Conformism and Cooperation in a Local Interaction Model

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Abstract

We present and analyze a local interaction model where agents play a bilateral prisoner's dilemma game with their neighbors. Agents learn about behavior through payoff-biased imitation of their interaction neighbors (and possibly some agents beyond this set). We find that the [Eshel, I., L. Samuelson and A. Shaked, 1998, Altruists, Egoists and Hooligans in a Local Interaction Model, Am. Econ. Rev 88] result that polymorphic states are stochastically stable in such a setting is not robust. In particular whenever agents use information also of some agents beyond their interaction neighbors the unique stable outcome is one where everyone chooses defection. Introducing a sufficiently strong conformist bias into the imitation process we find that full cooperation always emerges. Conformism is thus identified as a new mechanism that can stabilize cooperation.

Keywords: Cooperation, Imitation, Local Interaction, Conformism.
JEL - Classification: C72, C73, D85

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1 Introduction

Humans acquire much of their behavior through imitation, i.e. through copying or mimicking the action of others. Imitation can be payoff-biased, meaning that agents are more inclined to copy materially successful agents. This is the case most frequently studied in the Economics literature.[1] But it can also be conformist-biased, meaning that agents are more inclined to copy behavior that is frequent (or "popular") among the agents they observe.[2] Empirical evidence in anthropology or biology shows that humans imitate displaying both a payoff-bias and a conformist-bias.[3]

In this paper we study imitation learning in a local interaction environment where agents play a bilateral prisoner's dilemma with their interaction neighbors. We explicitly distinguish between the set of agents one interacts with (interaction neighborhood) and the set of agents one possibly imitates (information neighborhood). We do this because we want to allow for the fact that agents can be informed also about some agents beyond their interaction neighborhood (e.g. their friend's friends) and will use this information when imitating an action. We first analyze the more standard case of payoff-biased imitation in a circle network and find that whenever agents are allowed to hold such information the unique stochastically stable state is one where everyone chooses defection. The result from Eshel, Samuelson and Shaked (1998) that some cooperation survives in a stochastically stable state thus breaks down as soon as agents are allowed to use information about some agents beyond their interaction neighbors. Introducing a sufficiently strong conformist bias into the imitation process we find on the other hand that full cooperation always emerges irrespective of whether agents hold information beyond their interaction neighbors or not. Conformism is thus identified as an important and new mechanism that can stabilize cooperation in a local interaction environment.

We also show that the result from Eshel, Samuelson and Shaked (1998) does not extend to general networks irrespective of whether agents hold information about others beyond their interaction neighbors or not. In particular we give examples of asymmetric networks (where not all agents have the same number of neighbors) for which the unique stochastically stable state under payoff-biased imitation involves full defection. Conformism - on the other hand - stabilizes cooperation also in these networks.

The paper that is most closely related to ours is the already mentioned work by Eshel, Samuelson and Shaked (1998) who have analyzed cooperation in a circle network where agents rely on payoff-biased imitation and use only information about their interaction partners.[4] Previous literature has also ex-

[1]See Eshel, Samuelson and Shaked (1998), Basci (1999), Schlag (1998) or the textbooks by Vega-Redondo (2000) or Fudenberg and Levine (1999).

[2]Ellison and Fudenberg (1993) or Cartwright (2007) study such processes.

[3]See Boyd and Richerson (2005), Henrich and Boyd (1998) or Cavalli-Sforza and Feldmann (1981) among others.

[4]Alos-Ferrer and Weidenholzer (2006) use differing interaction and information neighborhoods to explain the emergence of the efficient convention in coordination games. See also Galeotti et al. (2006).

plained cooperation in networks through other mechanisms. Marsili, Slana and Vega-Redondo (2005) highlight the importance of the clustering degree for sustaining cooperation. Zimmermann, Eguiluz and San Miguel (2004) or Hanaki et al. (2007) explain cooperation through exclusion of non-cooperators in a dynamic network setting. The role of a conformist bias in imitation has been examined by Ellison and Fudenberg (1993) to study the spread of an efficient technology in a one person decision problem. Levine and Pesendorfer (2007) explain cooperation through an imitation process in a set up where agents get some information about the opponent's strategy prior to interaction. Imitation learning has been experimentally investigated by Apesteguía, Huck and Öchssler (2007). Kirchkamp and Nagel (2007) study imitation learning in networks in an experiment.

The paper is organized as follows. In section 2 the model is presented and in section 3 it is analyzed. Section 4 shows that our results are robust. Section 5 concludes. The proofs are relegated to an appendix.

2 The Model

2.1 The Local Interaction Game

There are $i = 1, ...n$ agents interacting in a 2×2 prisoner's dilemma game through a circle network. Interactions are not necessarily restricted to an agents first order neighbors. Denote N_i^Z the set of agents agent i interacts with or the "interaction neighborhood" of player i. Furthermore the set of agents i interacts with (N_i^Z) will in general not equal the set of agents i has information about. Denote the latter set - the information neighborhood of agent i - by N_i^I.[5] Assume $I \geq Z$. Let it be a convention that N_i^Z does not contain the player i herself while N_i^I does - i.e. while players do not interact with themselves they have information about themselves. As an illustration consider the circle with interaction radius $Z = 1$ and information radius $I = 2$ depicted below.

$$....\overbrace{(i-2) - \underbrace{(i-1)}_{N_i^Z} - i - \underbrace{(i+1)}_{N_i^Z} - (i+2)}^{N_i^I} - (i+3)\,...$$

Note that the relation "j is an element of N_i^I (N_i^Z)" is symmetric, i.e. $j \in N_i^I$ (N_i^Z) $\Leftrightarrow i \in N_j^I$ (N_j^Z).

Individuals play a 2×2 prisoner's dilemma with their interaction neighbors N_i^Z. The set of actions is given by $A = \{C, D\}$ for all players. For each pair of actions a_i,$a_j \in A$ the payoff $\pi_i(a_i, a_j)$ that player i earns when playing action

[5] When we say that i has information about j we mean that i is informed about j's action choice and payoff.

a_i against an opponent who plays a_j is given by the following matrix,

$$\begin{array}{|l|c|c|}\hline a_i \backslash a_j & C & D \\ \hline C & a & b \\ \hline D & c & d \\ \hline \end{array}. \tag{1}$$

We are interested in the case $c > a > d > b > 0$ i.e. the case where matrix (1) represents a Prisoner's dilemma. Assume also that $a > \frac{b+c}{2}$, i.e. that cooperation (C) is efficient. The payoffs at time t for player i from playing action a_i are given by[6]

$$\Pi_i^t(a_i^t, a_j^t) = \sum_{j \in N_i^Z} \pi_i(a_i^t, a_j^t). \tag{2}$$

Denote $\Pi^t(N_i^I(a)) = \frac{\sum_{k \in N_i^I | a_k^t = a} \Pi_k^t(\cdot)}{\text{card}\{k \in N_i^I | a_k^t = a\}}$ the average payoff of all agents in N_i^I that choose action a and let it be a convention that $\Pi^t(N_i^I(a)) = 0$ if $\text{card}\{k \in N_i^I | a_k^t = a\} = 0$.

2.2 Learning

At each point in time $t = 1, 2, 3....$ the state of the system is given by the action choices of all agents $s(t) = (a_i^t)_{i=1}^n$. Denote S the state space. Agents learn about optimal behavior through imitation. More precisely we assume that at each point in time a (small) number r of agents is randomly selected to revise their action choices. We consider two possible decision rules. First we consider the rule typically used in the literature where agents rely on payoff-biased imitation. Then we add a conformist-bias into the imitation process.

2.2.1 Payoff- biased Imitation

Under the basic process an agent (who is selected to revise her action choice) compares the average payoff in her information neighborhood of the action she is currently not choosing $\neg a_i$ and the action a_i she is currently choosing. If and only if

$$\Pi^{t-1}(N_i^I(\neg a_i)) - \Pi^{t-1}(N_i^I(a_i)) > 0 \tag{3}$$

she changes her action. With small probability ε she reverses her choice. This is the rule used for example by Eshel, Samuelson and Shaked (1998).[7]

[6] In equation (2) agents get the same payoff from all their interaction partners. One could easily generalize to a situation where - as in the connections model from Jackson and Wolinsky (1996) - payoffs are discounted in proportion to the geodesic distance between the two interaction partners.

[7] See also Schlag (1998). As the circle network is symmetric (i.e. all agents have the same number of links) it is immaterial whether one uses per-link payoffs or total payoffs of each agent to calculate the sum in the numerator of $\Pi^t(N_i^I(a))$.

2.2.2 Payoff- and Conformist- biased Imitation

The process with conformism takes into account the possibility that agents might be more inclined to make more "popular" choices. Decision rule (3) is substituted by the following rule,

$$\Pi^{t-1}(N_i^I(\neg a_i)) - \Pi^{t-1}(N_i^I(a_i)) > m(1 - 2x_{\neg a_i}). \tag{4}$$

$m \in \mathbb{R}^+$ is a finite conformity parameter and $x_{\neg a_i}$ the share of all agents that i knows about that use a different action then herself, i.e. $x_{\neg a_i} = (2I + 1)^{-1}\,\mathrm{card}\{j \in N_i^I | a_j \neq a_i\}$. Obviously $m = 0$ corresponds to the basic process. If both actions are equally popular, i.e. if $x_{\neg a_i} = 1/2$ the agent is not biased towards using either of them. If one of the actions is more popular on the other hand the agent will be ceteris paribus more inclined to use that action.[8]

2.3 Techniques used in the Analysis

The learning process described in subsections 2.2 (under either decision rule) gives rise to a finite Markov chain, for which the standard techniques apply. Denote $P^0(s, s')$ the transition probability for a transition from state s to s' whenever $\varepsilon = 0$ and $P^\varepsilon(s, s')$ the transition probability of the perturbed Markov process with strictly positive trembles. The evolutionary model is fully described by the triple $(S, P^0(\cdot), \varepsilon)$. An absorbing set under P^0 is a minimal subset of states which, once entered is never left. An absorbing state is a singleton absorbing set, or in other words,

Definition 1 State s is absorbing $\Leftrightarrow P^0(s, s) = 1$.

As trembles make transitions between any two states possible the perturbed Markov process is irreducible and hence ergodic, i.e. it has a unique stationary distribution denoted μ^ε. This distribution summarizes both the long-run behavior of the process and the time-average of the sample path independently of the initial conditions.[9] The limit invariant distribution $\mu^* = \lim_{\varepsilon \to 0} \mu^\varepsilon$ exists and its support $\{s \in S | \lim_{\varepsilon_a \to 0} \mu^{\varepsilon_a}(s) > 0\}$ is a union of some absorbing sets of the unperturbed process. The limit invariant distribution singles out a stable prediction of the unperturbed dynamics ($\varepsilon = 0$) in the sense that for any $\varepsilon > 0$ small enough the play approximates that described by μ^* in the long run. The states in the support of μ^* are called stochastically stable states.

Definition 2 State s is stochastically stable $\Leftrightarrow \mu^*(s) > 0$.

Denote ω the union of one or more absorbing sets and Ω the set of all absorbing sets. Define $X(\omega, \omega')$ the minimal number of mutations (simultaneous trembles) necessary to reach ω' from ω.[10] The stochastic potential $\psi(s)$ of a

[8] See Ellison and Fudenberg (1993) or Cartwright (2007).

[9] See Karlin and Taylor (1975) or Tijms (2003) for textbooks on stochastic processes or Young (1993,1998) and Ellison (2000) for applications of these techniques to economics.

[10] It is important to note that these transitions need not be direct (i.e. they can pass through another absorbing set).

state $s \in \Omega$ is defined as the sum of minimal mutations necessary to induce a (possibly indirect) transition to s from any alternative state $s' \in \Omega$, i.e. $\psi(s) = \sum_{s' \in \Omega} X(s', s)$.

Result (Young 1993) State s^* is stochastically stable if it has minimal stochastic potential, i.e. if $s^* \in \arg\min_{s \in \Omega} \psi(s)$.

The intuition behind Young's result is simple. In the long run the process will spend most of the time in one of its absorbing states. The stochastic potential of any state s is a measure of how easy it is to jump from the basin of attraction of other absorbing states to the basin of attraction of state s by perturbing the process a little.

3 Analysis

Throughout the analysis we assume that I is small relative to the number of players n. In particular we will assume that $I < \frac{n-2}{4}$ ensuring that for any agent at least one other agent can be found such that their information neighborhoods are disjoint.[11] Let us first take a brief look at the absorbing states. It should be clear that monomorphic states where all agents choose the same action are always absorbing. But also polymorphic states where some agents cooperate and some defect can be absorbing under certain conditions. The following proposition can be stated.

Proposition 1 *The monomorphic states where $a_i = a, \forall i \in G$ are always absorbing. Furthermore there exists $\overline{a}(m, Z, I) > 0$ such that a set of polymorphic states containing strings of cooperators separated by strings of defectors is absorbing whenever $a > \overline{a}(\cdot)$.*

Proof. Appendix. ■

The exact composition of the set of polymorphic absorbing states depends on the coefficient for conformism m as well as the information radius I and the interaction radius Z. In the following we will denote s^a the state where all agents play action a and ω^{CD} the set of polymorphic absorbing states.

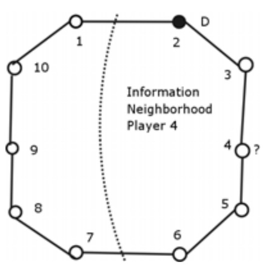
3.1 Payoff- biased Imitation

3.1.1 Case $I > Z$

We start with a situation where $I > Z$, i.e. with a situation where agents hold some information about other agents beyond their interaction neighborhood. Agents can have this information for example because their friends tell them about their friends or because in a physical neighborhood they observe the actions and payoff of all the neighbors on their street while they only interact

[11] We want to focus on both local interaction and local information. Without local interaction of course network analysis is pointless.

with some. One can think of many more examples. We will show that if agents rely on payoff-biased imitation only the unique outcome in these situations is full defection. As an illustration consider the network depicted in Figure 1 where $I = 2$ and $Z = 1$. Then from the absorbing state s^C where everyone cooperates one tremble by any player can induce a transition to the state s^D with full defection. To see this assume that starting from the fully cooperative state (s^C) player 2 trembles and switches to action D. Consider the decision of player 4. This player will imitate player 2 as the average defector payoff in his information neighborhood $\Pi^t(N_4^I(D)) = 2c$ exceeds the average cooperator payoff $\Pi^t(N_4^I(C))$ among his information neighbors (Figure 1).



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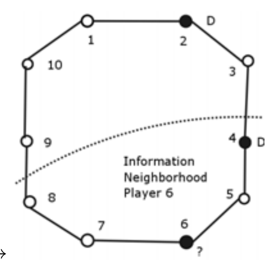

Figure 1. Figure 2.

What happens if next player 6 is drawn to revise his action choice ? Note that $N_6^I = \{4, 5, 6, 7, 8\}$. Consequently $\Pi^t(N_6^I(D)) = 2c > \Pi^t(N_6^I(C))$ and player 6 will switch to defection. Assume that next player 8 then player $10, 12$ etc... are drawn to revise their action choices. All of these will switch to defection and finally the remaining cooperating players will be surrounded by defectors that interact only with cooperators (Figure 2). Consequently $\Pi^t(N_i^I(D)) > 2b = \Pi^t(N_i^I(C)), \forall i \in G$ and the remaining cooperators will also want to switch to defection. We end up in a state of full defection. Such a transition after one tremble is always possible because of the fact that players have information about others beyond their interaction neighbors. This allows the deficient action to spread "across long distances", ensuring that cooperators always interact with more defectors than the defectors themselves during the transition. On the other hand it is clear that for a transition from s^D to a state characterized by some or full cooperation more than one tremble is needed as a single cooperator surrounded by defectors will have the minimum possible payoff and the cooperative action will never be imitated in this case. This sort of reasoning underlies the fact that the unique stochastically stable state is s^D, as is shown in the next proposition.

Proposition 2 *If $I > Z$ the unique stochastically stable state is s^D.*

Proof. Appendix. ■

The fact that agents are informed also about the actions and payoffs of relatively distant agents allows the deficient action to spread across long distances thereby destroying any possible clique of cooperators. Holding even slightly more information than just one's interaction neighbors is detrimental to cooperation. The reason is that this allows defection to spread across long distances thereby concealing the fact that joint defection is worse for a group of agents than joint cooperation. Next we will consider the case previously examined by Eshel, Samuelson and Shaked (1998) where there is less information, as now we will assume that agents are only informed about the agents they interact with.

3.1.2 Case $I = Z$

The case $I = Z$ reflects situations where agents' information is restricted to their interaction partners. Examples for these situations will be found in anonymous interactions, like for example the interaction between buyers and sellers in a supply chain. In these cases a transition from s^C to s^D is not always possible after one action tremble. As an illustration consider the network depicted in Figure 3 and assume that $I = Z = 1$. Let player 2 tremble and switch to action D. Her action will be imitated by one of her interaction partners, i.e. either player 1 or 3, as $N_2^I = \{1, 2, 3\}$.[12] N_2^I is depicted in Figure 3.

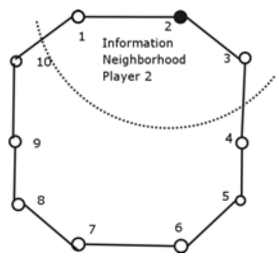

Figure 3.

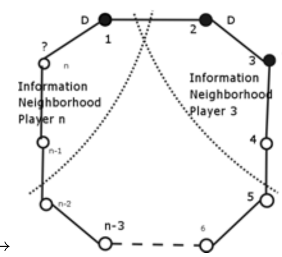

Figure 4.

Say for example that player 1 imitates the deficient action. Now both players 1 and 2 will interact with one defector and one cooperator each. As $N_1^I \cup N_2^I = \{n, 1, 2, 3\}$, the only players that might adopt the deficient action now are players n and 3. But their information neighborhoods also contain a cooperator who interacts only with other cooperators. The average payoff of cooperating agents in these information neighborhoods is given by $\Pi^t(N_n^I(C)) = \Pi^t(N_3^I(C)) = \frac{3a+b}{2}$. The average payoff of defectors is given by $\Pi^t(N_n^I(D)) = \Pi^t(N_3^I(D)) = c + d$. Defection will spread if and only if $a < \frac{2(c+d)-b}{3}$. If this is the case a transition to s^D can be induced via one tremble. If not no more actions will be imitated at this point and defection cannot spread through the whole graph.

What happens with the reverse transition from s^D to s^C ? Of course one action tremble will not suffice to induce such a transition as a single cooperator will never be imitated. Assume thus that players 2 and 3 simultaneously make a mistake and switch to action C. Player 4 will imitate whenever $\Pi^t(N_4^I(C)) > \Pi^t(N_4^I(D))$. If this is the case the cooperative action can spread through the graph until two defectors are left. These defectors will never want to imitate the cooperative action. Cooperation can survive in a polymorphic stochastically

[12]Remember that the relation $j \in N_i^I$ is symmetric.

stable state whenever the payoff to joint cooperation is "high enough". We can state the following proposition.

Proposition 3 *If $I = Z$, there exists $\overline{a} > 0$ such that if the game payoffs satisfy $a \geq \overline{a}$ state s is stochastically stable $\Rightarrow s \in \omega^{CD}$. If $a < \overline{a}$ the unique stochastically stable state is s^D.*

Proof. Appendix. ■

Less information does actually help cooperation. The reason is that defection now can only spread locally, forcing defectors to interact with each other. This reduces the average payoff of defectors revealing the social benefit of cooperation. Proposition 3 essentially generalizes the result from Eshel, Samuelson and Shaked (1998).[13]

3.2 Payoff- biased and Conformist- biased Imitation

3.2.1 Case $I > Z$

In this section we assume that agents display a conformist bias, i.e. that they are more inclined to imitate more popular actions. We show that if imitation is payoff-biased and (sufficiently) conformist-biased the unique stochastically stable state involves full cooperation. To illustrate the logic of our result consider the example from Figure 5 where $I = 2$ and $Z = 1$. Again consider first transitions from s^C to s^D and assume that only player 3 trembles and switches to action D. Player 1 will now imitate player 3 if and only if the payoff advantage of defection is high enough to make up for the "unpopularity" of this action.

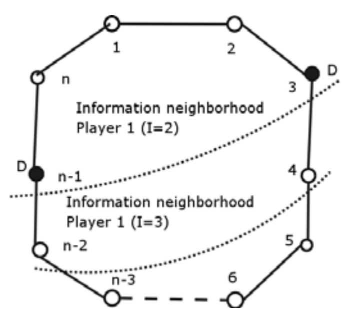

Figure 5.

It is shown in the appendix that a necessary condition for action D to spread through the whole graph after just one tremble is $m < \frac{(2I+1)[2IZ(c-a)+Z(a-b)]}{(2I-1)I}$. What happens if agents display a larger degree of conformism ? Then one needs (at least) a second tremble in agent 1's information neighborhood for him to imitate. Assume player 1 is willing to imitate action D after both players 3 and $n-1$ have trembled to action D. Can the deficient action leave N_1^I ? Consider

[13]They consider the cases where $I = Z = 1$ and $I = Z = 2$. Another difference is that in their model all agents revise their strategy each period (i.e. $r = n$). They claim though that their results are robust to a situation in which $r < n$ and indeed we show that this is the case.

the decision of player 4. His information neighborhood is given by $N_{n-2}^I = \{2, 3, 4, 5, 6\}$. Players $4, 5$ and 6 are cooperators. As one defector is not enough to induce imitation, both 2 and 3 have to play defect for defection to spread through the operation of the unperturbed dynamics alone. But then we have a string of interacting defectors $...1-2-3-4....$ A small amount of conformism can thus be enough to force transitions to be in which defectors interact mainly among each other. This reduces the payoff advantage of defectors compared to cooperators, revealing the social benefit of cooperation.

Let us consider now the reverse transition from s^D to s^C. As always transitions after one tremble are not possible as single cooperators will never be imitated. Depending on the degree of conformism more or less simultaneous trembles are needed in a given information neighborhood to induce a transition. Note though that there is a feedback effect, as more trembles of connected cooperators increase the payoff advantage of cooperation over defection. This in turn reduces the need for cooperation to be "popular" in order to spread. Higher degrees of conformism thus favor cooperative outcomes. We can state the following proposition.

Proposition 4 *Assume $I > Z$ and that agents display a conformist bias. There exist $\underline{m}(Z, I) > 0$ and $\overline{m}(Z, I) > 0$ s.th. if $m \geq \overline{m}(\cdot)$ the unique stochastically stable state is s^C. If $m \leq \underline{m}(\cdot)$ Proposition 2 applies. Furthermore $\overline{m}(\cdot)$ and $\underline{m}(\cdot)$ are strictly decreasing in I and increasing in Z.*

Proof. Appendix. ■

Conformism can stabilize cooperation as we have seen in Proposition 4. Furthermore if imitation is conformist-biased more information (larger I) actually helps cooperation. The intuition is as follows. Conformism is helpful to sustain cooperation because it requires the formation of strings of cooperators or defectors during any transition between absorbing states. But then - given that these strings exist - more information is helpful to achieve cooperation because it enables agents to "look deeper" into the strings. This increases the number of cooperators interacting with cooperators and of defectors interacting with defectors in any particular agents sample and makes more evident the higher payoff that cooperation yields to a community.

3.2.2 Case $I = Z$

The case $I = Z$ confirms the results from the case where $I > Z$. Again conformism is helpful to sustain cooperation as the following proposition shows.

Proposition 5 *Assume $I = Z$ and that agents display a conformist bias. There exist $\underline{m}(Z) > 0$ and $\overline{m}(Z) > 0$ s.th. if $m \geq \overline{m}(\cdot)$ the unique stochastically stable state is s^C.If $m \leq \underline{m}(\cdot)$ Proposition 3 applies.*

Proof. Appendix. ■

The intuition is similar to before. Conformism forces cooperators and defectors to interact with others that choose the same action during any given transition. This reveals the benefit of cooperation and stabilizes cooperative outcomes.

4 Robustness

In this section we would like to point out another dimension in which the Eshel, Samuelson and Shaked (1998) result is not robust, but where imitation with a conformist bias yields cooperative outcomes in the long run. In particular we want to discuss some asymmetric networks (where not all players have the same number of nodes) and show that while with decision rule (3) stochastically stable outcomes yield defection, cooperation is obtained with decision rule (4).

Consider first the interconnected star network depicted in Figure 6.

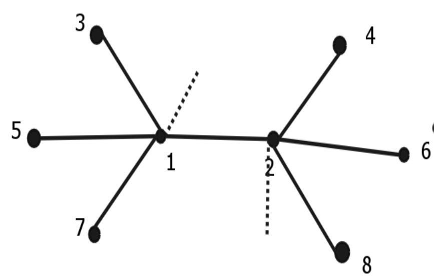

Figure 6.

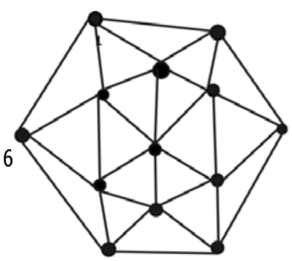

Figure 7.

In this network there are two types of agents - some "centers" with a very high degree (like agent 1 and 2) and some in the periphery with only one first-order neighbor. Assume $I = Z = 1$ as in the original model from Eshel, Samuelson and Shaked (1998). A transition from s^C to s^D can occur via one tremble by one of the centers, infecting first all the centers and then the agents in the periphery. Also a transition from a polymorphic state where in one star cooperation prevails and in others defection to the state s^D can occur after one such tremble irrespective of the payoff parameters. (Of course such a polymorphic state will only be absorbing under some parameter constellations.) On the other hand the reverse transitions need more than one tremble. Consequently with decision rule (3) the unique stochastically stable state will be one of full defection. What happens under decision rule (4) ? If the conformist bias is strong enough agents in the periphery will always conform to what the center does (their unique neighbor). A single action tremble by the center can infect any star. As soon as enough stars are infected the cooperative action will - starting from s^D - be able to spread to the whole network because the centers of other stars will want to switch to cooperation. On the other hand to induce a transition from s^C to s^D whenever the conformist bias is high all centers have to simultaneously make a mistake. That is why also in this network a high degree of conformism favors cooperative outcomes.

Of course the interconnected star is a network with extreme asymmetries in degree and maybe cooperation obtains even under (3) as long as the asymmetry is not too extreme. Consider thus the crystal network depicted in Figure 7. In this network there is an agent i with degree d (in Figure 7 $d = 6$) whose first-order neighbors have degree $d-1$, whose second-order neighbors have degree $d-2$ and so on until some minimal degree. Again for this network the unique stochastically stable state (with $I = Z = 1$) is one where everyone chooses defection. Defection can spread after a tremble by player i, infecting one player $j \in N_i^1$, then one player $k \in N_j^1 \cap \backslash N_i^1$ and so on.[14] Again decision rule (4) in this example leads to cooperation (whenever m is "large enough"), because conformism forces actions to spread locally thereby revealing the benefit of cooperation.

5 Conclusions

We have presented a model where agents interact in a prisoner's dilemma through a local interaction structure. Agents learn about optimal actions through imitation. The set of agents they possibly imitate (their information neighborhood) can differ from the set of agents they interact with (their interaction neighborhood). If agents rely on payoff biased imitation alone, choosing the action (cooperation or defection) that has yielded the higher payoff in the previous period we find the following results.

- If the information radius of agents exceeds their interaction radius the unique stochastically stable outcome is full defection. Only if information radius and interaction radius are the same some cooperation can be obtained in a stochastically stable state. In this sense more information hurts cooperation.

We then introduce a conformist bias into imitation, assuming that agents are more likely to adopt more "popular" actions and find the following.

- If the conformist bias is large enough all stochastically stable outcomes involve cooperation.

- If there is a conformist bias more information helps cooperation.

The intuition is as follows. Because joint cooperation is beneficial for a community but not individually optimal, strings of cooperators are better off than strings of defectors whereas single defectors are always better off than single cooperators. But then a larger information radius hurts in the standard case because it allows imitation across "long distances". This works against the formation of strings. Intuitively what conformism does is that it forces actions to spread "locally" thereby revealing the benefit of cooperation. A larger information radius helps in this case because it allows agents to "look deeper" into strings of cooperators and defectors.

[14] Note that this continues to be true if one uses per-link payoffs instead of total payoffs of each agent to calculate $\Pi^t(N_i^I(a))$.

Note that conformism alone cannot stabilize cooperation. It is the interaction of both conformist-biased and payoff-biased imitation that leads to cooperative outcomes. The model can thus provide a rationale for why humans seem to engage in both conformist-biased and payoff-biased imitation.[15] If one thinks of the network as a group of agents in a group-selection type environment, the success of conformism in sustaining cooperation can constitute an explanation for why people often display a conformist bias and why this bias is often directed towards agents they feel "close" to.

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A Appendix - Proofs

s−trees

For some of the proofs we will rely on the graph-theoretic techniques developed by Freidlin and Wentzell (1984).[16] They can be summarized as follows. For any state s an $s-$tree is a directed graph on the set of absorbing states Ω, whose root is s and such that there is a unique directed path joining any

[16] See also Young (1993, 1998).

other $s' \in \Omega$ to s. For each arrow $s' \to s''$ in any given $s-$tree the "cost" of the arrow is defined as the minimum number of simultaneous trembles necessary to reach s'' from s'.The cost of the tree is obtained by adding up the costs of all its arrows and the stochastic potential of a state s is defined as the minimum cost across all $s-$trees.

Notation

Denote ω_ρ^{CD} the set of polymorphic absorbing states with ρ strings of cooperators and defectors (note that in any polymorphic absorbing state there has to be an equal number of strings of cooperators and defectors). Furthermore denote $\overline{\gamma}(s) \in \mathbb{N}$ ($\overline{\delta}(s) \in \mathbb{N}$) the length of the longest cooperator (defector) string in state s and $\underline{\gamma}(s) \in \mathbb{N}$ ($\underline{\delta}(s) \in \mathbb{N}$) the length of the shortest such string.

Proof of Proposition 1:

Proof. It is clear that given our assumptions the two monomorphic states are always absorbing.[17]

(i) Consider polymorphic states (consisting of strings of cooperators separated by strings of defectors). Let us start with the case $m = 0$. Irrespective of the payoff parameters and the parameter I, there have to be at least $Z + 2$ cooperators in the largest string of any absorbing state, i.e. $\overline{\gamma}(s) \geq Z + 2$. If this is not the case the inequality $\Pi^t(N^I(D)) > \Pi^t(N^I(C))$ will always hold for the last cooperator (denote i) in any string and she will want to switch to defection. Now we will derive threshold levels $\widehat{a}_1(m, Z, I)$ and $\widehat{a}_2(m, Z, I)$ for the payoff parameters that determine under which conditions polymorphic absorbing states exist. For the cooperator i at the edge of a cooperator string the following holds.

$$
\begin{aligned}
&\Pi^t(N_i^I(C)) \\
= \quad &\frac{\sum_{l=0}^{I}(Z-l)b + (Z-l-\delta_j)1_+(a-b) + (Z+l)a + (Z+l-\gamma_i)1_+(b-a)}{I+1}
\end{aligned}
$$

and

$$
\begin{aligned}
&\Pi^t(N_i^I(D)) \\
= \quad &\frac{\sum_{l=0}^{I-1}(Z+l)d + (Z+l-\delta_j)1_+(c-d) + (Z-l)c + (Z-l-\delta_j)1_+(d-c)}{I},
\end{aligned}
$$

where γ_i is the length of the string containing i, δ_j is the length of the string containing i' s defecting neighbor j and 1_+ is the indicator function taking the value 1 if the preceding expression is positive and zero otherwise. Analogous expressions can be determined for the defecto jr at the edge of the defector string. Then in order for a state to be absorbing $\Pi^t(N_i^I(C)) \geq \Pi^t(N_i^I(D))$ and $\Pi^t(N_j^I(C)) \leq \Pi^t(N_j^I(D))$ have to hold for all strings. These inequalities define for any given length of a string two threshold level $\widehat{a}_1(m, Z, I)$ and $\widehat{a}_2(m, Z, I)$ ensuring that neither the cooperator i nor the defector j have incentives to change their actions. (It should be clear that if these agents don't have incentives to change the agents at the interior of a string don't have either).

[17] Note that we have assumed that $\Pi^t(N_i^I(a)) = 0$ if card$\{k \in N_i^I | a_k^t = a\} = 0$. The intuition simply is that no actions can be imitated that are not present in the population.

If $\gamma_i, \delta_j > Z + I$ and furthermore $I = Z$ then $\Pi^t(N_i^I(C)) \geq \Pi^t(N_i^I(D))$ can be rewritten as

$$\begin{aligned} & Z\left[\frac{Z(Z+1)}{2}b + \frac{2Z(2Z+1)-(Z-1)Z}{2}a\right] \\ \geq \quad & (Z+1)\left[\frac{Z(Z+1)}{2}c + \frac{(2Z-1)2Z-(Z-1)Z}{2}d\right] \end{aligned}$$

or equivalently

$$a \geq \frac{c - d + Z(3d + c - b)}{3Z} =: \widehat{a}_1(0, Z, Z).$$

Note that $\widehat{a}_1 \in (d, c)$. Also $\Pi^t(N_j^I(C)) \leq \Pi^t(N_j^I(D))$ can be rewritten as

$$a \leq \frac{Z(3d + c - b) - b}{3Z - 1} =: \widehat{a}_2(0, Z, Z).$$

Whenever $a \in [\widehat{a}_1(\cdot), \widehat{a}_2(\cdot)]$ these states will be absorbing. If $a < \widehat{a}_1$ no polymorphic states can be absorbing, as agents at the end of a cooperative string will always want to switch to defection. What happens if $a > \max\{\widehat{a}_1, \widehat{a}_2\}$? Then the defector at the end of the defector-string will want to switch to cooperation. The length of a cooperator string will increase and that of the defector string decrease. Whenever there are $Z + 1$ defectors left the payoff for defection will exceed that for cooperation in this neighborhood and consequently no more defectors will want to switch to cooperation.

(ii) In general $\widehat{a}_1(\cdot)$ and $\widehat{a}_2(\cdot)$ depend of course on I and Z. The higher I, Z the longer strings need to be and the larger will be the difference in length between cooperator and defector strings. Whenever I and Z are small enough though relative to n polymorphic absorbing states exist.

(ii) If $m > 0$ all polymorphic states with cooperator and defector strings larger than $I + 1$ that were absorbing for $m = 0$ are still absorbing. The reason is that for these strings cooperation (defection) is more popular in the information neighborhoods of all cooperators (defectors), i.e. $x_{\neg a_i} < \frac{1}{2}, \forall i \in G$. Furthermore as $\partial\ \widehat{a}_1(\cdot)/\partial m < 0$ whenever $\underline{\gamma}(s), \underline{\delta}(s) \geq I + 1$ additional such states can be absorbing. For states with strings shorter than $I + 1$ the threshold value a rises and thus some such states that were absorbing under the basic process may fail to be so whenever $m > 0$.

Proof of Proposition 2: ■

Proof. Transitions from s^C to s^D can occur via one action tremble, i.e. $X(s^C, s^D) = 1$. To see this assume cooperator i trembles and chooses D. Then $\Pi_i^t(\cdot) = 2Zc$. Consider agents $j \in \{N_i^I \cap \backslash N_i^Z\}$.[18] This set is non-empty as $I > Z$. As $\Pi^t(N_j^I(D)) = 2Zc > \Pi^t(N_j^I(C))$, these agents j will want to switch to action D. Next consider agents $k \in \{N_j^I \cap \backslash N_j^Z \cap \backslash N_i^Z\}$. Again $\Pi^t(N_k^I(D)) > \Pi^t(N_k^I(C))$ and agents k will switch to action D. In this way transitions can occur during which $D-$players always interact with less $D-$players than $C-$players do,

[18] $\backslash N_i^Z$ denotes the complement of the set N_i^Z.

ensuring that $\forall i \in G$ the inequality $\Pi^t(N_i^I(D)) > \Pi^t(N_i^I(C))$ always holds during the transition. The transition $s^D \rightarrow s^C$ on the other hand needs at least two simultaneous trembles as a single cooperator will never be imitated, i.e. $X(s^D, s^C) \geq 2$. But then also $X(s, \omega^C) \geq X(s, \omega^D), \forall s \in \omega_\rho^{CD}$ and consequently $\psi(s^C) > \psi(s^D)$.

Next we will show that $\psi(s^D) < \psi(s), \forall s \in \omega^{CD}$. Consider first states $s \in \omega_1^{CD}$ and assume wlg $\gamma(s)$ is impair. Assume that at time t the "median" cooperator in the cooperator string $\gamma(s)$ trembles and switches to D. If the deficient action then infects always the next node at distance $Z+1$ from the node last infected the whole cooperator string can be infected, i.e. $X(s, s^D) = 1$. To see this note first that initially defection can spread within the cooperator string as the unique defector that the cooperators in question observe has payoff $2Zc$. After some time τ the edge of the string will be reached (it is of course possible that $\tau = 0$). Note next that - for s to be absorbing - at time $t-1$ (before the mutation) the inequality $\Pi^{t-1}(N_k^I(D)) \geq \Pi^{t-1}(N_k^I(C))$ has to hold for the defector k at the edge of the defector string and the inequality $\Pi^{t-1}(N_h^I(D)) \leq \Pi^{t-1}(N_h^I(C))$ for the cooperator h at the edge of the cooperator string. But at time $t+\tau$ we have that $\Pi^{t+\tau}(N_h^I(D)) > \Pi^{t-1}(N_k^I(D)) \geq \Pi^{t-1}(N_k^I(C)) > \Pi^{t+\tau}(N_h^I(C))$. Consequently the cooperator h will want to switch to defection and consequently also the remaining cooperators. On the other hand of course $X(s^D, s) > 1$ as a single cooperator will never be imitated. Now starting from a minimal $s-$tree we can simply redirect the arrow $s \rightarrow s^D$, as the transitions with least resistance are direct. This yields an s^D-tree with $\psi(s^D) < \psi(s)$. Of course it follows from the same argument that a transition from a state $s' \in \omega_2^{CD}$ to some state $s \in \omega_1^{CD}$ can occur after one tremble only and we can show that $\forall s' \in \omega_2^{CD}, \exists s \in \omega_1^{CD}$ s.t. $\psi(s) \leq \psi(s')$ (again redirecting the arrow $s \rightarrow s'$). Replicating this argument one can show that $\forall s^j \in \omega_j^{CD}, j \geq 2, \exists s^{j-1} \in \omega_{j-1}^{CD}$ s.t. $\psi(s^{j-1}) \leq \psi(s^j)$. It then follows that $\psi(s^D) < \psi(s), \forall s \in \omega^{CD}$ and thus that s^D is the unique stochastically stable state. ■

Proof of Proposition 3:

Proof. (i) First note that whenever $a \leq \overline{a} := \min_{s \in S} \widehat{a}_1(0, I, Z|\delta(s), \gamma(s))$ the only absorbing states are the monomorphic states s^D and s^C. Furthermore in this case a transition from s^C to s^D can occur after one action tremble only, i.e. $X(s^C, s^D) = 1$. For the reverse transition though one tremble does not suffice, as a single cooperator is never imitated. Consequently the unique stochastically stable state is s^D.

(ii) Next consider the case where $a > \overline{a}$, in which polymorphic absorbing states do exist. We will first show that s^C is not stochastically stable under this conditions, in particular we will show that there exists a state $s' \in \omega^{CD}$ s.t. $\psi(s') < \psi(s^C)$. Consider the state $s \in \omega_1^{CD}$ that is obtained from s^C via one single tremble by an agent to defection, i.e. where $X(s^C, s) = 1$. Such a state of course always exists as an initial tremble by any defector will be imitated by at least one other defector (possibly more) and the unperturbed process will converge to a polymorphic absorbing state. Now take a minimal s^C-tree and add the arrow $s^C \rightarrow s$. Then consider the old path from s to s^C. For some

s' on this path two trembles will be needed to reach the next state (as two neighboring defectors will never want to switch to cooperation). Cutting the arrow that leads away from this state s' yields an $s'-$tree with $\psi(s') < \psi(s^C)$.

Now we will show that s^D cannot be stochastically stable. Consider transitions from s^C to s^D. The minimal number of trembles needed for this transition is proportional to $\left\lfloor \frac{n+1}{\underline{\gamma}(0,Z,Z)} \right\rfloor$ as all cooperator strings of size $\underline{\gamma}(\cdot)$ have to be broken. On the other hand the number of trembles needed to reach s^C from s^D does not increase with the number of players n.(Note that for a transition from s^D to s^C first a fixed number of cooperators have to tremble to start off the transition and then a fixed number of remaining defectors have to tremble to reach s^C). Now take a minimal s^D-tree and consider the path from s^C to s^D . Starting from s^D redirect arrows until at some state s a cost of $X(s^D, s^C)$ is saved (compared to the old path). Add the arrow $s^D \to s$. This arrow will have a cost strictly smaller than $X(s^D, s^C)$. Consequently we have constructed an $s-$tree with $\psi(s) < \psi(s^D)$ where $s \in \omega^{CD}$. ■

Proof of Proposition 4:

Proof. (i) Consider first transitions from s^C to s^D. Assume one player in N_i^I trembles and chooses D. Then $\Pi(N_i^I(D)) - \Pi(N_i^I(C)) = 2Zc - \frac{Z(a+b)+2Z(I-1)a}{I} = \frac{2IZ(c-a)+Z(a-b)}{I}$. Player i will want to imitate action D according to decision rule (4) whenever $\frac{2IZ(c-a)+Z(a-b)}{I} > m(1-2\frac{1}{I+1})$ or equivalently whenever $m < \frac{(2I+1)[2IZ(c-a)+Z(a-b)]}{(2I-1)I} =: \underline{m}$. Clearly $\frac{\partial \underline{m}}{\partial Z} > 0$ and

$$\frac{\partial \underline{m}}{\partial I} = \frac{(4I(I+1)-1)[(2IZ(a-c)-Z(a-b)]}{I^2(2I-1)^2} < 0.$$

Consequently $\underline{m}$ is strictly decreasing in $I-Z$. Now whenever $m < \underline{m}$ a transition from s^C to s^D can occur after a single action tremble whereas for the reverse transition at least two trembles are necessary and thus the analysis from Proposition 2 applies.

(ii) Consider again transitions from s^C to s^D and assume that $\kappa_D < \frac{I+1}{2}$ players in N_i^I tremble simultaneously and choose D. The following inequalities hold: $\Pi(N_i^I(D)) \leq 2Zc$ and $\Pi(N_i^I(C)) \geq \frac{2Z\kappa_D b+(I+1-2\kappa_D)2Za}{I+1-\kappa_D}$ with equality if $\kappa_D = 1$. Consequently $\Pi(N_i^I(D)) - \Pi(N_i^I(C)) \leq \frac{2Z(I+1-\kappa_D)(c-a)-2Z\kappa_D(a-b)}{I+1-\kappa_D}$. A necessary condition for a transition after κ_D trembles in N_i^I to be possible (for i to imitate action D) is $\frac{2Z(I+1-\kappa_D)(c-a)-2Z\kappa_D(a-b)}{I+1-\kappa_D} > m(1-2\frac{\kappa_D}{2I+1})$ or equivalently

$$m < \frac{(2I+1)\,[2Z(I+1-\kappa_D)(c-a)-2Z\kappa_D(a-b)]}{(I+1-\kappa_D)\,(2I+1-2\kappa_D)}. \tag{5}$$

Denote this threshold $\widehat{m}^D(\kappa_D)$. Next consider the reverse transition from s^D to s^C. Assume that $\kappa_C < \frac{I+1}{2}$ players tremble simultaneously in N_i^I and choose C. The following inequalities hold: $\Pi(N_i^I(C)) \leq \frac{2[Z(\kappa_C-2)+1]a+2(2Z-1)b}{2Z\kappa_C}$ and $\Pi(N_i^I(D)) \geq \frac{2(2Z-1)c+[2Z(I+1-\kappa_C)-2(2Z-1)]d}{2Z(I+1-\kappa_C)}$. Consequently a necessary condition is

$$\left(\begin{array}{c}\frac{2[Z(\kappa_C-2)+1]a+2(2Z-1)b}{2Z\kappa_C}\\ -\frac{2(2Z-1)c+[2Z(I+1-\kappa_C)-2(2Z-1)]d}{2Z(I+1-\kappa_C)}\end{array}\right) > m(1-2\frac{\kappa_C}{2I+1})$$

or equivalently

$$m < \frac{\left(\begin{array}{c}(2I+1)2Z(I+1-\kappa_C)[2[Z(\kappa_C-2)+1]a+2(2Z-1)b]\\ -Z\kappa_C[2(2Z-1)c+[2Z(I+1-\kappa_C)-2(2Z-1)]\,d]\end{array}\right)}{(2I+1-2\kappa_C)Z^2(I+1-\kappa_C)\kappa_C}. \quad (6)$$

Denote this threshold by $\widehat{m}^C(\kappa_C)$.

(iii) Now substitute κ for κ_D in (5) and for κ_C in (6) and consider the continuous extension (to $\kappa \in \mathbb{R}^+$) of the functions $\widehat{m}^C(\kappa)$ and $\widehat{m}^D(\kappa)$. Both $\widehat{m}^C(\kappa)$ and $\widehat{m}^D(\kappa)$ are strictly increasing in κ, as can be easily verified. Furthermore $\widehat{m}^D(0) > 0$, $\widehat{m}^C(0) < 0$ and $\left|\partial\widehat{m}^C(\kappa)/\partial\kappa\right| > \left|\partial\widehat{m}^D(\kappa)/\partial\kappa\right| > 0, \forall\kappa \in \mathbb{R}$. We are interested in the point κ^* where $\widehat{m}^C(\kappa)$ and $\widehat{m}^D(\kappa+1)$ intersect, i.e. κ^* s.th. $\widehat{m}^C(\kappa^*) = \widehat{m}^D(\kappa^*+1)$. This intersection defines the level of conformism such that always one mutation more is required to start off a transition from s^C to s^D than for the reverse transition. Of course eventually we have to focus on $\kappa \in \mathbb{N}$. Denote $\overline{m} = \widehat{m}^D(\lceil\kappa^*\rceil)$ where $\lceil\kappa^*\rceil$ denotes the smallest integer larger than κ^*.Then whenever $\widehat{m}(\frac{I+1}{2}) > m > \overline{m}$ strictly less simultaneous mutations are needed to start off a transition from s^D to s^C than to start off the reverse transition.[19] But if $m > \widehat{m}(\frac{I+1}{2})$ either less or equally many mutations are needed to start off either transition. But it is easy to see that in the latter case (where m is huge) the cooperative state can be reached easier from the polymorphic states.

(iv) Now we will show that whenever $m > \overline{m}$ only s^C is stochastically stable. We have already seen that $X(s^D, s^C) < X(s^C, s^D)$ whenever $m > \overline{m}$. What about polymorphic states ? We have seen in the proof of Proposition 1 that all such states have an equal number of strings of cooperators and defectors. Furthermore cooperator-strings are at least as long as defector-strings. But then of course even if additional mutations are needed for all agents in the circle to imitate the mutant less such mutations will be needed for a transition from s to s^C then from s to s^D. Consequently $X(s, s^C) \leq X(s, s^D), \forall s \in \omega^{CD}$ will always hold. Furthermore it should be clear that $X(s, s^C) \leq X(s^C, s), \forall s \in \omega^{CD}$. Consequently it follows that whenever $m > \overline{m}$ the state s^C has mimimal stochastic potential.

(v) Finally we will show that $\overline{m}$ is strictly decreasing in I. It is clear that sign $\left[\frac{\partial\overline{m}}{\partial I}\right]$ = sign $\left[\frac{\partial\kappa^*}{\partial I}\right]$ as $\widehat{m}^D(\kappa)$ is increasing in κ. The latter derivative can be computed using the implicit function theorem as

$$\frac{\partial\kappa^*}{\partial I} = -\frac{\partial(\widehat{m}^C - \widehat{m}^D)/\partial I}{\partial(\widehat{m}^C - \widehat{m}^D)/\partial\kappa}_{|\kappa=\kappa^*}. \quad (7)$$

[19]In fact one could also define κ^- such that $\widehat{m}^C(\kappa^-) = \widehat{m}^D(\kappa^- - 1)$ and $\underline{m}^{new} = \widehat{m}^D(\lfloor\kappa^-\rfloor)$. Of course $\kappa^- < \kappa^*$. Whenever $m < \underline{m}^{new}$ transitions to s^D need strictly less simultaneous mutations than the reverse transitions.

It is clear that the denominator of this expression is strictly positive. Furthermore we have seen that $\partial \widehat{m}^D / \partial I < 0$ and we have

$$\frac{\partial \widehat{m}^C}{\partial I} = \frac{(c-d)(2Z-1)}{Z(I+1-\kappa)} > 0.$$

Consequently the numerator of (7) is strictly positive and it follows that $\frac{\partial \overline{m}}{\partial I} < 0$. ■

Proof of Proposition 5:

Proof. The proof follows from the proof of Proposition 4 by substituting $I = Z$ into the relevant conditions. Now $\widehat{m}^D(\kappa_D)$ is given by

$$\widehat{m}^D(\kappa_D) = \frac{2I(2I+1)[c(\kappa_D - 1) - a + (c-a)I + b\kappa_D]}{(2I+1-2\kappa_D)(I+1-\kappa_D)}$$

and $\widehat{m}^C(\kappa_C)$ by

$$\widehat{m}^C(\kappa_C) = I^{-1}\left[\begin{array}{c} \frac{a(I(\kappa_C - 2)+1)+b(2I-1)}{\kappa_C} - \\ \frac{d(I(I-1-\kappa_C)+1)+c(2I-1)}{I+1-\kappa_C} \end{array}\right].$$

■