Nothing else matters: evolution of preference for social prestige

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Nothing else matters: evolution of preference for social prestige*†

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Abstract

This paper seeks answers to two questions. First, if a greater *social activity* of an individual enhances oblique (i.e. to non-relatives) transmission of her cultural traits, but reduces their vertical (i.e. to children) transmission as well as family size, which behavior is optimal from cultural evolution standpoint? I formalize a general model that characterizes evolutionarily stable behaviors. The proposed model replicates the theory of Newson et al. (2007) that fertility decline is caused by increasing role of oblique cultural transmission. Second, if social activity is a rational choice rather than culturally inherited trait, and if cultural transmission acts on preferences rather than decisions, which preferences survive the process of cultural evolution? I arrive at a very simple yet powerful result: under mild assumptions on model structure, only preferences which emphasize *exclusively* the concern for *social prestige*, i.e. extent to which one's cultural trait has been picked up by others, survive.

Keywords: Cultural transmission, demographic transition, social prestige, evolutionary steady state.

JEL Codes: C73,J11,Z13.

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1 Introduction

Culture may flow not only from parents to children, but also between non-relatives – this phenomenon is known as *oblique transmission* (Cavalli-Sforza and Feldman, 1981). Moreover, the probability that a cultural trait of an individual is picked up by others may depend on specific characteristics of that individual, a phenomenon called *biased transmission* by theorists of cultural transmission (Boyd and Richerson, 1985). Henrich and Gil-White (2001) define a concept of *prestige* of an individual, which affects the extent to which others are willing to pick up her cultural trait. In this paper, I ask a question: if individuals are able to affect their prestige level via costly effort, namely by reducing family size and vertical transmission of cultural traits (i.e. transmission from parents to children), what type of behavior would survive the process of cultural evolution?

In Section 2 of the paper, I specify a very general model that allows to characterize the evolutionarily stable behaviors, and analyze their dependence on model parameters. The proposed analysis is done under very loose assumptions about the model ingredients; in particular, for most of the analysis we do not require differentiability of the maximand functions, and analyze optimal behavior using derivative-free methods. The proposed model predicts that an increasing role of oblique transmission causes fertility decline.

The model should be viewed as a mathematical formalization of the idea by Newson et al. (2007), that fertility decline is caused by changing patterns of social interaction. According to their theory, fertility is high when individuals are surrounded mainly by close relatives; in such environments, high fertility cultural traits evolve because low-fertility communities become physically extinct. Fertility drops as cultural traits increasingly flow between non-relatives and across communities, because in such environment a low-fertility trait has a greater chance of survival. Newson (2009) provides empirical evidence in support of the theory.

In the field of Economics, theorization about cultural transmission usually builds on "imperfect empathy" preferences introduced by Bisin and Verdier (1998, 2001). There are several discrete cultural traits; members of each cultural trait derive utility from two ingredients: traditional instantaneous utility dependent on own actions, and instantaneous utility dependent on actions of children. Instantaneous utility function differs across cultural traits; parents apply own instantaneous utility to both own actions and those of children. If children pick up the "wrong" cultural trait, they act "wrong" from the point of view of parents, which makes the latter worse off. An expectation of such outcome causes parents to undertake costly "socialization" effort to increase the chance that children pick up parents' cultural trait.

Baudin (2010) develops a model of demographic transition that incorporates cultural transmission and imperfect empathy. There are two cultural types with ad-hoc assumptions about their fertility preferences and production mode; a productivity shock of one of types has multiple fertility effects on both types, which are analyzed in the paper.

A disadvantage of the approach to cultural transmission by Bisin and Verdier (2001) and most subsequent papers is an overly restrictive domain of possible cultural types: ad hoc assumptions about the utility function are made; the set of possible cultural types is usually limited to two elements. In Section 3 of this paper, we investigate which preferences are most successful, i.e. are capable to emulate the most successful behaviors characterized by Section 2. We arrive at a very simple, nearly tautological, yet powerful idea: on a very broad domain of possible preferences, the one that prescribes an individual to care *exclusively* about the social prestige, i.e. about the extent to which others have learned from that individual, is no less successful than any other preference map. Under mild regularity conditions, the preference for social prestige is strictly more successful than any alternative. An individual who cares not only about social prestige but also about anything else, e.g. genetic success, private consumption, leisure, absolute wealth, relative wealth, or any combination of these, makes suboptimal decisions, her cultural trait is picked up less frequently by the next generation and vanishes over time. A preference for consumption (and other things) may survive only as long as consumption (or other things) contributes to the ultimate purpose of social prestige.

2 Family versus social activity: evolution of behavior

In this section, individuals are viewed as dummies which inherit behaviors of the previous generation. Rational decision making is introduced in Section 3.

2.1 Setup

Consider a dynamic model of overlapping generations set up in discrete time on infinite time horizon. Each generation lives for two periods, such that generation t lives in period $t-1$ when young and period t when old. In each generation, there is a continuum of individuals of endogenous mass. Each individual $i \in G_t$ is characterized by her *social activity* $x(i) \in [0,1]$ that is culturally transmitted in a way specified below.

There is a parent-child relationship in the model: each young individual has one parent from the previous generation. Young individuals pick up the social activity level from the previous generation in the following exogenous way. With probability $1-q$, a young individual $i_t \in G_t$ picks up the social activity of own parent $i_{t-1} \in G_{t-1}$. With the remaining probability q, young individuals pick up the cultural trait of a randomly drawn old individual, such that the probability measure of drawing an individual $j_{t-1} \in G_{t-1}$ is proportional to the *social visibility* of the latter, a function of her social activity, $f(x(j_{t-1}))$. The social visibility function satisfies the following assumptions:

(i) boundedness: $f(0) = 0$ and $f(1) < \infty$;

(ii) monotonicity: $f(x) < f(x'), \forall x < x'$;

(iii) upper semi-continuity: for every $\alpha \geq 0$, the set of all x such that $f(x) \geq \alpha$ is closed.

When i_t becomes old, he implements the social activity level $x(i_t)$ that he has picked up from the previous generation when young. The time spend *not* on social activity, $1 - x(i_t)$, is devoted to family, which results in the family size of $n(x(i_t)) = \frac{1-x(i_t)}{\nu}$, where ν is the time cost per child born.

Which behaviors prosper and which perish? Denote by B the Borel σ-algebra on $[0,1]$, by $P_t : B \to R_+$ a probability measure of the number of old individuals at the beginning of time t on the set of all possible behaviors, such that $P_t([0,1]) = 1$, and by $L_t \equiv |G_t|$ a measure of generation-t population size. Then, the dynamics of $P_t(\cdot)$ is as follows:

$$L_{t+1}P_{t+1}(x) = (1-q)n(x)L_tP_t(x) + q\frac{f(x)}{\bar{f}_t}\bar{n}_tL_tP_t(x) \tag{1}$$

where

$$\bar{f}_t \equiv \int_{z\in[0,1]} f(z)dP_t([0,z]) \tag{2}$$

$$\bar{n}_t \equiv \int_{z\in[0,1]} n(z)dP_t([0,z]) \tag{3}$$

are average values of the visibility and family size in generation t.

2.2 Optimal behavior

The standard evolutionary theory prescribes that survival of a behavioral type necessitates maximization of the rate of growth of the population bearing the respective cultural trait:

$$\max_x \frac{L_{t+1}P_{t+1}(x)}{L_t P_t(x)} = (1-q)n(x) + q\frac{f(x)}{\bar{f}_t}\bar{n}_t \tag{4}$$

First, observe that $q = 0$ results in maximization of family size only, hence the evolutionarily stable strategy (ESS) is $x = 0$. Also observe that $q = 1$ results in maximization of prestige only, hence $x = 1$ is the ESS. Note however that such strategy implies zero family size and therefore extinction of the population.

Throughout the rest of the paper, we focus on the interior values of q. Maximization of (4) with respect to x, given $\bar{f}_t$ and $\bar{n}_t$, is equivalent to maximization of

$$f(x) - \frac{1}{\nu}\frac{1-q}{q}\frac{\bar{f}_t}{\bar{n}_t}x \tag{5}$$

with respect to x. Denote by $s_t \equiv \frac{1}{\nu}\frac{1-q}{q}\frac{\bar{f}_t}{\bar{n}_t} \geq 0$ the coefficient of the linear part of (5), and by $X(s_t)$ the set of all $x \in [0,1]$ that maximize $f(x) - s_t x$.

Proposition 1 *The set $X(s)$ has the following properties:*

(i) *existence: $X(s)$ is non-empty for every s;*

(ii) *boundary closedness:* $\inf(X(s)) \in X(s)$ *and* $\sup(X(s)) \in X(s)$*;*

(iii) *ordering: for every pair s_L and s_H such that $s_L < s_H$, for every $x_L \in X(s_L)$, for every $x_H \in X(s_H)$, we have that $x_L \geq x_H$;*

(iv) *completeness: for every $x \in [0,1]$, there exist s such that* $\inf(X(s)) \leq x \leq \sup(X(s))$.

A corollary of the ordering property is that for $s_H > s_L$, we have that $\sup X(s_H) \leq \inf X(s_L)$. Also note that the completeness property does *not* state that every x should belong to some $X(s)$. It only states that there are no "holes" between two consecutive sets, while holes within a set are possible. The proof of the proposition is in the Appendix.

The set of surviving strategies, $X(s)$ depends on parameter s which in turn itself depends on the distribution of behaviors. We now turn to characterization of the evolutionary steady state (ESS henceforth) where the distribution of behaviors does not change over time, and population grows at a constant rate.

2.3 Evolutionary steady state

By evolutionary steady state (ESS) we imply a time-invariant distribution of behavioral types, denoted $P_\infty(\cdot)$.[1] Time-invariant distribution of types also implies time-invariant average social activity behavior and average family size, which further implies a constant rate of population growth. Define by $\bar{x}_\infty$ the average social activity in a steady state:

$$\bar{x}_\infty \equiv \int_{z\in[0,1]} z dP_\infty([0,z]) \tag{6}$$

The following Lemma demonstrates that the set of optimal behaviors of an individual depends only on one moment of the distribution of optimal behaviors of others, namely on the average $\bar{x}_\infty$, which considerably simplifies the analysis of the ESS.

Lemma 1 *In evolutionary steady state, the parameter s_∞ can be presented as a function of q and of average social activity $\bar{x}_\infty$:*

$$s_\infty = s(q, \bar{x}_\infty) = \frac{1}{\nu}\frac{1-q}{q}\frac{f(\bar{x}_\infty)}{n(\bar{x}_\infty)} \tag{7}$$

Proof. Denote by $X \equiv X(s_\infty)$ the set of evolutionarily stable behaviors; by definition of ESS, every surviving individual chooses a strategy from X. If it has only one element $\tilde{x}$, then the entire mass of the population is concentrated at $\tilde{x}$ and therefore (i) $\bar{x}_\infty = \tilde{x}$ and (ii) $s_\infty = \frac{1}{\nu}\frac{1-q}{q}\frac{f(\tilde{x})}{n(\tilde{x})}$, which proves the claim. If X contains multiple types of behavior, then all of them must yield the same value of the function $f(x) - sx = c, \forall x \in X$ for some c, s. Then, the function f on the set of surviving strategies should be linear: $f = c + sx, \forall x \in X$, and can be shown to be equal to

$$f(x) = f(\inf(X)) + \frac{f(\sup(X)) - f(\inf(X))}{\sup(X) - \inf(X)}(x - \inf(X)), \forall x \in X \tag{8}$$

Given linearity of $f(x)$ on the domain of evolutionarily stable strategies, its average value $\bar{f}_\infty$ is equal to $f(\bar{x}_\infty)$. Function $n(x)$ is linear by construction on the entire domain of behaviors, therefore $\bar{n}_\infty = n(\bar{x}_\infty)$. Thus, s_∞ is defined by (7). ■

Given that $f(\cdot)$ is increasing in its argument while $n(\cdot)$ is decreasing in it, it is straightforward to observe that $s(q, x)$ is strictly increasing in x, and is strictly decreasing in q.

We now proceed to characterization of the evolutionarily stable average behavior, $\bar{x}_\infty$. Given that individuals are indifferent between elements of $X(s(q, \bar{x}_\infty)) \equiv X$, by definition

[1] Here and thereafter, all variables with an infinity subscript indicate their ESS values.

of the set, any probability distribution of strategies on that set may be implemented. By boundary closedness, both infinum and supremum of $X(s(q, \bar{x}_\infty))$ belong to the set; the average strategy may take any value between them. Define by $Y(q, x)$ the set of all possible values of the average strategy:

$$Y(q, x) \equiv [\inf X(s(q, x)), \sup X(s(q, x))] \tag{9}$$

Proposition 2 *The set-valued function $Y(q, x)$ has the following properties:*

(i) *existence: $Y(q, x)$ exists for every $q \in (0, 1)$ and $x \in [0, 1]$;*

(ii) *closedness: $Y(q, x)$ is closed for every $q \in (0, 1)$ and $x \in [0, 1]$;*

(iii) *q-ordering: for every $q_L, q_H \in (0, 1), x \in [0, 1]$ such that $q_L < q_H$, we have that $\sup Y(q_L, x) \leq \inf Y(q_H, x)$;*

(iv) *x-ordering: for every $q \in (0, 1), x_L, x_H \in [0, 1]$ such that $x_L < x_H$, we have that $\inf Y(q, x_L) \geq \sup Y(q, x_H)$;*

(v) *completeness: for any $y \in [0, 1]$, and for any $q \in (0, 1)$, there exist x such that $\inf(Y(q, x)) \leq y \leq \sup(Y(q, x))$.*

The proof of the proposition follows directly from the definition of $Y(q, x)$, Lemma 1, and the properties of $X(s)$. Note that $s(q, 1)$ is formally not defined; we assume $Y(q, 1) = \{0\}$. Intuitively, when average social activity is unity, the average family size is zero, so one's cultural trait can be picked up only by own children and therefore zero social activity is best. The properties of $Y(q, x)$ entail the following

Corollary 1 *The graph of $Y(q, x)$ as a function of x is closed, i.e. for any sequence x_N converging to some $x \in [0, 1]$ and for any sequence y_N converging to some y and such that $y_N \in Y(q, x_N), \forall N$, we have that $y \in Y(q, x)$.*

The proof is contained in the Appendix.

We now study the properties of the steady state average social activity, equal to the fixed point of $Y(q, x)$ as a function of x, and denoted $y(q)$. Mathematically, x is a fixed point of $Y(q, x)$ if it satisfies

$$x \in Y(q, x) \tag{10}$$

Theorem 1 **(i)** *The fixed point of $Y(q, x)$ as a function of x, $y(q)$, exists for any $q \in (0, 1)$;*

(ii) *$y(q)$ is unique for any $q \in (0,1)$;*

(iii) *$y(q)$ is weakly increasing in q;*

(iv) *$y(q) \in (0,1)$ (i.e. excludes boundaries) for any $q \in (0,1)$.*

Proof.

(i) Existence is established by the Kakutani fixed point theorem. The theorem requires that the set $Y(q,x)$ exists and is closed for every q, x, which is established by Proposition 2, and that the graph of $Y(q,\cdot)$ is closed, which is established by Corollary 1.

(ii) Uniqueness follows from x-ordering of Y: if there were two values $x_L < x_H$ that satisfied (10), that would directly contradict the x-ordering requirement that $\inf Y(q,\bar{x}_L) \geq \sup Y(q,\bar{x}_H)$.

(iii) Take any pair $q_L < q_H$. We have that

$$y(q_L) \underbrace{\leq}_{y(q_L)\text{ is fixed point}} \sup Y(q_L, y(q_L)) \underbrace{\leq}_{q\text{-ordering of }Y} \inf Y(q_H, y(q_L)) \tag{11}$$

If both hold with equality, by uniqueness of the fixed point $y(q_H)$ we must conclude that $y(q_H) = y(q_L)$, and proof is complete. If $y(q_L) < \inf Y(q_H, y(q_L))$, we have that $y(q_L) \neq y(q_H)$. Suppose $y(q_H) < y(q_L)$. Then,

$$\inf Y(q_H, y(q_H)) \underbrace{\geq}_{x\text{-ordering of }Y} \sup Y(q_H, y(q_L)) \geq \inf Y(q_H, y(q_L)) > y(q_L) > y(q_H) \tag{12}$$

and therefore $y(q_H)$ is not a fixed point. Therefore, $y(q_H) > y(q_L)$.

(iv) Suppose $y(q) = 0$. Then $s(q, y(q)) = 0$, and, by monotonicity of $f(\cdot)$, maximization of $f(x) - s(q,y(q))x$ with respect to x yields $x^\star = y(q) = 1$, which is a contradiction. The discussion following Proposition 2 establishes that $Y(q,1) = 0$, hence unity cannot be a fixed point either.

■

Thus, social activity is weakly increasing (and therefore family size is weakly decreasing) with the rate of oblique transmission, i.e. rate of interaction between non-relatives q. This is a mathematical formalization of the idea by Newson et al. (2007) that fertility decline is caused by an increase in such interaction.

3 Evolutionarily stable preferences

If social activity is a rational choice rather than a result of imitation, and if cultural transmission acts on preferences that govern the choice of social activity, which preferences will survive in the process of cultural evolution? In this section, we introduce the notion of two alternative incentives, and let individuals choose an optimal social activity by maximizing a mix of the two incentives. A cultural trait in this section is the importance of one incentive over the over; our objective is to characterize the cultural traits that survive.

3.1 The incentives

Throughout the rest of the paper, we fix $q \in (0, 1)$, and limit our analysis to the steady state distribution of behaviors.

Denote by $g(x)$ the *social prestige incentive* of the decision maker in the steady state, as a function of social activity x:

$$g(x) = (1 - q)n(x) + q\frac{f(x)}{\bar{f}_\infty}\bar{n}_\infty \tag{13}$$

Note that (13) is mathematically equivalent to the ESS version of (4), and therefore the set of behaviors that maximize $g(x)$ is X, i.e. the same set that maximizes (4) in the ESS. Denote by $h(x)$ the *alternative incentive*, i.e. anything that does *not* include the social prestige, such as consumption or family size, as a function of own social activity. To ensure that there indeed exists a tradeoff between social prestige and the alternative, and that individuals with different preferences (defined below) indeed make different choices, we impose the following regularity conditions on $h(\cdot)$ and on the ingredients of $g(\cdot)$:

(i) X belongs to the interior of the behavioral domain: $X \subset (0, 1)$;

(ii) The function $f(x)$ is differentiable on X, in addition to properties assumed in section 2.

(iii) The function $h(x)$ is differentiable on X, such that $\frac{dh(x)}{dx} \neq 0, \forall x \in X$.

A direct corollary of (ii) is that $\frac{dg(x)}{dx}$ exists on X and is equal to zero.

3.2 The utility

We now proceed to the definition of the individual objective function. We assume that individuals derive utility from social prestige g and from the alternative h; the "importance" of each of these alternatives may vary across individuals, and is labeled by $a \in [0,1]$. We make the following natural assumptions about the utility function:

(i) more is better: $\frac{\partial u(g,h,a)}{\partial g}$ and $\frac{\partial u(g,h,a)}{\partial h}$ exist, such that $\frac{\partial u(g,h,a)}{\partial g} \geq 0, \frac{\partial u(g,h,a)}{\partial h} \geq 0$;

(ii) a is the comparative importance of g over h: $\frac{\partial^2 u(g,h,a)}{\partial g \partial a}$ exists and is strictly positive; $\frac{\partial^2 u(g,h,a)}{\partial h \partial a}$ exists and is strictly negative. Moreover, $\frac{\partial u(g,h,a)}{\partial g} = 0$ iff $a = 0$, and $\frac{\partial u(g,h,a)}{\partial h} = 0$ iff $a = 1$.

An example of the utility function that satisfies the above assumptions is the constant elasticity of substitution function:

$$u(g,h,a) = \left(ag^{\frac{\sigma-1}{\sigma}} + (1-a)h^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \sigma > 0$$

3.3 Cultural transmission

Unlike in section 2, we assume that cultural transmission acts on the preference parameter a rather than on choice of social activity x. For this reason, we redefine the distribution of surviving cultural traits. Denote by $Q(\cdot) : B \to R_+$ the probability measure of the steady-state distribution of a, such that $Q([0,1]) = 1$. Denote also by $P : B \times [0,1] \to R_+$ the probability measure of the steady-state distribution of behaviors for every possible cultural trait, such that $P([0,1], a) = 1, \forall a$. Individual rationality implies that $P(x,a) > 0$ only if $u(g(\cdot), h(\cdot), a)$ achieves its maximum at x.

With above notations, the growth rate of a given cultural trait in the steady state is characterized by the following (cf.(4)):

$$\frac{L'Q'(a)}{LQ(a)} = (1-q)\int_{x\in[0,1]} n(x)dP([0,x],a) + q\frac{\int_{x\in[0,1]} f(x)dP([0,x],a)}{\bar{f}_\infty}\bar{n}_\infty \tag{14}$$

where L' and Q' denote future values of the corresponding variables, and where

$$\bar{f}_\infty = \int_{a'\in[0,1]}\int_{x\in[0,1]} f(x)dP([0,x],a')dQ(a') \tag{15}$$

$$\bar{n}_\infty = \int_{a'\in[0,1]}\int_{x\in[0,1]} n(x)dP([0,x],a')dQ(a') \tag{16}$$

We can now formulate the main result of this section.

Theorem 2 *The only evolutionarily stable cultural trait is $a = 1$.*

In other words, only those cultural traits that prescribe their hosts to care exclusively about social prestige may survive in the process of cultural evolution.

Proof. Standard evolutionary theory necessitates maximization of the rate of growth of one's cultural trait which, in turn, requires that every behavior chosen belongs to the set of evolutionarily stable behaviors: $P(x, a) > 0$ only if $x \in X$, for every surviving a. Individual rationality requires that the utility is maximized at every point where $P(x, a) > 0$. Since, by assumption, every ingredient of the utility is differentiable on X, we have the following first-order condition of individual rationality for surviving cultural traits:

$$\frac{du(g(x), h(x), a)}{dx} = \frac{\partial u(g, h, a)}{\partial g}\frac{dg(x)}{dx} + \frac{\partial u(g, h, a)}{\partial h}\frac{dh(x)}{dx} = 0, \forall x, a : P(x, a) > 0, Q(a) > 0 \tag{17}$$

From the properties of $g(x)$ and $h(x)$, we have that $\frac{dg(x)}{dx} = 0$ and $\frac{dh(x)}{dx} \neq 0$ for every x that satisfies (17). The first-order condition is therefore satisfied only if $\frac{\partial u(g,h,a)}{\partial h} = 0$, which is in turn only possible if $a = 1$ due to assumptions imposed on the utility function. ■

4 Conclusion

Mainstream Economics traditionally assumes that personal consumption is the major goal of a rational decision maker. Such assumption is at odds with theoretical biology. A carnivore animal would gladly accept an offer of meat, yet theoretical biologists do not think that meat consumption is an ultimate purpose of an animal; meat is viewed as an intermediate good that helps to achieve the ultimate purpose of reproduction. In this paper, I argue that humans are no different: the apparent desire for private consumption simply masks other, more fundamental, goals, and consumption *per se* should enter utility with zero weight. This idea offers new theoretical insights about the optimal motivation schemes, specifically motivation of high-ranked employees whose efforts are potentially observable by large number of people, e.g. those in military, political, religious, or scientific occupations.

This finding is reminiscent of the ideas put forth by the proponents of "memetics," in particular Blackmore (2000), that humans may be "infected" by the "memes," discrete invisible entities that may alter human behavior. Blackmore (2000) argues that, once the cost of oblique cultural transmission is low enough, the most successful meme is the one that induces its host to spread itself as much as possible.

This paper does not assume the presence of the "memes" as discrete entities, yet conveys a very similar idea. Moreover, we show that the exclusive concern for social prestige emerges with *any non-zero* probability of oblique cultural transmission.

Despite the simplicity of the idea, and its powerful implications for microeconomic theory, it has been surprisingly overlooked in the field of Economics. This paper is an attempt to fill the gap.

A Proofs

Proof of Proposition 1

(i) Since $f(x)$ is upper semi-continuous and is defined on closed support, so is $f(x) - sx$ with respect to x. A property of such function is that it has a maximum, therefore $X(s)$ is never empty.

(ii) For arbitrary s, consider any sequence of $x \in X(s)$ that converges to $\inf(X(s))$. Upper-semicontinuity of $f(x) - sx$ with respect to x implies that $\lim_{x \to \inf(X(s))} f(x) - sx \leq f(\inf(X(s))) - s\inf(X(s))$. Since every $x \in X(s)$ maximizes $f(x) - sx$ by definition of $X(s)$, so does $\inf(X(s))$. Therefore, $\inf X(s) \in X(s)$. Likewise, we can prove that $\sup X(s) \in X(s)$.

(iii) Take arbitrary s_L and $s_H > s_L$, $x_L \in X(s_L), x_H \in X(s_H)$. By definition of $X(\cdot)$, we have that

$$\begin{aligned} f(x_L) - s_L x_L &\geq f(x_H) - s_L x_H \\ f(x_H) - s_H x_H &\geq f(x_L) - s_H x_L \end{aligned}$$

By manipulating with these expressions, we can obtain $(s_H - s_L)(x_H - x_L) \leq 0$. Therefore, if $s_H > s_L$, it must be that $x_H \leq x_L$.

(iv) Suppose the contrary: there is no such s that $\inf(X(s)) \leq x \leq \sup(X(s))$. Then, pick the highest possible s_L such that $x \leq \inf(X(s_L))$, and lowest possible s_H such that $\sup(X(s_H)) \leq x$. If $X(s_L) = X(s_H)$, we have that both contain only one element, namely x, and therefore $\inf(X(s_L)) = x = \sup(X(s_L))$, which proves the claim. If $X(s_L) \neq X(s_H)$, we have that $s_L \neq s_H$ and, by the ordering property, $s_L < s_H$. Take any $s \in (s_L, s_H)$; by the ordering property, $\sup(X(s_H)) \leq \inf(X(s))$ and $\sup(X(s)) \leq \inf(X(s_L))$. If $\inf(X(s)) \leq x \leq \sup(X(s))$, proof is complete. If $x < \inf(X(s)) \leq \inf(X(s_L))$, that contradicts our assumption that s_L was the highest possible with

$x \leq \inf(X(s_L))$. If $\sup(X(s_H)) \leq \sup(X(s)) < x$, that contradicts our assumption that s_H was the lowest possible with $\sup(X(s_H)) \leq x$.■

Proof of Corollary 1 Suppose the opposite, i.e. that $y \notin Y(q, x)$. Since $Y(q, x)$ is a closed set, y must be either below its infinum or above its supremum. Suppose, without loss of generality, $y < \inf Y(q, x)$. By completeness and x-ordering of Y, $\exists x' > x$ such that $y \in Y(q, x')$. If $y < \sup Y(q, x')$, then every $y_N < \sup Y(q, x')$ for sufficiently large N, and therefore by x-ordering we have that $x_N \geq x' > x$ for sufficiently large N which contradicts the assumptions. If $y = \sup Y(q, x')$, pick any $y'' \in (y, \inf Y(q, x))$; $\exists x'' \in (x, x')$ such that $y'' \in Y(q, x'')$. Then, every $y_N < y''$ for sufficiently large N, and therefore by x-ordering we have that $x_N \geq x'' > x$ for sufficiently large N which contradicts the assumptions.■

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