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16 July 2012

Online at <https://mpra.ub.uni-muenchen.de/40096/>  
MPRA Paper No. 40096, posted 16 Jul 2012 13:33 UTC

# Modeling Risk in a Dynamically Changing World: From Association to Causation

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First Draft, July 2012

## Executive summary

The current crisis causes numerous economic uncertainties, such as a break-up of the European currency union, and a Greek exit from the euro area to boost the competitiveness by means of devaluation of national currency. When a factor such as exchange rate is expected to have a significant effect on the borrowers' creditworthiness or a shift in risk regime may have occurred, risk management models based on backward-looking statistical methods are inadequate.

Unlike the other approaches to risk modeling, the discussed approach for dynamic risk modeling doesn't ignore causation in favor of correlation and thus it is far more proactive. In contrast to existing risk models, FX rate is considered as a causal factor, which induces a negative correlation among default realizations and reveals *ex ante* dangerous risk concentrations with the clear economic and behavioral content.

Keywords: Correlation, causation, dynamic risk modeling, credit portfolio management, factor modeling, competitiveness, exchange rate, FEBA approach

*JEL* classification: E37, E47, G20, G21, G28, G30, G32.

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I would like to thank Philip Halperin, Eugene Trubin, Vladimir Shalagin, Vladimir Manaev, Elisa Sturaro, Ivan Sokolov, Oleg Morya, participants of the conference "Market Risk: Methodology and practice, Moscow, 2012" and the conference "Business Solutions: Banking & Finance, Moscow, 2012" for useful comments and conversations.

I also would like to thank Galina Guravleva and Scientific school "Economic Theory" of Plekhanov University for their helpful discussions and Skyline Risk Solutions, LLC for research assistance. As usual, all remaining mistakes are mine.

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## 1. Introduction

The current crisis causes numerous economic uncertainties, such as a break-up of the European currency union, and a Greek default and exit from the euro area. The main driver to leave the currency union for Greece and other PIGS countries is the need to boost their competitiveness by means of devaluation of national currency.<sup>1</sup> In case of Greece it could be the return to drachma.

The expected market opportunities for economy with devalued currency entail a new system of risks because the exchange rate depreciation creates not only winners but losers too. When a shift in risk regime may have occurred and a factor such as exchange rate is expected to have a significant effect, normal statistical approaches for risk modeling will fail due to non-equilibrium and nonlinear settings.

In general, risk models can be decomposed into two types: statistical and causal, based on economic reasoning to understand cause and effect. Causal models are little used in financial risk modeling, with the exception of the assessment of contagion risk (the risk that, the credit deterioration of counterparty causes the credit deterioration of other counterparties).

Over the last two decades of "the great moderation", economists got sure the business cycle was tamed. The massive amount of historical market data has powered the creation of statistical models built on a concept of association, which imply a worldview that ignores causation in favor of correlation. In this realm, it doesn't matter whether PDs increased because the S&P500 decreased or vice versa. As the old maxim goes, "correlation does not imply causation." The same is with other risk factors such as GDP growth, unemployment, inflation, real trade variables, exchange rate and so on, which influences all obligors in the same way.

At the same time, well – known statistical risk models often implicitly account for causality, albeit in a form that does not account for the direction of cause and effect. The analysts at CSFB circumvented the calculation problems associated with default correlation: "The defaults are correlated but there is no causal link between them - the correlation effect observed is due to a background factor, the state of the economy, which changes the rates of default... This approach is similar to that taken in market risk management, where no assumptions are made about the causes of market price movements."<sup>2</sup>

It's worth noting that, market risk can be defined as the risk that a financial position changes its value due to the change of an underlying market risk factor. Credit risk can be defined as the risk of not receiving the promised payment on an outstanding claim. At the same time, some risk factors may influence both market

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<sup>1</sup> For popular policy writings on the topic see Roubini in the Financial Times (<http://www.economonitor.com/nouriel/2011/06/13/the-eurozone-heads-for-break-up/>), Krugman in the New York Times (<http://krugman.blogs.nytimes.com/2010/05/01/why-devalue/>)

<sup>2</sup> See Credit Suisse Financial Products (1997)

and credit risk. Russian currency lost 12 percent in May 2012 and we are interested to determine not only risk metrics under previous years, but also the dynamics of beliefs under the changing market conditions. It is the aim of causal analysis where a more appropriate question in case of Russia might be: “What impact does the current devaluation have on risk profiles of Russian banks?”

Causal analysis is the process by which causality is determined. The impact of exchange rates on economy is well-known phenomenon in the economic - large movements in the real exchange rate away from medium-run equilibrium have opposite effect on economic agents (balance-sheet effects, dynamic Dutch disease).

As a matter of fact, borrowers may differ in their degree of sensitivity to market impact, but few firms are completely unresponsive to the market conditions in which they operate. Low-rated companies, being closer to default already, are more likely to be pushed into default because of an economic downturn. As new economic conditions affect all low-rated<sup>3</sup> credits simultaneously, defaults among these credits are likely to be correlated.<sup>4</sup> The high degree of correlation indicates a high level of unexpected losses and dangerous concentration risk.

Kealhofer (1997) has an interesting discussion on risk concentration. First he states that “there has been no method for actually measuring the amount of diversification in a debt portfolio”, and that “*ex ante*, no method has existed which could quantify concentrations”; concentrations have only been detected *ex post*.

The possibility to pin down forward-looking correlations are very important in risk management because the risk of a portfolio depends not on what the correlations were in the past, but on what they will be in the future. The subportfolios formed in line with FX causal factor can reveal *ex ante* dangerous risk concentration with the clear economic and behavioral content. Because of its economic substance this factor provides the needed negative correlations.<sup>5</sup>

Unlike the other approaches to risk modeling, the discussed approach for dynamic risk modeling doesn't ignore causation in favor of correlation and thus it is far more proactive and allows the introduction of prior knowledge. The dynamic risk modeling (DRM) can be used to mitigate as well as measure risk.

In our opinion, the DRM approach could be used by financial institutions in the countries where a shift in risk regime might have occurred and where FX rate is expected to have a significant impact. The intuition behind this is that FX rate plays a more important role in emerging market economies (EME) than advanced economies, where most domestic and foreign transactions are in local currency, markets are deeper, and the private sector is better equipped to absorb exchange

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<sup>3</sup> In contrast, defaults of highly-rated companies, besides being rare, are typically the result of company-specific problems.

<sup>4</sup> See FitchRatings (2008)

<sup>5</sup> By correlated credit and exchange rate risk we do not necessarily imply a linear relationship, but that we model the two risks' dependence.

rate changes.<sup>6</sup> Pass-through from the exchange rate to inflation is typically higher in EMEs, often reflecting more open economies, the currency denomination of trade, and, at times, less credible monetary policies. As a consequence, the default correlations and levels of unexpected losses are typically higher in emerging market economies than in developed economies.<sup>7</sup>

The remainder of the article is structured as follows: In section 2 we deliberate positive and negative types of default correlations, in section 3 we discuss the macro- and micro-structural channels of default correlations. Afterwards, in section 4 we discuss the dynamic risk modeling approach based on taking account of the direct effect of FX risk on credit risk. Finally, in section 5, the main conclusions are summarized.

## **2. Types of default correlations**

Correlation in credit risk is a well-recognized phenomenon, and understanding the sources of correlated credit losses is crucial for many purposes, such as setting capital requirements for banks and pricing of credit products. The concept of correlation lies at the heart of all statistical risk models.<sup>8</sup> Unfortunately, as noted in the literature, the estimating default correlation is the most difficult part of statistic modeling.

The task force of the Market operations committee of the ECB (2007) stated “Correlation measures the extent to which assets default or migrate together. In the credit risk literature the parameter often (but loosely) referred to is default correlation, formally defined as the correlation between default indicators (1 for default, 0 for non-default) over some period of time, typically one year.”

Such default correlation, i.e., default correlation in narrow sense, is difficult to estimate directly, simply because defaults are rare events. Besides, some default events are strategic decisions, and therefore, they may not relate to economic default. Some companies may be able to negotiate debt restructuring to avoid defaults. For these reasons, risk models estimates correlations of asset returns rather than of defaults.<sup>9</sup>

In this work we use term default correlation in a broader sense. A broader definition of default correlation includes changes in the value of net assets due to changes in credit spreads, downgrades as well as defaults. And all these factors are important when assessing the impact of credit risk on the economic value and profitability of a portfolio.

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<sup>6</sup> See IMF discussion note (2012)

<sup>7</sup> See Fitch Ratings (2008)

<sup>8</sup> The other critical quantities of credit risk in a portfolio involves: the probability of default for each individual position and the magnitude of financial loss in the event of each possible default.

<sup>9</sup> Default correlation is related non-linearly to asset correlation and tends to be considerably lower (in absolute value).

It is especially difficult to estimate default correlations when it is recognized that these correlations vary over time, so that a forward-looking correlation estimator is needed. Li (1999) specified “the independence assumption of the credit risks is obviously not realistic, in reality, the default rate for a group of credits tends to be higher in a recession and lower when the economy is booming. This implies that each credit is subject to the same set of macroeconomic environment and that there exists some form of *positive dependence* among the credits“.

In general, as detailed in ECB (2007) default correlation can be either positive – for instance because firms in the same industry are exposed to the same suppliers or raw materials, or because firms in one country are exposed to the same exchange rate<sup>10</sup> – or negative, when for example the elimination of a competitor increases a company’s market share.

### **3. Sources and channels of default correlations**

Default correlations may be caused by both macro- and micro-structural channels. First, as previously discussed, a firm’s financial success is sensitive to common macro-economic factors. For portfolios that are not well diversified in their common factors, such dependence could cause defaults to be positively correlated.

Commonly applied credit risk models rely on some form of the conditional independence assumption, under which default correlation is assumed to be captured by the dependence of all firms in the portfolio on the macro factors determining firms’ default intensities.

Well known examples of models which rely on some form of the conditional independence assumption include the Asymptotic Simple Risk Factor (ASRF) model as developed by the BCBS or applications of the structural Merton model like the KMV or the CreditMetrics model. In the context of reduced-form models the conditional independence assumption is also often referred to as the doubly stochastic property.

Secondly, there are direct business/legal links between firms in a portfolio, which provide a channel for the spread of financial distress within a portfolio. Such micro-structural interdependencies go beyond the borrowers’ exposure to macro factors and can lead to so-called “contagion.”

#### *Macro-structural channels*

There are two alternative branches of modern academic methods of credit risk measurement: the structural approach and the reduced form approach. Structural models measure the cyclical impact on credit risk by specifying the stochastic asset diffusion process. Because asset volatility increases (decreases) during economic downturns (expansions), credit risk also shows a cyclical pattern. The reduced form

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<sup>10</sup> The portfolio credit risk models considered by the task force don’t take into account the causation effect of FX rate.

approach models the cyclical component of credit risk as a function of a systematic risk factor.

In CreditMetrics, the credit transition matrix is conditioned on a credit cycle index, which shifts down when economic conditions deteriorate. Among the proprietary models, KMV considers the systematic factors using a three level approach: 1) a composite market risk factor, 2) an industry and country risk factor, and 3) regional factors and sectoral indicators. The factor loading for any individual firm for each of the factors are estimated using asset variances obtained from the option theoretical model, and the factor loadings are then used to calculate covariances for each pair of firms.

The asymptotic version of the Vasicek model that is used in the Basel II AIRB model focuses on a large diversified portfolio in which idiosyncratic risk is fully diversified and the only source of portfolio default rate uncertainty is the realization of a single common latent Gaussian factor<sup>11</sup>.

Das *et al.* (2007) question whether systematic risk factors are the sole source underlying corporate default correlations and whether they can explain the extent of default clustering observed in empirical data. The work (*op. cit.*) provides empirical evidence for the importance of micro-structural dependencies between debtors in risk models.

#### *Micro-structural channels*

Default contagion can increase the credit risk in a portfolio since the default of one borrower can cause the default of a dependent second borrower. As mentioned above, the microstructural interdependencies can be either positive – for instance because firms in the same industry are exposed to the same suppliers – or negative, when for example the elimination of a competitor increases a company's market share. Incorporating micro-structural information in the latent variable framework leads to a change in the idiosyncratic terms.

Contagion risk is then the risk that, the credit deterioration of counterparty causes the credit deterioration of other counterparties. Contagion can be defined as a transmission effect that underlies microstructure interdependence.

Egloff *et al.* (2006) complement the present literature on large homogeneous credit portfolios, which neglects credit contagion and instead focuses on cyclical default dependence. The authors set out an approach based on Markov chain models and extend existing approaches by modeling a microstructural dependence in addition to the usual macroeconomic dependence among the debtors. Micro-structural effects are modeled by using weighted graphs where the nodes represent the firms and the edges characterize business partner relationships between firms. Each edge is associated with a weight that represents the strength of the business partner relationship.

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<sup>11</sup> The AIRB assumes that the PD, exposure at default, and loss rates in default (LGD) are known nonstochastic quantities for all obligors.

Notwithstanding the mentioned above theoretical premises about sign of correlation, the authors (*op. cit.*) defined the micro-structural dependence in terms of a positive correlation. It seems reasonable, since a situation, when the borrower default has a direct positive effect on the creditworthiness of another borrower is rather infrequent.

#### 4. Cause and effect vs. risk factors

Risks arising from micro-structural dependencies in couple with from macro-structural dependencies constitute the systemic risk. Bank for International Settlements defines systemic risk as "the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties".<sup>12</sup> This definition stresses causation as well as correlation (*correlation with causation*). In the dynamically changing economy correlation and causation can decouple, making analysis and forecasting difficult.

The justification of correlations in the loan portfolio is part of the concept of risk concentration management, but when dealing with loan portfolios, no formal methodology for measuring concentration seems to have emerged.

Kealhofer (1997) point out "generally, banks partition loan portfolios into subportfolios or 'buckets' according to some practical criterion which is somehow related to the way in which they do business. For the purpose of credit risk in general and concentration in particular, it may be desirable to adopt a different criterion. ... One of the most difficult problems is to determine *ex ante* potentially dangerous dimensions of concentration, and these may have nothing to do with the organizational structure of the bank."

From our point of view *ex ante* potentially dangerous concentrations have little to do with the organizational approach for industry's classifications in periods of market turbulence too. Besides, there is no unique standard of dividing firms by industries, as well as there is no unique standard of what should be considered as industry or sector. One can define a sector in a way that each obligor in the sector should be an equally good substitute for each other obligor within the same sector of industry.

Consider a very simple although rather extreme example of a portfolio composed of two borrowers, "E" and "I", each with equal PDs. Each of the borrowers refers to the industry "motor trade". What does this tell us about the behavior of the portfolio as a whole? Not much, it turns out, unless we also understand the default correlation among credits in the portfolio.

Suppose a borrower "E" specializes in car export and the borrower "I" is an importer of cars. The companies belong to the same industry and are in the same

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<sup>12</sup> See BIS (1994)



region of the country. However, according to the definition given above, each of the debtors can't replace the other.

Consider an event where the exchange rate vis-à-vis the U.S. dollar should drop by 50 percent.<sup>13</sup> In such a risk regime shifting the level of competitiveness of the borrower “*I*” will worsen dramatically. During periods of declining demand for import the importers usually reduce purchases, delay delivery of ordered products and/or cancel orders, resulting in reduced revenues, delays in revenue recognition and excess inventory. It causes pressure on borrowers’ margin and net income.

Simultaneously, the creditworthiness of the borrower's “*E*” improves as a result of increasing its price competitiveness. That is, the same force moves up and down the borrowers’ creditworthiness. Such an effect of simultaneous motion in the opposite direction is a prerequisite for the perfect negative correlation.

The other important component of the FX rate impact on the debt service capacity is the direct effect on the borrower’s balance sheet.<sup>14</sup> In line with Sokolov (2009), one can distinguish six main types of portfolios in line with FX impact on borrowers’ debt service capacity through competitiveness and balance sheet channels:

- DL – loan to enterprises of Domestic market in Local currency;
- DF – loan to enterprises of Domestic market in Foreign currency;
- IL – loan to Importers in Local currency;
- IF – loan to Importers in Foreign currency;
- EL – loan to Exporters in Local currency;
- EF – loan to Exporters in Foreign currency.

A relatively low level of foreign currency - denominated loans in Russia<sup>15</sup> (*DF* and *IF* types of portfolios) means that balance sheet effect’s component in Russia is limited. At the same time Russia has emerged with the more flexible regime<sup>16</sup>, and as a result Russian currency lost 12 percent in the last month.

The Basel Committee's Working Group on the Interaction of Market and Credit Risk assessed<sup>17</sup> the FX factor’s impact on the of foreign currency - denominated loans of domestic borrowers (*DF* type of portfolio): For a B+ rated portfolio<sup>18</sup>, an integrated approach leads to an overall risk that is 1.5 to 7.5 times higher than an approach that tackles the credit risk and FX risk in isolation, i.e. without taking account of the FX factor’s effect. The scale of the effect is influenced by

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<sup>13</sup> The Russian ruble jumped from 25 rubles per US dollar in July 2008 to nearly 36 rubles per US dollar in February 2009, a 48% depreciation rate in a matter of six months.

<sup>14</sup> The borrowers acting both as importers and as exporters are less vulnerable to exchange rate movements by creating natural hedges. See Sokolov (2010).

<sup>15</sup> According S&P (2011), share of loans in foreign currency issued by Russian banks to Russian companies in 2011 is 20% which is twice as less than in the Ukraine (50%) and in Kazakhstan (40%).

<sup>16</sup> In comparison with Ukraine and Kazakhstan. For details see S&P (2011).

<sup>17</sup> See BCBS (2009b)

<sup>18</sup> Standard & Poor's rating methodology

idiosyncratic properties of individual obligors. Low-rated companies, by definition, are more likely to be pushed into default because of impact of causal factors. Accordingly, as the ratings decrease, the underestimation risk increases.

In light of the Merton's key idea of structural credit risk models, regarding a loan as short put on the payment ability, credit risk has the payoff profile of a short put on the payment ability with FX rate strike.

Since the debt service capacity of a borrower is a function of the exchange rate, market factor changes has "*direct effect*" on the credit risk (see *inter alia* Breuer *et al.*, 2008). The term "direct effect" is meant to quantify an effect that is not mediated by other variables in the model or, more accurately, the sensitivity of "E" and "I" borrowers to changes in FX rate while all other factors are held fixed.

It's worth noting that, if a cause and direct effect relationship is not established, yet there seems to be a statistical association between the two, the economic phenomenon suspected of being associated with an effect is called a risk factor. That is, a *risk factor* is an economic affluence, or behavior that has an association with but has not been proven to impact on default risk.

Furthermore, when a bank in our sample bases its credit decision on a normal statistical model, "E" and "I" debtors would be treated as perfect substitutes. In this case, the correlation between borrowers "E" and "I" due to FX risk factor should be define as positive (although not perfect), i.e., the both borrowers are expected to provide exactly the same marginal risk under any market impact. In this case, an incorrect specification of the input data related to default correlation can cause a distorted assessment of credit risk.

Implementation of the dynamic risk modeling implies cause and effect's priority over default correlation. Applying this risk modeling approach, one can analyze the impact of exchange rate on borrower by considering it as a set of "pure" subjects. Among them the following can be highlighted: exporter; importer; business in the domestic market having foreign competitors; business in the domestic market, which doesn't have foreign competition.

In order to clarify the effect of exchange rate risk on a real business entity, we just "expand" it to a number of "pure" subjects, and then summarize the impact on them.<sup>19</sup> As shown by Sokolov (2009) it is reasonable that an ERM system and the currency control system should be combined to expand the universe of necessary data.

The fact that the needed data for borrowers clustering in line with *ex ante* dangerous risk concentration are not available in a database explains why the competitiveness factor, despite its apparent obviousness, did not appear in well-known statistical models. The situation whereby a statistical model is based only on

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<sup>19</sup> The proprietary solution for active credit portfolio management "PortfolioBalance" is heavily related to information processing of foreign trade and other financial clients' transactions.

available data, rather than on economic reality, is called “conditioning on the data”. This enhances convenience but at the cost of accuracy.

Finally, each bank’s business line might be assessed on the basis of its sensitivity to changes in the FX rate. These sensitivities can be summed to provide a bank-wide assessment of the aggregate sensitivity to this exchange rate in line with direction of the sensitivity.

At both the business line level and at the aggregate level, such sensitivities could be fed into a VaR or stress test model to determine the amount of economic capital corresponding to this exposure. Overall, if some business lines are positively exposed to increases in the value of the currency, while others are negatively exposed, the aggregate economic capital for this risk will be less than the sum of the individual business line calculations.

## **5. Conclusions**

Current risk management models based on historical losses and backward-looking statistical methods are inadequate in the non-equilibrium and nonlinear world. Innovative thinking and incentives for additional methods are needed for more proactive risk management that discovers risk exposures as they build up. One approach is to supplement statistical models with the cause-and-effect ingredients in order to make models more dynamic and accurate.

If a cause and direct effect relationship can’t be established, correlations can be used to explain the company’s economic success by means of some global underlying influence. In contrast, when a factor such as exchange rate is expected to have a significant impact, cause and effects should have priority over concept of statistical association.

The present paper, unlike previous papers on risk modeling, considers FX rate as a causal risk factor, which induces a negative correlation among default realizations. The portfolio clustering in line with competitiveness criterion can reveal *ex ante* dangerous risk concentration with the clear economic and behavioral content.

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