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Abstract

In this note, we develop a monetary Schumpeterian growth model to explore the effects of monetary policy on endogenous market structure, economic growth and social welfare. We find that an increase in the nominal interest rate reduces the equilibrium number of firms. Although long-run economic growth is independent of the nominal interest rate due to a scale-invariant property of the model, a higher nominal interest rate leads to lower growth rates of innovation, output and consumption during the transition path. Taking into account transition dynamics, we find that social welfare is decreasing in the nominal interest rate; therefore, Friedman rule is socially optimal in this economy.

JEL classification: O30, O40, E41
Keywords: monetary policy, economic growth, R&D, endogenous market structure

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1 Introduction

In this note, we develop a monetary Schumpeterian growth model to explore the effects of monetary policy on economic growth, social welfare and endogenous market structure (EMS). Specifically, we set up an R&D-based growth model with two dimensions of technological progress: variety expansion (i.e., horizontal innovation) and quality improvement (i.e., vertical innovation). In the horizontal dimension, entrepreneurs create new firms by introducing new products to the market, and the number of firms in equilibrium determines two important elements of market structure: market concentration and firm size. In the vertical dimension, each incumbent firm performs in-house R&D to improve the quality of its products, and the return to in-house R&D is determined by the size of the firm. In this economy, technological progress and market structure are jointly determined in equilibrium: market structure is measured by the number of firms, whereas technological progress is determined by the growth rate of vertical innovation. In this growth-theoretic framework, we find that an increase in the nominal interest rate reduces labor supply via a cash-in-advance (CIA) constraint on consumption, and the reduced supply of labor decreases the number of firms in equilibrium. Although long-run economic growth is independent of the nominal interest rate due to a scale-invariant property of the model,[1] a higher nominal interest rate leads to lower growth rates of vertical innovation, output and consumption during the transition path. Intuitively, when the nominal interest rate increases, the supply of labor decreases causing lower employment per firm in the short run, which in turn reduces economic growth temporarily. In the long run, the number of firms adjusts such that employment per firm returns to the initial level; therefore, long-run economic growth is independent of the nominal interest rate. Furthermore, taking into account transition dynamics, we find that social welfare is decreasing in the nominal interest rate implying that Friedman rule is socially optimal in this economy.[2] Intuitively, the supply of labor is suboptimally low in equilibrium, so that a positive nominal interest rate that reduces labor supply is suboptimal.

This study relates to the literature on inflation and economic growth; see Tobin (1965) for a seminal study and Wang and Yip (1992) for a discussion on different approaches of modelling money. Studies in this literature analyze the effects of monetary policy mostly in variants of the Neoclassical growth model or the overlapping generations model. The present study instead relates to a more recent subbranch of this literature that analyzes the effects of monetary policy in R&D-based growth models; see for example, Marquis and Reffett (1994), Chu and Lai (2012), Chu *et al.* (2012) and Chu and Cozzi (2012). The present study differs from these studies by analyzing the effects of monetary policy in a more recent vintage of R&D-based growth models based on Peretto (2007) in which both the number of firms and the growth rate of vertical innovation are endogenous; in other words, we consider a scale-invariant Schumpeterian growth model with EMS. Unlike previous studies, we find that monetary policy has no effect on long-run economic growth; instead, monetary policy affects economic growth during the transition path. Furthermore, the abovementioned studies do not analyze the effects of monetary policy on EMS; see Peretto (1996, 1999) for seminal studies in R&D-based growth models with EMS and Etro (2012) for an excellent textbook

[1] See Jones (1999) and Laincz and Peretto (2006) for a discussion of scale effects in R&D growth models.
[2] See Mulligan and Sala-i-Martin (1997) for a discussion of Friedman rule.

treatment. Therefore, this study also contributes to the literature with a novel result that the long-run effects of monetary policy in an R&D-based growth model with EMS are reflected in the number of firms instead of the growth rate. Furthermore, we analyze optimal monetary policy by analytically deriving the complete changes in welfare on the transition path.

The rest of this note is organized as follows. Section 2 presents the model. Section 3 analyzes the effects of monetary policy. The final section concludes.

2 A monetary Schumpeterian growth model with EMS

In this section, we present the monetary Schumpeterian growth model. In summary, the theoretical framework is based on the Schumpeterian model with in-house R&D and EMS in Peretto (2007). We incorporate money demand into the model via a CIA constraint on consumption, under which monetary policy affects the economy by distorting households' tradeoff between consumption and leisure. Furthermore, we provide a complete closed-form solution for the economy's balanced growth path as well as its transition dynamics.

2.1 Households

There is a unit continuum of identical households, who have a lifetime utility function.

$$U = \int_0^\infty e^{-\rho t} \ln u_t dt = \int_0^\infty e^{-\rho t} [\ln c_t + \gamma \ln(L - l_t)] dt, \tag{1}$$

where c_t denotes consumption of final goods (numeraire) at time t and l_t denotes labor supply. The parameters $\rho > 0$ and $\gamma > 0$ determine respectively subjective discounting and leisure preference. Each household maximizes (1) subject to the following asset-accumulation equation.

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t + \tau_t - c_t - \pi_t m_t. \tag{2}$$

a_t is the real value of assets owned by each household, and r_t is the real interest rate. Each household has a labor endowment of L units and elastically supplies l_t units to earn a real wage rate w_t. The household also faces a lump-sum transfer (or tax) τ_t from the government. π_t is the inflation rate that determines the cost of holding money, and m_t is the real money balance held by each household to facilitate purchases of consumption goods. The CIA constraint is given by $\xi c_t \leq m_t$, where the parameter $\xi \in (0, 1]$ determines the importance of the CIA constraint.

The optimality condition of consumption is

$$\frac{1}{c_t} = \eta_t (1 + \xi i_t), \tag{3}$$

where η_t is the Hamiltonian co-state variable on (2) and $i_t = r_t + \pi_t$ is the nominal interest rate. The optimality condition of labor supply is

$$w_t (L - l_t) = \gamma c_t (1 + \xi i_t). \tag{4}$$

The intertemporal optimality condition is

$$-\frac{\dot{\eta}_t}{\eta_t} = r_t - \rho. \tag{5}$$

In the case of a constant nominal interest rate i, combining (3) and (5) yields the familiar Euler equation $\dot{c}_t/c_t = r_t - \rho$.

2.2 Final goods

Final goods Y_t are produced by competitive firms using the following production function.

$$Y_t = \int_0^{N_t} X_t^{\theta}(j)[Z_t^{\alpha}(j)Z_t^{1-\alpha}l_{y,t}(j)]^{1-\theta}dj, \tag{6}$$

where $\theta, \alpha \in (0,1)$ and $X_t(j)$ denotes intermediate goods $j \in [0, N_t]$. The productivity of $l_{y,t}(j)$ workers using intermediate goods $X_t(j)$ depends on its quality $Z_t(j)$ and on average quality $Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_t(j)dj$. The conditional demand function for $l_{y,t}(j)$ is

$$l_{y,t}(j) = \left(\frac{1-\theta}{w_t}\right)^{1/\theta} X_t(j)[Z_t^{\alpha}(j)Z_t^{1-\alpha}]^{(1-\theta)/\theta}, \tag{7}$$

whereas the conditional demand function for $X_t(j)$ is

$$X_t(j) = \left(\frac{\theta}{p_t(j)}\right)^{1/(1-\theta)} Z_t^{\alpha}(j)Z_t^{1-\alpha}l_{y,t}(j), \tag{8}$$

where $p_t(j)$ denotes the price of $X_t(j)$ denominated in units of Y_t. Perfect competition implies that the final goods producers pay $\theta Y_t = \int_0^{N_t} p_t(j)X_t(j)dj$ to intermediate goods firms and pay $(1-\theta)Y_t = \int_0^{N_t} w_t l_{y,t}(j)dj$ to workers.[3]

2.3 Intermediate goods

There is a continuum of industries producing differentiated intermediate goods $X_t(j)$ for $j \in [0, N_t]$. Each industry is dominated by a monopolistic firm that has price-setting power. In the following subsections, we describe the behaviors of incumbents and entrants. In the vertical dimension, each incumbent performs in-house R&D to improve the quality of its products as an attempt to obtain a larger share of the market. In the horizontal dimension, entrepreneurs make entry decisions and compete with incumbents for market share. Through the entry of firms, the number of firms and the size of each firm are endogenously determined in equilibrium.

[3]Free movement of workers across firms implies that wages must be equal across firms.

2.3.1 Incumbents

Existing intermediate goods firms produce differentiated goods with a technology that requires one unit of final goods to produce one unit of intermediate goods. The firm in industry $j \in [0, N_t]$ incurs $\phi Z_t^{\alpha}(j) Z_t^{1-\alpha}$ units of final goods as a fixed operating cost. To improve the quality of its products, the firm invests $R_t(j)$ units of final goods in R&D, and the innovation process is

$$\dot{Z}_t(j) = R_t(j). \tag{9}$$

The cash flow of firm $j \in [0, N_t]$ is

$$F_t(j) = [p_t(j) - 1] X_t(j) - \phi Z_t^{\alpha}(j) Z_t^{1-\alpha}, \tag{10}$$

and the flow profit is

$$\Pi_t(j) = F_t(j) - R_t(j). \tag{11}$$

Therefore, the value of the monopolistic firm in industry $j \in [0, N_t]$ is

$$V_t(j) = \int_t^{\infty} \exp\left(-\int_t^u r_s ds\right) \Pi_u(j) du. \tag{12}$$

Taking the conditional demand function (8) as given, the firm sets its own price and devotes resources to in-house R&D to maximize $V_t(j)$. The current-value Hamiltonian for this optimization problem is[4]

$$H_t(j) = \Pi_t(j) + q_t(j) \dot{Z}_t(j). \tag{13}$$

In Lemma 1, we derive the return to in-house R&D and find that it is increasing in firm size measured by employment per firm l_t/N_t.

Lemma 1 *The return to in-house R&D is given by*

$$r_t^I = \alpha \left[\theta^{(1+\theta)/(1-\theta)} (1-\theta) \frac{l_t}{N_t} - \phi\right]. \tag{14}$$

Proof. See Appendix A. ■

2.3.2 Entrants

A firm that is active at time t must have been born at some earlier date. A new firm pays a sunk setup cost $\beta X_t(j)$ at time t to set up its operation and introduce a new variety of products to the market.[5] We refer to this process as entry. Suppose entry is positive (i.e., $\dot{N}_t > 0$). Then, it must be the case that

$$V_t(j) = \beta X_t(j). \tag{15}$$

[4]See Appendix A for the solution of this optimization problem.

[5]It is useful to note that the setup cost is proportional to the new firm's initial output. This assumption captures the idea that the setup cost depends on the amount of productive assets required to start production; see Peretto (2007) for a discussion.

Following the standard approach in this class of models, we consider a symmetric equilibrium in which $Z_t(j) = Z_t$ for $j \in [0, N_t]$.[6] Under symmetry, $V_t(j) = V_t$, and the familiar Bellman equation implies that the return to entry is

$$r_t^E = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{16}$$

2.4 Monetary authority

The nominal money supply is denoted by M_t, and its growth rate is $\mu_t \equiv \dot{M}_t/M_t$. The real money balance is $m_t = M_t/P_t$, where P_t is the price of final goods. The monetary policy instrument that we consider is i_t.[7] Given an exogenously chosen i_t by the monetary authority, the inflation rate is endogenously determined according to $\pi_t = i_t - r_t$. Then, given π_t, the growth rate of the nominal money supply is endogenously determined according to $\mu_t = \pi_t + \dot{m}_t/m_t$. Finally, the monetary authority returns the seigniorage revenue as a lump transfer $\tau_t = \dot{M}_t/P_t = \dot{m}_t + \pi_t m_t$ to households.

2.5 Aggregation

Under symmetry, the labor market clearing condition becomes

$$l_t = N_t l_{y,t}. \tag{17}$$

The resource constraint of final goods is

$$Y_t = c_t + N_t(X_t + \phi Z_t + R_t) + \beta X_t \dot{N}_t. \tag{18}$$

Substituting (8) into (6) and imposing symmetry yield the aggregate production function.

$$Y_t = \left(\frac{\theta}{p_t(j)}\right)^{\theta/(1-\theta)} Z_t N_t l_{y,t} = \theta^{2\theta/(1-\theta)} Z_t l_t, \tag{19}$$

where the second equality uses (17) and markup pricing $p_t(j) = 1/\theta$.

2.6 Equilibrium

The equilibrium is a time path of allocations $\{m_t, a_t, c_t, Y_t, l_t, l_{y,t}(j), X_t(j), R_t(j)\}$, prices $\{r_t, w_t, p_t(j), V_t\}$ and policy $\{i_t\}$. Also, at each instance of time, the following holds:

- Households maximize utility taking $\{r_t, w_t, \pi_t\}$ as given;

[6] See Peretto (1998, 1999, 2007) for a discussion of the conditions under which the symmetric equilibrium is a reasonable equilibrium concept in this class of models.

[7] It is useful to note that on the balanced growth path, $i = r + \pi = \rho + \mu$.

- Competitive final goods firms maximize profits taking $\{w_t, p_t(j)\}$ as given;
- Incumbents in the intermediate goods sector choose $\{p_t(j), R_t(j)\}$ to maximize the present value of profits taking $\{r_t\}$ as given;
- Entrants make entry decision to the intermediate goods sector taking $\{V_t\}$ as given;
- The monetary authority balances budget such that $\tau_t = \dot{m}_t + \pi_t m_t$;
- The value of all existing monopolistic firms adds up to the value of households' assets such that $a_t = N_t V_t$;
- The market-clearing condition of labor holds;
- Finally, the market-clearing condition of final goods holds.

3 Growth and welfare effects of monetary policy

In this section, we analyze the effects of monetary policy on the equilibrium number of firms, economic growth and social welfare. Specifically, we consider the effects of the nominal interest rate i. In Section 3.1, we analyze the effects of monetary policy on the balanced growth path. In Section 3.2, we analyze effects of monetary policy on the transition path.

3.1 Effects of monetary policy on the balanced growth path

In Appendix A, we show that the consumption-output ratio c_t/Y_t jumps to a unique and stable steady-state value.

Lemma 2 *The consumption-output ratio c_t/Y_t jumps to a unique and stable steady-state value given by*

$$(c/Y)^* = 1 - \theta + \rho\beta\theta^2. \tag{20}$$

Proof. See Appendix A. ■

Given a constant nominal interest rate i and a stationary consumption-output ratio, one can use (4) to show that the supply of labor l_t also jumps to its steady-state value given by

$$l^* = \left[1 + \gamma(1 + \xi i)\left(1 + \frac{\rho\beta\theta^2}{1 - \theta}\right)\right]^{-1} L. \tag{21}$$

Equation (21) shows that the equilibrium supply of labor is decreasing in the nominal interest rate i. Intuitively, an increase in the nominal interest rate increases the cost of consumption due to the CIA constraint; as a result, households reduce consumption and increase leisure. Given that labor supply is stationary, (19) and (20) imply that

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{22}$$

where the last equality uses the Euler equation. Setting $r_t^I = r_t$, one can then use (14) and (22) to derive the equilibrium growth rate given by

$$g_t \equiv \frac{\dot{Z}_t}{Z_t} = \max\left\{\alpha\left[\theta^{(1+\theta)/(1-\theta)}(1-\theta)\frac{l^*}{N_t} - \phi\right] - \rho, 0\right\}, \tag{23}$$

which is increasing in firm size measured by employment per firm l^*/N_t; see Laincz and Peretto (2006) for empirical evidence. The growth rate g_t would be strictly positive if and only if

$$N_t < \overline{N} \equiv \frac{\theta^{(1+\theta)/(1-\theta)}(1-\theta)}{\phi + \rho/\alpha} l^*.$$

In Appendix A, we provide the derivations of the dynamics of N_t.

Lemma 3 *The growth rate of N_t is given by*

$$\frac{\dot{N}_t}{N_t} = \left\{ \begin{array}{ll} \frac{1-\theta}{\beta\theta} - \left(\phi + \frac{\dot{Z}_t}{Z_t}\right)\frac{N_t}{\beta\theta^{2/(1-\theta)}l^*} - \rho & \textit{if } N_t < \overline{N} \\ \frac{1-\theta}{\beta\theta} - \phi\frac{N_t}{\beta\theta^{2/(1-\theta)}l^*} - \rho & \textit{if } N_t > \overline{N} \end{array} \right\}. \tag{24}$$

Proof. See Appendix A. ■

In the following Lemma, we derive the steady-state values of $N_t = N^*$ and $g_t = g^*$. Also, we provide the parameter restrictions that ensure $N^* \in (0, \overline{N})$ and $g^* > 0$.[8]

Lemma 4 *Under the parameter restrictions that $\frac{1-\theta}{\beta\theta} - \alpha\phi < \rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}$, the economy is stable and has a positive and unique steady-state value of N_t as well as a positive and unique steady-state growth rate given by*

$$N^* = \left[\frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho\right]\frac{\beta\theta^{2/(1-\theta)}l^*}{\phi(1-\alpha) - \rho} > 0, \tag{25}$$

$$g^* = \alpha\left[\theta^{(1+\theta)/(1-\theta)}(1-\theta)\frac{l^*}{N^*} - \phi\right] - \rho = \frac{(\rho + \alpha\phi)\beta\theta - (1-\theta)}{(1-\alpha)(1-\theta)/\rho - \beta\theta} > 0. \tag{26}$$

[8] In this model, we have assumed zero population growth, so that N_t converges to a steady state. If we assume positive population growth, it would be the number of firms per capita that converges to a steady state instead, and our main results would be unchanged.

Proof. See Appendix A. ■

In the following proposition, we provide our first main result. We find that an increase in i reduces the steady-state equilibrium number of firms but does not affect the steady-state equilibrium growth rate. Intuitively, an increase in i reduces the supply of labor l^* in (21), which in turn leads to a decrease in the steady-state equilibrium number of firms N^*. However, due to the scale-invariant property of the model, steady-state employment per firm l^*/N^* remains unchanged; as a result, the steady-state equilibrium growth rate in (26) is independent of the nominal interest rate.

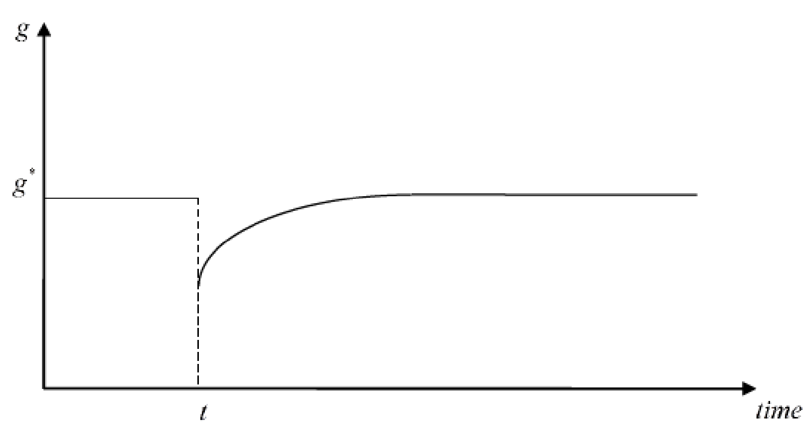
Proposition 5 *The steady-state equilibrium number of firms is decreasing in the nominal interest rate, whereas the steady-state equilibrium growth rate is independent of the nominal interest rate.*

Proof. Note (21), (25) and (26). ■

3.2 Effects of monetary policy on the transition path

In this subsection, we analyze the effects of monetary policy on the transition path. The model features transition dynamics because N_t is a state variable that gradually converges to its state-state value N^*. When the monetary authority increases the nominal interest rate, the equilibrium supply of labor l^* adjusts instantly, but the equilibrium number of firms adjusts slowly. Given that the equilibrium growth rate is determined by employment per firm l^*/N_t, monetary policy could have an effect on economic growth during transition dynamics. Indeed, in the following proposition, we show that an increase in the nominal interest rate reduces the growth rates of vertical innovation, output and consumption on the transition path. Figure 1 illustrates the transitional effects of an increase in the nominal interest rate at time t.

Figure 1: Transitional effects of the nominal interest rate on economic growth



Proposition 6 *An increase in the nominal interest rate reduces the growth rates of vertical innovation, output and consumption on the transition path.*

Proof. Note (21), (22) and (23). Also, recall that N_t is a state variable. ■

Finally, we analyze the welfare effects of monetary policy. Specifically, we consider the effects of i on flow utility $\ln u_t$ and show that $\partial \ln u_t / \partial i < 0$ for any arbitrary t, which is sufficient for $\partial U / \partial i < 0$ because $U = \int_0^\infty e^{-\rho t} \ln u_t dt$. Taking the log of (19), we obtain

$$\ln Y_t = \frac{2\theta}{1-\theta} \ln \theta + \ln Z_t + \ln l_t = \frac{2\theta}{1-\theta} \ln \theta + \int_0^t g_s ds + \ln l^*, \tag{27}$$

where we have normalized $Z_0 = 1$. Taking the log of (20), we obtain

$$\ln c_t = \ln(1 - \theta + \rho\beta\theta^2) + \ln Y_t. \tag{28}$$

Substituting (27) and (28) into flow utility $\ln u_t$ in (1) and then differentiating it with respect to i yield

$$\frac{\partial \ln u_t}{\partial i} = \underbrace{\int_0^t \frac{\partial g_s}{\partial i} ds}_{-} + \underbrace{\frac{\partial \ln l^*}{\partial i}}_{-} + \gamma \underbrace{\frac{\partial \ln(L - l^*)}{\partial i}}_{+}. \tag{29}$$

In other words, an increase in the nominal interest rate has the following three effects on social welfare. First, it reduces welfare by decreasing the growth rate of vertical innovation, output and consumption. Second, it reduces welfare by decreasing labor supply l^*, which in turn decreases the levels of output and consumption. Third, it improves welfare by increasing leisure $L - l^*$. Although the overall welfare effects appear to be ambiguous, we find that $\partial \ln u_t / \partial i < 0$ because the equilibrium supply of labor is suboptimally low, so that any increase in i that reduces l^* is welfare-worsening. To see this result,

$$\frac{\partial \ln l^*}{\partial l^*} + \gamma \frac{\partial \ln(L - l^*)}{\partial l^*} = \frac{L - (1+\gamma) l^*}{l^*(L - l^*)} > 0 \tag{30}$$

because $L/(1+\gamma) > l^*$ in (21). Finally, we summarize this result in the following proposition.

Proposition 7 *Social welfare is decreasing in the nominal interest rate; therefore, Friedman rule (i.e., a zero nominal interest rate) is socially optimal in this economy.*

Proof. Note (29) and (30). Also, recall that $\partial l^* / \partial i < 0$. ■

4 Conclusion

In this note, we have analyzed the effects of monetary policy on economic growth, social welfare and endogenous market structure in a scale-invariant Schumpeterian growth model. A novel result is that monetary policy affects economic growth only in the short run; in the long run, the effects of monetary policy are reflected in market structure measured by the number of firms in equilibrium. This result differs from previous studies that analyze the effects of monetary policy in R&D-based growth models with either horizontal or vertical innovation, but not both. Furthermore, we analyze optimal monetary policy by analytically deriving the complete changes in welfare on the transition path.

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Appendix

Proof of Lemma 1. Substituting (8), (10) and (11) into (13) yields

$$H_t(j) = \theta \left[Z_t^{\alpha}(j) Z_t^{1-\alpha} l_{y,t}(j)\right]^{1-\theta} [X_t(j)]^{\theta} - X_t(j) - \phi Z_t^{\alpha}(j) Z_t^{1-\alpha} - R_t(j) + q_t(j) R_t(j). \quad \text{(A1)}$$

The first-order conditions include

$$\frac{\partial H_t(j)}{\partial X_t(j)} = 0 \Leftrightarrow p_t(j) = \theta \left[\frac{Z_t^{\alpha}(j) Z_t^{1-\alpha} l_{y,t}(j)}{X_t(j)}\right]^{1-\theta} = \frac{1}{\theta}, \quad \text{(A2)}$$

$$\frac{\partial H_t(j)}{\partial R_t(j)} = 0 \Leftrightarrow q_t(j) = 1, \quad \text{(A3)}$$

$$\frac{\partial H_t(j)}{\partial Z_t(j)} = \alpha(1-\theta) \frac{\theta \left[Z_t^{\alpha}(j) Z_t^{1-\alpha} l_{y,t}(j)\right]^{1-\theta} [X_t(j)]^{\theta}}{Z_t(j)} - \alpha \phi Z_t^{\alpha-1}(j) Z_t^{1-\alpha} = r_t^I q_t(j) - \dot{q}_t(j). \quad \text{(A4)}$$

Substituting (A2) and (A3) into (A4) yields

$$r_t^I = \alpha \left[(1-\theta)\theta^{(1+\theta)/(1-\theta)} l_{y,t} - \phi\right], \quad \text{(A5)}$$

where we have applied $Z_t(j) = Z_t$. Finally, substituting (17) into (A5) yields (14). ■

Proof of Lemma 2. Substituting $\tau_t = \dot{m}_t + \pi_t m_t$ into (2) yields

$$\dot{a}_t = r_t a_t + w_t l_t - c_t. \quad \text{(A6)}$$

Then, substituting (15) into $a_t = V_t N_t$ yields

$$a_t = \beta X_t N_t = \beta \frac{p_t X_t N_t}{p_t} = \beta \theta^2 Y_t, \quad \text{(A7)}$$

where the last equality uses (A2) and $p_t X_t N_t = \theta Y_t$. Substituting (A7) into (A6) yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{a}_t}{a_t} = r_t + \frac{w_t l_t - c_t}{\beta \theta^2 Y_t}. \quad \text{(A8)}$$

Substituting the Euler equation and $w_t l_t = (1-\theta) Y_t$ into (A8) yields

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{Y}_t}{Y_t} = \frac{c_t / Y_t}{\beta \theta^2} - \left(\frac{1-\theta}{\beta \theta^2} + \rho\right). \quad \text{(A9)}$$

Therefore, the dynamics of c_t/Y_t is characterized by saddle-point stability such that c_t/Y_t must jump to its steady-state value in (20). ■

Proof of Lemma 3. Substituting (10), (11), (15) and (A2) into (16) yields

$$r_t^E = \frac{1-\theta}{\beta\theta} - \frac{\phi Z_t + R_t}{\beta X_t} + \frac{\dot{X}_t}{X_t}, \quad \text{(A10)}$$

where we have applied $Z_t(j) = Z_t$ and $\dot{V}_t/V_t = \dot{X}_t/X_t$. Substituting (17) and (A2) into (8) yields

$$X_t = \theta^{2/(1-\theta)} Z_t \frac{l^*}{N_t}. \tag{A11}$$

Substituting (9) and (A11) into (A10) yields

$$r_t^E = \frac{1-\theta}{\beta\theta} - \left(\phi + \frac{\dot{Z}_t}{Z_t}\right) \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} + \frac{\dot{Z}_t}{Z_t} - \frac{\dot{N}_t}{N_t}, \tag{A12}$$

where we have used $\dot{X}_t/X_t = \dot{Z}_t/Z_t - \dot{N}_t/N_t$. Setting $r_t^E = r_t$ and substituting (22) into (A12) yield the dynamics of N_t given by[9]

$$\frac{\dot{N}_t}{N_t} = \frac{1-\theta}{\beta\theta} - \left(\phi + \frac{\dot{Z}_t}{Z_t}\right) \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} - \rho. \tag{A13}$$

Equation (A13) describes the dynamics of N_t when $N_t < \overline{N} \equiv \frac{\theta^{(1+\theta)/(1-\theta)}(1-\theta)}{\phi+\rho/\alpha} l^*$. When $N_t > \overline{N}$, $\dot{Z}_t/Z_t = 0$ as shown in (23). ■

Proof of Lemma 4. This proof proceeds as follows. First, we prove that under $\rho < \min\left\{\phi(1-\alpha), \frac{(1-\alpha)(1-\theta)}{\beta\theta}\right\}$, there exists a stable, unique and positive steady-state value of N_t. Then, we prove that under $\rho > \frac{1-\theta}{\beta\theta} - \alpha\phi$, the growth rate of vertical innovation is strictly positive. Finally, the above parameter conditions can be merged into $\frac{1-\theta}{\beta\theta} - \alpha\phi < \rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}$, which ensures $\frac{(1-\alpha)(1-\theta)}{\beta\theta} < \phi(1-\alpha)$. We consider the equilibrium under which there is positive in-house R&D. Substituting (23) into the first equation of (24) yields

$$\frac{\dot{N}_t}{N_t} = \frac{\rho - \phi(1-\alpha)}{\beta\theta^{2/(1-\theta)} l^*} N_t + \frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho. \tag{A14}$$

Because N_t is a state variable, the dynamics of N_t is stable if and only if $\rho < \phi(1-\alpha)$. Solving $\dot{N}_t = 0$, we obtain the steady-state value of N_t in an economy with positive in-house R&D.

$$N^* = \left[\frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho\right] \frac{\beta\theta^{2/(1-\theta)} l^*}{\phi(1-\alpha) - \rho}. \tag{A15}$$

Given $\rho < \phi(1-\alpha)$, (A15) shows that $N^* > 0$ if and only if

$$\rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}. \tag{A16}$$

Combining $\rho < \phi(1-\alpha)$ and (A16) yields

$$\rho < \min\left\{\phi(1-\alpha), \frac{(1-\alpha)(1-\theta)}{\beta\theta}\right\}. \tag{A17}$$

Substituting (A15) into (23) yields (26). Given (A16), (26) shows that $g^* > 0$ if and only if $\rho > \frac{1-\theta}{\beta\theta} - \alpha\phi$. ■

[9] It is useful to note that we have followed the standard approach in this class of models to treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also βX_t); therefore, $V_t = \beta X_t$ always holds. Otherwise, there would be an infinite number of either entries or exits.