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Abstract

We develop a game-theoretical model to examine the implications of the introduction of a non-profit "public option" in the U.S. health insurance market, in which a continuum of heterogeneous consumers, each facing unknown medical expenditures and differing in their expectations of such expenditures, have to choose between a profit-maximizing private insurance plan and a social-welfare-maximizing public plan. We then estimate and calibrate the model based on the U.S. data and quantify the Nash equilibrium of the market structure. Empirical results suggest that private insurer will still represent a significant part of the insurance market and generate a substantially positive profit.

JEL Clasiffication: L10, L21, L32, I11

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1 Introduction

One of the most controversial issues in the recent debate over health care reform in the United States is whether the reform should include a "public option," i.e., a non-profit insurance plan managed by the federal government that would compete with the private, for-profit insurance plans. Advocates of the public option argue that a non-profit insurance plan, with lower administrative costs and without profitability pressure, will not only provide a less expensive option to the general public, but will also discipline the private insurance companies because of the competition it brings to the insurance market.

Opponents of the public option, on the other hand, warn that the public option may eventually drive out the private insurers who cannot compete, and take over the whole insurance market. For instance, in its comments submitted to the U.S. Senate Finance Committee on June 10, 2009, the American Medical Association opposed the creation of the public option, stating that health services should be "provided through private markets, as they are currently," because "the introduction of a new public plan threatens to restrict patient choice by driving out private insurers, which currently provide coverage for nearly 70 percent of Americans," and that "the corresponding surge in public plan participation would likely lead to an explosion of costs that would need to be absorbed by taxpayers" (New York Times, June 10, 2009).

How to reform the existing health care system is apparently one of the most important challenges the United States faces, in particular at this moment when the health care expenditure is accounting for more than 16% of the GDP while continuing to increase at a faster pace than the GDP itself, and when the U.S. federal government is running an alarmingly high level of public debt. However, despite the heated debate over the legitimacy and feasibility of the "public option," there seems to be a lack of rigorous economic and quantitative analysis of the consequences of introducing such an option. For instance, the report by the Council of Economic Advisors published in 2009 (CEA 2009) mainly focuses on the impacts of overall medical reforms on long-run economic growth, employment, and government deficits, through international and states comparisons and under hypothetical scenarios, but does not provide a model-based quantitative analysis of the market equilibrium.

In this paper, we tackle this issue by examining the following questions: is it true that a non-profit public option in the health insurance market would drive out the private insurers? If not, what is the minimum market share of the public insurer that would allow attaining complete market coverage while not sustaining a budget deficit? We answer these two questions by first developing a theoretical model in a stochastic environment, in which a continuum of heterogeneous consumers, each facing unknown medical expenditures, but differing in their expectations of these expenditures, have to choose between two plans: one plan is offered by a profit-maximizing private insurance company, while the other by a public insurer that aims to maximize social welfare while not sustaining a budget deficit. We then estimate and calibrate the model and provide an empirical characterization of the Nash equilibrium of the market structure.

The model is calibrated using empirical data on U.S. medical expenditures. In particular, based on a data set from the U.S. Medical Expenditure Panel Survey, we first estimate a Bayesian hierarchical model to obtain the underlying distribution of expected medical expenditures, and then numerically solve the model and calibrate the market equilibrium, adopting standard choices of risk-preference parameter values and the estimated distribution of medical expenditures. Our model suggests that the public plan will not drive out the private insurers from the market. This is because the private insurers can skim the market by offering higher deductibles and lower premiums to attract the relatively healthier consumers, who do not expect high medical expenses but are mandated to purchase an insurance policy. The public plan takes the residual part of the market, having to demand higher premiums to cover the expected higher expenditures. In equilibrium, both the private insurer and the public option stay in the market. Consequently, the private insurance plan will still account for a significant part of the insurance market with a substantially positive profit. This suggests that the worries about a government takeover are unwarranted. We also find that relaxing the public plan's zero profit constraint and allowing it to run at a limited deficit will increase the public plan's market share, forcing the private plan to substantially lower its premiums and profit, but the social welfare improves. A more cost-effective public insurer, or an upper limit on private insurer's profit margin, will also lead to a decline in private insurer's profit and to an improvement of social welfare.

Traditionally, private entities have been the main suppliers of health insurance coverage for working age individuals in the United States. However in a few instances, the federal or state governments have intervened to improve the provision of health care to certain groups of patients. As in the current policy debate over the universal health care, concerns from the private sector have always arisen whenever the government enters the market as an alternative insurance provider, and such concerns have promoted several previous empirical studies to try to quantify the potential crowding out effect of those government programs. Yet so far researchers have not been able to reach a consensus. For instance, Cutler and Gruber (1996) estimate the effect of the Medicaid expansion to pregnant women and children on private insurance coverage, and conclude that on average 50% of the individuals who were previously covered by private insurers have switched to the new public program.[1] Lo Sasso and Buchmueller (2004) obtain a similar estimate when examining the implications of the Supplemental Children's Health Insurance Program. An even larger estimate is obtained by Brown and Finkelstein (2008), in which they study the Medicaid's

[1] When revisited 10 years later, Gruber and Simon (2008) obtain an even higher estimate at 60%.

crowding out effect on long-term care private insurers, and conclude that even if the private insurers choose to offer comprehensive policies at actuarially fair prices, the bottom two-thirds less wealthy patients would still prefer Medicaid. On the other hand, some other studies such as Rask and Rask (2000), Lo Sasso and Meyer (2010), and Ham and Shore-Shepard (2005) suggest that the crowding out effect of Medicaid on private insurance coverage may be very small or insignificant. A recent study by Miller and Yeo (2011) examines the potential crowding out effect in the Medicare Part D prescription drug market, and suggests that if the government plan operates at the same cost as the private insurers, it will have a negligible effect on the market structure; however, a 25 percent cost advantage would enable the government plan to capture one-fourth of the market.

In this paper, instead of focusing on some specific government programs that only target certain groups of citizens as in previous studies, we analyze and quantify the market equilibrium in the presence of a universal public health insurance plan for the whole U.S. population. Thus our study provides an alternative and much more comprehensive answer to the question and has a direct bearing on the relevant policy debates. Moreover, methodologically, most of the existing studies have relied on a reduced-form or statistical approach, and our paper is the first one that examines the market equilibrium in a structural game-theoretical framework. Another novelty of our study is the estimation of a Bayesian hierarchical model of a continuum of heterogeneous health "types" or expected medical expenditures for the whole U.S. population. This captures the heterogeneity of health conditions of individual consumers as well as the uncertainties that they face when purchasing insurance policies, both of which are very important characteristics of consumers in the medical insurance market in reality but have largely been ignored by existing studies.

The rest of the paper is organized as follows. Section 2 presents our theoretical model and discusses the strategy to solve and calibrate the model. Section 3 estimates a Bayesian hierarchical model to obtain the underlying distribution of expected medical expenditures. Section 4 calibrates the model and presents the empirical results, and Section 5 concludes.

2 A Model of Health Insurance Markets

Consumers are assumed to possess a constant absolute risk aversion utility function over final wealth $u(x) = -e^{-\theta x}$, with coefficient of risk aversion $\theta > 0$. Each consumer incurs health expenses over the period covered by the policy that are distributed exponentially with parameter γ; the corresponding probability density function is $f(t) = \frac{1}{\gamma} e^{-\frac{t}{\gamma}}$ for $t \in [0, \infty)$.[2] Thus, the expected health expenses

[2] The exponential distribution, that we employ for its analytical tractability and versatility, is a particular case of the Weibull distribution with shape parameter equal to 1. These distributions are employed in the literature to model the health expenses distribution with a parametrized hazard function that describes the likelihood of any possible

of an individual of type γ are exactly γ. The expenditure type γ is private information of each consumer, and is thus unknown to the public or private insurers. The types are distributed in the population with cumulative distribution function $H(\cdot)$ on $[0, \infty)$, which will be estimated from the data. All consumers are mandated to purchase medical insurance.

The private insurer offers a policy characterized by two parameters (p_p, d_p), where p_p is the insurance premium and d_p the deductible. A consumer insured by the private insurer will pay the first d_p dollars of the realized health expenses, and the insurance plan will cover the rest. The public insurer offers an insurance policy (p_g, d_g) with a premium p_g and deductible d_g. In practice, insurance contracts are defined by a larger number of provisions instead of only a premium and a deductible.[3] However, all these plan characteristics serve to describe the cost and various features of a risk sharing mechanism between insurance companies and individuals, and for simplicity here we use the deductible as a measure of the overall risk share allocated to the consumer. Finally, in our model the public insurer must cover all consumers not covered by the private insurer, and must not run a budget deficit (although later on we will relax this non-negative profit constraint and explore the market equilibrium with a limited deficit level). The private insurer maximizes expected profits, while the public insurer aims at maximizing the expected social welfare defined as the sum of the consumer and the producer surpluses.

2.1 Consumer's Problem

Consider two generic insurance plans characterized by the premium-deductible pairs (p_1, d_1) and (p_2, d_2). Clearly, when $p_i > p_j$ and $d_i > d_j$, plan (p_j, d_j) will dominate plan (p_i, d_i) and all consumers will select the former. In the following, we assume that $d_1 > d_2$ and $p_1 < p_2$. Denote by

$$\Psi(d, \theta, \gamma) \equiv \begin{cases} \frac{\theta\gamma e^{\left(\theta - \frac{1}{\gamma}\right)d} - 1}{\theta\gamma - 1}, & \text{when } \gamma \neq \frac{1}{\theta} \\ 1 + \theta d, & \text{when } \gamma = \frac{1}{\theta} \end{cases} \tag{1}$$

The next proposition describes the choice of the individual of type γ between the two contracts.

Proposition 1 *Assume $d_1 > d_2$ and $p_1 < p_2$. Then, a consumer of type γ will choose the plan (p_1, d_1) if and only if*

$$p_2 + \frac{1}{\theta} \ln \Psi(d_2, \theta, \gamma) \geq p_1 + \frac{1}{\theta} \ln \Psi(d_1, \theta, \gamma) \tag{2}$$

claim level. See Basu, Manning and Mullahy (2004) for an overview of these models.

[3] These provisions may include coinsurance rates, copayments, etc. In addition, current health insurance plans may also specify annual or lifetime coverage limits. According to the new health care law, insurers are no longer allowed to include coverage limits in the design of their plans. Finally, health insurance plans may specify covered benefits, drug formularies, exclusions, in-network providers, etc. Since these characteristics do not have clear financial implications, we do not include them in the model.

Proof. See Appendix A1.

Thus, when choosing a plan (p, d), a consumer of type γ trades off the flat up-front premium p, with an *indirect premium* $\frac{1}{\theta} \ln \Psi(d, \theta, \gamma)$, which is the certainty equivalent of the random prospect induced by the cost sharing mechanism, i.e., by the deductible. $\Psi(d, \theta, \gamma)$ is clearly increasing in the deductible d and, as shown in appendix A2, is also increasing in the coefficient of risk aversion θ and in the type γ.

The next proposition describes the resulting separation of types among the two insurance plans.

Proposition 2 *Assume $d_1 > d_2$ and $p_1 < p_2$. Then, there exists a cutoff $\overline{\gamma}(p_1, d_1, p_2, d_2)$ such that a consumer of type γ chooses plan (p_1, d_1) if and only if $\gamma \leq \overline{\gamma}(p_1, d_1, p_2, d_2)$.*

Proof. See Appendix A3.

Thus, the healthier types, who expect low expenses, assign a lower probability to the event that the deductible will be reached, and select the plan with the lower premium and higher deductible. It can be shown that $\overline{\gamma}(p_1, d_1, p_2, d_2)$ is increasing in p_2 and d_2, and decreasing in p_1 and d_1; as expected, an increase in either the premium or the deductible of an insurance plan will make that plan less attractive, shrinking its market share.

2.2 Insurers' Problems

For any possible combination of the private and public plan characteristics (p_g, d_g, p_p, d_p), denote the set of consumers who choose the private and public plan by $\Omega^p(p_g, d_g, p_p, d_p)$ and $\Omega^g(p_g, d_g, p_p, d_p)$, respectively. As elicited by Proposition 2, the insurer with the higher deductible and lower premium will attract the healthier individuals. When, for instance, when $p_p < p_g$, the private plan captures the healthier part of market and $\Omega^p(p_g, d_g, p_p, d_p) = \{\gamma : \gamma \leq \overline{\gamma}(p_g, d_g, p_p, d_p)\}$.

Now, given the public plan's premium and deductible (p_g, d_g), the private insurer's decision is to choose p_p and d_p in order to maximize its profit $\pi^P(p_g, d_g, p_p, d_p)$, defined as

$$
\begin{aligned}
\pi^P(p_g, d_g, p_p, d_p) &= \int_{\Omega^p(p_g, d_g, p_p, d_p)} \left(p_p - \int_{d_p}^{\infty} \frac{t - d_p}{\gamma} e^{-\frac{t}{\gamma}} dt \right) dH(\gamma) \\
&= \int_{\Omega^p(p_g, d_g, p_p, d_p)} \left(p_p - \gamma e^{-\frac{d_p}{\gamma}} \right) dH(\gamma)
\end{aligned}
\tag{3}
$$

On the other hand, given the private plan's premium and deductible (p_p, d_p), the public insurer's problem is to choose p_g and d_g so as to maximize the social welfare $SW(p_g, d_g, p_p, d_p)$, subject to a non-negative constraint on its profit $\pi^G(p_g, d_g, p_p, d_p) \geq 0$, where

$$\pi^G(p_g, d_g, p_p, d_p) = \int_{\Omega^g(p_g,d_g,p_p,d_p)} \left(p_g - \gamma e^{-\frac{d_g}{\gamma}}\right) dH(\gamma) \tag{4}$$

To construct the social welfare function, we first compute the consumer surplus of an individual of type γ who purchases a generic insurance contract (p, d). For this, we compute the value of the premium p_0 that would make the individual indifferent between paying the premium p_0 for a contract with deductible d, and having no insurance at all. Then, the consumer surplus will be $p_0 - p$. Since the value of the expected utility experienced by the individual with no insurance does not depend on any of the choice variables (p_g, d_g, p, d_p), and thus does not affect the equilibrium game play between the two insurers, we forgo computing it and instead denote it by $U_0(\gamma)$. Therefore, p_0 will be the solution to the equation:

$$\int_0^d u(w - p_0 - t) f(t)dt + \int_d^\infty u(w - p_0 - d) f(t)dt = U_0(\gamma)$$

Solving it, we obtain:

$$p_0 + \frac{1}{\theta} \ln \Psi(d, \theta, \gamma) = w + \frac{1}{\theta} \ln U_0(\gamma)$$

Therefore,

$$\begin{aligned} CS(\gamma, p, d) &= p_0 - p \\ &= A(\gamma) - \frac{1}{\theta} \ln \Psi(d, \theta, \gamma) - p \end{aligned} \tag{5}$$

where $A(\gamma)$ is a function that does not depend on p or d. On the other hand, the expected profit that the seller of an insurance contract (p, d) would make on an individual of type γ is

$$PS(\gamma, p, d) = p - \int_d^\infty \frac{t-d}{\gamma} e^{-\frac{t}{\gamma}} dt = p - \gamma e^{-\frac{d}{\gamma}} \tag{6}$$

Thus, from (5) and (6) it follows that:

$$\begin{aligned} SW(p_g, d_g, p_p, d_p) &= A - \int_{\Omega^p(p_g,d_g,p_p,d_p)} \left[\frac{1}{\theta} \ln \Psi(d_p, \theta, \gamma) + \gamma e^{-\frac{d}{\gamma}}\right] dH(\gamma) \\ &\quad - \int_{\Omega^g(p_g,d_g,p_p,d_p)} \left[\frac{1}{\theta} \ln \Psi(d_g, \theta, \gamma) + \gamma e^{-\frac{d_g}{\gamma}}\right] dH(\gamma) \end{aligned} \tag{7}$$

where A is a constant. Note that $SW(p_g, d_g, p_p, d_p)$ does not depend directly on the two premiums p_p and p_g since they represent transfers from the consumers to the insurers that do not affect the social welfare. However, the two premiums affect social welfare through their effects on the market share of the two insurers, as elicited by $\overline{\gamma}(p_g, d_g, p_p, d_p)$. On the other hand, the two deductibles also affect social welfare directly through their effects on the consumer and producer surpluses associated with each type γ. This is because a higher deductible reduces the consumer surplus, as the individual bears more of the risk, and increases the insurer's profit by reducing its expected costs. We assume that the public and private insurers compete over market share by adjusting premiums and deductibles, and we solve the model and characterize the Nash equilibrium of the market in the empirical sections below.

So far, we have assumed that the public option competes against a single monopolistic private insurer. Although the presence of the public insurer does induce some degree of competition in the market, the public insurer aims at maximizing social welfare instead of profit, and the private insurer is thus the only profit-maximizing player in the market. In reality, while the health insurance market does exhibit a significant and increasing degree of concentration and monopoly power, the market is not perfectly monopolistic.[4] To acknowledge this in a manageable way under our model framework, we simulate an alternative scenario in which the private insurer is constrained by a maximum profit margin τ over actuarial cost, corresponding to the case where several monopolistic competitors in the private segment of the insurance market compete with each other and bid down their average profit margin. The parameter τ will be a measure of the competitiveness of the market, and will be one of the key variables in the comparative statics analysis.

In the later simulation analysis, we will also investigate claims that the public option may either have a stronger bargaining power with health care providers and is thus more cost effective, or alternatively, that a lack of efficiency may actually lead to higher administrative costs of the public plan. Both of these claims have been frequently argued by both sides of the health care reform debate. We proceed by introducing an additional parameter, μ, to measure the cost effectiveness of the public insurer. When $\mu = 1$, the public plan is running with the same efficiency as the private plan, and the costs the public insurer faces are identical to the actual medical expenditures as drawn from the exponential distribution by individual patients. $\mu < 1$ corresponds to the case that the public plan is more cost effective and can thus cover the patients at a discounted rate, and $\mu > 1$ refers to the opposite case when the public insurer faces a higher operating and management cost. The private insurer always faces the same costs as the patients actually incur (i.e., as drawn from the exponential distribution).

The stylized model that we have constructed makes several strong albeit very interesting assumptions. First, we assume that everyone is required by law to purchase medical insurance, i.e., an "individual mandate", an assumption not satisfied in the current U.S. legislation, but that has been signed into law ("Patient Protection and Affordable Care Act" of March 23, 2010) and

[4] Dafny (2010) elicits the presence of market power in most insurance markets by identifying increases in premiums for employer-sponsored plans following a positive profitability shock to an employer.

will take effect in 2014.[5] Secondly, we assume that individuals purchase medical insurance by themselves, whereas in the U.S., most non-elderly people purchase medical insurance through their employer-sponsored plans, i.e., through a two-tier decision making process in which the employers first negotiate with insurance companies and choose several plans to sponsor, and then employees make their purchase decisions within the limited set of the chosen plans, or opt out.[6] However, in our model there are only two insurers, and we thus assume that individuals will make their choice entirely by themselves. Moreover, whether employer-based health insurance should continue to be the format going forward is again up for debate (Reinhardt 2009), and our simulation study based on this assumption is still very relevant. Finally, for simplicity we assume that individuals have the same level of risk aversion and are homogenous on all dimensions of their economic and social status except for their health conditions. This is a useful and very standard assumption in the literature, yet we should still note that the quantitative results of our simulation study in Section 4 need to be interpreted with caution.

3 Estimation of the Underlying Distribution of Expected Medical expenditures

To examine the empirical characteristics of market equilibrium as implied by the model, we need to obtain a realistic calibration of the underlying distribution $H(\cdot)$ of the *expected* medical expenditures γ_i for the U.S. consumers. This is because when purchasing medical insurance, consumers are unable to observe the actual medical expenditures in the future. Rather, they have to make their decisions based on the probabilistic distributions of the actual medical expenses that they may incur over the following year. Therefore, they face uncertainty when making purchasing decisions, even with the private knowledge of their own medical "types" γ_i.

Most of the existing studies in the literature, however, have been concentrated on analyzing a group of consumers' *actual* medical expenditures, for instance, by regressing the actual expenditures on various observable characteristics of the consumers in a given sample, such as age, gender, and income. Such a methodology can generate a model-based prediction of the actual expenses, i.e., a fixed number for each individual, conditional on the observed consumer characteristics. However,

[5] The legislation is currently being widely challenged in federal courts as "unconstitutional," and the U.S. Supreme Court is expected to rule on the case soon.

[6] According to a survey conducted by Kaiser Family Foundation (2010), individual, or non-group, health insurance covers about 14 million nonelderly people in the U.S., making it the least common source of health insurance. In contrast, about 157 million nonelderly people are covered by employer-sponsored insurance.

it is unable to capture the uncertainty or the probabilistic distribution that each individual faces when making purchasing decisions, and is only able to provide a group of different predicted values for a given set of consumers, rather than the distribution of a continuum of heterogeneous "types" or expectations of medical expenditures for the whole U.S. population. We thus decide to take a more structural approach and estimate a Bayesian hierarchical model of conjugate likelihood distribution of the expected medical expenditures, details to be explained later.

We estimate this distribution using data from the Medical Expenditure Panel Survey (MEPS), which is the most comprehensive survey on medical service of the U.S. households, their medical providers, and employers. The Household Component of the MEPS collects detailed data from a sample of families and individuals in selected communities across the U.S., including information such as demographic characteristics, health conditions, use of medical services, charges and source of payments, etc.

For our purpose, we extract the medical expenses incurred by all individuals included in the database and estimate the underlying distribution of expected medical expenditures for the U.S. consumers. We use the data from the year of 2007, the latest year for which the MEPS had expenditure data on when we estimated the distribution. Among the $30,964$ individuals, we focus on the ones that are of age 64 and under, since those of age 65 and above are covered by the U.S. Medicare program, and are thus not the focus of the current medical policy debates or our study. Therefore our estimation sample consists of $27,238$ individuals of age 64 and under in the U.S., with 35% are under the age of 18, 23% are between 18 and 34, 25% between 35 and 50, and 17% between 51 and 64. The mean medical expenditure in 2007 was \$2,600, and the median \$434. Figure 1 shows the empirical histogram of the observed medical expenditures, which is heavily skewed to the right.

Since the MEPS only reports the *actual* or *ex-post* expenditures of the survey respondents but not their *ex-ante expectations* of medical expenditures, we need to infer the distribution of the latter from the observations of the former with the help of some statistical modeling. Because of this hierarchical structure, we estimate a Bayesian hierarchical model of conjugate likelihood distributions (George, Makov and Smith, 1993) to uncover the distribution of the individual's expectations of their own medical expenditures, γ_i, whose cumulative distribution function is denoted as $H(\cdot)$ as in Section 2.

We assume that γ_i follows the conjugate likelihood prior distribution of a negative exponential distribution, i.e., an Inverse Gamma distribution of parameters α and β, which is also a highly skewed distribution to the right, similar to the histogram of the realized medical expenditures in Figure 1. Thus, conditional on α and β, the individual parameter distributions γ_i are independent and identically distributed as:

$$s(\gamma_i|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\gamma_i^{-\alpha-1}e^{-\frac{\beta}{\gamma_i}} \text{ for } \gamma_i \geq 0 \tag{8}$$

For α and β , rather than treating both of them as fixed as in Gaver & O'Muircheartaigh (1987), we proceed by conducting a Bayesian estimation, similar to the strategy in Gelfand and Smith (1990). In particular, we put a Gamma prior on β in the form of $s(\beta) = \beta^{k_{\beta1}-1}\exp(-\beta/k_{\beta2})$, i.e., a Gamma distribution with parameters $(k_{\beta1}, k_{\beta2})$, and estimate the *distribution* of β using Markov Chain Monte Carlo (MCMC) method in a Gibbs Sampling algorithm. α will be estimated using a Method of Moment estimator in each round of Gibbs sampling.[7]

Conditional on α and β, from (8) and the underlying assumption of exponentially distributed individual heath expenses, (i.e., $s(t_i|\gamma_i,\alpha,\beta) = \frac{1}{\gamma_i}e^{-\frac{t_i}{\gamma_i}}$), it follows that the posterior for γ_i upon observing the individual i's health expenses t_i is:

$$s(\gamma_i|t_i,\alpha,\beta) = \frac{s(\gamma_i,t_i|\alpha,\beta)}{s(t_i|\alpha,\beta)} = \frac{s(t_i|\gamma_i,\alpha,\beta)s(\gamma_i|\alpha,\beta)}{s(t_i|\alpha,\beta)}$$

and

$$s(\gamma_i|t_i,\alpha,\beta) \propto \left(\frac{1}{\gamma_i}e^{-\frac{t_i}{\gamma_i}}\right)\left(\gamma_i^{-\alpha-1}e^{-\frac{\beta}{\gamma_i}}\right) = \gamma_i^{-\alpha-2}e^{-\frac{\beta+t_i}{\gamma_i}}$$

Therefore, the posterior distribution of γ_i, $s(\gamma_i|t_i,\alpha,\beta)$, is an Inverse Gamma distribution with parameters $\alpha+1$ and $\beta+t_i$.

Since the likelihood function of an Inverse Gamma distribution is:

$$L(\beta|\alpha,\gamma_1,...,\gamma_n) \propto \left[\frac{\beta^\alpha}{\Gamma(\alpha)}\right]^n \left(\prod_{i=1}^n \gamma_i^{-1}\right)^{\alpha+1} e^{-\beta\sum_{i=1}^n \gamma_i^{-1}} \tag{9}$$

the posterior of β given α and $\gamma_1,...,\gamma_n$ is of the form:

$$s(\beta|\alpha,\gamma_1,...,\gamma_n) \propto L(\beta|\alpha,\gamma_1,...,\gamma_n)\, s(\beta) \tag{10}$$

Furthermore, we can derive the posteriors for β as

$$s(\beta|\alpha,\gamma_1,...,\gamma_n) \propto \beta^{\alpha n}e^{-\beta\sum_{i=1}^n \gamma_i^{-1}} s(\beta) \tag{11}$$

Thus the posterior distribution for β, $s(\beta|\alpha,\gamma_1,...,\gamma_n)$, is indeed a Gamma distribution with parameters $\alpha n + k_{\beta1}$ and $(\sum_{i=1}^n \gamma_i^{-1} + k_{\beta2}^{-1})^{-1}$.

[7] We have also experimented a full-scale Bayesian estimation by putting an exponential prior on α as well. The posterior probability density function of α turns out to be quite complicated and is not a known distribution, and cannot be directly simulated. Thus an Adaptive Rejection Sampling (ARS) algorithm is adopted to numerically draw α in each round of the MCMC. Unfortunately, the ARS algorithm is very time-consuming, and the MCMC converges quite slowly and the results are not very stable. Therefore, we finally chose to estimate α using a Method of Moment estimator.

With the analytical posterior distributions of γ_i and β derived above, we are now ready to estimate the distribution $H(\cdot)$ using Markov Chain Monte Carlo (MCMC) method in a Gibbs Sampling algorithm. The parameters of the Bayesian hierarchical model to be estimated are α and β. To obtain the *joint posterior distribution* of (α, β), we simulate $100,000$ iterations of the Gibbs samplers, with the first $99,000$ observations as the initial burn-in period, and report the inferences of the last $1,000$ random draws of the joint posterior distribution of α and β. For robustness we start with four different prior distributions of β, $s(\beta) = Gamma(1, 0.1)$; $s(\beta) = Gamma(1, 1)$; $s(\beta) = Gamma(0.1, 1)$, and $s(\beta) = Gamma(0.1, 100)$.

In particular, for each Bayesian prior, the following steps are conducted:

1). Upon observing the actual health expense $\{t_i\}_{i=1}^n$, we draw the posterior $\{\gamma_i\}_{i=1}^I$ from an Inverse Gamma distribution $(\alpha + 1, \beta + t_i)$;

2). Conditional on the simulated $\{\gamma_i\}_{i=1}^I$ and α obtained from last round, draw the posterior β from a Gamma distribution $(\alpha n + k_{\beta 1}, [\sum_{i=1}^n \gamma_i^{-1} + k_{\beta 2}^{-1}]^{-1})$;

3). Estimate α using a method-of-moments empirical Bayes argument based on $E(\gamma_i) = \beta/(\alpha - 1) \approx \sum_{i=1}^n \gamma_i / n$.

And then we go back to the step 1 and iterate the Gibbs sampler for $100,000$ times.

The MCMC converges very quickly, within the first few thousands of Bayesian draws. Table 1 presents the median and 95% error bands of the marginal distributions of α and β for each of the four priors. As displayed in the table, the final convergence points of the four different priors are very close, and the error bands are quite tight around the median estimates, both supporting the accuracy of the MCMC estimation. Thus the following empirical calibration of market equilibrium will be based on the median estimates of α and β as reported.

Figure 2 displays the probability density function of the estimated posterior distribution of γ_i, with expenses normalized to units of \$1,000 in model estimation and simulation. It can be immediately seen that the estimated Inverse Gamma distribution of γ_i "fits" the observed frequency of realized expenditures t_i in Figure 1 reasonably well, and that the parameter estimates are very similar across the four different priors. For instance, the mean of the posterior distribution of γ_i under prior 1 is \$2,585, compared with the mean of the actual medical expenditure of \$2,600. However, one shall note that t_i is *not* the same as γ_i, but rather the realization of some random variable which follows an exponential distribution with a *mean* of γ_i.

4 Empirical Results

Based on the estimated posterior distribution of expected medical expenditures, we then solve the model for the market equilibrium. We use the NIRA (Nikaido-Isoda/Relaxation Algorithm) as developed by Krawczyk and Zuccollo (2006) to numerically solve for a Nash Equilibrium over possible combinations of (p_g, d_g, p_p, d_p) in the parameter space, in which the two players, the private insurer and the public insurer, choose their optimal strategies (p_p, d_p) and (p_g, d_g) independently, with the goal of maximizing private profit and social welfare in equations (3) and (7), respectively. With a starting point, the NIRA algorithm conducts an iterative search process for a fixed point that represents a Nash Equilibrium, and the process converges when the fixed point is obtained.[8]

We begin our numerical search at different starting points of (p_g, d_g, p_p, d_p), and once the NIRA algorithm converges and a convergence point is obtained, we conduct a grid search around it, in order to make sure that the convergence point represents the best responses for each insurer, given the other insurer's strategy as reflected in the convergence point. Through this procedure, we can verify that the convergence point is indeed a Nash equilibrium.

On the other hand, when selecting the starting points of (p_g, d_g, p_p, d_p), we use points that are evenly sparsed in the parameter space, and conduct the NIRA numerical algorithm from each of these points. Most starting points lead to non-convergence or converge to unreasonable solutions such as negative premiums or deductibles going to infinity, and are thus eliminated. The starting points that lead to convergence to a reasonable solution determine the same Nash Equilibrium for a given specification of the parameters of the model. These ensure the uniqueness of the obtained Nash equilibrium..

The model is highly non-linear and involves integrals without analytical solutions in several equations. Therefore, when solving the model and calculating each player's payoffs, a numerical quadrature algorithm is adopted to numerically integrate equations (3) and (7).[9] We also employ

[8]The NIRA procedure is based on the observation that the problem of finding the fixed point of a correspondence is equivalent to that of maximizing a properly defined induced function. Thus, using the standard notation from a normal form game of a player i's strategy by $s_i \in S_i$, and of the resulting payoffs $u_i : \mathbf{S} \to \mathbb{R}$ by $u_i(\mathbf{s})$, one defines the Nikaido-Isoda function $\Psi : \mathbf{S} \times \mathbf{S} \to \mathbb{R}$ by $\Psi(\mathbf{s}', \mathbf{s}) \equiv \sum_{i=1}^{n} [u_i(s_i', s_{-i}) - u_i(s_i, s_{-i})]$. It is straightforward to see that $\Psi(\mathbf{s}, \mathbf{s}) = 0$ and that $\Psi_0(\mathbf{s}) \equiv \max_{\mathbf{s}' \in \mathbf{S}} \Psi(\mathbf{s}', \mathbf{s}) \geq 0$ for all $\mathbf{s} \in \mathbf{S}$. Moreover, $\mathbf{s}$ is a Nash equilibrium of the game if and only if $\Psi_0(\mathbf{s}) = 0$. When the set of available actions for a player is endogenously defined by the actions of the other players according to some constraint function, the Nikaido-Isoda function is defined in terms of the resulting Lagrangian that includes the constraint function. For more details, see Krawczyk and Zuccollo (2006).

[9]To ensure the accuracy of numerical integration, we adopt a composite trapezoidal quadrature rule and divide the integral interval at a very refined level, for instance, when calculating an insurer's profit over $(0, \overline{\gamma})$, the interval is divided into $1,000$ sub-intervals; when calculating an integral over $(\overline{\gamma}, \infty)$, the integral is first transformed into a finite interval $(0, 1)$, and the numerical quadrature is then conducted.

a non-linear equation solver to numerically solve for $\overline{\gamma}(p_g, d_g, p_p, d_p)$, the cut-off type between choosing public and private insurance plans for all possible (p_g, d_g, p_p, d_p). Details of our numerical methodology are available upon requests.

We first calibrate the value of the CARA risk preference parameter θ to the value estimated by Cohen and Einav (1997) of $1*10^{-3}$, but we also experiment with higher values of θ, as the literature, in particular macroeconomic literature, has generally calibrated with, or obtained through estimation, some higher values for the risk aversion parameter. Based on the posterior estimates of α and β, we find the Nash Equilibrium as described above, and report in Table 2 the market equilibrium $\overline{\gamma}$, the cut-off type between choosing public and private insurance plans. The implied private and public insurers' premium, deductible, and market share are also reported in the table. The results corresponding to different calibration values for the risk aversion parameter θ are reported in the same table.

Calibration results confirm that private insurer will still be able to capture a significant part of the insurance market. As suggested by (7), because the public insurer maximizes social welfare instead of its profit, it is indifferent between capturing the healthier or the less healthy individuals as long as the budget constraint is satisfied. Thus, at Nash equilibrium, the private insurer will always choose to offer a lower premium and capture the healthier part of market, generating a positive profit. The less healthy consumers will enroll in the public plan, paying a higher premium but enjoying a low deductible.

For instance, when θ is set to $1 * 10^{-3}$, the market equilibrium is characterized by a private plan charging a premium of $\$3,228$ and a deductible of \$548, along with a public plan with a higher premium of $\$3,535$ and an almost zero deductible. Even though with a higher premium, the public plan is still more attractive for consumers with an expected expenditure more than \$385, accounting for 66% of the population. Covering these relatively less healthy consumers at such premium and deductible generate zero profit for the public insurer, i.e., no fiscal burden for the government. In contrast, a lower premium combined with a higher deductible makes the private insurance plan more attractive to healthier consumers, as they expect to incur less medical expenditures during the year, and thus would prefer a plan with lower premium despite the higher deductible. The private plan will cover 34% of the whole population, generating a positive profit of $\$1,082$.

If the CARA parameter θ is set at $2 * 10^{-3}$, corresponding to the case when consumers are less willing to take the chance to bet on a low medical expense during the year, more consumers will choose the public plan, as the cut-off $\overline{\gamma}$ declines from \$385 to \$366 (column 2 of Table 2), and the market share of the private plan shrinks from 34% to 32%. With a higher market share, the public insurer is able to offer a lower premium, at $\$3,440$. This is because the increased market share, i.e., those expecting an expenditure from \$366 to \$385, is composed of consumers who are healthier than the public insurer's existing pool of consumers in column 1 of Table 2, and therefore the public insurer is able to lower the premium and still run a balanced budget. Accordingly, with a smaller market share and a lower premium, the private profit declines from $\$1,082$ to $\$987$. As we increased the CARA parameter, the public plan's market share will continue to rise. For instance, as shown in columns 3 and 4 of Table 2, when the CARA parameter θ is calibrated to $5 * 10^{-3}$ and $10 * 10^{-3}$, the public plan's market share will increase to 73% and 80%, respectively.

Next we explore how the market equilibrium may change if the non-negative profit constraint imposed on the public insurance plan is relaxed. In particular, we assume that, instead of having to run a non-negative profit, the public insurer is now allowed to run a loss, with the maximum amount of $250, i.e., now the government needs to subsidize the public insurance plan at no more than $250 per capita, which would amount to a maximal annual loss of $75 billion for the whole U.S. population of 301 million in 2007, or less than 2.7% of the U.S. federal expenditures in the same year.

Simulation results as reported in Table 3 suggest that, when the non-negative profit constraint is relaxed, the public insurer can substantially lower its premium, thus capturing a higher market share. Moreover, facing a lower public premium, the private insurer will also have to lower its premium and deductible, eventually leading to a social welfare improvement. For instance, in column 1 of Table 3 where the risk-preference parameter θ is set to $1 * 10^{-3}$, the public plan is charging a premium of $\$2,958$, $577 lower than in the same column of Table 2 when a non-negative profit constraint is imposed. The private plan responds by substantially cutting its premium and deductible as well, by $534 and $53, respectively. At equilibrium, the market share of the public insurer increases from 66% to 71%. The private insurer's profit declines significantly as well, from $\$1,082$ to $769. Social welfare improved from lower deductibles and an expansion of the public plan.

When the CARA parameter θ is set to higher levels, the public plan is capturing an even higher market share, as shown in columns 2 to 4 and explained above. On the other hand, compared with the equilibrium under a non-negative profit constraint in Table 2, both the public and private insurers are charging substantially lower premiums — indeed the reductions in both public and private premiums substantially exceed the maximal fiscal subsidy provided by the government of $250 per consumer. Both plans run at lower profits, yet the social welfare and consumer welfare improve.

How shall we compare the lower profitability and high premiums of the public plan with the high profitability and lower premiums of the private plan? Notice that up till now, we have assumed that both private and public insurance plans have the same management efficiency and operating costs, and the only reason why the public plan generates a lower profit at equilibrium is that, as a social welfare maximizer, the public insurer is providing insurance to the less healthy group of consumers, yielding the healthier, more profitable segment of the market to the private insurer, which turns out to run a substantial positive profit. On the other hand, as shown in Table 2, the public plan does not necessarily run on a loss. Yet providing fiscal subsidy to a limited extent will enable both the public and private insurers to substantially lower their premiums — even larger than the amount of fiscal subsidy the government provides — and improve social welfare.

A more cost-effective public insurer will drive both the public and private premiums down, and increase its market share, as shown in columns 1 and 2 of Table 4. For instance, when the risk-preference parameter θ is set to $1 * 10^{-3}$, a public plan that is 5% more cost effective than before will be able to lower premium by approximately \$380 and still run on a balanced budget, and occupy a larger market share of 71%, 5 percentage points higher than in our benchmark case in Table 2. Facing more competitive pressure from the public insurer, the private insurer will again be forced to lower its premium, from $\$3,223$ to $\$2,876$, and its profit shrinks from $\$1,082$ to \$830. Social welfare improves, as now the public plan with lower deductible has a higher market share. Simulation based on a higher value of the CARA parameter generates qualitatively similar results.

On the other hand, if the public plan becomes less competitive than the private plan, for instance due to a lack of efficiency and higher administrative costs, then both the public premium and private premium will increase, and public plan's market share, because it becomes less competitive, tends to decline. As shown in column 3 of Table 4, when the risk-preference parameter θ is set to $1*10^{-3}$, a public plan 5% less cost effective than private plan will have to increase its premium to avoid a profit loss, from $\$3,535$ in Table 2 to $\$3,914$, and the cut-off $\overline{\gamma}$ rises from \$385 to \$421, implying a significantly smaller market share of 62%, compared at the 73% in Table 2. Facing a less competitive public insurer, the private insurer can now increase its premium yet still occupying a larger market share, and its profit rises from $\$1,082$ to $\$1,335$. Social welfare declines as more consumers are now paying the higher private deductibles.

Finally, we investigate the scenarios in which the private segment of the insurance market faces more intense internal competition and thus the average private profit rate is lower. In particular, we impose a restriction that the private insurer's profit margin τ cannot exceed half of its equilibrium level in our benchmark case in Table 2. With this upper limit on the profit margin, the private insurer will substantially lower its deductible, by more than 50%, as shown in Table 5. Facing the enhanced competition, the public plan responds by lowering its premium, and at equilibrium its market share will slightly increase, by one to two percentage points. This is because, if at the equilibrium the public plan only lowers its premium to the extent that ensures the same market share as before, it will run a profit loss. Bounded by a zero-profit constraint, the public insurer will now be forced to lower its premium further, by more than enough to maintain the same market share as before, and ends with a slightly higher market share. With the upper limit over its profit margin, the private insurer has to run at a lower profit, yet social welfare improves significantly (Table 5).

5 Concluding Remarks

This paper develops a stochastic game-theoretical model to analyze the consumers' behavior in choosing between a private medical insurance plan and a public insurance plan, with the former a profit-maximizer and the latter a social-welfare-maximizer who faces different profit constraints. The model is calibrated based on the data on medical expenditure from the U.S. Medical Expenditure Panel Survey and estimation of a Bayesian hierarchical model using a Markov Chain Monte Carlo (MCMC) method. The Nash Equilibrium is solved using a numerical algorithm.

Calibration results reveal that the private insurer will not be completely driven out of the insurance market, and will still capture a significant part of the market. Moreover, at equilibrium, the public insurer will choose to cover the less healthy group of consumers, leaving the healthier, more profitable section of the market to the private insurer. Consequently, the private insurance plan generates a substantially positive profit, and the public plan runs at a balanced budget. When the risk-aversion coefficient is set to higher levels, the public plan's market share will rise, the public and private insurance premiums will decline, and private profit will fall. We also find that when the non-negative profit constraint imposed on the public plan is relaxed, both the public and private insurance insurers will substantially lower their premiums, and the social welfare improves. A more cost-effective public insurer, or an upper limit on private insurer's profit margin, will both lead to a decline in private insurer's profit and to an improvement of social welfare.

Appendix

Appendix A1. Proof of Proposition 1

A consumer of type γ chooses the plan (p_2, d_2) if and only if

$$\int_0^{d_1} u\left(w-p_1-t\right)f(t)dt + \int_{d_1}^{\infty} u\left(w-p_1-d_1\right)f(t)dt \leq$$
$$\int_0^{d_2} u\left(w-p_2-t\right)f(t)dt + \int_{d_2}^{\infty} u\left(w-p_2-d_2\right)f(t)dt \Longleftrightarrow$$
$$-\int_0^{d_1} e^{-\theta(w-p_1-t)}\frac{1}{\gamma}e^{-\frac{t}{\gamma}}dt - \int_{d_1}^{\infty} e^{-\theta(w-p_1-d_1)}\frac{1}{\gamma}e^{-\frac{t}{\gamma}}dt \leq$$
$$-\int_0^{d_2} e^{-\theta(w-p_2-t)}\frac{1}{\gamma}e^{-\frac{t}{\gamma}}dt - \int_{d'}^{\infty} e^{-\theta(w-p_2-d_2)}\frac{1}{\gamma}e^{-\frac{t}{\gamma}}dt \Longleftrightarrow$$
$$e^{\theta p_1}\left[\frac{1}{\gamma}\int_0^{d_1} e^{\left(\theta-\frac{1}{\gamma}\right)t}dt + \frac{1}{\gamma}e^{\theta d_1}\int_{d_1}^{\infty} e^{-\frac{t}{\gamma}}dt\right] \geq e^{\theta p_2}\left[\frac{1}{\gamma}\int_0^{d_2} e^{\left(\theta-\frac{1}{\gamma}\right)t}dt + \frac{1}{\gamma}e^{\theta d_2}\int_{d_2}^{\infty} e^{-\frac{t}{\gamma}}dt\right] \tag{12}$$

When $\gamma \neq \frac{1}{\theta}$, equation (12) can be written as $e^{\theta p_2}\frac{\theta\gamma e^{\left(\theta-\frac{1}{\gamma}\right)d_2}-1}{\theta\gamma-1} \leq e^{\theta p_1}\frac{\theta\gamma e^{\left(\theta-\frac{1}{\gamma}\right)d_1}-1}{\theta\gamma-1}$. When $\gamma = \frac{1}{\theta}$, (12) becomes $e^{\theta p_2}\left(1+\theta d_2\right) \leq e^{\theta p_1}\left(1+\theta d_1\right)$. Using the definition of $\Psi(d,\theta,\gamma)$ from (1), these can be written concisely as $e^{\theta p_2}\Psi(d_2,\theta,\gamma) \leq e^{\theta p_1}\Psi(d_1,\theta,\gamma)$, and this completes the proof. ■

Appendix A2.

We will show that $\Psi(d,\theta,\gamma)$ is increasing in θ and γ. Denote by $b\left(\lambda,\theta\right) \equiv \frac{\theta e^{(\theta-\lambda)d}-\lambda}{\theta-\lambda}$ and note that $\Psi(d,\theta,\gamma) = b(\frac{1}{\gamma},\theta)$. We first show that $\frac{\partial}{\partial\lambda}b(\lambda,\theta) < 0$ for $\lambda \in [0,\infty)\setminus\{\theta\}$. This would then immediately imply that $\frac{\partial}{\partial\gamma}\Psi(d,\theta,\gamma) = -\frac{1}{\gamma^2}\frac{\partial}{\partial\lambda}b(\frac{1}{\gamma},\theta) > 0$. Thus, $\frac{\partial}{\partial\lambda}b(\lambda,\theta) = \frac{-d\theta e^{(\theta-\lambda)d}(\theta-\lambda)+\theta e^{(\theta-\lambda)d}-\theta}{(\theta-\lambda)^2}$, for $\lambda \neq \theta$. Therefore, when $\lambda \neq \theta$, we have

$$\frac{\partial}{\partial\lambda}b(\lambda,\theta) < 0 \Longleftrightarrow de^{(\theta-\lambda)d}\left(\theta-\lambda\right) > e^{(\theta-\lambda)d}-1 \tag{13}$$

Define $B_1(\theta) \equiv e^{(\theta-\lambda)d}$ and note that $B_1'(\theta) \equiv de^{(\theta-\lambda)d} > 0$ and $B_1''(\theta) \equiv d^2e^{(\theta-\lambda)d} > 0$. Thus, B_1 is increasing and strictly convex. Consider now some $\lambda < \theta$, and note that we can rewrite (13) as $de^{(\theta-\lambda)d} > \frac{e^{(\theta-\lambda)d}-1}{\theta-\lambda}$, that is: $B_1'(\theta) > \frac{B_1(\theta)-B_1(\lambda)}{\theta-\lambda}$. Since B_1 is convex and $\theta > \lambda$, this condition is satisfied. On the other hand, for $\lambda > \theta$, we rewrite (13) as $de^{(\theta-\lambda)d} < \frac{1-e^{(\theta-\lambda)d}}{\lambda-\theta}$, that is: $B_1'(\theta) < \frac{B_1(\lambda)-B_1(\theta)}{\lambda-\theta}$. Since B_1 is convex and $\theta < \lambda$, this condition is again satisfied. Therefore, $\frac{\partial}{\partial\lambda}b(\lambda,\theta) < 0$ as desired.

Next, we show that $\frac{\partial}{\partial\theta}b(\lambda,\theta) > 0$ for $\theta \in [0,\infty)\setminus\{\lambda\}$, which would imply immediately $\frac{\partial}{\partial\theta}\Psi(d,\theta,\gamma) > 0$. We have $\frac{\partial}{\partial\theta}b(\lambda,\theta) = \frac{e^{(\theta-\lambda)d}(1+\theta d)(\theta-\lambda)-\left[\theta e^{(\theta-\lambda)d}-\lambda\right]}{(\theta-\lambda)^2}$, for $\theta \neq \lambda$. Denote by $B_2(\theta) \equiv \theta e^{(\theta-\lambda)d}$ and note that $\frac{\partial}{\partial\theta}b(\lambda,\theta) > 0$ if and only if $B_2'(\theta) > \frac{B_2(\theta)-B_2(\lambda)}{\theta-\lambda}$. Since $B_2'(\theta) \equiv (1+\theta d)\, e^{(\theta-\lambda)d} > 0$ and $B_1''(\theta) \equiv \left(2d+\theta d^2\right) e^{(\theta-\lambda)d} > 0$, the result follows using the same argument as for $\frac{\partial}{\partial\lambda}b(\lambda,\theta)$. ■

Appendix A3. Proof of Proposition 2

By Proposition 1, a consumer of type γ will chose the insurance plan (p_1, d_1) if an only if $e^{\theta(p_2-p_1)} \geq k(\gamma, d_1, d_2)$ where when

$$k(\gamma, d_1, d_2) \equiv \frac{\Psi(d_1,\theta,\gamma)}{\Psi(d_2,\theta,\gamma)} = \begin{cases} \frac{\theta\gamma e^{\left(\theta-\frac{1}{\gamma}\right)d_1}-1}{\theta\gamma e^{\left(\theta-\frac{1}{\gamma}\right)d_2}-1}, & \text{when } \gamma \neq \frac{1}{\theta} \\ \frac{1+\theta d_1}{1+\theta d_2}, & \text{when } \gamma = \frac{1}{\theta} \end{cases} \tag{14}$$

Note that $\lim_{\gamma\to\frac{1}{\theta}} k(\gamma, d_1, d_2) = k(\frac{1}{\theta}, d_1, d_2)$, so $k(\cdot, d_1, d_2)$ is continuous at $\frac{1}{\theta}$, and thus everywhere. Next, we will show that $\frac{\partial}{\partial\gamma}k(\gamma, d_1, d_2) > 0$ for $\gamma \in [0,\infty)\backslash\left\{\frac{1}{\theta}\right\}$. This would complete the argument because it would imply that the condition $e^{\theta(p_2-p_1)} \leq k(\gamma, d_1, d_2)$ is satisfied for all the $\gamma \geq \overline{\gamma}(p_1, d_1, p_2, d_2)$ where $\overline{\gamma}(p_1, d_1, p_2, d_2)$ is defined implicitly by $e^{\theta(p_2-p_1)} = k(\overline{\gamma}, d_1, d_2)$.

Denote by

$$k_1(z, \lambda_1, \lambda_2) = \begin{cases} \frac{e^{(1-z)\lambda_1}-z}{e^{(1-z)\lambda_2}-z}, & \text{when } z \neq 1 \\ \frac{1+\lambda_1}{1+\lambda_2}, & \text{when } z = 1 \end{cases}$$

and note that $k(\gamma, d_1, d_2) = k_1(\frac{1}{\theta\gamma}, \theta d_1, \theta d_2)$. Thus, to show $\frac{\partial}{\partial\gamma}k(\gamma, d_1, d_2) > 0$, when $d_1 > d_2 > 0$, it is enough to show that $\frac{\partial}{\partial z}k_1(z, \lambda_1, \lambda_2) < 0$, when $z > 0$ and $\lambda_1 > \lambda_2 > 0$. The arguments are different for the cases when $z < 1$ and $z > 1$, and so we will consider the two cases separately.

Lemma 3 *$\frac{\partial}{\partial z}k_1(z, \lambda_1, \lambda_2) < 0$ for $z > 1$ and $\lambda_1 > \lambda_2 > 0$.*

Proof. We have

$$\frac{\partial}{\partial z}k_1(z, \lambda_1, \lambda_2) = \frac{\left[1+\lambda_1 e^{(1-z)\lambda_1}\right]\left[z-e^{(1-z)\lambda_2}\right] - \left[1+\lambda_2 e^{(1-z)\lambda_2}\right]\left[z-e^{(1-z)\lambda_1}\right]}{\left[e^{(1-z)\lambda_2}-z\right]^2}$$

so it is enough to show that $G(\lambda_1) < 0$ for $\lambda_1 > \lambda_2 > 0$ where

$$G(\lambda_1) \equiv \left[1+\lambda_1 e^{(1-z)\lambda_1}\right]\left[z-e^{(1-z)\lambda_2}\right] - \left[1+\lambda_2 e^{(1-z)\lambda_2}\right]\left[z-e^{(1-z)\lambda_1}\right]$$

is a function of λ_1. Since $G(\lambda_1)|_{\lambda_1=\lambda_2} = 0$, it is enough to show that $G'(\lambda_1) < 0$ for $\lambda_1 > \lambda_2$. Now, $G'(\lambda_1) = z\,(1-z)\, e^{(1-z)\lambda_1} + e^{(1-z)\lambda_1} - e^{(1-z)(\lambda_1+\lambda_2)} - (1-z)\,(\lambda_1-\lambda_2)\, e^{(1-z)(\lambda_1+\lambda_2)}$, so to show $G'(\lambda_1) < 0$, it is enough to show that

$$e^{(z-1)\lambda_2}\left[1 - z\,(z-1)\,\lambda_1\right] < 1 - (\lambda_1-\lambda_2)\,(z-1) \tag{15}$$

Denote by $LHS\,(z)$ and $RHS\,(z)$ the left and right hand sides of (15) as functions of z and note that $LHS\,(1) = RHS\,(1) = 1$ so it is enough to show that $LHS'\,(z) < RHS'\,(z)$ for $z > 1$. Now,

$$\begin{aligned} RHS'\,(z) &= \lambda_2 - \lambda_1 \\ LHS'(z) &= \lambda_2 e^{(z-1)\lambda_2}\left[1 - z\,(z-1)\,\lambda_1\right] + e^{(z-1)\lambda_2}\left[-2z+1\right]\lambda_1 \end{aligned}$$

Note that $LHS'\,(1) = RHS'\,(1) = \lambda_2 - \lambda_1$ so to prove that $LHS'\,(z) < RHS'\,(z)$ for $z > 1$, it is enough to show that $LHS''\,(z) < RHS''\,(z)$ for $z > 1$. We have

$$\begin{aligned} RHS''\,(z) &= 0 \\ LHS''(z) &= \lambda_2^2 e^{(z-1)\lambda_2}\left[1 - z\,(z-1)\,\lambda_1\right] + \lambda_2 e^{(z-1)\lambda_2}\left[-2z+1\right]\lambda_1 + \lambda_2 e^{(z-1)\lambda_2}\left[-2z+1\right]\lambda_1 - 2e^{(z-1)\lambda_2}\lambda_1 \\ &= \left[\lambda_2^2 e^{(z-1)\lambda_2} - \lambda_2 z e^{(z-1)\lambda_2}\lambda_1\right] + \\ &\quad \left[-\lambda_2^2 e^{(z-1)\lambda_2} z\,(z-1)\,\lambda_1 + \lambda_2 e^{(z-1)\lambda_2}\left[-z+1\right]\lambda_1 + \lambda_2 e^{(z-1)\lambda_2}\left[-2z+1\right]\lambda_1 - 2e^{(z-1)\lambda_2}\lambda_1\right] \end{aligned}$$

Note that $\lambda_2^2 e^{(z-1)\lambda_2} - \lambda_2 z e^{(z-1)\lambda_2}\lambda_1 < 0$ because $z > 1$ and $\lambda_2 < \lambda_1$. The rest of the terms in $LHS''(z)$ are all negative since $z > 1$ and therefore $LHS''(z) < 0 = RHS''(z)$. This completes the proof of the lemma. ■

Lemma 4 *$\frac{\partial}{\partial z} k_1(z, \lambda_1, \lambda_2) < 0$ for $0 < z < 1$ and $\lambda_1 > \lambda_2 > 0$.*

Proof. With a change of variable $y = -z$, we can rewrite $k_1(z, \lambda_1, \lambda_2)$ as $k_2(-z)$ where

$$k_2(y) \equiv \frac{e^{(1+y)\lambda_1} + y}{e^{(1+y)\lambda_2} + y}$$

so it is enough to show that $k_2'(y) > 0$ for $y \in (-1, 0)$. Now,

$$k_2'(y) = \frac{\left[\lambda_1 e^{(1+y)\lambda_1} + 1\right]\left[e^{(1+y)\lambda_2} + y\right] - \left[\lambda_2 e^{(1+y)\lambda_2} + 1\right]\left[e^{(1+y)\lambda_1} + y\right]}{\left[e^{(1+y)\lambda_2} + y\right]^2}$$

so it is enough to show that $J(y) > 0$ for $y \in (-1, 0)$ where

$$\begin{aligned} J(y) &\equiv \left[\lambda_1 e^{(1+y)\lambda_1} + 1\right]\left[e^{(1+y)\lambda_2} + y\right] - \left[\lambda_2 e^{(1+y)\lambda_2} + 1\right]\left[e^{(1+y)\lambda_1} + y\right] \\ &= (\lambda_1 - \lambda_2)\, e^{(1+y)(\lambda_1+\lambda_2)} + y\lambda_1 e^{(1+y)\lambda_1} - y\lambda_2 e^{(1+y)\lambda_2} + e^{(1+y)\lambda_2} - e^{(1+y)\lambda_1} \end{aligned}$$

$J(-1) = 0$, so it is enough to show that $J'(y) > 0$ for $y \in (-1, 0)$. But $J'(y) = \left(\lambda_1^2 - \lambda_2^2\right) e^{(1+y)(\lambda_1+\lambda_2)} + y\lambda_1^2 e^{(1+y)\lambda_1} - y\lambda_2^2 e^{(1+y)\lambda_2}$. By writing $J'(y)$ as a function of λ_1, we denote by

$$I\,(\lambda_1) \equiv J'(y) = \left(\lambda_1^2 - \lambda_2^2\right) e^{(1+y)(\lambda_1+\lambda_2)} + y\lambda_1^2 e^{(1+y)\lambda_1} - y\lambda_2^2 e^{(1+y)\lambda_2}$$

Note that $I(\lambda_1)|_{\lambda_1=\lambda_2} = 0$, so it is enough to show that $I'(\lambda_1) > 0$ for $\lambda_1 > \lambda_2$. But $I'(\lambda_1) = 2\lambda_1 e^{(1+y)(\lambda_1+\lambda_2)} + \left(\lambda_1^2 - \lambda_2^2\right)(1+y)\, e^{(1+y)(\lambda_1+\lambda_2)} + 2y\lambda_1 e^{(1+y)\lambda_1} + y(1+y)\lambda_1^2 e^{(1+y)\lambda_1}$, so it is enough to show that

$$\left[2\lambda_1 + \left(\lambda_1^2 - \lambda_2^2\right)(1+y)\right] e^{(1+y)\lambda_2} > (-y)\left[2\lambda_1 + (1+y)\lambda_1^2\right] \tag{16}$$

Denote by $LHS(y)$ and $RHS(y)$ the left and right hand sides of (16) and note that $LHS(-1) = RHS(-1) = 2\lambda_1$. Therefore, to prove the inequality, it is enough to show that $LHS'(y) > RHS'(y)$ for $y \in (-1, 0)$. Now, $LHS'(y) = \left[2\lambda_1\lambda_2 - \lambda_2^2 + \left(\lambda_1^2 - \lambda_2^2\right)(1+y)\lambda_2 + \lambda_1^2\right] e^{(1+y)\lambda_2}$ and $RHS'(y) = -2\lambda_1 - (2y+1)\lambda_1^2$. Note that, $2\lambda_1\lambda_2 > \lambda_2^2$ because $\lambda_1 > \lambda_2$, $\left(\lambda_1^2 - \lambda_2^2\right)(1+y)\lambda_2 > 0$ because $1 + y > 0$, $\lambda_1^2 > -(2y+1)\lambda_1^2$ because $y \in (-1, 0)$ and $e^{(1+y)\lambda_2} > 1$ because $1 + y > 0$. Therefore, indeed $LHS'(y) > RHS'(y)$ for $y \in (-1, 0)$ which completes the proof of the lemma. ■

This completes the proof of proposition 2. ■

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Table 1: Posterior Distribution of γ_i

	Prior 1	Prior 2	Prior 3	Prior 4
α				
Median	1.1956	1.1944	1.1949	1.1954
95% band	(1.1778,1.2063)	(1.1772,1.2066)	(1.1780,1.2052)	(1.1802,1.2066)
β				
Median	0.5056	0.5032	0.5044	0.5055
95% band	(0.4917,0.5186)	(0.4893,0.5178)	(0.4904,0.5170)	(0.4926,0.5193)

Table 2: Nash Equilibrium with A Non-negative Public Profit Constraint

	$\theta = 1 * 10^{-3}$	$\theta = 2 * 10^{-3}$	$\theta = 5 * 10^{-3}$	$\theta = 10 * 10^{-3}$
Public Plan				
Premium	$3,535	$3,440	$3,234	$2,988
Deductible	$4	$3	$15	$1
Profit	$0	$0	$0	$0
Private Plan				
Premium	$3,228	$3,133	$2,937	$2,613
Deductible	$548	$520	$465	$525
Profit	$1,082	$987	$792	$527
Market Share				
$\gamma^* * 1000$	$385	$366	$326	$269
Public Share	65.9%	68.0%	72.5%	79.5%
Private Share	34.1%	32.0%	27.5%	20.5%
Social Welfare	-$2,419.7	-$2,423.4	-$2,430.4	-$2,441.6

Table 3: Nash Equilibrium with a Maximal Public Loss of $250

	$\theta = 1 * 10^{-3}$	$\theta = 2 * 10^{-3}$	$\theta = 5 * 10^{-3}$	$\theta = 10 * 10^{-3}$
Public Plan				
Premium	$2,958	$2,900	$2,755	$2,274
Deductible	$6	$3	$6	$5
Profit	-$250	$-250	-$250	-$249
Private Plan				
Premium	$2,689	$2,625	$2,473	$2,020
Deductible	$495	$482	$445	$483
Profit	$769	$708	$563	$102
Market Share				
$\gamma^* * 1000$	$339	$325	$292	$151.4
Public Share	70.9%	72.5%	76.8%	94.8%
Private Share	29.1%	27.5%	23.2%	5.2%
Social Welfare	-$2,413.8	-$2,421.2	-$2,426.9	-$2,419.9

Table 4: Nash Equilibrium with Alternative Cost Effectiveness of Public Option

	$\mu = 0.95$		$\mu = 1.05$	
	$\theta = 1 * 10^{-3}$	$\theta = 10 * 10^{-3}$	$\theta = 1 * 10^{-3}$	$\theta = 10 * 10^{-3}$
Public Plan				
Premium	\$3,156	\$2,734	\$3,914	\$3,259
Deductible	\$4	\$0	\$1	\$5
Profit	\$0	\$0	\$0	\$0
Private Plan				
Premium	\$2,876	\$2,403	\$3,584	\$2,860
Deductible	\$525	\$475	\$562	\$550
Profit	\$830	\$404	\$1,335	\$672
Market Share				
$\gamma^* * 1000$	\$342	\$245	\$421	\$297
Public Share	70.7%	82.9%	62.2%	76.1%
Private Share	29.3%	17.1%	37.8%	23.9%
Social Welfare	-\$2,418.6	-\$2,433.7	-\$2,420.5	-\$2,448.9

Table 5: Nash Equilibrium with an Upper Limit on Private Insurers' Profit Margin

	$\theta = 1 * 10^{-3}$	$\theta = 2 * 10^{-3}$	$\theta = 5 * 10^{-3}$	$\theta = 10 * 10^{-3}$
Public Plan				
Premium	$3,490	$3,362	$3,149	$2,814
Deductible	$0	$0	$0	$0
Profit	$0	$0	$0	$0
Private Plan				
Premium	$3,385	$3,253	$3,022	$2,694
Deductible	$123	$129	$155	$148
Profit	$1,064	$935	$722	$390
Market Share				
$\gamma^* * 1000$	$375	$350	$304	$230
Public Share	64.9%	69.8%	75.0%	84.9%
Private Share	33.1%	30.2%	25.0%	15.1%
Social Welfare	-$2,415.0	$-2,415.3	-$2,416.5	-$-2,416.9

Figure 1. Sample from the U.S. Medical Expenditure Panel Survey

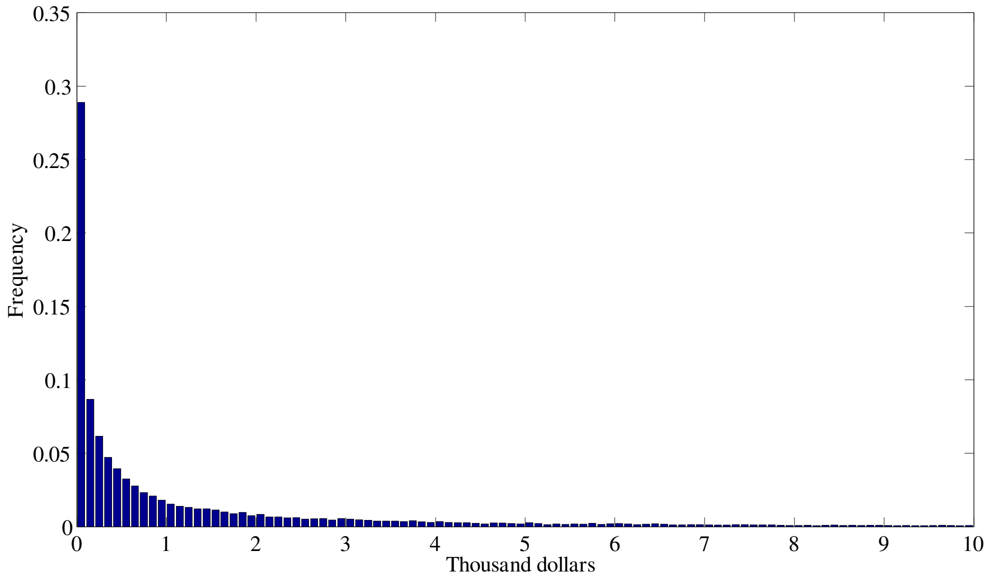

Figure 2. Posterior Distribution of Expected Medical Expenditure