. The third outlier type is a *level shift*, where the time series moves suddenly to a different level and does not move back over the next few reference periods. Additionally, a *seasonal level shift* (Kaiser and Maravall, 1999) is a level shift which occurs at the same period in each year i.e. 2000-Q1, 2001-Q1, 2002-Q1... and does not move back over the next few reference periods.

In this method, *level shift* variables used to capture differences in trend components, and *seasonal level shift* used to capture differences in seasonal components are included in the ARIMA model. To combat the problem of under-identification, a Turkish calendar effect variable, created by Atabek et al. (2009), is added to the ARIMA model because Koçak (2008) suggested that there is a significant calendar effect in Turkish ESA-GDP series. The Model-based-link method can be implemented in TRAMO via TSW (TRAMO SEATS for Windows). The application of the method is defined below step by step:

Step 1. A consolidated time series is created by the series-to-be-recalculated (which is brought after the old version of the series. For instance, in this application the full period covers between Q1-1987 and Q3-2009. The period between Q1-1987 and Q4-1997 is represented by SNA-GDP, and the period between Q1-1998 and Q3-2009 is represented by ESA-GDP.

Step 2. The deterministic effects to be used in ARIMA model are specified used to capture the impact of changes in definitions and concepts.

- Step 2.1. The first effect is related to the trend component of the series. In this application, there is a structural break at Q1-1998 in the consolidated series (column B in Table 2) since both the definitions and the calculation unit changed. Since this type of effect is classified as a level shift in time series analysis, there should be a level shift variable at Q1-1998 in the ARIMA model.
- Step 2.2. The second effect is related to the seasonal component of the series. Seasonal fluctuations in the SNA-GDP are larger than the ones in the ESA-GDP (variances in the Table 1). According to these findings, three seasonal level shifts capture differences in the second, third and fourth seasons. i.e. starting from Q2-1998, Q3-1998, and Q4-1998 should be added to the ARIMA model[††].
- Step 2.3. The third and last effect is related to the calendar effect assigned to seasonal components and it is estimated separately as a regressor in TRAMO. This calendar effect is added to the model as a control variable since it does not have a role in the linking process. In the application, the calendar variable is used for Q1-1987 and 2011-Q3 which covers the forecast horizon .

Step 3. RSA=3 with IMEAN=1 to TSW and the regression variables defined are entered in order to be included in the estimatation with the ARIMA model[‡‡].

Step 4. In the output file of TSW, the effects of the level shift and seasonal level shifts on the series can be seen. Table 2 shows how the backward recalculation process is applied using outputs of TRAMO. Finally, a comparable backward recalculated series for the forthcoming data is obtained.

The Model-based-link method has one constraint that can be seen as a disadvantage as it reduces the application to real cases: the need for additional information consisting of a previous version of the time series that has to be backward recalculated. Where no previous version of the series is available, the Model-based-link model cannot be used because it would lack information about past economic events and trend/seasonal movements. On the other hand, the Model-based-link method can be extended to data of different frequencies (such as annual data, see section 5 for possible extensions).

[††] When a fourth seasonal level shift is added to capture seasonal differences of the first quarter on the same observation which has already included a level shift at Q1-1998 to capture differences in the trend components, the estimation process deteriorates. Therefore, the ARIMA model must include a mean (constant) term to capture seasonal differences of first quarters with the logic of dummy variables (Menezes, 1971).

[‡‡] For the commands and detailed user instructions of TSW, please see Caparello and Maravall (2004).

Table 2. Linking process using by results of TRAMO

No	Date	Consolidated time series (B)	Level shift effect (C)	Level shift adjusted series (D)	SO2 Effect of seasonal level shift starting at Q2-1998 (E)	Converted SO2 (F)	SO2 adjusted series (G)	SO3 Effect of seasonal level shift starting at Q3-1998 (H)	Converted SO3 (I)	SO3 adjusted series (J)	SO4 Effect of seasonal level shift starting at Q4-1998 (K)	Converted SO4 (L)	Backward recalculated series (M)
1	Q1-1987	13.464.440.200.000	152.108.409	=B1/C1*100	100	=F4	=D1/F1*100	100	=I4	=G1/I1*100	100	=L4	=J1/L1*100
…	…	…	…	…	…	…	…	…	…	…	…	…	…
38	Q1-1996	20,290,105,000,000	152,108,409	=B38/C38*100	100	=F42	=D38/F38*100	100	=I42	=G38/I38*100	100	=L42	=J38/L38*100
39	Q2-1996	24,071,522,000,000	152,108,409	=B39/C39*100	100	=F43	=D39/F39*100	100	=I43	=G39/I39*100	100	=L43	=J39/L39*100
40	Q3-1996	34,245,901,000,000	152,108,409	=B40/C40*100	100	=F44	=D40/F40*100	100	=I44	=G40/I40*100	100	=L44	=J40/L40*100
41	Q4-1996	26,137,624,000,000	152,108,409	=B41/C41*100	100	=F45	=D41/F41*100	100	=I45	=G41/I41*100	100	=L45	=J41/L41*100
42	Q1-1997	21,692,722,000,000	152,108,409	=B42/C42*100	100	=F46	=D42/F42*100	100	100	=G42/I42*100	100	100	=J42/L42*100
43	Q2-1997	26,110,722,000,000	152,108,409	=B43/C43*100	100	=E43*F47/E47	=D43/F43*100	100	100	=G43/I43*100	100	100	=J43/L43*100
44	Q3-1997	36,655,365,000,000	152,108,409	=B44/C44*100	100	100	=D44/F44*100	100	=H44*I48/H48	=G44/I44*100	100	100	=J44/L44*100
45	Q4-1997	28,172,396,000,000	152,108,409	=B45/C45*100	100	100	=D45/F45*100	100	100	=G45/I45*100	100	=K45*L49/K49	=J45/L45*100
46	Q1-1998	15265677,8	100	=B46/C46*100	100	100	=D46/F46*100	100	100	=G46/I46*100	100	100	=J46/L46*100
47	Q2-1998	16484807,7	100	=B47/C47*100	92,0627078	100	=D47/F47*100	100	100	=G47/I47*100	100	100	=J47/L47*100
48	Q3-1998	20346607,7	100	=B48/C48*100	100	100	=D48/F48*100	80,6944864	100	=G48/I48*100	100	100	=J48/L48*100
49	Q4-1998	18106053,9	100	=B49/C49*100	100	100	=D49/F49*100	100	100	=G49/I49*100	97,1849924	100	=J49/L49*100
50	Q1-1999	14436128,6	100	=B50/C50*100	100	100	=D50/F50*100	100	100	=G50/I50*100	100	100	=J50/L50*100
51	Q2-1999	16217898,6	100	=B51/C51*100	92,0627078	100	=D51/F51*100	100	100	=G51/I51*100	100	100	=J51/L51*100
52	Q3-1999	19361768,2	100	=B52/C52*100	100	100	=D52/F52*100	80,6944864	100	=G52/I52*100	100	100	=J52/L52*100
52	Q4-1999	17824774,3	100	=B53/C53*100	100	100	=D53/F53*100	100	100	=G53/I53*100	97,1849924	100	=J53/L53*100
…	…	…	…	…	…	…	…	…	…	…	…	…	…
91	Q3-2009	27129335,8	100	=B53/C53*100	100	100	=D91/F91*100	100	100	=G91/I91*100	97.1849924	100	=J91/L91*100

In Table 2, column B shows the SNA-ESA-GPD created in Step 1, column "C" shows the estimated effects of level shift by TRAMO created in Step 2.1. Estimated effects of seasonal level shifts by TRAMO created in Step 2.2 are shown in the columns "E", "H" and "K". In a default situation, TRAMO applies these seasonal level shift effects to series-to-be-recalculated to linearise the consolidated series. But, in the columns "F", "I" and "L", we invert seasonal level shift effects to apply to old part (i.e. between Q1-1987 and Q4-1997) of the consolidated series while the other part (after Q1-1998) of the consolidated series is fixed. Consolidated series adjusted from the effects of seasonal level shifts are shown in columns "G", "J" and "L". The order in which seasonal level shifts are adjusted from the consolidated series is not important since this calculation is linear. Then, backward recalculated series is computed in column "M" by adjusting all the effects from the consolidated series.

4. Application

Firstly, we present the results of recalculation by ARIMA forecasts in Table 3. The identified seasonal ARIMA model is $(0,1,0)(1,1,0)_4$ and only a seasonal auto-regressive parameter is estimated significantly. A seasonal autocorrelation test is defined in Pierce (1978) which is calculated using seasonal frequencies (for quarterly data, 4). The linearity test suggested by Ljung and Box (1978) shows whether there is an indication of autocorrelation in squared residuals. According to the diagnostics of the ARIMA model, all of the test statistics values are below the critical value. Then, forecasts obtained for the period between 2009-Q4 and 2020-Q3 (actually, 1987Q1-1997Q4) are shown in Figure 2.

Table 3. Diagnostics and estimation results of ARIMA with inverted series

Name of the series	Number of Observations	Properties of ARIMA Residuals							
		Standard error	BIC value***	Autocorrelation test	Jarque-Bera test	Skewness (t)	Kurtosis (t)	Seasonal Autocorrelation test	Linearity test
ESA-GDP (inverted)	47	0.4662	-1.1	13.4	0.05	-0.21	-0.07	0.274	10.6
		Critical Values:		**>30.6**	**>9.2**	**>|2.5|**	**>|2.5|**	**>9.2**	**>32.0**

	Coefficients and t-test values	
	Seasonal AR	**(t)**
	0.49	3.6
Critical Value:	**(t) >|2.5|**	

Note. Critical values are valid for %1 significance level. Calendar regressor is also used in ARIMA model estimation.
*** **B**ayesian **I**nformation **C**riteria.

Secondly, diagnostics of the signal extraction process are presented in Table 4. In the procedure, when the seasonal AR parameter ">0", SEATS (is modified accordingly, for example it estimates a seasonal MA parameter instead of seasonal AR parameter. Therefore, in our application we used an ARIMA model which has degrees of $(0,1,0)(0,1,1)_4$. The estimated ARIMA model is decomposed into three separate models which show the properties of trend, seasonal and irregular components. Models for the components are shown in

Table 5. It should be noted that variance of irregular components is maximized in the signal extraction as assumed in Burman (1980). Then, forecasts covered by the periods between 1987-Q1 and 1997-Q4 obtained from signal extraction are shown in Figure 2.

Table 4. Estimated ARIMA coefficients in the signal extraction process

Name of the series	Coefficients and t-test values	
	Seasonal MA	**(t)**
ESA-GDP (inverted)	-0.52	3.8
Critical Value:	**(t) >|2.5|**	

Note. Critical values are valid for %1 significance level. Calendar regressor is also used in ARIMA model estimation.

Table 5. Models for the components

Trend (p)	$\nabla^2 p_t = (1 + 0.15B \quad 0.892B^2)a_{p,t}$,	$var(a_{p,t}) = 0.129$*
Seasonal (s)	$(1+B+B^2+B^3)s_t = (1+0.99B+0.34B^2-0.46B^3)a_{s,t}$	$var(a_{s,t}) = 0.023$*
Irregular (e)	-	$var(a_{e,t}) = 0.145$*

* in units of variance of main ARIMA model

Lastly, the Model-based-link method estimation results which are given in different aspects are shown in Table 6, Table 7, Table 8 and Table 9. According to Table 6, the series is estimated in logarithms and first seasonal differences. The ARIMA model includes both regular AR and seasonal AR parameters. According to model diagnostics, residuals are normally distributed and there is no indication of autocorrelation and seasonal autocorrelation in the residuals. As shown in Table 7, the regular AR parameter is statistically significant and close to 1 (-0.79) which means that the series has a fairly stochastic trend. On the other hand, the seasonal AR parameter also shows that the series has moderately stochastic seasonal component.

Automatically detected outliers (by TRAMO) and the calendar effect added to model are in expected signs and also statistically significant (Table 8 and Table 9). There are two outliers in Q2-1994 and Q1-2009 which correlate with economic crisis periods in Turkey. Seasonal outliers for second and third quarters (SO2 and SO3) have negative signs (meaning that the seasonal fluctuations of SNA-GDP are bigger than ESA-GDP's) and are statistically significant while SO4 which is added to capture differences in fourth quarters is not statistically significant. So, there is no significant difference between fourth quarters of SNA-GDP and ESA-GDP's. However, SO4 should not be dropped from the model since it controls seasonal differences between fourth quarters, even if it is not significant.

The short part of the linking process is given in Table 10. As shown in the table, the series is adjusted from level shift, and seasonal outliers. The calendar effect and other detected outliers are not adjusted from the series since the nature of its structure must be protected in the past and present. In other words, it only takes into account differences between SNA-GDP and ESA-GDP which are caused by trend and seasonal components in the linking process. A backward recalculated ESA-GDP series is given in Figure 2 for the period between Q1-1987 and Q4-1987 (the series covering the full period is also given in Annex as a table).

The results of three approaches can be summarized in Figure 2. As shown in the figure, there are three different time series which are differentiated between the 1987-Q1 and 1997-Q4 periods. The series obtained from signal extraction (square dots line) shows a horizontal shape for these periods. The ARIMA backcasts (triangle dots line) and the Model-based-link method results have almost the same trend and seasonal components. However, the main difference between these two series shows itself especially in periods of economic crises. For example, the Turkish GDP series decreased sharply in the years between 1994 and 1995. In this period, the Model-based-link method managed to integrate this effect to the GDP series. In contrast, in the same period, the ARIMA forecast shows no reflection of the effects of the economic crisis.

Table 6. Estimation results of ARIMA model

Name of the series	Number of Observations	Specification of ARIMA							Properties of ARIMA Residuals							
		ln logarithms	AR	Regular dif. order	MA	SAR*	Seasonal dif. order	SMA**	Standard error	BIC value***	Autocorrelation test	Jarque-Bera test	Skewness (t)	Kurtosis (t)	Seasonal Autocorrelation test	Linearity test
SNA-ESA-GDP	91	Yes	1	0	0	1	1	0	0.0293	-6.7	9.9	0.4	-0.5	-0.4	0.2	12.6
									Critical Values:		**>36.0**	**>9.2**	**>|2.5|**	**>|2.5|**	**>9.2**	**>37.0**

* Seasonal AR; ** Seasonal MA; *** **B**ayesian **I**nformation **C**riteria
Note. Critical values are valid for %1 significance level.

Table 7. Coefficients of ARIMA model

Name of the series	Coefficients and t-test values					
	Mean	(t)	AR(1)	(t)	SAR	(t)
SNA-ESA-GDP	0.039	4.01	-0.79	-11.0	0.52	4.7
Critical Value:	**(t) >|2.5|**					

Note. Critical values are valid for %1 significance level.

Table 8. Automatically detected outliers

Name of the series	Outliers		Coefficients		
	Type*	Date	Value	Standard error	(t)
SNA-ESA-GDP	TC	Q2-1994	-0.13	0.024	-5.54
	TC	Q1-2009	-0.11	0.029	-3.74
				Critical Value:	**>|2.5|**

* TC: Transitory change
Note. Critical values are valid for %1 significance level.

Table 9. Regressors in the ARIMA model

Name of the series	Regressors		Coefficients		
	Type*	Date	Value	Standard error	(t)
SNA-ESA-GDP	LS	Q1-1998	-14.24	0.029	-494.96
	SO2	Q2-1998	-0.08	0.026	-3.22
	SO3	Q3-1998	-0.21	0.031	-7.02
	SO4	Q4-1998	-0.03	0.029	-0.99
	Calendar	Q1-1987:Q3-2011	0.01	0.002	3.48
				Critical Value:	**>|2.5|**

*LS: Level shift, SO: Seasonal outlier, Calendar: Calendar effect variable
Note. Critical values are valid for %1 significance level.

Table 10. Converting the effects and linking process

Date	Consolidated series (SNA-ESA-GDP)	Level shift effect	Level shift effect adjusted series	SO2 (starting from Q2-1998) Seasonal Level Shift	Converted S02	SO2 adjusted series	SO3 (starting from Q3-1998) Seasonal Level Shift	Converted SO3	SO3 adjusted series	SO4 (starting from Q4-1998) Seasonal Level Shift	Converted SO4	Backward recalculated ESA-GDP
Q1-1987	**13,464,440,200,000**	152,108,409	8,851,871	100	100	8,851,871	100	100	8,851,871	100	100	**8,851,871**
…	**…**	…	…	…	…	…	…	…	…	…	…	**…**
Q1-1996	**20,290,105,000,000**	152,108,409	13,339,240	100	100	13,339,240	100	100	13,339,240	100	100	**13,339,240**
Q2-1996	**24,071,522,000,000**	152,108,409	15,825,241	100	108.621615	14,569,145	100	100	14,569,145	100	100	**14,569,145**
Q3-1996	**34,245,901,000,000**	152,108,409	22,514,141	100	100	22,514,141	100	123.9242041	18,167,670	100	100	**18,167,670**
Q4-1996	**26,137,624,000,000**	152,108,409	17,183,550	100	100	17,183,550	100	100	17,183,550	100	102.8965456	**16,699,831**
Q1-1997	**21,692,722,000,000**	152,108,409	14,261,356	100	100	14,261,356	100	100	14,261,356	100	100	**14,261,356**
Q2-1997	**26,110,722,000,000**	152,108,409	17,165,864	100	108.621615	15,803,359	100	100	15,803,359	100	100	**15,803,359**
Q3-1997	**36,655,365,000,000**	152,108,409	24,098,184	100	100	24,098,184	100	123.9242041	19,445,906	100	100	**19,445,906**
Q4-1997	**28,172,396,000,000**	152,108,409	18,521,261	100	100	18,521,261	100	100	18,521,261	100	102.8965456	**17,999,886**
Q1-1998	15,265,678	100	15,265,678	100	100	15,265,678	100	100	15,265,678	100	100	15,265,678
Q2-1998	16,484,808	100	16,484,808	92.0627078	100	16,484,808	100	100	16,484,808	100	100	16,484,808
Q3-1998	20,346,608	100	20,346,608	100	100	20,346,608	80.6944864	100	20,346,608	100	100	20,346,608
Q4-1998	18,106,054	100	18,106,054	100	100	18,106,054	100	100	18,106,054	97.1849924	100	18,106,054
Q1-1999	14,436,129	100	14,436,129	100	100	14,436,129	100	100	14,436,129	100	100	14,436,129
Q2-1999	16,217,899	100	16,217,899	92.0627078	100	16,217,899	100	100	16,217,899	100	100	16,217,899
Q3-1999	19,361,768	100	19,361,768	100	100	19,361,768	80.6944864	100	19,361,768	100	100	19,361,768
Q4-1999	17,824,774	100	17,824,774	100	100	17,824,774	100	100	17,824,774	97.1849924	100	17,824,774
…	…	…	…	…	…	…	…	…	…	…	…	…
Q3-2009	27,129,335	100	27,129,335	100	100	27,129,335	80.6944864	100	27,129,335	100	100	27,129,335

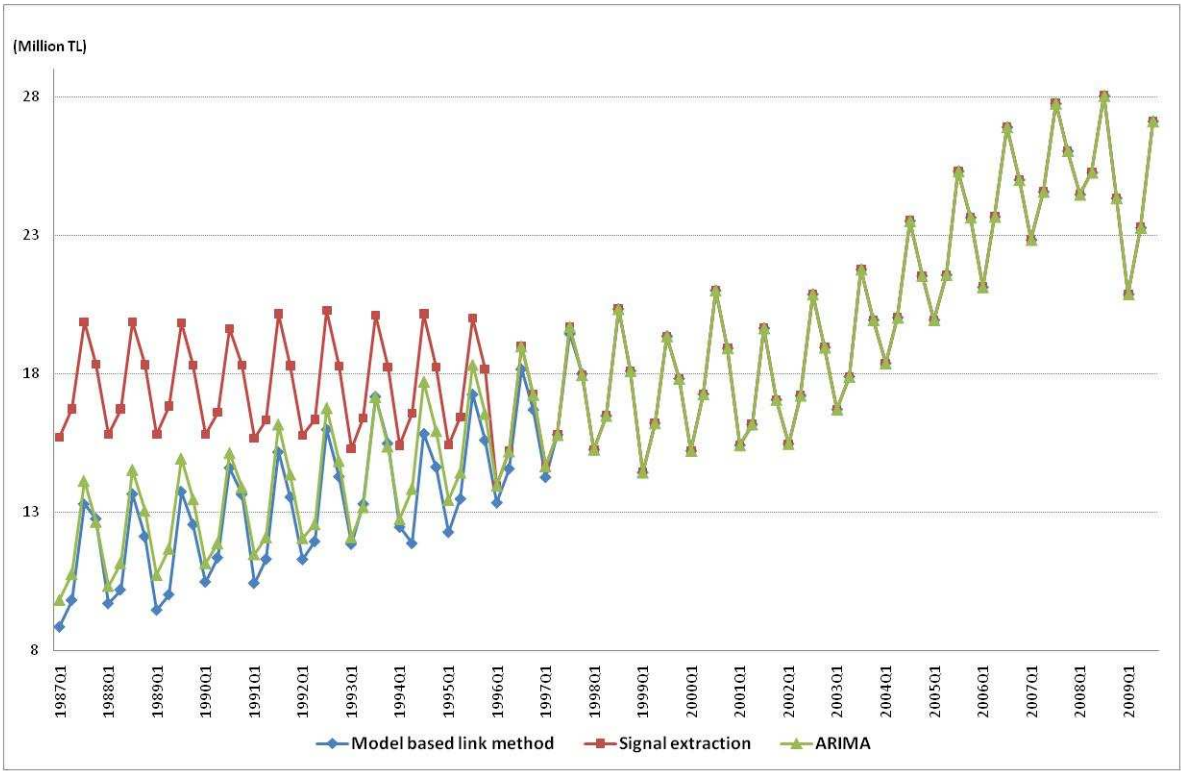

Figure 2. Comparison of the approaches to obtain backward recalculated ESA-GDP

5. Extension from higher to lower frequency data (annual)

So far, this paper has discussed backward recalculation only for quarterly data. However, monthly and annual data also may be the subject of a recalculation process. In these cases, the Model-based-link method can easily be adapted according to the frequency of the data. In the case of monthly data, like the quarterly example, a level shift variable used to capture differences in trend components is included in the ARIMA model. But also, eleven seasonal level shift variables (and a constant term) are used to capture differences in seasonal components and should be included in the monthly ARIMA model. On the other hand, for annual data, the ARIMA model contains only the level shift variable used to capture differences in trend components in the backward recalculating series since it has two components, trend and irregular. Therefore, there is no limitation to the frequencies of the series to be backward recalculated for applying the Model-based-link method.

Moreover, if the Model-based-link method is used for both monthly/quarterly and annual versions of same data, an aggregation-disaggregation problem is possibly revealed. For instance, when an annual version of consolidated data is backward recalculated, it would be the same as the annual total of quarterly backward recalculated data for the periods from 1998 up to the present. But, this would not be true for the period between 1987 and 1997 since this part of the annual backward recalculated data would also be changed.

As an illustrative example, the Model-based-link method is applied to the annual version of consolidated SNA-ESA-GDP. The results are shown in Table 11. It should be noted that the available data covers the time span from 1987 to 2008. Data for 2009 are not used since only three quarters are available. Detailed diagnostics with the results of the ARIMA model estimated are not presented here but are available on request from the authors. Nevertheless, it is worth mentioning that the identified order of the ARIMA is (1,1,0) given that seasonal components cannot be considered in the annual data. The hypothesis of the coefficient of the level shift variable used to capture the trend effect being statistically significant is not rejected. But, the coefficient of calendar effect is not statistically significant in the ARIMA model for annual data.

Table 11. Linking process of annual data

Date	Consolidated annual series (SNA-ESA-GDP)	Level shift effect factors	Level shift effect adjusted series	Backward recalculated annual linked ESA-GDP (A)	Yearly total of the backward recalculated quarterly ESA-GDP (B)	C=A-B
1987	7,472,192,520,000	168,648,151	44,306,401	**44,306,401**	44,727,234	-420,833
1988	7,630,629,210,000	168,648,151	45,245,852	**45,245,852**	45,669,236	-423,384
1989	7,649,831,100,000	168,648,151	45,359,709	**45,359,709**	45,776,917	-417,208
1990	8,357,846,410,000	168,648,151	49,557,890	**49,557,890**	50,079,472	-521,582
1991	8,435,283,010,000	168,648,151	50,017,050	**50,017,050**	50,458,183	-441,133
1992	8,940,074,500,000	168,648,151	53,010,214	**53,010,214**	53,505,959	-495,745
1993	9,659,037,130,000	168,648,151	57,273,306	**57,273,306**	57,797,740	-524,434
1994	9,132,072,600,000	168,648,151	54,148,667	**54,148,667**	54,801,080	-652,413
1995	9,788,779,900,000	168,648,151	58,042,616	**58,042,616**	58,611,808	-569,192
1996	10,474,515,200,000	168,648,151	62,108,687	**62,108,687**	62,775,886	-667,199
1997	11,263,120,500,000	168,648,151	66,784,726	**66,784,726**	67,510,508	-725,782
1998	7,020,315	100	70,203,147	**70,203,147**	70,203,147	0
1999	6,784,057	100	67,840,570	**67,840,570**	67,840,570	0
2000	7,243,640	100	72,436,399	**72,436,399**	72,436,399	0
2001	6,830,935	100	68,309,352	**68,309,352**	68,309,352	0
2002	7,251,983	100	72,519,831	**72,519,831**	72,519,831	0
2003	7,633,819	100	76,338,193	**76,338,193**	76,338,193	0
2004	8,348,559	100	83,485,591	**83,485,591**	83,485,591	0
2005	9,049,973	100	90,499,731	**90,499,731**	90,499,731	0
2006	9,673,832	100	96,738,320	**96,738,320**	96,738,320	0
2007	10,125,463	100	101,254,625	**101,254,625**	101,254,625	0
2008	10,216,397	100	102,163,974	**102,163,974**	102,163,974	0

Column A of Table 11 shows the annual Turkish ESA-GDP series obtained by applying the Model-based-link method at annual levels. Column B shows the annual Turkish ESA-GDP series obtained by summing the quarterly data obtained by applying the Model-based-link method at quarterly levels. Then, the data in column B respects the temporal constraint (the sum of the quarters equals the annual value per each year) by definition. Since, for the specific case of Turkey, no separate information set is available for the annual GDP, the data in column B can be considered as the more appropriate for publication purposes.

For the periods from 1987 to 1997, it can be seen in column C of Table 11 that the annual total of the backward recalculated quarterly series and the backward recalculated annual series are not same.

6. Conclusions

It is important to use comparable long time series data in the economic analysis. On the other hand, it is also important to consider whether a long time series truly respects crises periods. In the recalculation process, the classic question is "Does the recalculated series include economic events and seasonal behaviours in the past?".

In this paper, we tried three approaches to find an answer this question. Firstly, the classic ARIMA approach gives more or less the same tendency with old series. But, it could not include past economic crises since additional information could not be used. Secondly, we used the signal extraction method (forecast of trend and seasonal component). It fit well with the seasonal component, but the trend component had not same movements compared to the old series. It showed a horizontal trend in the past.

The Model-based-linking process can be suggested as offering a good solution to the comparability problem caused by changes in definitions or calculation units of the data produced by statistical agencies. The Model-based-link method uses more sophisticated statistical and econometric tools. However, it is useful to apply this method through the available version of TSW. The point of the Model-based-link method is that differences between two time series are appropriately classified as related with trend and seasonal components. Following this classification, the ARIMA model is estimated with the variables required. Finally, an up-to-date series is provided with effects on trend and seasonal components applied to the old (not updated) part of the series. As an extension, it is also shown that the Model-based-linked method can be applied to annual data. Consequently, it can be said that a comparable GDP series can be obtained between the periods of 1987 and 2009 using the Model-based-link method.

7. Topics for discussion and challenges for future research

Challenges for the continuation of the research would include at least the following topics:

- If the Model-based-link method is applied at the level of the GDP components (indirect approach), the process must be complemented by an opportune temporal disaggregation technique plus benchmarking (such as the Denton multivariate that allows the obtaining of a balanced data set that respects the accounting constraints for all the periods considered and the annual constraints for the past years),
- Where annual National Accounts are calculated according to an information set separate to that of the Quarterly National Accounts, the Model-based-linked method applied at annual level should be complemented by the opportune temporal disaggregation technique, such as Chow and Lin's procedure (Chow and Lin, 1971) to be applied to provide consistency between annual and quarterly backward recalculated data,
- A possible alternative extension is the recalculation of each component time series, such as trend, of the aggregated GDP,
- An application of the Model-based–link method to index series and/or growth rates,
- The issue of calendar adjustment for annual series.

Annex

Table 12. Backward recalculated quarterly and annual Turkish ESA-GDP by Model-based-link method

	Quarterly linked series						Annual linked series
Date	**ESA-GDP(TL)**	**Date**	**ESA-GDP(TL)**	**Date**	**ESA-GDP(TL)**	**Year**	**ESA-GDP(TL)**
Q1-1987	8,851,871.0	Q1-1997	14,261,356.2	Q1-2007	22,844,200.3	1987	44,306,400.6
Q2-1987	9,820,349.6	Q2-1997	15,803,358.8	Q2-2007	24,581,028.3	1988	45,245,851.7
Q3-1987	13,294,506.1	Q3-1997	19,445,906.2	Q3-2007	27,772,166.8	1989	45,359,709.4
Q4-1987	12,760,506.9	Q4-1997	17,999,886.5	Q4-2007	26,057,230.0	1990	49,557,889.5
Q1-1988	9,699,257.3	Q1-1998	15,265,677.8	Q1-2008	24,482,883.3	1991	50,017,050.2
Q2-1988	10,191,551.2	Q2-1998	16,484,807.7	Q2-2008	25,279,413.7	1992	53,010,213.7
Q3-1988	13,654,078.0	Q3-1998	20,346,607.7	Q3-2008	28,048,888.8	1993	57,273,306.0
Q4-1988	12,124,349.4	Q4-1998	18,106,053.9	Q4-2008	24,352,788.4	1994	54,148,667.3
Q1-1989	9,463,568.7	Q1-1999	14,436,128.6	Q1-2009	20,879,186.1	1995	58,042,616.3
Q2-1989	10,021,095.9	Q2-1999	16,217,898.6	Q2-2009	23,285,107.4	1996	62,108,687.0
Q3-1989	13,741,438.0	Q3-1999	19,361,768.2	Q3-2009	27,129,334.8	1997	66,784,725.8
Q4-1989	12,550,814.4	Q4-1999	17,824,774.3			1998	70,203,147.2
Q1-1990	10,480,941.2	Q1-2000	15,217,908.0			1999	67,840,569.8
Q2-1990	11,356,000.2	Q2-2000	17,269,135.1			2000	72,436,398.9
Q3-1990	14,600,021.2	Q3-2000	21,019,480.9			2001	68,309,352.1
Q4-1990	13,642,509.2	Q4-2000	18,929,874.9			2002	72,519,831.0
Q1-1991	10,434,754.4	Q1-2001	15,419,915.4			2003	76,338,192.5
Q2-1991	11,301,626.8	Q2-2001	16,173,158.2			2004	83,485,590.6
Q3-1991	15,176,320.2	Q3-2001	19,650,703.8			2005	90,499,730.9
Q4-1991	13,545,481.9	Q4-2001	17,065,574.7			2006	96,738,320.2
Q1-1992	11,291,600.4	Q1-2002	15,469,976.6			2007	101,254,625.0
Q2-1992	11,941,614.1	Q2-2002	17,214,452.3			2008	102,163,974.0
Q3-1992	15,988,360.3	Q3-2002	20,876,687.0				
Q4-1992	14,284,384.3	Q4-2002	18,958,715.1				
Q1-1993	11,846,605.8	Q1-2003	16,716,746.3				
Q2-1993	13,293,621.8	Q2-2003	17,898,517.4				
Q3-1993	17,173,581.5	Q3-2003	21,774,717.8				
Q4-1993	15,483,931.1	Q4-2003	19,948,211.1				
Q1-1994	12,461,445.2	Q1-2004	18,380,246.8				
Q2-1994	11,872,973.0	Q2-2004	20,035,371.7				
Q3-1994	15,833,446.6	Q3-2004	23,528,095.5				
Q4-1994	14,633,215.1	Q4-2004	21,541,876.6				
Q1-1995	12,274,935.4	Q1-2005	19,947,282.9				
Q2-1995	13,480,209.5	Q2-2005	21,577,563.3				
Q3-1995	17,254,612.8	Q3-2005	25,323,570.1				
Q4-1995	15,602,050.6	Q4-2005	23,651,314.5				
Q1-1996	13,339,239.5	Q1-2006	21,133,291.1				
Q2-1996	14,569,145.2	Q2-2006	23,678,188.1				
Q3-1996	18,167,670.1	Q3-2006	26,916,390.2				
Q4-1996	16,699,831.4	Q4-2006	25,010,450.8				

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