

Envy can promote more equal division in alternating-offer bargaining

Kohler, Stefan

Charité University Medical Center, Berlin

10 September 2012

Online at https://mpra.ub.uni-muenchen.de/40761/ MPRA Paper No. 40761, posted 12 Sep 2012 12:50 UTC

Envy can promote more equal division in alternating-offer bargaining

Stefan Kohler*

Abstract

Bargainers in an open-ended alternating-offer bargaining situation may perceive envy, a utility loss caused by receiving the smaller share that is modeled in some social preferences in addition to self-interest. I extend Rubinstein (1982)'s original solution of the bargaining problem for two self-interested bargainers to this strategic situation. Bargainers still reach agreement in the first period and their bargaining shares increase in the strength of their own envy. As both bargainers' envy diminishes, the agreed partition converges to the Rubinstein division. If equally patient bargaining parties exhibit similar envy, then the agreed partition is tilted away from the Rubinstein division towards the equal division. Notably, the potential sensation of envy also boosts the share of the eventually envy-free party who leaves the bargaining with the larger share under the agreed partition. This gain in bargaining strength through envy can result in a bargaining outcome that is more unequal than predicted by the Rubinstein division.

Keywords: alternating offers, bargaining, bargaining power, behavioral economics, envy, equity, fairness, inequality aversion, negotiation, social preferences *JEL classifications:* C72, C78, C91, D03, D31, D63

1 Introduction

Bargaining encounters are a frequent interaction in economic life. They share the feature that a mutually beneficial outcome can be realized if the parties participating in the bargaining process reach agreement. Rubinstein (1982) proposed a seminal framework to investigate strategic behavior in open-ended alternating-offer bargaining situations. It predicts a unique outcome of this bargaining process, reveals the interdependence of bargainers' intertemporal strategic considerations and has been widely used to forecast bargaining outcomes of self-interested parties. The bargaining problem was narrowed down by Rubinstein to the following situation and question: "Two individuals have before them several possible contractual agreements. Both have interests in reaching agreement but their interests are not entirely identical. What 'will be' the agreed contract, assuming that both bargainers behave rationally?" [...] "Two other problems that may be asked about the bargaining situation, namely: (A) the positive question - what is the agreement reached in practice; (B) the normative question - what is the just agreement" were left outside the original investigation. Thus, its contribution is a selection

^{*}September 10, 2012. Institute for Social Medicine, Epidemiology and Health Economics, Charité University Medical Center, Free University and Humboldt University of Berlin, Germany; *Email:* stefan.kohler@charite.de, *Homepage:* http://epidemiologie.charite.de. I have benefited from helpful and constructive comments by Karl Schlag, Pascal Courty, Simon Gächter, Jordi Brandts and anonymous reviewers. I gratefully acknowledge the financial support provided by the European University Institute and the German Academic Exchange Service at an early stage of this research.

mechanism to decide which of numerous individually rational and Pareto optimal contracts will be agreed. As the framework was not intended as a positive theory, unsurprisingly, subsequent bargaining experiments confirmed deviations between the theoretical predictions and observed outcomes.

Building on Rubinstein's plausible and instructive model, this paper studies the additional impact of a psychological element, suggested, for instance, by Von Neumann & Morgenstern (1944), on the intertemporal utility maximization in the bargaining of two rational bargainers. Ample evidence suggests that at least some people show regard for others, which includes aspects of envy (e.g., Camerer 2003; Cooper & Kagel n.d.; De Bruyn & Bolton 2008; Kohler 2012c; Smith 2008; Zwick et al. 1992). Neural Correlates of envy were shown in functional magnetic resonance imaging studies that associated stronger envy with stronger anterior cingulate cortex activation Takahashi et al. (2009). I extend Rubinstein's solution of the alternating-offer bargaining problem to envious bargainers. Envy is modeled as a utility loss from receiving the smaller share by assuming an asymmetric preference of linear inequality aversion. I show that there is an unique subgame perfect equilibrium, in which bargainers agree in the first period (as in Rubinstein 1982), and calculate the equilibrium payoffs. Deriving the subgame perfect equilibrium stepwise by backwards induction uncovers how either bargainer's envy affects the strategic decision-making of the bargainers over time and, thus, codetermines the agreed contract. The equilibrium payoffs depend on the envy parameter of each bargainer and their discount factors. If bargainers have the same envy and discount factor, then the difference between their equilibrium payoffs is smaller than the difference in equilibrium payoffs predicted by Rubinstein. More envious bargainers can ensure higher shares. If the bargainer who has the first move is sufficiently more envious than the other bargainer, then he obtains a share that is higher than in the Rubinstein solution, but this advantage can be offset by impatience expressed though a lower discount factor in the model. Notably, the relative strengths of envy in comparison to bargainers' self-interest impact the alternating-offer bargaining outcome through two channels: First, there is a direct effect of envy, imposed by its weight in the utility function, due to which the disadvantaged bargainer directly suffers from an unequal division. Second, there is an indirect effect of envy since the credible threat of realizing a fairer allocation in the future, in case of disagreement, strengthens the bargaining position of the envious bargainer, who receives the smaller share, as well as the bargaining position of the envy-free bargainer asserting the large share. As in the Rubinstein solution, the more patient the bargainers, the smaller the difference between their payoffs. If bargainers are free of envy then bargaining proceeds as predicted by Rubinstein.

Section 2 reviews evidence of envy and fairness in infinite horizon alternating-offer bargaining and related theory. Section 3 introduces the bargaining problem with envious bargainers. In section 4, I derive the optimal bargaining behavior and the selected contract. Section 5 concludes.

2 Related literature

Envy may refer to interpersonal comparisons of well-being in its colloquial use or to an intrapersonal comparison of different consumption bundles, for instance, in the fair division literature (see, Herreiner & Puppe 2009). Herreiner and Puppe distinguish interpersonal (the maximin principle or inequality aversion) and intrapersonal criteria (envy freeness) by experiments on fair division problems. They find a limited role of intrapersonal comparisons in the bargaining and evidence in support of "inequality aversion as an empirically relevant fairness criterion", concluding that interpersonal criteria seem to be "deeply ingrained in human behavior". Weg et al. (1990) and Zwick et al. (1992) assess the predictive accuracy of the Rubinstein solution with respect to variations in the discount factor and uncertainty about the bargaining horizon, respectively. Weg et al. limit the number of trials until agreement to 20 rounds, resulting in a termination of 7 out of 324 bargaining games in their experiment. First period demands were accepted in 48.6 to 75 percent of the games, but agreements differed from those predicted by the Rubinstein and models that invoke "notions of equity or equality accounted for a substantial percentage of the agreements". Zwick et al.'s experiment was motivated by the formal similarity between discounting and the probability of continuing the bargaining. In the experiment, bargaining could be randomly terminated after each period. The discount factor is interpreted as a probability of continuing the game after rejection of a proposal. Their results reject both the subgame perfect equilibrium and equal division solutions. First period agreement increased with the probability of termination and, in all cases, mean demands were closer to the equal division than the Rubinstein division. Other experiments on infinite horizon bargaining test the Rubinstein solution for the case in which bargainers have a fixed cost of bargaining (e.g., Rapoport et al. 1990). Several studies were conducted changing Rubinstein's original framework. For instance, Binmore et al. (1991, 1989) impose outside options or an optional or forced breakdown and document the sensitivity of subjects to the bargaining structure. A survey of infinite horizon bargaining theory including the aforementioned and further classic experiments is provided by Weg & Zwick (1999).

Binmore et al. (2007) offer another experiment to test Rubinstein's bargaining model with bargainers who face unequal discount factors. The computer interrupted games in which proposals keep being refused after 3 to 7 rejections. Considering that the Rubinstein model is highly stylized, they test a perturbed version that takes into account "some of the peculiarities of human psychology." The perturbed model predicts the sign of deviations in the opening proposal from the final undiscounted agreement in the previous period. Learning, rationality and fairness are all significant in determining the outcome. Binmore et al. (2007) confirm that subjects tend to exploit the first mover advantage and that the final outcomes are shifted away from the Rubinstein prediction towards a more equal division. They conclude that "the underlying structure of Rubinstein's solution to the bargaining problem holds up unexpectedly well [while] the precise form of the Rubinstein solution is fragile [and that] future research [...] needs to focus on the nature of the psychological quirks that perturb Rubinstein's basically sound model in real bargaining situations."

Focusing on finite horizon alternating-offer bargaining, Bolton (1991) incorporated money and fairness (relative money) into bargainer utility functions.¹ His comparative model is consistent with five enumerated bargaining regularities, including partitions that deviate from the narrow self-interest equilibrium in the direction of the equal division. In the Rubinstein bargaining problem, the influence of regard for others is investigated, for instance, by Driesen et al. (2012) under the assumption of loss averse bargainers. The reference point associated with loss

¹Ståhl (1972) proposed a model similar to Rubinstein's framework for finite horizon bargaining problems. The majority of experiments tests the former theory.

aversion equals the highest proposed share turned down by the opponent in the past. Agreement is immediate and, in equilibrium, the bargainers' strategies depend only on the current reference points. Higher loss aversions leads to a lower share of the surplus in equilibrium. Loss aversion with a fixed reference point at the equal division becomes an alternative interpretation of envy and vice versa.² Also Miettinen (2010) studies open-ended alternating-offer bargaining with bargainers who have history dependent reciprocity preferences. Preferences in the highly stylized model only depend on the history of rejected proposals, rather than also on the bargainers' beliefs. The framework is related to Li (2007), Fershtman & Seidmann (1993) or Compte & Jehiel (2004), which assume that the bargaining history itself influences the bargainers' preferences. With reciprocal aspirations, the proposing bargainer faces a trade-off between the share he receives if the proposed partition is accepted and the worsened bargaining position due to the increase in the opponent's aspiration if the proposed share is rejected. Miettinen's model illustrates that endogenous aspirations on their own do not imply delay, but the bargainer who starts the bargaining can become disadvantaged when both bargainers are sufficiently patient due to the reciprocal motivations' inverse relation to previous proposals.

Relaxing the assumption of complete information, Kohler (2012b) studies uncertainty about the strength of the second bargainer's envy in a finite alternating-offer bargaining problem. Similar to the influence of envy in infinite horizon bargaining, the agreed contract can contain an equal division with or without delay, which results from the need to incur costs in order to credibly signal a psychological element in the preferences. In open-ended alternating-offer bargaining with complete information, envy, qualitatively, has the opposite effects of low guilt that are studied in a model of similar first and second mover preferences by Kohler (2012a). Montero (2008) studies bargaining when there is competition for bargaining partners. In a model with irrevocable choice of partner, neither altruism nor spite is unambiguously beneficial. In a theoretical analysis of bargaining games, in which unanimity is not required, Montero (2007) shows two effects of inequality aversion: The payoff division inside a coalition can be more unequal once responders prefer to accept a lower share rather to the risk of being left out. As bargainers become more impatient, the advantage of the proposer can be reduced; i.e., inequality aversion may also revers the effect of impatience.

3 Bargaining model

Two bargainers $i, j \in \{b, s\}$, called seller and buyer, have to reach an agreement on the partition of a surplus of size one which depreciates after any disagreement. Bargaining takes place at periods of time t = 1, 2, ..., T. Depreciation is modeled by assigning discount factors δ_s and δ_b to the two bargainers. By naming a partition $p_t \in (0, 1]$ in odd periods, the seller demands share p_t and offers share $(1 - p_t)$ that the buyer can accept or reject. In even periods, the buyer proposes a partition p_t to the seller that he can accept or reject. If a partition is accepted the game ends in period *T*. This bargaining outcome is denoted (p_T, T) .

Assuming complete information in this bargaining problem, Rubinstein (1982) has shown

²The model of loss aversion represents bargaining with envy as the special case with reference point $r_i = 0.5$ and loss aversion coefficient $\lambda_i = 2\alpha_i$ under the additional assumption of a common discount factor δ . The bargaining model with envy evolved separately adding an alternative interpretation to the corresponding equilibrium outcome.

the existence of a unique subgame perfect equilibrium (SPE) under generic preference assumptions.³ For preferences $u_i(x_i) = x_i$, where utility is derived from own payoff x_i , Rubinstein derived an explicit solution, in which the seller proposes and the buyer accepts partition $p^* = \frac{1-\delta_s}{1-\delta_s\delta_b} \in (0,1]$ in period 1. The equilibrium outcome is supported by the bargainers' similar strategies: Bargainer *i* always demands the equilibrium share p^* , when it his turn to make a proposal, otherwise accepts any share equal or greater than $\delta_i p^*$ and refuses any smaller share. The demand of p^* is the highest share that is accepted by the other bargainer *j*. Bargainer *i* cannot gain by asking a lower share, for it too will be accepted. Stipulating a higher (and rejected) share and waiting to accept bargainer *j*'s counteroffer in the next period hurts bargainer *i* as $\delta_i (1 - p^*) = \delta_i \delta_j p^* < p^*$.

This paper harnesses the generality of Rubinstein's framework and investigates the strategic behavior of bargainers who care, to some extend, about relative as well as absolute payoff in the described bargaining process. Relative payoff hereby means bargainers compare their own benefit x_i from accepting a certain partition to the benefit of the other bargainer x_j , and put weight $\alpha_i \ge 0$ on the difference whenever the own benefit is lower. This relative concern is interpreted as envy. Explicitly, I assume that the utility function of the bargainers is given by:

$$u_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\}$$

These preferences are an asymmetric version of inequality aversion as originally put forward in Fehr & Schmidt (1999) and extended by altruism in Kohler (2011). Inequality aversion consistently predicts a rich set of stylized experimental behavior (e.g., Cooper & Kagel n.d.; Fehr & Schmidt 1999). Throughout, $u_s(p_t) := u_s(p_t, 1 - p_t)$ denotes the seller's utility and $u_b(p_t) := u_b(1 - p_t, p_t)$ the corresponding buyer's utility if a proposed partition p_t is accepted in period t.

4 Subgame perfect equilibrium

Proposition 1. The alternating-offer bargaining problem with envious and discounting bargainers has a unique SPE. If $\delta_s \leq \frac{1+2\alpha_b}{2+2\alpha_b-\delta_b}$ and $\delta_b \leq \frac{1+2\alpha_s}{2+2\alpha_s-\delta_s}$, then the seller immediately receives:

$$p^{*} = \frac{\left(1 + 2\alpha_{s}\right)\left(1 + \alpha_{b}\right) - \delta_{b}\left(1 + \alpha_{s}\right)}{\left(1 + 2\alpha_{s}\right)\left(1 + 2\alpha_{b}\right) - \delta_{s}\delta_{b}}$$

The proof of proposition 1 is based on Shaked & Sutton (1984) who applied backwards induction in a truncation of the infinite horizon game: The beginning of the infinite horizon game is equal to its subgame in the third round, should it be reached. In odd periods, the seller

³(*i*) Trade is desirable: $\frac{\partial u(x)}{\partial x} > 0$. This assumption allows for altruistic but can exclude envious bargainers. As shown in the paper, Rubinstein's result of a unique SPE that includes immediate acceptance of the initial proposal holds for envious bargainers as well (see also Montero 2008), (*ii*) Time is valuable: $\delta < 1$, (*iii*) Continuity: $\lim_{x\to\overline{x}} u(x) = \lim_{\overline{x}\to x} u(x)$, (*iv*) Stationarity: preferences are time independent, (*v*) The larger the share the more compensation a bargainer needs for a delay of one period to be immaterial to him. Strategies are said to constitute a SPE if, in every subgame, the strategies relating to that subgame form a Nash equilibrium. In a SPE, a bargainer will agree to a proposal if it offers at least as much as he will obtain in the future given the strategies of both bargainers. Rubinstein (1982) states the precise definition.

is proposing and then bargainers alternate in making subsequent offers until an agreement is reached.

Proof. Suppose the above strategies induce a backwards induction outcome $(p^*, 1)$ of the game as a whole. It is possible to use the partition p^* in the subgame starting in the third period assuming it was reached and, then, to work back to the first period. In the backward induction outcome of the whole game the seller will propose $p_1 = \pi (p_3)$ in period 1 and the buyer will accept. If π (.) is a monotone function then the equilibrium partition is uniquely defined by $p^* = \pi (p^*)$. In order to determine π (.), the periods in which envy influences the bargainers' decisions need to be identified. For now, assume the payoff distribution uniquely favors the proposing bargainer who demands at least half of the surplus, i.e., $p_1 \ge 0.5$, $p_2 \le 0.5$, $p_3 \ge 0.5$. The existence of is advantage of the proposing bargainer is established after deriving the equilibrium outcome.

The period 3 subgame begins with a successful proposal of partition $p_3 \in [0, 0.5]$ by an envy-free seller. Consequently, the lowest share $p_2 = \frac{\alpha_s(1+2\alpha_b)+\delta_s(1+\alpha_b)-\delta_s\delta_b}{(1+2\alpha_s)(1+2\alpha_b)-\delta_s\delta_b}$ that is accepted by the seller in period 2 gives him the equivalent of his outside option, the discounted period 3 utility. Similarly, the highest share $p_1 = \frac{1+\alpha_b}{1+2\alpha_b} - \delta_b \frac{1+\alpha_s-\delta_sp_3}{(1+2\alpha_s)(1+2\alpha_b)}$ that is accepted by the envy-perceiving buyer in period 1 gives him the equivalent of his outside option, the discounted period 2 utility. Indifferent bargainers are assumed to accept the proposed share. As $u_b(p_2) \ge \delta u_b(p_3)$ and $u_s(p_1) \ge \delta u_s(p_2)$, the buyer and seller prefer proposing the just agreeable shares p_2 and p_1 to disagreement with the subsequent counteroffer.

Since the game in period 3 is identical to the game in period 1, the unique fixed point $p^* := p_1(p_3) \equiv p_3$ defines the equilibrium partition:

$$p^* = \frac{\left(1 + 2\alpha_s\right)\left(1 + \alpha_b\right) - \delta_b\left(1 + \alpha_s\right)}{\left(1 + 2\alpha_s\right)\left(1 + 2\alpha_b\right) - \delta_s\delta_b}$$

As only p_2 and p_1 maximize the utility of the bargainer proposing the partition, there is no other SPE.

The advantage of the proposing bargainer on the equilibrium path requires $p_2 \leq 0.5$, which implies $\delta_s \leq \frac{1+2\alpha_b}{2+2\alpha_b-\delta_b}$, and $p^* \geq 0.5$, which implies $\delta_b \leq \frac{1+2\alpha_s}{2+2\alpha_s-\delta_s}$. This completes the derivation of the Rubinstein result for a model with envy concerned bargainers.

The conditions $\delta_s \leq \frac{1+2\alpha_b}{2+2\alpha_b-\delta_b}$ and $\delta_b \leq \frac{1+2\alpha_s}{2+2\alpha_s-\delta_s}$ also ensure a unique sign of the partial derivatives of the equilibrium partition $\frac{\partial p^*}{\partial \alpha_s}$, $\frac{\partial p^*}{\partial \delta_s} \geq 0$ and $\frac{\partial p^*}{\partial \alpha_b}$, $\frac{\partial p^*}{\partial \delta_b} \leq 0$ with respect to the seller's and buyer's envy and discount factor, respectively. If bargainers value time similarly but differ in their strengths of envy, then the partial derivatives of $p^*(\alpha_s, \alpha_b, \delta)$ with respect to envy have these unique signs unconditionally and the partial derivatives with respect to the common discount factor is negative. The partial derivatives of the equilibrium partition $p^*(\alpha, \delta) = \frac{1+\alpha}{1+2\alpha+\delta}$ with respect to common envy or a common discount factor $\frac{\partial p^*}{\partial \alpha}$, $\frac{\partial p^*}{\partial \delta} \leq 0$ are negative. These results are derived in the appendix.

Note that the assumption $p^* \ge 0.5$ and $p_2 \le 0.5$ under which the equilibrium is derived is always true for a common discount factor as $\frac{\partial p^*}{\partial \alpha_s} \ge 0$, $\frac{\partial p^*}{\partial \alpha_b}$, $\frac{\partial p^*}{\partial \delta} \le 0$ and, hence, the infimum $\lim_{\alpha_b\to\infty} p^*(0,\alpha_b,0)$ of $p^*(\alpha_s,\alpha_b,\delta)$ is 0.5. Similarly, $p_2 \le 0.5$ as $\frac{\partial p_2}{\partial \alpha_b} \le 0$, $\frac{\partial p_2}{\partial \alpha_s}$, $\frac{\partial p_2}{\partial \delta} \ge 0$ and, hence, the supremum $\lim_{\alpha_s\to\infty} p_2(\alpha_s,0,1)$ of $p_2(\alpha_s,\alpha_b,\delta)$ equals 0.5.

5 Conclusion

Bargainers may incur a loss of utility if they receive the smaller share of a surplus to be divided in a bargaining process. Loss aversion with respect to disadvantageous deviations from the equal division can be interpreted as envy. Envy reinforces the bargaining position of each bargainer in open-ended alternating-offer bargaining with two parties: The non-credible threat of a non-envious bargainer to reject unequal contracts becomes credible in the case with envy. If the two bargainers are similarly envious, then the partition agreed between envious bargainers departs from the Rubinstein solution converging towards an equal division. Therefore, being symmetrically envious is not equivalent to being without envy in this strategic context.

The signs of the partial derivatives of the equilibrium partition imply: First, the more envious the bargainers who share similar preferences, the more fair the immediately agreed outcome of the bargaining in comparison to the Rubinstein division. Second, ceteris paribus, each bargainer's share increases in his own envy and patience. Taken separately the bargainers' distributional concerns have opposite effects. If the bargainer, who starts the bargaining, is substantially more envious than the second bargainer, he can realize a share greater than predicted in the Rubinstein outcome because the threat to reject uneven counteroffers primarily increases his bargaining position. Third, also in the extended equilibrium outcome the agreement is immediate. If bargainers are equally patient, like in the Rubinstein solution, the higher a common discount factor the more equal will be the offer. As both bargainers' discount factors approach unity, bargainers divide the surplus equally. Taking the limits of the equilibrium partition of equally discounting bargainers shows that the equilibrium partition is strictly greater than half and that, for common envy and discounting, the equilibrium partition lies between an equal division and the Rubinstein division.⁴

Envious bargainers agree on the equal division if they are both perfectly patient but also if the first bargainer's marginal utility from reducing a disadvantageous situation by one increment scaled by his impatience equals the second bargainer's utility of receiving this additional increment in the next period scaled by his impatience. In other words, if the rate of the first bargainer's marginal utility from reducing an disadvantageous situation by one increment and the second bargainer's utility of receiving this additional increment in the next period equals his relative impatience. Equally patient and envious bargainers split equally if the first bargainer's marginal utility from reducing a disadvantageous situation by one increment equals the second bargainer's utility of receiving this additional increment in the next period.⁵ As in a heterogeneous population of envious and non-envious bargainers the envious types realize higher shares in the bargaining than their non-envious counterparts, each bargaining party prefers to bargain with a non-envious type.

⁴The limits of the equilibrium partition are derived in appendix.

⁵Equal division in equilibrium $p^*(\alpha_s, \alpha_b, \delta_s, \delta_b) = 0.5$ implies $\frac{1+2\alpha_s}{1-\delta_s} = \frac{\delta_b}{1-\delta_b}$ or $\frac{1+2\alpha_s}{\delta_b} = \frac{1-\delta_s}{1-\delta_b}$. For equal bargainers $p^*(\alpha, \delta) = 0.5$ implies $1 + 2\alpha_s = \delta_b$.

6 Appendix

6.1 Partial derivatives and limits

6.1.1 Individual envy and discounting

The equilibrium partition $p^*(\alpha_s, \alpha_b, \delta_s, \delta_b)$ is given by

$$p^{*} = \frac{\left(1 + 2\alpha_{s}\right)\left(1 + \alpha_{b}\right) - \delta_{b}\left(1 + \alpha_{s}\right)}{\left(1 + 2\alpha_{s}\right)\left(1 + 2\alpha_{b}\right) - \delta_{s}\delta_{b}} =: \frac{N}{D}$$

The partial derivatives of the equilibrium partition are given by

$$\begin{split} \frac{\partial p^*}{\partial \delta_s} &= \frac{\delta_b}{D^2} \left[\alpha_s \left(1 + \alpha_b \right) + \left(1 + \alpha_b - \delta_b \right) \left(1 + \alpha_s \right) \right] \ge 0 \\ \frac{\partial p^*}{\partial \delta_b} &= -\frac{1 + 2\alpha_s}{D^2} \left[\left(1 + \alpha_s \right) \alpha_b + \left(1 + \alpha_s - \delta_s \right) \left(1 + \alpha_b \right) \right] \le 0 \\ \frac{\partial p^*}{\partial \alpha_s} &= \frac{\delta_b}{D^2} \left[\left(1 - \delta_s \right) \left(1 + 2\alpha_b \right) - \delta_s \left(1 - \delta_b \right) \right] \ge 0 \text{ if } \delta_s \le \frac{1 + 2\alpha_b}{2 + 2\alpha_b - \delta_b} \\ \frac{\partial p^*}{\partial \alpha_b} &= -\frac{1 + 2\alpha_s}{D^2} \left[\left(1 - \delta_b \right) \left(1 + 2\alpha_s \right) - \delta_b \left(1 - \delta_s \right) \right] \le 0 \text{ if } \delta_b \le \frac{1 + 2\alpha_s}{2\alpha_s - \delta_s + 2} \end{split}$$

The respective signs follow from evaluating the derivatives. The limits of the equilibrium offer $p^*(\alpha_s, \alpha_b, \delta_b, \delta_s)$ are given by $\lim_{\alpha_s \to \infty} p^* = \frac{(1+2\alpha_b)+(1-\delta_b)}{2(1+2\alpha_b)} \in [0.5, 1]$, $\lim_{\delta_s \to 1} p^* = \frac{(1+2\alpha_s)(1+\alpha_b)-\delta_b(1+\alpha_s)}{(1+2\alpha_s)(1+2\alpha_b)-\delta_b} \in [0.5, 1]$, $\lim_{\alpha_b \to \infty} p^* = 0.5$, $\lim_{\delta_b \to 1} p^* = \frac{(1+2\alpha_s)(1+\alpha_b)-(1+\alpha_s)}{(1+2\alpha_s)(1+2\alpha_b)-\delta_s} \in [0, 0.5]$; $\lim_{\alpha_s, \alpha_b \to 0} p^* = \frac{1-\delta_s}{1-\delta_s\delta_b} \in [0, 1]$, $\lim_{\alpha_s, \alpha_b \to \infty} p^* = 0.5$ and $\lim_{\delta_s, \delta_b \to 0} p^* = \frac{1+\alpha_b}{1+2\alpha_b} \in [0.5, 1]$, $\lim_{\delta_s, \delta_b \to 1} p^* = 0.5$. The limiting values follow from evaluating the limits or $\lim_{\alpha_s \to a} \lim_{\alpha_b \to a} p^*$, $\lim_{\alpha_b \to a} \lim_{\alpha_s \to a} p^*$ for $a \in \{0, \infty\}$ and $\lim_{\delta_s \to d} \lim_{\delta_b \to d} p^*$, $\lim_{\delta_b \to d} \lim_{\delta_s \to d} p^*$ for $d \in \{0, 1\}$, respectively.

6.1.2 Individual envy and common discounting

The equilibrium partition $p^*(\alpha_s, \alpha_b, \delta)$ is given by

$$p^{*}=rac{\left(1+2lpha_{s}
ight)\left(1+lpha_{b}
ight)-\left(1+lpha_{s}
ight)\delta}{\left(1+2lpha_{s}
ight)\left(1+2lpha_{b}
ight)-\delta^{2}}=:rac{N}{D}$$

The partial derivatives of the equilibrium partition are given by

$$\begin{aligned} \frac{\partial p^*}{\partial \delta} &= -\frac{1}{D^2} \left[\left(1 + \alpha_s \right) \left(1 + 2\alpha_s \right) \left(1 + 2\alpha_b \right) + \delta^2 \left(1 + \alpha_s \right) - 2\delta \left(1 + 2\alpha_s \right) \left(1 + \alpha_b \right) \right] \le 0 \\ \frac{\partial p^*}{\partial \alpha_s} &= \frac{1}{D^2} \delta \left(1 - \delta \right) \left(1 + 2\alpha_b - \delta \right) \ge 0 \\ \frac{\partial p^*}{\partial \alpha_b} &= -\frac{1}{D^2} \left(1 - \delta \right) \left(1 + 2\alpha_s \right) \left(1 + 2\alpha_s - \delta \right) \le 0 \end{aligned}$$

The optimal partition $p_2(\alpha_s, \alpha_b, \delta)$ to be offered in bargaining period 2 of the truncated game

is $p_2 = \frac{\delta p^* + \alpha_s}{1 + 2\alpha_s}$. Its partial derivatives and their respective signs are

$$\begin{aligned} \frac{\partial p_2}{\partial \delta} &= \frac{p^*}{1 + 2\alpha_s} \ge 0\\ \frac{\partial p_2}{\partial \alpha_b} &= \frac{\delta}{1 + 2\alpha_s} \frac{\partial p^*}{\partial \alpha_b} \le 0\\ \frac{\partial p_2}{\partial \alpha_s} &= \frac{\delta \frac{\partial p^*}{\partial \alpha_s} \left(1 + 2\alpha_s\right) + 1 - 2\delta p^*}{\left(1 + 2\alpha_s\right)^2} \ge 0 \end{aligned}$$

The respective signs follow from evaluating the derivatives. The limits of the equilibrium offer $p^*(\alpha_s, \alpha_b, \delta)$ are given by $\lim_{\delta \to 0} p^* = \frac{1+\alpha_b}{1+2\alpha_b} \in [0.5, 1)$, $\lim_{\delta \to 1} p^* = \frac{\alpha_s + \alpha_b + 2\alpha_s \alpha_b}{2\alpha_s + 2\alpha_b + 4\alpha_s \alpha_b} = 0.5$; $\lim_{\alpha_s \to 0} p^* = \frac{1+\alpha_b - \delta}{1+2\alpha_b - \delta^2} \in [0.5, 1)$, $\lim_{\alpha_s \to \infty} p^* = \frac{1+\alpha_b - 0.5\delta}{1+2\alpha_b} \in [0.5, 1)$; $\lim_{\alpha_b \to 0} p^* = \frac{(1+\alpha_s)(1-\delta)+\alpha_s}{1+2\alpha_s - \delta^2} \in (0.5, 1)$, $\lim_{\alpha_s \to \infty} p^* = \frac{1+\alpha_b - 0.5\delta}{1+2\alpha_b} \in [0.5, 1)$; $\lim_{\alpha_b \to 0} p^* = \frac{(1+\alpha_s)(1-\delta)+\alpha_s}{1+2\alpha_s - \delta^2} \in (0.5, 1)$, $\lim_{\alpha_s, \alpha_b \to \infty} p^* = \frac{1+2\alpha_s}{2(1+2\alpha_s)} = 0.5$ and $\lim_{\alpha_s, \alpha_b \to 0} p^* = (1+\delta)^{-1}$, $\lim_{\alpha_s, \alpha_b \to \infty} p^* = 0.5$. The limiting values follow from evaluating the limits.

6.1.3 Common envy and discounting

The equilibrium partition $p^*(\alpha, \delta)$ is given by

$$\frac{1+\alpha}{1+2\alpha+\delta} =: \frac{N}{D}$$

The partial derivatives of the equilibrium partition are given by

$$\frac{\partial p^*}{\partial \alpha} = -\frac{1}{D^2} (1 - \delta) \le 0$$
$$\frac{\partial p^*}{\partial \delta} = -\frac{1}{D^2} (1 + \alpha) \le 0$$

The respective signs follow from evaluating the derivatives. The limits of the equilibrium offer $p^*(\alpha, \delta)$ are given by $\lim_{\delta \to 0} p^* = \frac{1+\alpha}{1+2\alpha+\epsilon} \in [0.5, 1)$, $\lim_{\delta \to 1} p^* = 0.5$ and $\lim_{\alpha \to 0} p^* = \frac{1}{1+\delta} \in [0.5, 1]$, $\lim_{\alpha \to \infty} p^* = 0.5$. The limiting values follow from evaluating the limits.

6.2 Model calibration

Bargaining experiments that try to implement infinite horizon style bargaining are forced to terminate after a finite number of bargaining periods, typically reached only exceptionally. As players are aware of this condition, an alternative approach introduced by Zwick et al. (1992) implements the common discount factor as a fixed probability of exogenous breakdown rather than as cost of delay. Mean demands in Zwick et al.'s experiment were closer to the equal split than to the Rubinstein division. In games with continuation probability of $\frac{9}{10}$, $\frac{2}{3}$ and $\frac{1}{6}$, the Rubinstein division allocates 52.6, 60.00 and 85.71 percent of the surplus to be divided to the proposer, respectively. Previous studies conducted by Binmore et al. (1989) and Weg et al. (1990) examined bargaining with comparable discount factors. I use the results of all three studies (as summarized by Zwick et al.) to impute the average strength of envy α in the model of common envy and discounting. According to the SPE solution, first period demands of the Rubinstein division should be accepted. As delay occurs but is not studied, I report envy values from model calibrations based on the partitions observed in the first and final periods of the

		Treatment							
	$\delta =$	$\delta = \frac{9}{10}$		$\delta = \frac{2}{3}$			$\delta = \frac{1}{6}$		
	BSS	ZRH		WRS	ZRH	-	WRS	ZRH	
First period partition (%)	55.3	54.2		52.02	53.30		57.33	54.84	
	(0)	(0)		(3.292)	(1.692)		(2.258)	(3.721)	
Final period partition (%)	50.2	50.1		51.22	50.82		54.77	53.61	
	(11.6)	(24.1)		(5.997)	(9.329)		(3.784)	(5.187)	

Table 1: Mean first period and accepted final period partition, imputed envy

Notes: Binmore et al. (1989) (BSS), Weg et al. (1990) (WRS), Zwick et al. (1992) (ZRH). δ denotes the discount factor (BSS, WRS) or a probability of continuation (ZRH). Imputed strength of envy α in parentheses. No envy is the border solution when non-admitted negative envy values are imputed. Rubinstein division (%): 52.6, 60.00, 85.71.

Figure 1: Observed and predicted partition at different discount factors



Notes: Circles indicate first period partitions. Boxes indicate final period partitions. Discount factor δ from top to bottom $\frac{1}{6}$, $\frac{2}{3}$ and $\frac{9}{10}$.

experiments (table 1).

The strength of envy varies approximately from 2 to 5 in the low patience treatment, from 2 to 9 in the medium patience treatment and from 0 to 24 in the high patience treatment. In the high patience treatment, the model assumes the border solution of no envy and predicts a more equal division than observed in the first period. As the discount factor increases, the Rubinstein division (without envy) converges to the equal split and, therefore, the additional impact of the other-regarding motive decreases (figure 1). Thus, small differences in the observed partition have a high impact on the imputed envy in the high discount factor treatment.

Strengths of envy imputed in the low discount factor treatment are in line with earlier calibrations of the full inequality aversion model including envy and guilt. Stylized ultimatum game behavior, for instance, is reproduced by Fehr & Schmidt (1999) assuming strengths of envy from 0.5 to 4 for 70 percent of the players. This comparison is flawed, however, if an ambivalent impact of guilt (see Kohler 2012a) is unduly neglected in the alternating-offer bargaining model. The equilibrium partition candidate in a model of common envy, guilt and discount factor, under the assumption that the equilibrium payoff distribution uniquely favors the proposing bargainer, is $\frac{(1+\alpha)(1+2\alpha-\delta)+\delta\beta(1-2\beta+\delta)}{(1+2\alpha)^2-\delta^2(1-2\beta)(1+2\beta)}$, but alternating-offer bargaining amongst inequality averse bargainers that perceive envy and guilt is outside the scope of this paper.

References

- Binmore, K., Morgan, P., Shaked, A., & Sutton, J. (1991). Do people exploit their bargaining power? an experimental study. *Games and Economic Behavior*, *3*, 295–322.
- Binmore, K., Shared, A., & Sutton, J. (1989). An outside option experiment. *Quarterly Journal of Economics*, 104, 753–770.
- Binmore, K., Swierzbinski, J., & Tomlinson, C. (2007). An experimental test of Rubinstein's bargaining model.
- Bolton, G. E. (1991). A comparative model of bargaining: Theory and evidence. *American Economic Review*, *81*, 1096–1136.
- Camerer, C. F. (2003). *Behavioral Game Theory. Experiments in Strategic Interaction*. The roundtable series in behavioral economics. Princeton: Princeton University Press.
- Compte, O., & Jehiel, P. (2004). Gradualism in bargaining and contribution games. *The Review* of *Economic Studies*, 71, 975–1000.
- Cooper, D. J., & Kagel, J. H. (n.d.). Other regarding preferences: A selective survey of experimental results. In J. H. Kagel, & A. E. Roth (Eds.), *Handbook of Experimental Economics, Volume* 2. Princeton: Princeton University Press.
- De Bruyn, A., & Bolton, G. E. (2008). Estimating the influence of fairness on bargaining behavior. *Management Science*, *54*, 1774–1791.
- Driesen, B., Perea, A., & Peters, H. (2012). Alternating offers bargaining with loss aversion. *Mathematical Social Sciences*, 64, 103–118.
- Fehr, E., & Schmidt, K. M. (1999). A theory of fairness, competition, and cooperation. *Quarterly Journal of Economics*, 114, 817–868.
- Fershtman, C., & Seidmann, D. (1993). Deadline effects and inefficient delay in bargaining with endogenous commitment. *Journal of Economic Theory*, *60*, 306–321.
- Herreiner, D., & Puppe, C. (2009). Envy freeness in experimental fair division problems. *Theory and Decision*, 67, 65–100.
- Kohler, S. (2011). Altruism and fairness in experimental decisions. *Journal of Economic Behavior* & Organization, 80, 101–109.
- Kohler, S. (2012a). Guilt causes equal or unequal division in alternating-offer bargaining. *MPRA Paper 40760, University Library of Munich, Germany.*
- Kohler, S. (2012b). Incomplete information about social preferences explains equal division and delay in bargaining. *Games*, *3*, 119–137.
- Kohler, S. (2012c). More fair play in an ultimatum game after resettlement in Zimbabwe: A field experiment and a structural model. *MPRA Paper 40248, University Library of Munich, Germany.*
- Li, D. (2007). Bargaining with history-dependent preferences. *Journal of Economic Theory*, 136, 695–708.
- Miettinen, T. (2010). History-dependent reciprocity in alternating offer bargaining. *Finnish Economic Papers*, 23, 1–15.
- Montero, M. (2007). Inequity aversion may increase inequity. Economic Journal, 117, C192–C204.
- Montero, M. (2008). Altruism, spite and competition in bargaining games. *Theory and Decision*, 65, 125–151.
- Rapoport, A., Weg, E., & Felsenthal, D. S. (1990). Effects of fixed costs in two-person sequential

bargaining. Theory and Decision, 28, 47–71.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. Econometrica, 50, 97–109.

- Shaked, A., & Sutton, J. (1984). Involuntary unemployment as a perfect equilibrium in a bargaining model. *Econometrica*, *52*, 1351–1364.
- Smith, R. (Ed.) (2008). Envy: Theory and Research. New York: Oxford University Press.
- Ståhl, I. (1972). Bargaining Theory. Stockholm School of Economics.
- Takahashi, H., Kato, M., Matsuura, M., Mobbs, D., Suhara, T., & Okubo, Y. (2009). When your gain is my pain and your pain is my gain: Neural correlates of envy and Schadenfreude. *Science*, *323*, 937–939.
- Von Neumann, J., & Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. Princeton, New Jersey, USA: Princeton University Press.
- Weg, E., Rapoport, A., & Felsenthal, D. S. (1990). Two-person bargaining behavior in fixed discounting factors games with infinite horizon. *Games and Economic Behavior*, *2*, 76–95.
- Weg, E., & Zwick, R. (1999). Infinite horizon bargaining games: Theory and experiments. In D. V. Budescu, I. Erev, & R. Zwick (Eds.), *Games and Human Behavior: Essays in Honor of Amnon Rapoport*. New Jersey: Lawrence Erlbaum Associates.
- Zwick, R., Rapoport, A., & Howard, J. C. (1992). Two-person sequential bargaining behavior with exogenous breakdown. *Theory and Decision*, *32*, 241–268.