The Conflict Between General Equilibrium and the Marshallian Cross

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Abstract. There is a conflict in the mechanism for price determination used in a Marshallian partial equilibrium supply and demand framework and the Walrasian general equilibrium framework. It is generally thought that partial equilibrium is a simplified approximation to the complexities of the general model. The goal of this paper is to show that there is a strong conflict between the two models – intuitions and heuristics suggested by partial equilibrium are contradicted by extensions to the general equilibrium case. We review the literature on the conflict and also provide a very simple model where partial equilibrium analysis fails completely. Several intuitively plausible remedies fail to restore partial equilibrium results. We show that Marshallian analysis can be made to work only under rather stringent conditions requiring joint production with low fixed costs and decreasing returns resulting in identical production proportions by all producers.

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Introduction

Today most economic analysis as well as the majority of elementary textbooks explain price determinations via the partial equilibrium method, which gained its distinction with the publication of the first edition of *Principles of Economics* in 1890 by Alfred Marshall. It is widely believed that this is a pedagogically convenient simplification in harmony with the deeper and more complex general equilibrium analysis, first proposed by Leon Walras in his *Elements of Pure Economics* (1874). However, Piero Sraffa in two seminal papers, one published in 1925 in *Annali* and the other in 1926 in *Economic Journal* showed that a critical condition for perfect competition is violated if production costs (and consequently economic returns) change with output. In a diminishing returns industry using only a part of a factor of production that is in limited supply, an expansion in the output increases the price of this particular factor, this will impact on all industries using the same factor as input in their production. Sraffa (1926: 539) argued that "since commodities using a common special factor of production are often substitutes for one another (for example, various kinds of agricultural produce), the modification in their price will not be without appreciable effects upon demand in the industry concerned." This means that the supply and demand curves cannot be conveniently separated and studied in isolation, as is required for a Marshallian analysis. The goal of this paper is to show, via a simple model, that this incompatibility is fundamental, and cannot be easily removed. Thus, Marshallian

supply and demand analysis can only be an exceptional special case, and price determination in typical markets cannot be analyzed via partial equilibrium analysis. Below, we first provide some historical background of the debate regarding partial equilibrium analysis introduced by Marshall.

Sraffa's analysis shows that the conditions of perfect conditions assumed as necessary background for Marshall's partial equilibrium require very special types of cost structures within an industry. Incompatibility with perfect competition can occur under increasing returns resulting from *internal* or *external* economies. Internal economies due to an increase in the scale of production lead to a monopolistic industry. Monopolies violate the price taking behavior assumed for derivation of a supply curve in the Marshallian analysis. Similarly, external economies due to the general economic progress are unlikely to result from a small increase in the scale of output of a single industry, hence they are inconsistent with a stable supply curve in a partial equilibrium. Thus, environments that are compatible with partial equilibrium analysis under increasing returns only involve a thin class of economies that are "external from the point of view of the individual firm, but internal as regards the industry in its aggregate" (Sraffa 1926: 540). On the other hand, under diminishing returns partial equilibrium analysis will be valid only for those commodities in the production of which the whole of a factor is used. If a factor of production is shared with other industries, then changes in its cost will impact on supply of alternatives, and hence on

the demand, as argued earlier. Given these extremely restrictive results, Sraffa argued that "the supply schedule with variable costs cannot claim to be a general conception applicable to normal industries" (Sraffa 1926: 540) and "in normal cases the cost of production of commodities produced competitively [...] must be regarded as constant in respect of small variations in the quantity produced" (Sraffa 1926: 541).

This conclusion of Sraffa that constant costs define the whole set of relevant environments compatible with Marshallian competitive equilibrium was criticized by Samuelson (1987: 458-9) on the basis that it was produced by an ideological bias towards the classical cost-based price theories where demand plays no role. But, Sraffa (1926) did not deny the possibility of industries with non-constant supply curves and proposed two remedies for the problems with Marshallian analysis, one in terms of a simultaneous determination of interdependent prices in a general equilibrium system and the other in terms of the monopolistic competition.

The controversy generated by Sraffa's critique reached a peak during the *Symposium on Increasing Returns and the Representative Firm* published in March 1930 by *the Economic Journal*, where he concluded his reactions to the criticisms of Dennis Robertson to his earlier paper in 1926 by saying that "in the circumstances, I think it is Marshall's theory that should be discarded" (Sraffa 1930: 93). Consistent with this extreme view, which was found to be "negative and destructive" by Keynes in his Editorial opening of the Symposium, Sraffa ignored the route on monopolistic com-

petition he proposed to correct Marshallian equilibrium analysis.[3] Indeed, after his *Economic Journal* (1926) article, Sraffa was mostly interested in constructing a general equilibrium analysis of competitive markets and eventually published in 1960 his *Production of Commodities by Means of Commodities*, in which he studied - using the classical approach of Ricardo - the problems of value and distribution as well as issues such as joint production and switch in methods of production.

While some Cambridge economists, including Dennis Robertson and Gerald Shove were highly critical of Sraffa's early contribution, Arthur Pigou soon came to agree with one of the conclusions of Sraffa (1926) by arguing that "it is impossible for production anywhere to take place under conditions of increasing costs" (Pigou 1927: 193). However, Pigou did not assume away the possibility of external economies; thus according to him "only the laws of constant or decreasing supply price, as so conceived, are admissible" (Pigou 1928: 256). Consequently, he assumed an industry supply curve in the form of a rectangular hyperbola. Studying the same problem, Viner (1931) allowed for the possibility of positively sloped industry supply curves and U-shaped industry average cost curves, by assuming external economies as well as diseconomies pecuniary

[3]Shortly after Sraffa's (1926) proposal, this route of research was pursued by two American economists Edward Chamberlin, who is known to be influenced by the early work of Arthur Pigou, and Robert Triffin as well as a young generation of Cambridge economists including Austin Robinson, Joan Robinson, and Richard Kahn, among many others.

to the changes in the prices of services and materials. He also distinguished between short-run and long-run in developing the cost function of a single firm.

Sraffa showed that only special kinds of industry cost structures are compatible with perfect competition and Marshallian supply and demand. The constant costs required create the problem that aggregate supply by the industry as well as firm size become indeterminate, as noted by Pigou (1928) and Viner (1931). Austin Robinson's coordination failure model and the factor proportions model of Joan Robinson provided potential solutions to these problems. Austin Robinson (1931) suggested that diseconomies of scale caused by managerial, marketing, and risk and fluctuation forces would determine an optimal firm size beyond which firms would not grow. Joan Robinson (1941) showed that individual firms facing infinitely elastic supplies of inputs would have constant returns to scale, while diseconomies would arise at the level of the industry when aggregate supplies of inputs were imperfectly elastic. This potentially allows for a determinate aggregate supply function for the industry as a whole.

A milestone in this route of research is Stigler's (1942) textbook entitled *The Theory of Competitive Price* that integrated the previous works of Viner, Austin Robinson and Joan Robinson, constructing a new Marshallian theory of cost and supply curves. Among his several contributions to the price theory, Stigler (1942) distinguished between the short-run and long-run in which the size of the plant is fixed and variable respectively and showed that the long-run cost curves can be obtained as an enve-

lope of the short-run cost curves. Since Stigler (1942) assumed - like Viner (1931) - that the short-run cost curves are U-shaped, he found - using the coordination failure argument of Austin Robinson (1931) - that the induced long-run cost curves for the firm are also U-shaped. Studying the equilibrium of a competitive industry, Stigler (1942) then proposed that a firm would be able to operate in the long run only if it produced an output consistent with the minimum point on its long-run cost curve. Like Viner (1931), he assumed for the firm an upward-sloping short-run marginal cost curve, inducing an upward-sloping supply curve, and argued that scale diseconomies or coordination failure would not lead to rising industry costs, which according to him could only arise due to an expansion of the industry by the entry of new firms. Assuming that factors of production used by the industry are heterogenous and then using the factor proportions argument of Joan Robinson (1941), Stigler (1942) further proposed that such an expansion in the number of firms would result in diseconomies, leading to rising industry costs and supply curves.

A prompt challenge to this new theory "was developed during the 1930s by London School of Economics (LSE) scholars, such as Robbins, Kaldor, Hicks, and Allen, and was later refined by Samuelson" (Steedman 2008: 247). This challenge determined output of a commodity only as a function of its price assuming all other prices constant, opposing the earlier definition of the supply curve in the Marshallian tradition where "a movement along the supply curve involves a change in many economic variables,

such as input prices and other product prices" (Opocher and Steedman 2008a: 247). Thus, the supply curve of an individual firm was defined in the LSE approach as a symmetric counterpart of the Marshallian demand curve of an individual consumer. On the other hand, the industry supply curve was obtained by the aggregation of individual firm supply curves. While the LSE approach led to the conventional short-run supply curves of the firm we see in current microeconomics textbooks, it did not solve the main problems in the Marshallian tradition, such as the indeterminacy of long-run equilibrium, the incoherence of comparative statics in long-run equilibrium, and the presence of supply-side interdependencies via changes in factor prices when the industry expands due to entry of new firms.

After the perceived perfection of the competition theory by Stigler's (1942) synthesis, Sraffian (or neo-Ricardian) economists at Cambridge, England involved in 1950s and 1960s in a (Cambridge) capital controversy with neoclassical economists at (MIT) Cambridge, Massachusetts as to the nature of capital as a means of production in the aggregate production function. While MIT economists claimed that capital can move as an input seeking the highest return from one production technique to another in a competitive market, economists in the other side of the Atlantic argued against it using the *re-switching* argument of Sraffa (1960) that a production technique may be cost minimizing for both low and high rates of profits. Although this controversy has never been settled, "when theories of endogenous growth and real business cycles took

off in the 1980s using aggregate production functions, contributors usually wrote as if the controversies had never occurred and the Cambridge, England contributors had never existed" (Cohen and Harcourt, 2003). Absolutely, the same criticism can also be made of Stigler's (1942) synthesis of perfect competition theory, a significant part of which has been in use for the last 70 years. The reason is that this synthesis "fails to respond to Sraffa's criticism: It makes the demand for and the supply of industry output interdependent" (Aslanbeigui and Naples 1997: 528), since upward sloping supply curves could arise only if the supplies of inputs were imperfectly elastic, implying that an increase in the demand for a commodity could not be met with decreased supply of other commodities in the economy.

Despite economists' general ignorance of Sraffian challenge to the Marshallian competitive analysis, a number of studies have in the last three decades attempted to revive the debate. Importantly, a partial equilibrium model studied by Steedman (1988: 95) showed that "the interdependence between industries which is implied by the existence of produced means of production, which was so strongly emphasised by Sraffa in 1960, is quite sufficient to produce 'collateral effects' which undermine the results of partial analysis." In a related study, Ozanne (1996), using simulations based on an econometric model of UK agriculture between 1978-1982, showed that the existence of produced inputs in the production process may yield, due to the implied *collateral effects*, in-

dustry supply curves that are downward sloping.[4] In the same literature, Opocher and Steedman (2008b), using a theoretical model, established that whether a long-run industry supply curve is upward or downward sloping depends on the numéraire in terms of which the price of the output is measured.

While most of the related literature since Sraffa (1925, 1926) has focused on the incompatibility of Marshallian supply curves and general equilibrium supply curves under perfect competition, a recent strand of papers, involving Vives (1987, 1999), Miyake (2006), and Hayashi (2009), studied the conditions for the set of commodities or utility functions under which a Marshallian demand function is well behaved and the partial equilibrium analysis is applicable.

Pursuing the same issue of applicability from a similar angle, we consider in our

[4]The pioneers of a handful of empirical studies supporting Sraffian challenge date back to the late 1930s. While economists in Cambridge, England were dealing with solving the deficiencies of Marshallian analysis of perfect competition during 1930s, a group of economists in Oxford headed by Sir Huber Henderson pursued a series of empirical research between 1938-1940 based on surveys with businessmen, reaching the conclusion "that the Marshallian framework was inconsistent with the empirical evidence" (Lee 1981: 339). In more detail, this research showed that price decisions of businessmen did not depend upon the state of demand whereas the effect of the interest rate on their investment decisions was negligible (Lee 1981: 340). The definite fruit of this research was the development of "full cost pricing and the kinked demand curve" (Lee 1981: 349), but unfortunately the criticisms of the research group as to the Marshallian cross has not received due consideration by academic scholars.

paper a two-good production economy where individuals who face both production and consumption decisions have demand schedules that are dependent on supply schedules. In this simple environment, we aim to identify restrictions on the economic domain that will ensure the coherence of the partial equilibrium analysis. Our results show that many restrictions one can meaningfully propose to this end are ineffective. Interestingly, we show that the conflict between the partial and general equilibrium analysis can be resolved in environments with decreasing returns if the joint production possibility is present and fixed costs of joint production are sufficiently small.

The organization of the paper is as follows: In the next section, we introduce the model and the problems with the Marshallian competitive analysis. After that, we provide some explanations for the incoherence of the Marshallian thought experiment. Then, we discuss several conjectures which fail to eliminate this incoherence and give a remedy. Next, we give a brief account of how partial equilibrium analysis is accommodated in current reputable microeconomics textbooks. Finally, we present some concluding remarks.

Model

We will consider a simple example of a market in which there is no equilibrium in the sense that no configuration of prices will lead to a stable configuration of decentralized

decisions. Our model allows the sole factor of production, land, to shift across industries. The existence of this possibility makes partial equilibrium analysis impossible, as we demonstrate.

Suppose that there are 100 farmers each of whom owns a single plot of land of equal size which is capable of growing either one unit of rice (R) or one unit of wheat (W) but not both. Let us assume for simplicity that production takes place at zero fixed cost. Each of these producers is also a consumer of W or R or both and has the utility function $U(W, R) = WR$, satisfying standard assumptions.

We will now try to calculate the aggregate demand and supply functions in this economy. Let us take wheat as the numéraire, and set the price of wheat to one. We will use p to denote the price of rice. The aggregate supply function of rice is straightforwardly given by

$$S^R(p) \in \begin{cases} \{0\} & \text{if } p < 1, \\ [0, 100] & \text{if } p = 1, \\ \{100\} & \text{if } p > 1. \end{cases} \tag{1}$$

If $p > 1$, then all farmers will produce rice, while if $p < 1$, then all will produce wheat. If $p = 1$, then the farmers will have equal profits from producing either rice or wheat.

Nonexistence of Marshallian Market Demand Function

Unlike the supply function, the thought experiment which defines the demand curve is not clear in this example. We contemplate changing the price of rice and asking consumers how much rice they would consume. However, the *ceteris paribus* assumption cannot be fulfilled in this example.[5] Since a representative consumer is also a producer, changes in the price of rice will affect his production decision and hence his income. So, we consider the following two cases separately:

A wheat farmer is somehow committed to producing wheat. In this case, his income is one, and his budget constraint is $W + pR = 1$. Subject to this budget constraint, the utility function $U(W, R)$ is maximized at $W^* = 1/2$, and $R^* = 1/(2p)$.

A rice farmer is similarly committed to producing rice. In this case, his budget constraint is $W + pR = p$, since his income is p - the thought experiment in which the price of rice is altered cannot keep the income of the rice farmer constant. Maximizing utility subject to this constraint, we find that $R^* = 1/2$ and $W^* = p/2$.

What is the Marshallian aggregate demand function? This question cannot be answered in this model, because the number of wheat and rice farmers is itself endogenous. Nonetheless, let us fix the quantity of wheat and rice farmers; perhaps at

[5]This is a general problem with partial equilibrium analysis. Lee and Keen (2004: 188) write that "The existence of collateral effects invalidates the ceteris paribus, partial equilibrium methodology [...]". In their footnote 29, they provide additional reference supporting theoretical, empirical and methodological evidence against the law of demand.

historical levels. Let N_W be the number of wheat farmers and N_R be the quantity of rice farmers. The demand for rice by each wheat farmer is $1/(2p)$, so the aggregate demand will be $N_W/(2p)$. On the other hand, for each rice farmer the demand for rice is $1/2$, pure and simple; so the total aggregate demand for rice will be

$$D^R(p) = \frac{N_R}{2} + \frac{N_W}{2p}. \tag{2}$$

Using this hypothetic demand, we will now attempt to find a partial competitive equilibrium.

A Symmetric Equilibrium

Because of the model's complete symmetry in R and W, a natural equilibrium with many demonstrable optimality properties is one in which there are 50 rice farmers and 50 wheat farmers, the two products have the same price and every farmer consumes $1/2$ units of both wheat and rice. If $N_R = 50$ and $N_W = 50$, this desirable symmetric equilibrium somehow emerges at a Marshallian cross at which the demand curve for rice, obtained using (2), is given by the function $D^R(p) = 25 + 25/p$ for any $p > 0$, intersecting the supply curve represented by (1) at $p^* = 1$ and $S^R(p^*) = 50$.

We want to know if this historical equilibrium will perpetuate itself. We announce to the world that this is the best equilibrium - there will be 50 rice farmers and 50 wheat farmers and prices of the two products will be equal. Let every farmer maximize his

profit. Then every farmer is indifferent between growing rice and wheat and therefore we cannot predict what output will emerge. That is, announcing the equilibrium prices does not decentralize production decisions in the way that economic theory predicts. Because the production decision is arbitrary, 70 farmers may decide to grow wheat and 30 to grow rice. Any set of decisions is ex-ante compatible with profit maximization. There is no way to decentralize the production decisions and produce 50 units of wheat and 50 of rice. If some arbitrary quantities are produced, like (70,30), then rice will be more expensive and wheat less so. The wheat producers will suffer welfare loss, while the rice farmers will gain. At any non-symmetric equilibrium, the gainers are fewer than the losers, so that a 'democratic' equilibrium would be symmetric.

What will happen in the next period? If farmers are naïve and believe that these prices will continue, then they will all plant rice. This will lead to severely problematic equilibria. Some partial and probabilistic adjustment mechanisms would lead towards (50,50), the optimal equilibrium; but it can never be achieved and price fluctuations will persist since there is no decentralized way to arrive at the symmetric equilibrium. Obviously, assuming rational expectations on the part of farmers would not help matters, either.

The knife-edge equilibrium we have described above ties in and illustrates the contention of Aslanbeigui and Naples (1997) that under conditions required for demand and supply to exist, firm outputs are unstable, although the model, and the reasons

for the fluctuations are different.

An Incoherent Thought Experiment

At some level, it is clear that the general equilibrium story conflicts with the Marshallian supply and demand. General equilibrium theory tells us that all markets are in general interdependent, while Marshallian models seek to explain equilibrium in one market without reference to what is happening in others. The above model of ours shows that the conflict is much more dramatic than is realized. The concepts used in a Marshallian model cannot even be defined within a general equilibrium framework. One cannot explain equilibrium price formation within a market in isolation. Since all our elementary textbooks rely on the Marshallian framework, and our intuitions about equilibrium price and quantities are shaped by these textbooks, some radical rethinking is required. The problem with the standard textbook argument is hidden in the income "Y". Our elementary micro models do not have money. Thus we cannot consider a thought experiment in which we fix the income in nominal terms and ask consumers what they would purchase. But any notion of real income must go through prices, which are considered to be varying in constructing the demand.

Another explanation of the incoherence of the Marshallian thought experiment in more general terms is as follows. The Marshallian framework takes tastes and

technologies (represented by utility and production functions, respectively) as well as factor costs as given, and produces equilibrium prices and quantities. The demand function is based on asking the question: what would a consumer purchase if the price of the good being purchased changed? This thought experiment is incoherent, because within a Marshallian framework, a price cannot change. An endogenous variable can only change when some exogenous variable does. In a Marshallian world, if I am asked what I will do when the price changes, I must ask 'why' did the price change? Being endogenous, prices are not free to change on their own. There are three possible reasons, each of which has their own and different effects: i. prices changed because of shifts in the supply function; ii. prices changed because of shifts in the demand function; iii. there was some exogenous shock.

This last possibility is the one that is in the back of the mind of a partial equilibrium economist. However, exogenous shocks are delicate and subtle and require very careful treatment. it is not enough to talk about exogenous shocks to prices; the exact and specific nature of shock must be specified before we can ask about responses. In particular, we must know whether or not this new price being quoted at us, in order to elicit a demand, is a disequilibrium price resulting from a temporary or permanent shock. To give a good response, we will need a lot more information about the nature of the disequilibrium and also the mechanisms which come into effect following disequilibrium. If it is an equilibrium price on the other hand, then *ceteris paribus*

cannot hold, and we must know what other changes have occurred to cause this shift in equilibrium. This means that the thought experiment required to define a Marshallian demand function is internally incoherent or at least incomplete.

Several Conjectures and a Remedy

Responding to earlier drafts of this article, several economists presented conjectures about conditions which would lead to the supply and demand framework holding as an approximation in one of the submarkets in a general equilibrium economy. Five conjectures are in order:

A conjecture (due to George Judge) is that if there was a large number of goods, and the good in question was a small proportion of the total budget, then supply and demand would be good approximation. In our model above, if we have n goods, $G_1, G_2, \ldots, G_n$ any one of which can be produced by the farmers, and the utility functions are the product of all consumptions, then the example goes through exactly as before.

A second conjecture (due to Jeffrey M. Perloff) is that additively separable utility functions were required. If $U(R, W) = W + R$, then wheat and rice are in effect identical goods and all production and consumption decisions are equivalent. Except for this trivial special case, separable utilities do not help with the two problems that

production decisions are endogenous and that varying the prices leads to variations in the income.

A third conjecture (due to Debraj Ray) is that we need to separate the consumers from the producers, to get the Marshallian framework. There are several levels of separation possible, but none that we have tried could succeed in producing Marshallian supply and demand. For example, we have tried the separation where the farms are run by foreign firms which repatriate all profits to home countries (so variations in firm income do not impact on the market) and do not hire domestic labor, while consumers have their own endowments which they use to purchase rice and wheat from these foreign producers.

As a fourth conjecture, one can argue that the conflict between the partial and general equilibrium frameworks and the problem with defining a Marshallian demand curve can be resolved if consumption, instead of consumers, is completely separated from production. To disprove this claim, take a pure exchange economy with goods $x_1, x_2, \ldots, x_k$. Suppose good one is numéraire and let $p_2, \ldots, p_k$ be the price of the others. Let $e(j) = \langle e_1(j), ..., e_k(j) \rangle$ be the endowment bundle of the jth consumer. At any vector of prices $p = \langle 1, p_2, ..., p_k \rangle$, let $D(j, p)$ be the vector of demand of the jth consumer. Consider the problem of defining a Marshallian demand curve for the second good.

In the first instance, this can be done by varying p_2, keeping all other prices fixed.

Then we may have a well defined demand function for good 2, but it does not represent the concept of Marshallian demand since incomes of all consumers would vary as p_2 is changed.

Alternatively, we may try the following. Let $I(j) = \sum_{i=1}^{k} p_i^* e_i(j)$ be the income of the consumer j at some fixed vector of prices p^*. Now ignore this origin of income and fix this income as a number for each consumer. Ask consumer j to maximize utility derived from the consumption vector $\langle x_1(j), x_2(j), \ldots, x_k(j)\rangle$ subject to the budget constraint $\sum_{i=1}^{k} p_i x_i(j) \leq I(j)$. This should be the Marshallian demand. The demand function is now well-defined, but does not have anything to do with consumer j's behavior because it artificially fixed the value of his endowment vector $e(j)$ at p^*.

One can finally argue that the incoherence of Marshallian competitive analysis would vanish if the producers did not face constant returns to scale. To check this last conjecture, we assume that each producer has a divisible labor endowment of one unit as the single input of production, and a decreasing returns to scale technology[6] represented by the production function f that converts any labor input $L > 0$ to the output $f(L) > 0$. We assume that $f'(L) > 0$ and $f''(L) < 0$ for all $L > 0$, and $f'(0) = \infty$. Let $K \geq 0$ denote the fixed costs of producing any of the two goods separately. We further assume that when joint production possibility is present, the

[6]We simply disregard the case of increasing returns to scale in which the general equilibrium usually does not exist.

fixed costs of producing wheat and rice jointly is $K' \geq 0$.

We are now ready to present a positive result.

Proposition 1: *The conflict between the partial and general equilibrium vanishes in the described production economy with decreasing returns under sufficiently small values of fixed costs of joint production*[7] *if the following two conditions both hold: i) the joint production possibility is present, ii) the joint production of commodities is desirable at the market clearing price in the absence of any fixed costs.*

Proof. Suppose the conditions in the hypothesis of the proposition hold. Then each farmer will, by condition (i), jointly produce wheat and rice at any given price $p > 0$ if and only if

$$\begin{aligned} &\max_{L\in[0,1]}\{pf(L) + f(1-L) - K'\} = pf(\hat{L}(p)) + f(1-\hat{L}(p)) - K' \\ &> \max\{pf(1) - K, f(1) - K\}, \end{aligned} \tag{3}$$

where

$$pf'(\hat{L}(p)) = f'(1-\hat{L}(p)). \tag{4}$$

At any price $p > 0$ where wheat and rice are jointly produced, the market supply of

[7] In situations, where fixed costs are doubled under joint production ($K' = 2K$), Proposition 1 would be valid when fixed costs of producing any good separately are sufficiently low.

rice is

$$S^R(p) = 100f(\hat{L}(p)), \tag{5}$$

whereas the market demand for rice is

$$D^R(p) = 50f(\hat{L}(p)) + \frac{50f(1-\hat{L}(p))}{p}, \tag{6}$$

by equation (2) and the fact that $N_W = 100f(1-\hat{L}(p))$.

Obviously, $S^R(.)$ is an upward sloping schedule by equation (4) and the assumption that f is strictly concave. On the other hand, $D^R(.)$ is downward sloping since

$$\frac{dD^R(p)}{dp} = -\frac{50f(1-\hat{L}(p))}{p^2}, \tag{7}$$

using equation (4). Then, the unique price $\hat{p}$ that clears the market for rice satisfies

$$\hat{p}f(\hat{L}(\hat{p})) = f(1-\hat{L}(\hat{p})), \tag{8}$$

which is obtained by equating (4) and (5).

By condition (ii), inequality (3) is satisfied at $\hat{p}$ under the fixed costs $(K', K) = (0, 0)$; therefore we have $\hat{p}f(\hat{L}(\hat{p})) + f(1-\hat{L}(\hat{p})) > \max\{\hat{p}f(1), f(1)\}$. Now choose $K' = \hat{K} \geq 0$ such that

$$\hat{K} - K < \hat{p}f(\hat{L}(\hat{p})) + f(1-\hat{L}(\hat{p})) - \max\{\hat{p}f(1), f(1)\}. \tag{9}$$

Thus, the inequality (3) is satisfied at $\hat{p}$ under the fixed costs $(\hat{K}, K)$ of joint and separate production of the two goods. □

While the presence of decreasing returns and small fixed costs of joint production does not cure the interdependency of the demand with the supply, it eliminates the discontinuity of the supply curve around the equilibrium price, rendering the conventional Marshallian cross, defined by an upward sloping supply curve (eqn. 5) and a downward sloping demand curve (eqn. 6), stable around the equilibrium price. Heuristically, we can argue that joint production joins the two industries into one, and solves the problem of Sraffa by eliminating the possibility of shifting factors of production from one industry to the other.

Partial Equilibrium Analysis in Current Microeconomic Textbooks

A comprehensive study by Lee and Keen (2004) examined the coherence of neoclassical microeconomic theory in 74 textbooks in the undergraduate or graduate level focusing on many key subjects involving choices, preferences, utility functions, demand curves, production, costs, factor input demand functions, partial equilibrium, perfect competition, and the supply curve. This study showed that the generality of the examined textbooks handle these subjects incoherently or lack adequate empirical support. For example, while the short or long run market supply is derived in all textbooks by the horizontal aggregation of the individual firm supply curves, the underlying conditions

for successful aggregation, such as homothetic production functions and equality of the input prices for the same factor input, are not mentioned. Neither, there exists a reference to perverse supply curves when the output is among the factor inputs or to collateral-effects the upward sloping market supply curve generates when production is carried out by produced means of production. The conclusion of Lee and Keen (2004: 190) upon these observations was that "possible problems with consistent and representational aggregation, perverse outcomes, and violation of the partial equilibrium methodology clearly suggest that the market supply curve (both short and long run) is a unsustainable theoretical concept" in microeconomic textbooks.[8]

Current textbook treatments of supply and demand based on Stigler and Samuelson have been strongly shaped by the debate introduced by Sraffa. Nonetheless, for the most part, textbooks show very little awareness of this historical background, and of the existing critiques of supply and demand framework. Below, we provide further details, supporting the previous conclusions of Lee and Keen (2004) as to the general ignorance of the potential problems with partial equilibrium analysis, based upon our examination of a number of microeconomics textbooks assigned as primary class material in many prestigious undergraduate or graduate programs in Economics[9].

[8] A similar conclusion as to the accommodation of supply curves in current textbooks is reached by Opocher and Steedman (2008a: 271) who argue that "the analytical foundations of the long-run curves are not self-evident".

[9] As anecdotal evidence, we may add that both authors went through graduate education without

Of the three most reputable textbooks used in graduate programs, Varian (1992: 313) contains a single short paragraph to address the partial equilibrium model. The textbook that devotes the lengthiest discussion on the partial equilibrium framework is Mas-Colell et al (1995), where pages 316-43, nearly the whole tenth chapter, study competitive and welfare analysis under partial equilibrium, while pages 538-40 discuss using the tax incidence example of Bradford (1978) "how a naïve application of partial equilibrium analysis leads us seriously astray" Mas-Colell et al (1995: 538). Although the book emphasizes in a number of places the significance of the assumed absence of wealth effects and substitution effects for the validity of welfare analysis and competitive analysis respectively, it contains no explicit account of the potential problems due to supply side interdependencies. A brief, yet more balanced, discussion is available in Kreps (1990), where a subsection (in pages 279-83) of the chapter that introduces partial equilibrium analysis of perfect competition addresses two main problems with partial equilibrium, namely the interdependencies between the demand curves of goods that are substitutes and the interdependencies of the supply curves of goods using a common factor of production.

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receiving any hint of problems with or even any limitations on supply and demand framework – we were taught to think that it was universally applicable to all markets. Informal surveys of recent graduate students also show that they believe that price determination can be analyzed by breaking up the economy into separate markets for each good, where the price of each good is determined in its own separate market in isolation via forces of supply and demand.

As the above discussion shows, some authors of graduate texts do show awareness of problems with partial equilibrium supply and demand models. However, all of them do so on the general grounds that this ignores potential interactions with other markets, and not that there is a conflict between the PE story of price determination and the GE version of the same story.

The ignorance of the problems with Marshallian competitive theory is significantly more evident in undergraduate textbooks. For example, in Mansfield (1988: 439), Pindyck and Rubinfeld (2001: 565) and Varian (2002: 540) the reference to the partial equilibrium model is confined to a couple of sentences. Nicholson (2002: 14) addresses the distinction between the partial equilibrium and general equilibrium models, as well as demand side and supply side interdependencies that require general equilibrium analysis. Similarly, an explicit but brief discussion on "when partial equilibrium analysis will do" is available in a couple of paragraphs in Stiglitz and Walsh (2002: 216-7).

In an overall evaluation, while nearly a half of the most reputable textbooks in Microeconomics, especially those in the graduate level, address demand and supply side interdependencies that would require a general - instead of a partial - equilibrium analysis, though only briefly and without any reference to the related literature grown up in almost a century, the remaining half contains neither explicit nor implicit reference to the potential problems with Marshallian cross, rendering Microeconomic teaching

extremely biased in favour of partial equilibrium models. To give just one example of the serious misunderstandings that result, we note that several textbooks discuss the supply and demand of housing, and derive policy implications based on this partial equilibrium analysis. Rent is a substantial portion of a typical household's budget, and variations in rent would have not only have substantial income effects, they would also impact on all markets by increasing or decreasing income available. Nonetheless, none of the elementary textbooks mention potential problems with a partial equilibrium analysis, which could be seriously misleading.

Conclusions

The complexity of general equilibrium, where everything depends on everything else, does not help us in understanding price formation. Marshall followed a standard scientific strategy of dividing a complex problem into smaller components which could be understood easily in isolation and combined to yield the answer to the bigger problem. Unfortunately, the two components are not independent of each other. There are a large number of channels through which supply and demand interact with each other. This makes supply and demand analysis unreliable. A large number of misleading results from supply and demand are discussed in the final section of Aslanbeigui and Naples (1997).

Our analysis provides a concrete demonstration of the incoherence of supply and demand analysis in the context of a elementary economic model conforming to neoclassical assumptions. We show on a simple model of production economy that the concepts of Marshallian supply and demand can be in conflict with the general equilibrium, and because of certain logical contradictions, these conflicts cannot be resolved for a majority of restrictions on economic environments. We are able to obtain a compatibility result under decreasing returns and in the presence of joint production possibility provided that the fixed costs of joint production are sufficiently small. Looking from the opposite angle, it is interesting to note that the failure of equilibrium problem studied in this paper does not hinge upon the type of production technology that farmers employ, if the joint production possibility is absent or undesirable or if the fixed costs of joint production are too large. Thus, in many economic environments the essence of the matter may reduce to a coordination problem, and would generalize far beyond the simple context discussed. At equilibrium, all activities bring equal profits, so entrants can choose arbitrarily among different activities. But this leads to quantities being indeterminate. An artificial solution can be created by allowing for joint production under decreasing returns to scale and small fixed costs. Under suitable hypotheses, this eliminates the coordination problem created by the indeterminacy of the profit maximizing production levels. But, then, in the situation described in the first conjecture in the previous section, all farmers would produce all goods in identical proportions.

This does not seem like a realistic scenario, and many plausible assumptions (such as fixed costs, or constant returns to scale) can be used to rule it out. Thus the conflict between the partial and general equilibrium models would appear to be generic and widespread.

This study also shows that several intuitions of ours may not be justified even by very simple models. Our intuitions as economists, and our policy advice, are based on intuitions generated by the models we learn in universities. One of the simplest of these is that there are competitive equilibria, and that markets achieve these equilibria. Our very simple model does not have an equilibrium for a wide range of economic restrictions. That is, there is no configuration of decentralized decisions which is self-replicating and self-sustaining.

A second widely believed and strong intuition is that fixing prices above equilibrium values would lead to a reduction in trades. The controversy about the Card and Kreuger (1997) findings that minimum wages did not lead to increased unemployment arose because it conflicts with this intuition. In an interview, Card stated that he thought that elementary supply and demand models did not apply to the labor market, as our analysis also suggests. He felt that more complex search theoretic frameworks could explain their findings. The emotional attachment of economists to "supply and demand" framework is reflected in Card's (2006) statement that "[...] (economists)

became very angry or disappointed.[10] They thought that in publishing our work we were being traitors to the cause of economics as a whole."

A third intuition is that planning leads to inefficiencies. Suppose that in the model described in Section 2 the government designates 60 people to be wheat farmers and the rest to be rice farmers. This will be inefficient (as discussed earlier) but will lead to a stable configuration with predictable incomes. Suppose Jeffrey Sachs comes in and advises that transition to a free market will produce better results. Since farmers cannot coordinate production decisions, chaos is likely to result. Different models of farmers expectations regarding price formation will lead to different results, but rational expectations cannot prevent substantial random variation in decision making driven solely by maximization of profits. The uncertainty and random variations in incomes generated by decentralized decision making would lead to substantial loss of efficiency relative to the planned economy.

Sraffa (1925, 1926) argued that partial equilibrium ideas we use in elementary textbooks today to explain formation of prices in markets are only applicable under very restrictive assumptions. Our example illustrates in a very simple model that interactions between demand and supply are ubiquitous, and even render the concept of demand function un-intelligible. The vast majority of current textbooks assume that

[10] "Interview with David Card." Federal Reserve Bank of Minneapolis, The Region, December 1, 2006. http://www.minneapolisfed.org/publications_papers/pub_display.cfm?id=3190

Marshallian supply and demand is automatically applicable to all markets. Given the centrality of this concept in economics, research on conditions for the validity of this approximation would be a high priority. Preliminary results by Hayashi (2009) and Vives (1987) in this direction show that very stringent conditions are needed. Even these results only establish the possibility of obtaining a Marshallian demand function for a single consumer - the requirements for being able to aggregate this demand would be even more restrictive, and are not addressed. Instead of attempting to derive a theory of price from assumptions of maximization for consumers and firms, a more promising approach appears to be more empirically oriented. Behavioral economics has made significant advances in explaining consumer behavior as well as financial anomalies. Similarly empirical examination of how prices are set by firms, as reviewed in Lee (2011), provides a promising alternative to neoclassical theories of price formation.

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