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Kaizoji, Taisei

Graduate School of Arts and Sciences, International Christian University

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Taisei Kaizoji
Graduate School of Arts and Scineces,
International Christian University
3-10-2 Osawa, Mitaka, Tokyo 181-8585, Japan
e-mail: kaizoji@icu.ac.jp

Abstract

The purpose of this note is to demonstrate a sufficient condition for a discrete tâtonnement process to lead to chaos in a general equilibrium model with multiple commodities. The result indicates that as the speed of price adjustment increases, the discrete tâtonnement process is complex in a general equilibrium economy in which there are multiple equilibria.

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1. Introduction

Over the past decades a considerable number of studies have been made on the tâtonnement process. The papers by Arrow and Hurwicz (1958), Arrow and Hahn (1971), and Negishi (1958) have proved that the continuous tâtonnement process converges to the unique equilibrium price under global gross substitutability. Recent major contributions to the tâtonnemnt process have been devoted to instability of discrete tâtonnemnt process, The papers by Bala and Majumdar (1992), Day and Pianigiani (1991), Day (1994) and Mukherji (1999) show that the discrete tâtonnemnt process may lead to chaotic dynamics under global gross substitutability. Further, Goeree et al. (1998), Tuinstra (1997, 1999), Saari (1985), and Weddepohl (1995) show that the discrete tâtonnement processes become unstable and exhibit chaos as the speed of price adjustment increases.

This paper further examines the dynamics of the discrete-time tâtonnemnt process in a competitive economy in which there are multiple equilibria[1]. We demonstrate that as the adjustment speeds of prices are sufficiently fast, the discrete tâtonnement process is chaotic in the competitive economy in which there are multiple equilibria. The result can be demonstrated by Hatta's theorem (Hatta 1982).

In section 2, we introduce the theorem on the existence of equilibria in Warlasian economy proposed by Dierker (1972) and Varian (1975). In section 3, we demonstrate sufficient conditions for tâtonnement process to lead to chaos in the economy which has multiple equiribria by applying the Hatta theorem (Hatta (1982)). A few concluding remarks are given in section 4.

2. Multiple equilibria

Let us consider a competitive economy with k commodities labeled $j = 1,...,k.$ Let

$$S := \left\{ p = (p_1,..., p_j,..., p_{k-1}) \in R^{k-1} \,\middle|\, all\ p_j > 0,\ \sum_{j=1}^{k} p_j = 1 \right\} \qquad (1)$$

denote the open price simplex. Let $\varsigma = (\varsigma_1,...,\varsigma_j,....,\varsigma_k) : S \to R^k$ be the aggregate excess demand function which is continuously differentiable. Walras' law, i.e., $\sum_{j=1}^{k} p_j \varsigma_j(p) = 0$ for all $p \in S$ holds. By Walras' law these zeros coincide with the zero of $z = (\varsigma_1,...,\varsigma_j,....,\varsigma_{k-1}) : S \to R^{k-1}$. Assume that the Jacobian of the aggregate excess demand function z is non-zero at all equilibria. It implies that the competitive economy is regular in the sense of Debreu (1972). Dierker (1972) demonstrates that a regular economy has an odd number of equilibria. Further, under this assumption of desirability of all commodities, that is, one assume that as the price of a good goes to 0, its excess demand becomes positive, Varian (1975) demonstrates that if the Jacobian matrix $\det(-Dz(p))$ of the excess supply function $-z$ is positive at all equilibria, there is exactly one equilibrium. Their theorems mean the following:

[1] Kaizouji (1994) demonstrates sufficient conditions for the discrete-time tâtonnement process to lead to chaos in the competitive economy with only two commodities by applying the Yamaguchi-Matano theorem (Yamaguchi and Matano 1981). This paper extends the theorem to the economy with n commodities.

If the Jacobian matrix $\det(-Dz(p))$ of the excess supply function $-z$ is negative at all equilibria, there are at least three equilibria.

3. The tâtonnemant process

In this paper we focus attention to the discrete tâtonnemnet process. The discrete tâtonnement process can be generally formalized as

$$p_{j,t+1} - p_{j,t} = \lambda z_j(p_t), \quad j = 1, 2, ..., k-1, \tag{2}$$

where λ denotes the speed of adjustment. The dimension of the tâtonnemnet process is a discrete $k-1$ dimensional system. Suppose that the Jacobian matrix of the excess supply function is negative, that is, there are multiple equilibria. Under a large value of the speed of the adjustment, it is shown that two of the equilibria are snapback repeller (Marotto 1978). Here we present the result more formally.

Proposition. For all regular economies which the Jacobian matrix of the excess supply function is negative, there exists a finite λ_0 such that for any $\lambda > \lambda_0$ the discrete tâtonnement process (2) is chaotic in the sense of Li and Yorke (Li and York 1975).

The proof of the proposition is given by Hatta (1982) (see Appendix).

4. Concluding remarks

One should note that the condition which we present is a sufficient condition for chaos of the tâtonnement process, but not a necessary condition. The condition on the existence of multiple equilibria can be weakened. One can show that in regular economics which have the unique equilibrium, the discrete tâtonnment process is chaotic for a large value of the speed of adjustment while the continuous tâtonnemnt process converges to the unique equilibrium.

5. Appendix: the theorem of Hatta

Consider an n-dimensional difference equations,

$$x_{t+1} = x_t + \lambda f(x_t), \quad x \in R^n. \qquad \text{(A)}$$

Let f be continuous differentiable in R^n. Suppose there exist $\bar{u} \neq \bar{v}$ such that $F(\bar{u}) = F(\bar{v}) = 0$, $\det F(\bar{u}) \neq 0$ and $\det F(\bar{v}) \neq 0$. Then there exists a positive constant c such that for any $\lambda > c$ the difference equation (A) is chaotic in the sense of Li and Yorke (Li and Yorke 1975).

The proof of the theorem is given by Hatta (1982).

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