FUTURE OF OPTION PRICING: USE OF LOG LOGISTIC DISTRIBUTION INSTEAD OF LOG NORMAL DISTRIBUTION IN BLACK SCHOLES MODEL

Raja, Ammar

University of Hertfordshire

November 2009

TITLE: FUTURE OF OPTION PRICING: USE OF LOG LOGISTIC DISTRIBUTION INSTEAD OF LOG NORMAL DISTRIBUTION IN BLACK SCHOLES MODEL

SUBMITTED BY: AMMAR AFTAB RAJA

Acknowledgements

I am very thankful to my supervisors Dr. Willem Buiter and Dr. Rudra Sensarma for their kind help and support throughout the research paper. At first I thought that this is a difficult topic and I won't be able to do it but they always pushed me in the right direction.

I am also helpful to Professor Desmond J. Higham of University of Strathclyde and Professor Aurélie Thiele of Leigh University, work of whom inspired me to do mine and who always answered my question no matter how stupid they were.

Abstract

Options are historically being priced using Black Scholes option pricing model and one of the prominent features of it is normal distribution. In this research paper I will calculate European call options using log logistic distribution instead of normal distribution. My argument is that a model with logistic distribution reflects better fit of option prices as compared to normal distribution. In this research paper I have used historic data on stocks, value European call options using both logistic and normal distribution and then finally compare the results in order to check the validity of my argument. What I have found is that European call options prices based on log logistic distribution better reflect stock prices on expiry date and Black Scholes Model based on normal distribution tend to overprice European call options. Another interesting fact is that before 1987 stock market crash, Black Scholes model valued options more correctly on average. But with time as the volatility of stocks increased and with more and more crashes normal distribution tend to underestimate the probability of default and thus generally overpriced options. At this point of time log logistic distribution is better serving the purpose but all depends on volatility of the stocks. If volatility levels further increase then fat tails of log logistic distribution have to become even fatter, that's why keeping an eye on facts and incorporating all relevant variables in your model is very important. In finance there is never a universal truth every thing depends on what's happening in the market.

TABLE OF CONTENTS

CHAPTER ONE: INTRODUCTION

Options belong to the family of derivatives and derivatives are guilty of playing a huge role in the current financial crises. Options did not exacerbate the crises because they were traded on exchanges like Chicago Board of Option Exchange (CBOE) in US and London International Financial Futures and Options Exchange (LIFFE). These exchanges are very transparent and have well defined rules for trading which resulted in options saved from the ills of Credit Default Swaps (CDS).

Every body knows that Credit Default Swaps (CDS) were lethal and brought the demise of American International Group and Lehman brothers. Lehman was hugely exposed to sub prime lending but AIG story had more to do with CDS. Here I want to discuss what role options played?

Derivatives came from the term derive i.e. they derive their value from other assets. Options derive their value from whatsoever is the underlying asset on which the value of options depends. Options are a contract which gives an option to the buyer of options to buy the underlying asset at a certain price on a certain date. There are two type of options i.e. Call options and put options. Call options are the right to buy the asset by the buyer and put option is right to sell the asset by the buyer at a certain price at a certain time. So its makes sense so far. Buyers buy options to hedge their risks and sellers sell options because they get an upfront payment to enter the contract.

So our option equation has so many unknowns from future and that's where uncertainty comes into equation. As far as the hedging is concerned everything is fine speculators can create problems who buy and sell options systematically to make a profit. Many models are used to value options but the most famous models are Black Scholes model and Binomial model.

Apart from these models many others are used and with the advent of cheap computing powers these days every one can easily use a different model. That's why nowadays simulation based complex option pricing models are common.

In spite of all these improvement in computing powers Black Scholes model is by far the most famous and widely used model. Black Scholes model came into lime light in early 70s and was glorified because of its use of Chicago Board of Option Exchange (CBOE). It came under some criticism after the October 1987 market crash but the criticism eased off soon. Then the Asian crash of 1998 resulted in the failure of Long Term Capital Management and default of Russia. Mr Scholes and Mr. Merton was chairman of Long Term Capital management (LTCM) then which defaulted that year and was bailed out by Wall Street giants. Incidentally same year they were awarded the Nobel Prize by Nobel foundation (1997) in economics. Black Scholes model sustained that crises as well along with the dot come crises in early parts of this century.

Derivative trading increased by phenomenal numbers after 2002 and the use of options as a hedging and speculative tool and Black Scholes model was used mostly to price them. The most prominent critic of BSM was Mr. Nicholas Nassim Taleb and he emphasized on the point that BSM overstates prices of options because of its understatement of risk. How is risk understated in BSM? Mr. Taleb was so furious that he uttered comments like "Myron Scholes should be in a retirement home doing Sudoku" (New York times, 2009).

Mr. Nicholas Nassim Taleb gave me the inspiration to do this work but surprisingly as far as I have seen most of his work is theoretical rhetoric and don't purpose any alternative to Black Scholes. He is more concerned at proving that Black Scholes model was discovered by Louis Bachelier. (Taleb and Haug, 2009) There are many losers from the current financial crises but it made at least some celebrity economists like Professor Nassim Nicholas Taleb and Professor Nouriel Roubini.

There are many problems with Black Scholes model and I cannot evaluate each of them in this research paper alone. Here I will attack one of its assumptions i.e. use of log normal distribution. As we have seen from the start of this century that assets have become more volatile, here I am concentrating on stocks. The probability of default taken by normal distribution is low and any valuation based on it is flawed as I will prove later in the research paper.

I had a hard time actually brainstorming and ultimately getting to my research question. I found only one article (Najjab and Thiele, 2009) which purports this research but they only gave results of their findings with out any good explanation of how they got these results. This motivated me to work on this problem by providing better explanations of procedures used in the research and prove my point. I used FTSE 100 companies for the analysis as compared to NYSE listed companies used by the other article.

Every Statistic distribution takes a probability of default which gives us an idea of confidence intervals i.e. 95 % Confidence intervals mean that we are 95 percent sure that price of option will be between two values. According to normal distribution 99 % of the values lie with in three standard deviations from the mean. Normal Distribution is very famous and widely used in applications to forecast height of men in America etc. In this example if we see the probability of someone to be very short or very long is very less and can be modelled by normal distribution. But in the case of option pricing which is a bet on the stock price etc probability of default is much more than that.

That's how normal distribution overstates prices of options by understating the probability of default of underlying stock and hence any pricing based on this distribution overstates prices of options. This understatement of risk sometimes becomes the cause of excessive risk taking which can result in doom day scenarios like the credit crunch we are going through right now.

In this research paper, I will use log logistic distribution to prove my point. Log logistic distribution is similar to normal distribution but has fatter tails. Fatter tails means that the probability of default that we use to value options is more. My argument is that since Black Scholes Model uses normal distribution to value options its over estimates the price of options which creates risk free profits for option sellers who know this discrepancy and use more realistic distributions like Log Logistic Distribution.

Logistic distribution is a continuous probability distribution. The cumulative distribution function of logistic distribution is logistic function which appears in feed forward neural networks and logistic regressions. It resembles the normal distribution in shape but has heavier tails (higher kurtosis). The obvious way to check if logistic distribution is a better indicator of current price movements is to take historic data and then use both normal distribution and logistic distribution to forecast future stock prices. Then check the prices you get using these distributions against the actual prices from the stock exchange. If logistic distribution gives a better fit than normal distribution then it is better to use logistic distribution.

I used historic data for my analysis here. All the data is taken from http://uk.finance.yahoo.com/ and the period which is used to compute the parameters for both distributions for both distributions is two years. The reasons for this are explained later on in this research paper. The stocks used in the research paper are all taken from London Stock Exchange and are FTSE 100 companies. Details of the stocks are given in Appendix 1. The later analysis comprises of three stages. In the first stage the log returns for all the companies are fitted to both Logistic distribution and normal distribution. Then in the second stage some out of sample tests are done to check the validity of the arguments and finally option prices are calculated using first normal distribution and then logistic distribution.

Limitations of the study are that the data which we took in the study is of a period (01/03/07 to 28/02/09). This period is one of the most extraordinary periods in finance history. Nobody could have imagined the downfall of Bear sterns, Lehman brothers and AIG to name a few just a single period before the crises started. Crises were so big that it brought the Governments from US and Iceland to their knees. So the period is volatile plus has the effect of huge fiscal stimulus given by all the Governments. But my point is that these extreme events the so called black swans by Mr. Nicholas Nassim Taleb are becoming more frequent so they should become part of our option valuation and other financial valuation.

Today everybody knows that stock's probability of default is more and this view is based on current events which happened in recent past. The expectations have changed so should all the analysis which is based to capture them. If we use normal distribution we overstate prices of options. If sellers of options know this discrepancy and buyers' don't, then this creates reason for sellers to oversell options and make risk free profits because options are overpriced and underestimate the risk.

The problem above is an information asymmetry problem which is most probably what is happening because Black Scholes is so dominant and is used by large number of small investors i.e. buyers of options. And Sellers of options are mostly big companies which can better evaluate these discrepancies. Understatement of risk in sub prime lending market brought us to the brink of another great depression after the great depression of the 1930s and now we should look in other directions as well to find these problems and resolve them to save the system.

Another limitation of the study is that it is based on current data and it's not universal i.e. the probability of default in the 1980s was less and there normal distribution works better but not for today analysis. Logistic distribution is working today but it may not tomorrow so a keen look at the financial environment is required to come to pragmatic solutions. That's why as I said before there are no universal truths in finance.

CHAPTER TWO: CRITICAL LITRETURE REVIEW

The inspiration of doing work on this topic came from an article written by Pablo Triana (a professor at the Instituto de Empresa, Madrid, and author of Corporate Derivatives: Practical Insights for Real-Life Understanding) on in financial times (Financial Times, 2007). In that article he criticised various assumptions of Black Scholes model and concluded that Black Scholes Model was successful for so long because it was convenient for traders to use. They can get their required results from BSM very conveniently. He said "They don't trust Black-Scholes, but they like it".

Triana (2009) also wrote a new book "Lecturing birds on flying: Can mathematical Theories destroy the financial markets" in which he again criticised the complex mathematical models and our excessive faith in them. He mentioned in the article the problems because of normal distribution but never explained them enough. But by all means the above mentioned article stirred my interest in Black Scholes Model and its problems.

Comparatively, Mr. Nicholas Nassim Taleb (2007) in his book "The Black Swan" focused more on extreme events, their effect and how frequently they happened. When I started my research I focused more on extreme events and how to model them but came to the result that there is no way that I can model them. Extreme events by their nature are unpredictable and cannot be modelled. Work by Mr. Ariel Rubenstein (1998) in his book "Modelling Bounded Rationality" helped clearing my mind.

In a recent article "Why We Have Never Used the Black-Scholes-Merton Option Pricing Formula (fifth version)" by Espen Haug and Nicholas Nassim Taleb (2009) purported that option pricing formula which glorified Mr. Black, Mr. Scholes and Mr. R. Merton was present a long time ago by Louis Bachelier and Edward O. Thorp (1902). Both the authors say that Louis Bachelier and Edward O. Thorp (1902) allow a broad choice of probability distributions) and removed the risk parameter by using put-call parity in their model. The article is not complex but do not provide detailed explanations.

In another working article Derivatives, Prediction and *True* Fat Tails (i.e. Fractal), Part 1: The Fragility of Option Pricing by Nassim Taleb, he tried to tackle the tail problem of option pricing but this again is very mathematically complex and hard to grasp. Professor Nicholas Nassim Taleb gave me the idea of fat tails but for the implementation I had to look somewhere else which is shown in the next sections of literature review.

I once asked my supervisor that why to use these models which are so backward looking and if they cant predict these extreme events and got the answer that that's the best we can do (its better than doing nothing) and now I testify that that's the right approach to go about it. Later on I realised that the solution I was looking for was not very far from the modelling of extreme events. I realised that frequency of these extreme events do matter and there is actually a way to model it as well.

As I have explained in the introduction as well that probability of default of assets i.e. stocks is perceived to be more because of the events that started from the Asian crises in 1998 and still going on in the form of current credit crunch. So for any analysis, assumptions should use increased frequency of these events and ultimately resulted in me choosing log logistic distribution instead of log normal to value options because logistic distribution takes a probability of default which is more than the one taken in normal distribution.

I was amazed to find very few scholarly articles in which this kind of work was done. The articles which were present were either focused on too many problems at a single time and losing track of everything or mathematically too complex to read, understand and do analysis. So I had a hard time reading them and ultimately finding material which focused on the problem which I wanted to work at.

It is shown by Jackwerth and Rubinstein (1996) that the distribution of the S&P 500 before 1987 exert lognormal distributions, but since have deteriorated to resemble leptokurtosis and negative skew ness. This further increased my confidence in the work I was doing.

While McKenzie et al, (2007) illustrated the significance of Black Scholes model under logistic distribution is superior to log normal distribution, a more recent and extensive study by Muhannad R. Al Najjab and Aurélie Thiele (2008) better reflects my research which changes lognormal distribution to log logistic distribution to value options and compared results. McKenzie et al, (2007) used jump diffusion approach to increase the tail properties of lognormal distribution while Najjab and Thiele, (2009) completeley changed the log normal distribution to log logistic distribution.

The focus of my research paper is the article from Najjab and Thiele, (2009) and they provided foundations for my work. They used stocks from New york stock exchange where as I have used stocks from London Stock Exchange to see implementation of work on a diffrent exchange in a different country. While I found their work very interesting, I had troubles relating things becuase they gave very little explanation of how they reached at their conclusions.

I used a different statistic software to prove my results as compared to the one used by Najjab and Thiele, (2009) in their study. I had to work very hard to relate everything, used simulations and explained where they just skipped. The book by Benninga (2008) really helped here. Inspite of these differences the results we reached were more or less the same.

I used EasyFit Professional version 5.1 by MathWave Technologies downloaded from www.mathwave.com for quantitative analysis. The software has many distributions to choose from to fit the data and gives chi square tests results along with other important statistical tests of fit of data. I have also used the random number generator of EasyFit Professional version 5.1 to produce random numbers from logistic and normal distribution for analysis purposes. Apart from that I have used Microsoft excel predominantly for quantitative analysis.

I have tried to use simple analysis software and tried to keep everything very transparent so that every body can easily figure out what's happening. Apart from that I have used the simplest kind of statistical tests and explained them in the body of the research paper to keep things simple and pragmatic.

CHAPTER 3: RESEARCH METHODOLOGY

The results which I wanted to show required detailed quantitative analysis and the data of stocks which are used as underlying assets is widely available. I took the data from yahoo finance UK, details of stocks used are given in Appendix one. The actual data which I took is stock prices for a two year period (01/03/2007 to 28/02/2009). The data was used to find daily log returns on the stocks which were then used to calculate annualised standard deviation and annualised mean to use in further analysis.

Results for Anglo American are given in the Appendix. Same procedures are used for the rest of the stocks used it the analysis and not presented here because of space constraints. I am providing excel work sheets with this research paper as well which have all the workings.

The original research paper i.e. Najjab and Thiele, (2009) used Logistic distribution to value options took stocks mostly from New York stock exchange. But here I am taking stocks from London stock exchange stocks to check the validity of the argument on different set of stocks from across the globe. I took daily prices for all the stocks in the dataset from finance.yahoo.co.uk for a two year period (01/03/2007 to 28/02/2009). The stocks used are from different industries but I originally intended not using any financial or bank stocks because of all the distortions in them in the current financial crises but later on I decided to add some banks to the analysis to see effects of my analysis on them. Details of all the stocks used in the study are given in Appendix 1.

Before going any further I want to briefly explain what is mean and standard deviation. Mean is average returns over the period, it is usually denoted by Greek letter μ which is pronounced in English as Mu. The other parameter is standard deviation. Standard deviation is a measure of how far the data is spread from the mean. Standard deviation is high if data is well spread from the mean, standard deviation will be high and if the data is close to the mean standard deviation is low. Standard deviation is denoted by Greek letter σ which is called sigma.

In this case Log Normal distribution is characterised by two parameters, μ(Greek word Mu used for mean) and σ(Greek word sigma used for standard deviation), which can be estimated from historical prices by using following formulae

Where St is the current stock price and St-1 is stock price of one period earlier than St. Where as Log Logistic distribution is characterised by the following two parameters, μ and s, which can be estimated from historical prices by:

One of the key issues in estimating the parameters of distributions is to decide on how much historical data is relevant and if the distribution of stock price did not vary over time, then its best to take as much data as possible. But as can be seen from general stock market analysis, the probability of default is perceived to be more today than it was in the beginning of Bull

Run in 2003, changes in ownership, introduction of new stocks and threat of litigation all create non stationary effects. There fore it becomes very critical to remove obsolete data points when computing parameters of both Log Normal and Log Logistic Distribution.

I made the following observations after varying the periods from three months to five years:

- σ and s are much less volatile than μ. Specifically, the estimates of σ and s do not vary significantly if the time horizon between 1 and 4 years.

- Smaller time horizon (up to 1 year of trading) induces higher volatility.

- The estimates appear to stabilize for time horizons of about 2 years (500 trading days).

- This is true in particular for the mean, which as mentioned above is the more volatile parameter.

- Significant volatility occurs for longer time horizons. From a practical standpoint, five-year-old data points have little value in helping the decision maker understand current stock prices.

These observations suggest that the parameters are estimated most accurately with about two years of data. (Najjab and Thiele, 2009)

This is the reason I have used a two year period to estimate parameters to use in my research. The focus of this research paper is quantitative analysis rather than qualitative analysis because of the nature of the research question and wide availability of the required input i.e. stock prices returns in my case. All the research is based on these stock prices i.e. log returns were found from these stock prices which helped finding standard deviation and mean and so on.

Here I want to explain the reasons for using log returns rather than discrete returns. One of the main benefits of log returns instead of discrete returns is that log returns are time additive. Time additive means two period log returns are identical to the sum of the each periods log returns. This is very convenient and apart from that to get n period log return, we can simply add the consecutive single period log returns. Simple return is not time additive.

Using log returns is also mathematically convenient as logs and exponents are easier to manipulate with calculus. Theoretical models tend to assume, unrealistically but conveniently, continuously compounded rates of return. For example, if Log Return = LN (A1/A0), then exp(log return) = A1/A0. Apart from that for short periods (e.g. daily), the log return approximates the discrete return very well.

There are some drawbacks of using log returns i.e. portfolio return is weighted sum of components but we cannot say this under log returns: the log return is not linearly additive across portfolio components, but the discrete return in linearly additive. It's unrealistic as well because market tends to quote discrete returns. But in our case we are take daily returns of which log returns are a better approximation and the advantages way out performs these minor drawbacks so I am using log returns.(Harper,2009)

Finally I want to emphasize on the role of literature review in transforming my research methodology. Since there is very little work done on changing distributions in option pricing I had to concentrate more on Najjab and Thiele, (2009) article since it just tackled the distribution problem in their whole article. Najjab and Thiele, (2009) helped me estabilishing the right length of period to choose for the data i.e. two years.

I took data for a period of three months to five years and checked their conclusions myself and found them right. They give me a good idea of what to do but I had to do all the calculations using my own intellect as they never provided detailed analysis. Detailed Litreture review gave me a detailed understanding of the problem and helped me resolving it pragmatically.

Now I will move forward to the actual tests done to prove my point.

CHAPTER FOUR: DATA PRESENTATION, ANALYSIS AND INTERPRETATION OF FINDINGS

This section comprises of two main sections for the kind of analysis I want to do. First is comparing log normal and log logistic distribution fit of log daily returns. Second part of first section is out of sample test to check which distribution better predicts stock price on average and finally in the second section I will find option prices based on lognormal and log logistic distribution and then interpret the results.

COMPARING THE DISTRIBUTIONS

In this section I will compare log logistic model with lognormal using chi square test and out of sample test. The distribution which will better fit the data will be better to use in any analysis involving stock price log returns because that's what the data is i.e. daily log returns.

Chi-Square Test

Chi square test is a tool used to find goodness of fit of a specific distribution for the data set considered. When an analyst attempts to fit a statistical model to observed data, he or she may wonder how well the model actually reflects the data. How "close" are the observed values to those which would be expected under the fitted model? One statistical test that addresses this issue is the chi-square goodness of fit test. This test is commonly used to test association of variables in two-way tables, where the assumed model of independence is evaluated against the observed data. In general, the ***chi-square test statistic*** is of the form

Definition	The chi-square test is defined for the hypothesis:	
	H_0:	The data follow a specified distribution.
	H_a:	The data do not follow the specified distribution.
	Test Statistic:	For the chi-square goodness-of-fit computation, the data are divided into *k* bins and the test statistic is defined as where is the observed frequency for bin *i* and is the expected frequency for bin *i*. The expected frequency is calculated by

Where F is the cumulative Distribution function for the distribution being tested, $\boldsymbol{Y_u}$ is the upper limit for class ***i***, $\boldsymbol{Y_l}$ is the lower limit for class ***i***, and *N* is the sample size. If the computed test statistic is large, then the observed and expected values are not close and the model is a poor fit to the data. In our case of comparison of log logistic model with log normal model, the distribution which gets smaller number from chi square distribution is better fit of sample data of log stock returns under consideration here. (NIST/ SEMATECH e-Handbook of Statistical Methods)

For data analysis in this article I am using MathWave data analysis and simulation tool along with Microsoft excel to apply statistical tools to my data to reach an outcome. I am giving all the results which I got by running simulations on data using MathWave tools in excel in Appendix 2. According to chi square test run by MathWave on all companies used in the study, log logistic distribution outperformed log normal distribution in eight out of nine cases (Appendix 2).

I have included the out put of probability density functions of both log normal and log logistic distribution along with the pictures of fit of data of both the distributions. It's very evident with even with the naked eye if you look at the graphic output in the Appendix 2 that logistic distribution is better fitting that data as compared to lognormal distribution.

Log logistic distribution also outperformed log normal distribution according to other tests like Kolmogorov Smirnov test and Anderson Darling test in eight out of nine cases considered. Results of these tests are also given in Appendix 2. This is strong support for my argument that log stock returns follow a log logistic distribution pattern with fatter tails, so log logistic distribution should be used instead of logistic distribution to value options on these stocks.

Out of Sample Test

For out of sample test only a subset of historical data is used consisting of closing prices between 28/02/2007 to 28/02/08. Distribution parameters are calculated based on that subset and then used to predict closing prices from 01/03/2009 to 30/08/2009. Forecasted prices by both distributions are then compared to actual closing prices in order to check which distribution better predicts stock price movements.

Distribution parameters are calculated as follows.

Lognormal Distribution:

Where Z is standard Normal random variable and r is the risk free rate of return.

Logistic Distribution:

Where L is a Logistic random variable calculated using the parameter estimates discussed above. (Najjab and Thiele, 2009)

Here again I have used MathWave data analysis and simulation tool along with Microsoft excel to fit distributions to data and then generate random numbers from both log normal and log logistic distribution in order to use them to get predicted stock prices. The Detailed process is given below.

The period used to calculate the parameter used in the analysis is of 28/02/2007 to 28/02/08. The parameters calculated are standard deviation i.e. volatility and mean return. As the return and standard deviation I first calculated is daily mean and daily standard deviation, I had to annualise them to use in the analysis.

There are 252 trading days in the year so to annualise the daily mean we multiplied it with 252 and to annualise daily standard deviation we multiplied it with square root of 252. As a result of this calculation we found annual mean and annual standard deviation.

After annualising I took actual stock prices of Anglo American from yahoo finance for the period 01/03/2009 to 28/08/09 and then used the following procedure to simulate stock prices using both normal and logistic distribution.

Suppose we denote by St the price of stock at time t. The lognormal distribution assumes that the natural logarithm of 1 plus the return from holding the share of stock between time t and time t + Δ t is normally distributed with mean μ and standard deviation **.** Denote the (uncertain) rate of return over an interval Δ t by r. Then we can write S (t + Δ t) = S (t) exp (r Δ t). In the lognormal distribution we assume that the rate of return r over short period Δ t is normally distributed with mean μ Δ t and variance ^{2} Δ t.

Another way of writing this relation is to write the stock price $S_{t+\Delta t}$ at time t + Δ t in the following way.

$$\frac{S_{t+\Delta t}}{S_t} = \exp(\mu \Delta t + \quad Z \sqrt{\Delta t})$$

Where Z is a standard normal variable (mean=0, standard deviation =1).To see what this means, suppose first that **=**0. In this case we have

$$S_{t+\Delta t} = \exp(\mu \Delta t)$$

This simply says that the stock price grows at an exponential rate with certainty. In this case the stock is like a reckless bond that bears interest rate μ, continuously compounded.

Now if > 0. In that case the lognormal assumption says that, although the tendency is for the stock price to increase, there is an uncertain element (normally distributed) that must be taken into account. The best way to think about this process is in terms of a simulation.

This is the detailed process for stock price simulation based on normal distribution. Z denotes random number generated from normal distribution with mean 0 and standard deviation 1. I have generated the random numbers through EasyFit software from Mathwave. This can be done through excel as well by using data analysis add in. Annualised standard deviation and mean were found for the two year period 28/02/2007 to 28/02/08 and then used to simulate daily stock price for a period 01/03/2009 to 28/08/09. The parameters used are annualised mean, annualised volatility, Δ t and initial stock price. Δ t is daily change in stock price and the number of business days in a year is approximately 250. Thus when we define Δ t = 1/250= 0.004. That's why we used Δ t =0.004 in our analysis. (Harper,2008)

The detailed simulations are given in the excel worksheets which I am providing with the research paper. Then instead of Z, I generated random numbers from logistic distribution and simulated stock price. The results were not very promising, simulated stock price followed the actual for the first few days but then huge differences remained between simulated and actual prices.

Then I computed simulated closing prices after five months starting from 01/03/2009. I used (121) iterations of randomly generated numbers from both logistic and normal distribution, which were used to compute 121 simulated closing prices and then computed mean and standard deviation of 121 closing prices found by using first normal distribution and then logistic distribution. After detailed simulations of closing prices using both normal and logistic distribution (results given in the appendix), the results show that both distributions don't predict the price perfectly.

I also computed simulated closing prices with closing price set after two months starting at 01/03/2009 rather than five months as in the case above. I used (121) iterations of randomly generated numbers from both logistic and normal distribution which were used to compute 121 simulated closing prices and then computed mean and standard deviation of 121 closing prices found by using different distributions.

Detailed calculations are given in excel worksheet. Out of eleven companies considered logistic distribution better predicted closing price in 8 cases which is impressive. But its standard deviation is higher than normal distribution predictions in every single case which means larger deviations from the mean are more common in logistic distribution.

As the forecasting period is decreased from six to two months results have improved but not a lot. Both models don't predict price perfectly here as well but as the period is decreased their results have gone closer to the actual. Apart from that Standard deviation has decreased in two month period as compared to six months. When two months period is used the general trend of results stayed the same i.e. logistic model still outperforms normal model in eight out of eleven cases and again with larger standard deviation.

Normally higher standard deviation is not considered good and the rationale given for it is that models with higher standard deviation give wayward results i.e. more extreme values and data is well speeded from the mean. But in this case this is a good thing because if we see last two years data stock prices have become more volatile and after the current financial crises, stock prices have a more tendency to move to extreme values. And every body now expects prices to move in a wider spread as compared to before and the probability of default taken by everybody is more. That's why even with high standard deviation of logistic distribution it better reflect movement of stock prices.

Seller of options know this that's why they are happy to sell options according to black Scholes model because they are overpriced because of less probability of default of stock which normal distribution takes. And the actual probability of default is high as proved by logistic distribution comparison with normal distribution. So this created an incentive for the option sellers to over sell as its evident any seller will sell more if the prices are high than they should have been if buyers also know this discrepancy. In the next section I will use both black Scholes model and simulated options prices using logistic distribution to prove my point.

I am repeating some points in the research paper and that's because these points are very important and I want the user to be very familiar with them.

Comparison of Option Valuation Based on Normal and Logistic Distribution

In this section I will calculate option prices using Black Scholes model which uses normal distribution and compare it with option prices which use logistic distribution instead of normal distribution. The mechanism of finding option prices is as follows.

I am using excel program to find option prices based on Black Scholes model. Existence of a closed end solution makes every thing very easy. The formula calculates option price as given by the model on the next page.

The Model:

$$C = SN(d_1) - Ke^{(-rt)}N(d_2)$$

C = Theoretical call premium

S = Current Stock price

t = time until option expiration

K = option striking price

r = risk-free interest rate

N = Cumulative standard normal distribution

e = exponential term (2.7183)

$$d_1 = \frac{\ln(S/K) + \left(r + s^2/2\right)t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

s = standard deviation of stock returns

$\ln$ = natural logarithm

In order to understand the model itself, we divide it into two parts. The first part, SN(d1), derives the expected benefit from acquiring a stock outright. This is found by multiplying stock price [S] by the change in the call premium with respect to a change in the underlying stock price [N(d1)]. The second part of the model, Ke(-rt)N(d2), gives the present value of paying the exercise price on the expiration day. The fair market value of the call option is then calculated by taking the difference between these two parts.

Assumptions of the Black and Scholes Model:

1) The stock pays no dividends during the option's life

Most companies pay dividends to their share holders, so this might seem a serious limitation to the model considering the observation that higher dividend yields elicit lower call premiums. A common way of adjusting the model for this situation is to subtract the discounted value of a future dividend from the stock price.

2) European exercise terms are used

European exercise terms dictate that the option can only be exercised on the expiration date. American exercise term allow the option to be exercised at any time during the life of the option, making american options more valuable due to their greater flexibility. This limitation is not a major concern because very few calls are ever exercised before the last few days of their life. This is true because when you exercise a call early, you forfeit the remaining time value on the call and collect the intrinsic value. Towards the end of the life of a call, the remaining time value is very small, but the intrinsic value is the same.

3) Markets are efficient

This assumption suggests that people cannot consistently predict the direction of the market or an individual stock. The market operates continuously with share prices following a continuous Itô process. To understand what a continuous Itô process is, you must first know that a Markov process is "one where the observation in time period t depends only on the preceding observation." An Itô process is simply a Markov process in continuous time. If you were to draw a continuous process you would do so without picking the pen up from the piece of research paper.

4) No commissions are charged

Usually market participants do have to pay a commission to buy or sell options. Even floor traders pay some kind of fee, but it is usually very small. The fees that Individual investor's pay is more substantial and can often distort the output of the model.

5) Interest rates remain constant and known

The Black and Scholes model uses the risk-free rate to represent this constant and known rate. In reality there is no such thing as the risk-free rate, but the discount rate on U.S. Government Treasury Bills with 30 days left until maturity is usually used to represent it. During periods of rapidly changing interest rates, these 30 day rates are often subject to change, thereby violating one of the assumptions of the model.

6) Returns are log normally distributed

This assumption suggests, returns on the underlying stock are normally distributed, which is reasonable for most assets that offer options. (Rubash,2006)

The only assumption which I am testing here is the last one that returns are log normally distributed as I have proved from the tests done in the previous two sections. So I will find options prices using Black Scholes model and another model which uses logistic distribution instead of normal. My argument is that options are overpriced because normal distribution has thin tails and don't truly reflect the possibility of default of a stock and logistic distribution which has fatter tails is more appropriate. So according to my argument options should price less with logistic distribution and this is the assertion I will test here.

The parameters of Black Scholes like volatility model are taken from historical data (01/03/2007 to 28/02/2009) which was calculated as part of previous calculations and given in the appendix for all the stocks used in the research paper. Straight after this interval (01/03/2007 to 28/02/2009) option prices are calculated with period taken as three months. Option prices are calculated with three different strike prices

1. Actual Closing price after three months rounded to nearest 5p.
2. 5p less than the actual closing price.
3. 5p more than the actual closing price.

As mentioned above all the parameters were found by historical data with the exception of interest rates. Interest rates are taken as 8 %. This is just taken for convenience purposes because as long as the interest rates are same for all the analysis its fine. My purpose was not to check effect of different interest rates on options so I kept it same in all the analysis for consistency. All these results are given in the Appendix 4. Here again I have used Microsoft excel to calculate Black Scholes option prices.

The calculator used to calculate option prices in Appendix 4 is of Professor Espen Haug and I am giving location of this calculator on internet in references. In the next section I have transformed this calculator to calculator option prices with logistic distribution. The calculator in the next section calculates both option prices with normal and logistic distributions and we can then see the difference in both valuations.

Logistic Distribution Option Pricing Model

The last part of analysis is the most important as here I want to prove that any option valuation model which uses normal distribution is flawed and rather logistic distribution option pricing valuation model should be used. I have provided strong support to the argument that logistic distribution better reflects stock price return already in this research paper.

Here again I am using Black Scholes model for consistency. My argument was that options are overpriced if calculated using Black Scholes model with normal distribution. For my argument to be right, any option value calculated by using a Black Scholes model with logistic distribution should be less than the one found with Black Scholes model with normal distribution.

I am keeping all the assumptions of Black Scholes model the same apart from the use of normal distribution. By doing this I am checking just the effect of change of distribution and not distorting it with any other assumption changes. I have already calculated option prices by using Black Scholes Model with normal distribution in the last section. Here I will calculate option prices using the same Black Scholes Model but a different distribution i.e. logistic distribution.

NORMSDIST function of Excel is used by Black Scholes Model (Normal Distribution) to translate the number of standard deviations (z) into cumulative probabilities.

To illustrate:

=NORMSDIST (-1) = 15.87%

=NORMSDIST (+1) = 84.13%

Therefore, the probability of a value being within one standard deviation of the mean is the difference between these values, or 68.27%. (Kyd, 2006)

The only place where distribution plays a part in Black Scholes model is when it is used to translate the number of standard deviations into cumulative probabilities.

The Model:

$$C = SN(d_1) - Ke^{(-rt)}N(d_2)$$

C = Theoretical call premium

S = Current Stock price

t = time until option expiration

K = option striking price

r = risk-free interest rate

N = Cumulative standard normal distribution

e = exponential term (2.7183)

$$d_1 = \frac{\ln(S/K) + \left(r + s^2/2\right)t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

s = standard deviation of stock returns

$\ln$ = natural logarithm

$C = S\,N(d_1) - K\,e^{(-rt)}\,N(d_2)$ where N is cumulative standard normal distribution. This is the only place in Black Scholes model where distribution plays a part. If we change normal distribution to Logistic distribution here, we will be able to find option prices based on logistic distribution and that's what I have done.

If we use Logistic distribution here to translate the number of standard deviations (z) into cumulative probabilities, we will be able to get option prices with Logistic distribution. So how did I manage to do this?

Here I used Kesian calculation library by Casio http://keisan.casio.com/ to translate the number of standard deviations (z) into cumulative probabilities of logistic distribution. The web site has comprehensive set of tools and the actual calculator to translate the number of standard deviations (z) into cumulative probabilities of logistic distribution can be found at http://keisan.casio.com/has10/SpecExec.cgi?path=07000000.Probability%20Function/01016000.Logistic%20distribution/10000100.Logistic%20distribution/default.xml&charset=utf-8 .

I am providing a screen shot of Kesian calculation library by Casio where calculator to translate the number of standard deviations (z) into cumulative probabilities of logistic distribution is shown. Here it is effectively doing the same thing which Microsoft does through normsdist function but with a different distribution i.e. logistic.

Standard normal distribution is used in Black Scholes model with normal distribution. One of the prominent features of standard normal distribution is that it uses Mean 0 and standard deviation 1. So for consistency I have used location parameter equal to zero and scale parameter equal to 1 in logistic distribution when used with Black Scholes model.

Appendix 5 provides the out put for Anglo Americans option valuation using Black Scholes model with logistic distribution and its comparison with the one using normal distribution. Here Black Scholes model with logistic distribution is valuing option at a price less than the one found by Black Scholes model with normal distribution. The price calculated is actually negative in this case and can be taken as zero because price cannot be taken negative. The important point is that the price calculated is less than the one found by Black Scholes model with logistic distribution.

In all the eleven cases considered in the study the prices of options calculated by Black Scholes model with logistic distribution is less than the price calculated by black Scholes model with logistic distribution. This is strong support for my argument. I am providing output of program calculating option prices using logistic distribution of some more companies to show this. So yes options were overpriced during the last few years and still are if we use Black Scholes model with normal distribution to calculate their value.

From the point of view of trader he would like to short overvalued options and make profits but if they use Black Scholes model naively they can get in trouble. As long as the buyers and sellers of options use the same model to value their options, the problem don’t become too absurd. But with so many participants in the market in which every body is striving to increase their profits by pennies, this can become a problem. Long term Capital management tried these high risk penny profit approach and got busted. I will comment on this more in the last section.

CHAPTER FIVE: CONCLUSIONS AND RECOMMENDATIONS

The main argument of my research paper was that stock returns follow a pattern better described by log logistic distribution instead of log normal distribution, which results in any analysis based on log normal distribution flawed and prone to errors. These errors can become very problematic when used in models where some sort of prediction is required as part of the analysis. Here I am not supporting the idea that asset prices can be predicted with hundred percent accuracy rate or options can be priced perfectly by the use of logistic distribution. This can never be done because of the nature of the problem; asset returns follow a Brownian motion i.e. by looking at the position of returns today you can never perfectly predict returns for tomorrow, returns can follow any path.

I have used real stock market data from London Stock Exchange to prove my argument. In the first part of analysis, I proved that logistic distribution provides a better fit of stock returns as compared to lognormal distribution by using real stock data and statistical tools. In the second part of analysis I completed out of sample tests to check the validity of my argument. I used both logistic and normal distribution and used a stock return prediction model to predict stock price movement pattern. Here too logistic distribution model gave better results on average. In Out of eleven companies considered logistic distribution better predicted closing price in eight cases which is impressive. But its standard deviation was higher than normal distribution predictions in every single case which means bigger deviations from the mean are more common in logistic distribution.

Normally higher standard deviation is not considered good and the rationale given for it is that models with higher standard deviation give wayward results i.e. more extreme values and data is well speeded from the mean. But in this case this is a good thing because if we see last two years data stock prices have become more volatile and after the current financial crises, stock prices have a more tendency to move to extreme values. And every body now expects prices to move in a wider spread as compared to before and the probability of default taken by everybody is more. That's why even with high standard deviation of logistic distribution it better reflect movement of stock prices.

Finally I used Black Scholes model incorporating different distributions and found that Black Scholes model with logistic distribution mostly gives option prices which are less than the one found by normal distribution. I have shown in the last two analyses that logistic distribution is a better predictor of stock price and stock returns; therefore stock option price calculated by Black Scholes model with normal distribution is overpriced.

This has implications for traders for options. If every body is using the same model to find prices of options then there is no problem. But in the current environment with so many kind of investors and speculators working to earn a living from all kind of markets and cheap computing powers, this can never be a case i.e. use of same model by everybody to price options.

If options are over priced and sellers are aware of the problem but the buyers are not, then sellers will oversell options i.e. if something's price is more than what it actually is it does the following things from the point of view of seller. It first of all gives seller opportunities of making risk free profit without taking the required amount of risk and this creates an incentive to oversell. This is not a merry situation considering what happened in the sub prime market because of over selling of sub prime loans. Yes the default rate ultimately became the main culprit but it was overselling which originally gave the potential defaulters access to sub prime loans.

From the point of view of buyer if they are aware of the discrepancy but the sellers are not, then the buyers can find option prices using both logistic and normal distribution. As the buyers are aware that normal distribution model prices are overvalued they can short the stock options priced with normal distribution and make returns. The problem is there as long as there is one party i.e. buyer or the seller who is using a different model. Yes this is an information asymmetry problem because if every body is using the same model to price options then there is no problem at all.

RECOMMENDATIONS

Black Scholes model is coming under increased pressure by every passing day and the opposing people are becoming more powerful as compared to Mr. R Merton and Mr. Scholes. I used to think why Black Scholes model is taught at every university if it's worthless but after researching so much on this topic I reached other conclusions. Black Scholes model is one of the best models which can be used to learn the basics of options engineering. It helps you grasp you the core of option mechanics and that is what is required.

But the problem is that it cannot stay true forever like gravity, theory of relativity or Newton's laws of motion. With the changing environment we have to consider current situation and change models accordingly and as long as we do that we can get better, useful and more robust models with wide applicability.

By all means there are other problems with Black Scholes Model but I concentrated on only one for a reason. During my literature review I came across so many articles in which people have tried to tackle all the problems in one go and made every thing even more complex. That's why I fully concentrated on the problem of distributions in this research paper. Once all the problem areas are resolved and clarified individually then every thing can be put into perspective to make sense and produce better models.

CHAPTER SIX: REFLECTION

The objectives of the papers were very well defined from the beginning as I was very clear about what I wanted to do. I encountered some problems while putting the problem in to perspective but literature review really helped here. After reading a lot of material on Black Scholes model and the respective problem about distributions which I wanted to work on, I had more confidence and clear mind to tackle the problem which I did ultimately.

The findings of the research paper compared very well with initial expectations as I have shown with through the analysis. My expectations were that logistic distribution better reflects stock price returns as compared to log normal distribution and this has been subsequently proved with the help of data analysis in part of analysis. Apart from that I expected logistic distribution to better predict stock prices and I did this in an out of sample test.

This has been proved as well because in eight out of eleven cases considered. The only problem is that logistic distribution better predicted prices on average but in all the cases it produced results with higher standard deviation. But in my case as I have explained before this is not a bad thing because higher standard deviation means that stocks price movements are well speeded from the mean.

This means that if we use logistic distribution to predict stock price, stock prices will have a higher tendency to go to extreme values and this is one of the core things which I include in my model. And lastly Black Scholes model with logistic distribution gave a price of options less than the price given by the Black Scholes model with log normal distribution. So my last point is proved as expected that Black Scholes model with log normal distribution over values option.

So the research paper was quite well executed as planned. The main theory behind it is the Black Scholes model and I used both quantitative and qualitative analysis to reach the conclusions but mainly quantitative because of the nature of the problem. As for as the model's implementation in the business is concerned it can improve the option pricing mechanism by making the model more realistic which better reflect asset price movements. One potential problem is that the data which I took was of a period in which a financial crisis started and stock became more volatile. But I wanted to compute a current option price that's why I used current parameters and as shown in the research paper before that the best period to consider when calculating parameters is two years.

The philosophy which I used from the beginning is that when using any model use all the current relevant variable and considers current circumstances. So keeping in view current circumstances Black Scholes model with logistic distribution is a better model but it may not be a case after some time. That's why we have to keep a questioning mind and consider all the relevant information when using any model.

Apart from that all the analysis is based on a sample of London Stock Exchange. I have tried to include all kind of stocks from the exchange but the results may not hold for each and every stock. This research paper can be improved by tackling other problems of Blacks Scholes model and make a model which incorporates all the significant factors. The only purpose of just attacking normal distribution was to completely focus on this problem and get robust results but obvious improvements can be made in Black Scholes by researching on a model which improves on all its problems.

Research paper really polished my quantitative analysis skills. Our course was very quantitative but this work really took my quantitative skills to a higher level. Understanding different statistic distributions and implementing all the work in Microsoft excel also improved my application of knowledge. Overall the learning experience was excellent.

I had problems grasping quite a few articles because the mathematics used was very complex. I had problems understanding them and I think I need more mathematical training to get better grasp of them. But complex mathematical finance is getting more unpopular these days and every body supports models which are both understandable and implement able with more transparency. So I want to work with more mathematical sophistication but only to make every thing clear and simple rather than complex models produced by physics PhDs.

Derivatives are my favourite topic and next time I want to work on Option pricing model which better tackles more problems of Black Scholes model and make a model which better incorporates all the problems and give robust results. Asset price is unpredictable because of its very nature; options are kind of bets on stocks to move in a direction if it's done by speculators. Other research themes can be to check effects of speculation on option pricing, effects of recessions on options, find solutions of the assumption that markets are efficient because we know that markets are not as efficient as we think of them, analysis when short selling is not allowed and impacts on options of dramatic actions taken by authorities to save the so called system.

So under this environment when all the major Governments are looking to increase regulation derivative trading is always a risky strategy but this will be better for the whole system. China's banks, still majority-owned by the state, were discouraged from meddling in complex products. At the start of the decade, Chinese academics warned Communist party leaders that derivatives were like mirrors reflecting other mirrors, a glimpse of infinity that is decidedly bad feng shui. Like Japanese banks during the take-off phase, Chinese banks have largely stuck to the dull task of recycling retail deposits into corporate loans.(Pilling,2009) This decision made Chinese Banks now the biggest banks according to market capitalisation. So care must be taken in using these instruments.

So I want to keep these themes in mind for my next work. I want to end the research paper by remembering the greatest crash of all times in finance Lehman Brothers. Government of America tried to tackle the problem of moral hazard by failing Lehman brothers but by doing this they made it very clear that how destructive this kind of event can be and thus now we are all sure that this kind of mistake will never be repeated. This has clarified to every body that yes the banks are too big to fail and will not be allowed to fail. Government of America tried to get rid of moral hazard but in due course made the problem even worse.

I just mentioned this because Lehman failure was allowed to deter others but it resulted in a different outcome. So before thinking about any kind of solution all the aspects of the solution should be kept in view in order to reach a robust solution. That’s how I want to focus on my next work.

Appendix 1

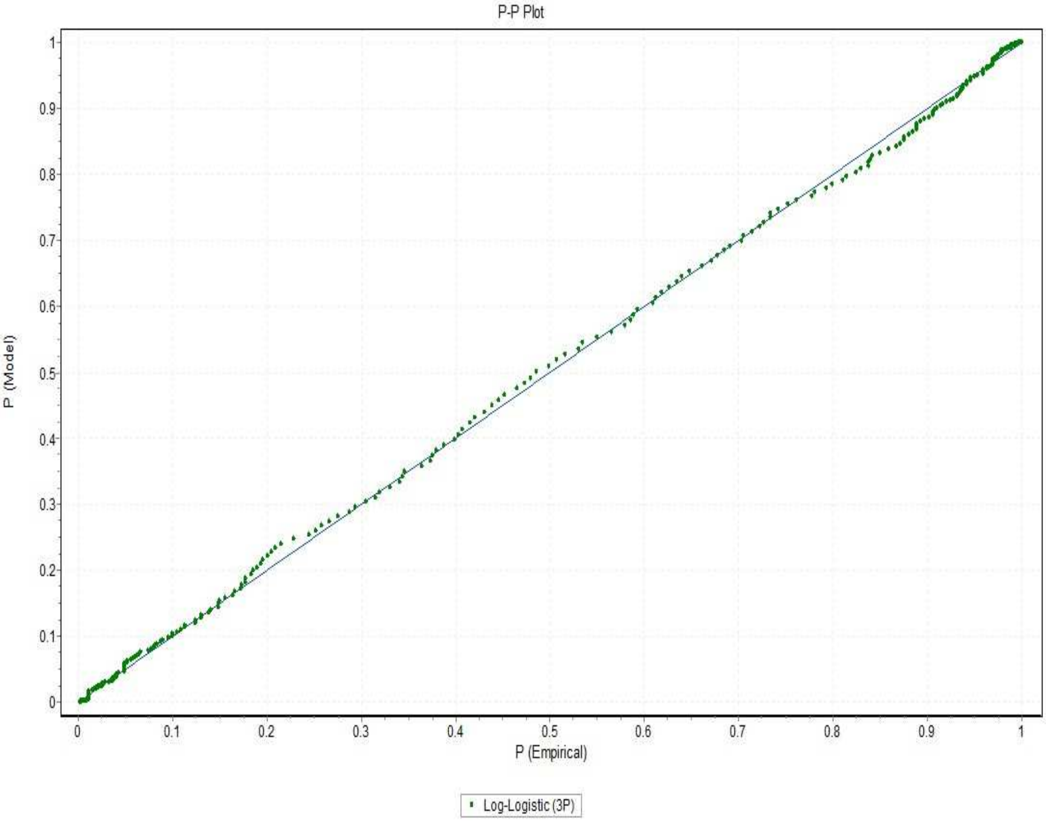
Stocks used in the research paper.

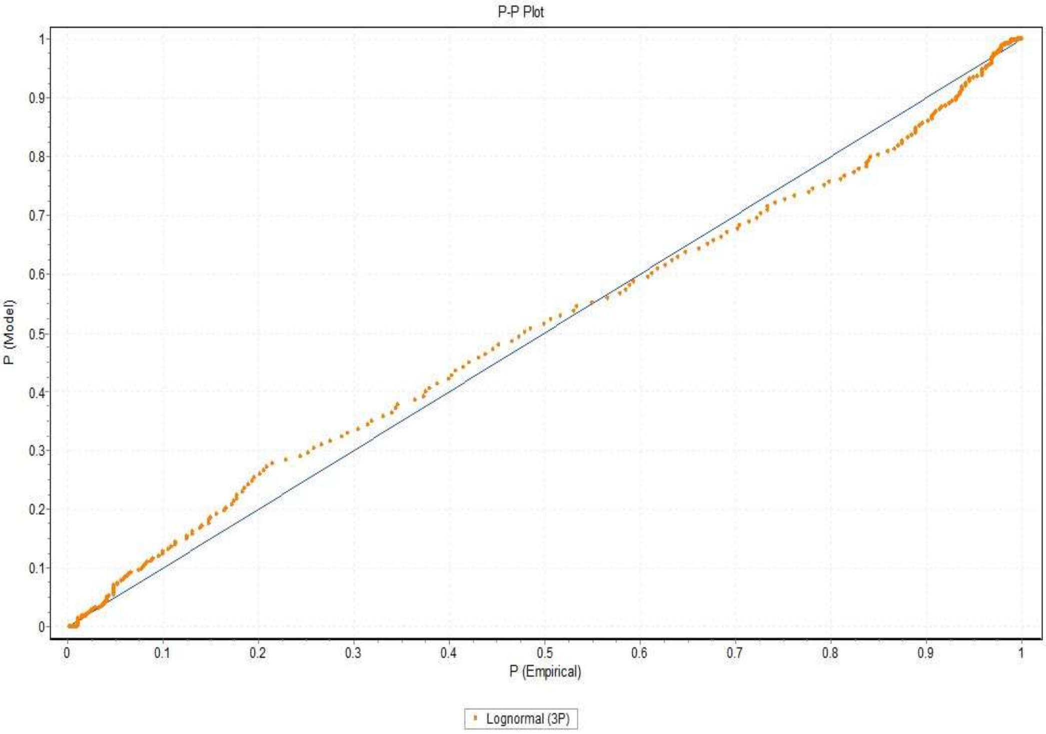
Company	Ticker Symbol
Anglo American	AAL.L
BAE Systems	BA.L
Cadberry	CBRY.L
Glaxosmithkline	GSK.L
HSBC	HSBA.L
Imperial Tobbaco	IMT.L

King Fisher Group	KGF.L
Morrisons Group	MRW.L
Royal Bank of Scotland	RBS.L
Rolls Royce	RR.L
Wpp	WPP.L

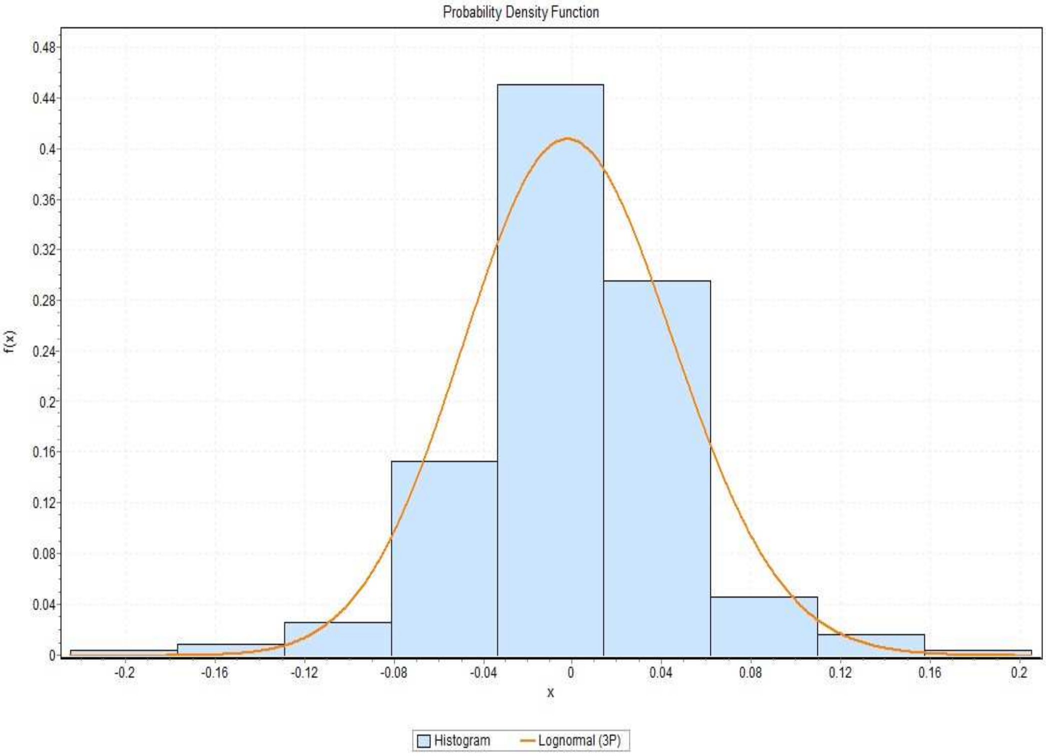
Appendix 2

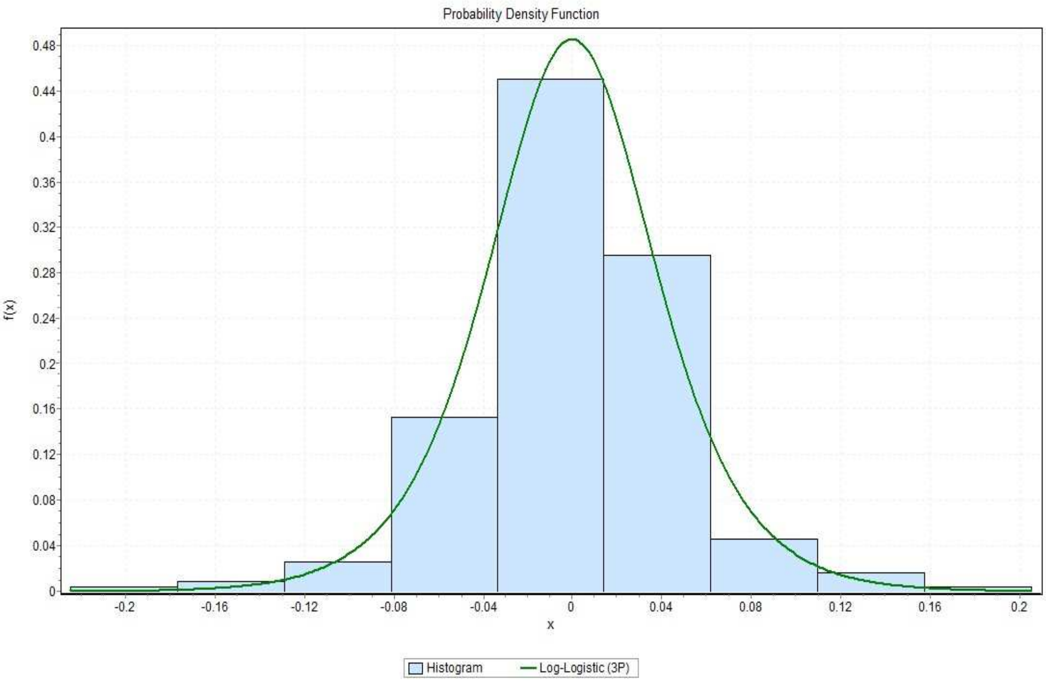
Results of Anglo American





PROBABILITY DENSITY FUNCTIONS OF LOGNORMAL AND LOG LOGISTIC DISTRIBUTION





RESULTS OF TEST OF FIT OF DISTRIBUTION

Goodness of Fit - Summary

#	Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Log-Logistic (3P)	0.02661	1	0.41173	1	9.1176	1
2	Logistic	0.03675	2	0.86982	2	9.297	2
3	Lognormal (3P)	0.06334	3	3.1984	3	23.21	3

Fitting Results

#	Distribution	Parameters
1	Log-Logistic (3P)	α=9.1805E+7 β=2.2607E+6 γ=-2.2607E+6
2	Logistic	σ=0.02575 μ=-3.8201E-4
3	Lognormal (3P)	σ=0.02815 μ=0.50938 γ=-1.6654
4	Log-Logistic	No fit (data min < 0)
5	Lognormal	No fit (data min < 0)

Appendix 3

Anglo American

Results and Findings

Paramaters Based on Historical data					
Mean	-0.50373				
Sigma	0.695903				
T	0.004				
Initial Stock Price	1001				
Results Based on Normal Distribution					
Period 01/03/ 2009 to 28/08/ 2009					
Average Price		755.783	Actual Average Price		1589.834
STDEV		114.6559	Actual STDEV		293.266
Actual Closing after 6 Months			2039.51		
Projected Closing Price after 6 months			909.3916		
Standard Deviation			482.2112		
Actual Closing after 2 months			1484		
Projected Closing Price after 2 months			973.2383		
Standard Deviation			246.1335		
Results Based on Logistic Distribution					

Period 01/03/2009 to 28/08/2009					
Average Price		757.0281	Actual Average Price		1589.834
STDEV		121.6025	Actual STDEV		293.266
Actual Closing after 6 Months			2039.51		
Projected Closing Price after 6 months			997.8773		
Standard Deviation			965.771		
Actual Closing after 2 months			1484		
Projected Closing Price after 2 months			1083.972		
Standard Deviation			594.833		

Appendix 4

Options prices Using Black Scholes Model with historical data

Black-Scholes	
S	**914**
X	**1905**
T	**0.25**
r	**8.00%**
v	**69.6%**
d1	**-1.8792**
d2	**-2.2272**
call value	**3.3033**
put value	**956.5818**

	3.30333658

Appendix 5

Comparison of option Valuation with Different Distributions

Anglo American

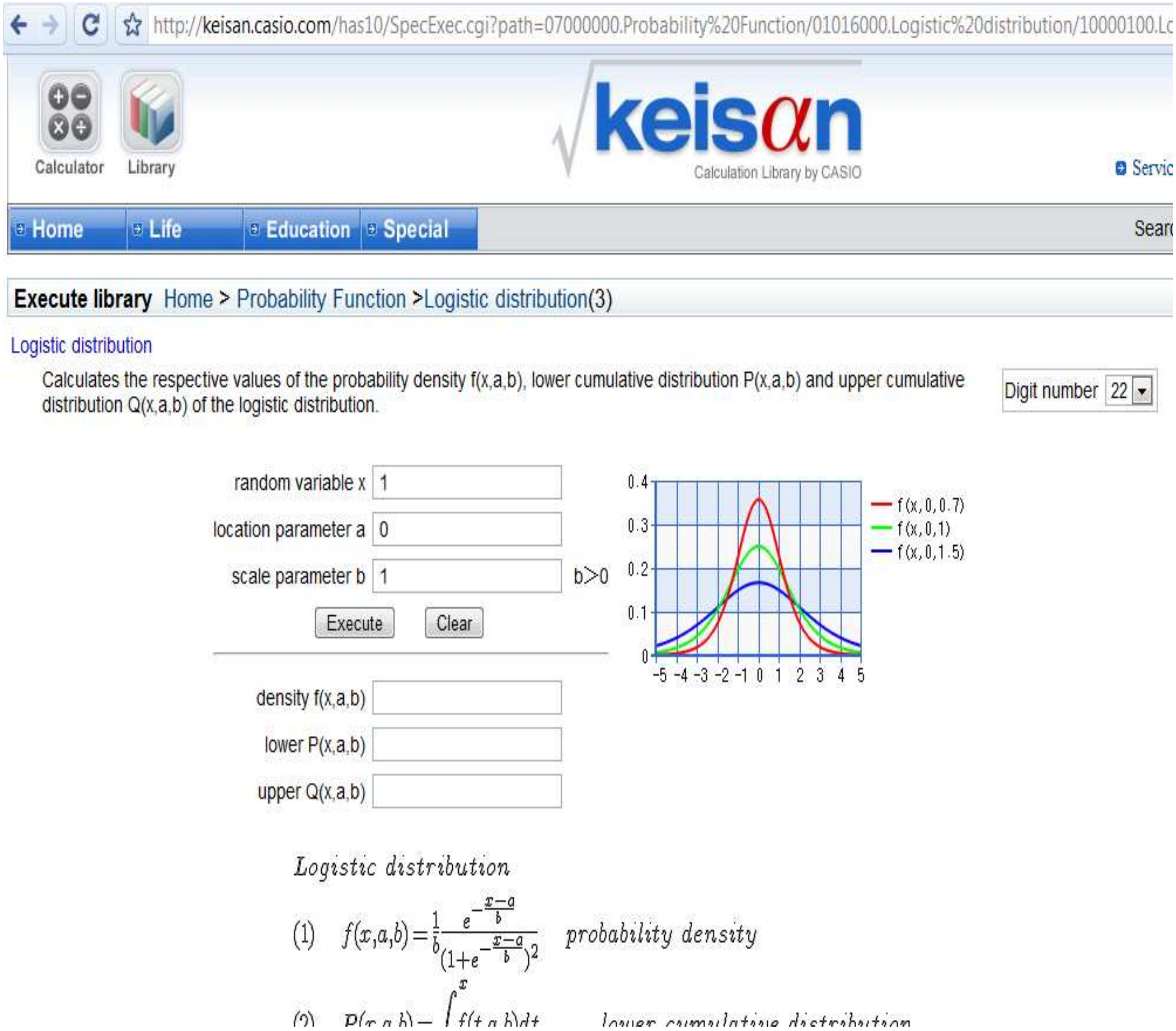
Black-Scholes Directly in a Excel Sheet ("keep it simple stupid"), by Espen Gaarder Haug							
S	**914**						
X	**1910**						
T	**0.25**						
r	**8.00%**						

v	**69.6%**						
d1	**-1.8867**		Normsdist (d1)	0.02959782		Logistic (d1)	0.1324808
d2	**-2.2347**		Normsdist (d2)	0.01271887		Logistic (d2)	0.0973344
Call value Normal Distribution	**3.2404**		Call Value Based on Logistic Distribution			-61.139972	

BAE Systems

Black-Scholes Directly in a Excel Sheet ("keep it simple stupid"), by Espen Gaarder Haug							
S	356.75						
X	**340**						
T	**0.25**						
r	**8.00%**						
v	**36.0%**						
d1	**0.4680**		Normsdist (d1)	0.6801145		Logistic (d1)	0.61491027
d2	**0.2879**		Normsdist (d2)	0.61327242		Logistic (d2)	0.57148192
Call value Normal Distribution	**38.2471**		Call Value Based on Logistic Distribution			28.9128601	

Screen shot of Keisan Calculation library by Casio


http://keisan.casio.com/has10/SpecExec.cgi?path=07000000.Probability%20Function/01016000.Logistic%20distribution/10000100.Lo
Calculator
Library
keisan
Calculation Library by CASIO
Servic
Home
Life
Education
Special
Searc
Execute library Home > Probability Function >Logistic distribution(3)
Logistic distribution
Calculates the respective values of the probability density f(x,a,b), lower cumulative distribution P(x,a,b) and upper cumulative distribution Q(x,a,b) of the logistic distribution.
Digit number 22
random variable x 1
location parameter a 0
scale parameter b 1
b>0
Execute
Clear
density f(x,a,b)
lower P(x,a,b)
upper Q(x,a,b)
f(x,0,0.7)
f(x,0,1)
f(x,0,1.5)
Logistic distribution
(1) $f(x,a,b)=\frac{1}{b}\frac{e^{-\frac{x-a}{b}}}{(1+e^{-\frac{x-a}{b}})^2}$ probability density