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Halkos, George and Papageorgiou, George

University of Thessaly, Department of Economics

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George E. Halkos and George J. Papageorgiou
University of Thessaly, Department of Economics,
Korai 43, 38333, Volos, Greece

Abstract
Traditional economic theory, up to the middle of the twentieth century, builds up the production functions regardless of the inputs' scarcity. In the last few decades it has become clear that in many cases inputs are depletable quantities and at the same time a lot of constraints are imposed in their usage in order to ensure economic sustainability. Furthermore, the management of exploitation and use of natural resources (either exhaustible or renewable) has been discussed by analyzing dynamic models applying methods of Optimal Control Theory. This theory provides solutions that are concerned with a single decision maker who can control the model's dynamics facing a certain performance index to be optimized. In this paper we consider some simple taxation schemes based both on price charged and on the stock size as well. As the feedback taxation rules are more efficient than the other (non feedback) rules we have constructed the simple taxation scheme and found the analytical expression of the tax function.

Keywords: Nonrenewable resources; differential games; Markov equilibrium.

JEL classification: C61; C62; Q32.

1. Introduction

In the literature of Environmental Economics, existing models often makes an assumption in which the involved agents exploit resources from a common pool area in a non-cooperative way. This approach yields inefficiency in the well-known sense of the "tragedy of commons" (Benchekroun, 2003). Tragedy of commons refers to a situation in which a producible asset is exploited jointly by several economic agents whose "non-cooperative" behavior results in overexploitation of the asset, i.e. an exploitation of the asset that is not jointly efficient (not Pareto optimal).

Considering the market structure of resources exploitation in fact, these markets are often oligopolistic, i.e., there are several decision makers whose policies influence each other. So, game theoretical approaches are introduced into the discussion. According to the theory of continuous time models of Optimal Control, the appropriate analogue of differential games is used. Roughly, this is an extension of Optimal Control, when there is an exactly one decision maker, to the case of N (N>1) decision makers interacting with each other.

Dynamic models of exploitation (or harvesting) and use of natural resources refer to two different systems of property rights: in the case of sole ownership, optimal extraction policies can be obtained by means of Optimal Control Theory (Clark, 1976); in the case of open access or common property exploitation, game theoretical models are applicable in the sense that all decision makers exploit a resource from a common pool without any restriction, looking only at their own profits over some time horizon, and without considering the stock of the resource, which is diminished due to the extraction policies of all the players of the game who share the common pool (Clark, 1980, Dockner *et al.*, 1989). Whenever decision makers are few, one cannot use models of perfect competition, but the appropriate framework for the discussion of these problems is given by theoretical approaches with special attention to the question of "how to play the game": cooperatively or non–cooperatively?.

In Natural Resources Economics there is a chain of externalities arisen by human activities, known as environmental externalities. Once a natural resource is explored and is ready for exploitation the first externality arises from the fact that the extraction cost increases not only with the current exploitation rate, but with the cumulative amount extracted to date. Consequently, a unit of resource extracted today will inflict an inter-temporal externality in the form of pushing up extraction costs at all future dates, assuming a twice continuously differentiable cost function.

The cost function, along the extraction path, must be an increasing function not only with respect to the extraction rate but also it must be an increasing function of the remainder stock. In such a way it is possible to assume that the marginal current exploitation cost is higher both at higher exploitation rate and, for a constant rate of exploitation, at higher depletion rates.

The second externality is related to the use of the extracted resource. The resource used not only damages the environment through the current flow of an externality, but also harms the environment indirectly by adding to the accumulated stock of an externality and pushing it toward to a critical level.

From the supply side point of view, resource-extracting oligopolists continually engage in the search for additional stocks or in finding new technologies to transform resources that are economically non-exploitable into resources that can be profitably extracted. If the demand curve facing the industry is elastic, the discovery of additional stocks will raise the industry's profit. It is not clear, however, if all firms will benefit from a windfall "gain" (discovery) that increases the stock of each firm.

When a given number of firms deplete an exhaustible resource with zero extraction costs and iso-elastic demand, it has been argued that the oligopoly and cartel outcomes are efficient and that firms deplete according to the Hotelling's rule (Dasgupta and Heal, 1979). This implies that dynamic oligopolies and cartels cannot be distinguished from perfect competition and that firms act as if there are well defined private property rights.

These results are somewhat counter intuitive and cannot explain the phenomena of 'wild–cutting'. One reason for excessive extraction rates in oligopolistic resource markets may be that firms are worried that, if they announce to extract efficiently, one of their rivals with access to current stock levels will have an incentive to deplete more rapidly, therefore yielding inefficiency.

In this paper, we consider oligopolistic equilibrium in sub-game perfect strategies in continuous time, and investigate the effect of a tax factor imposed by the regulator on the price charged. For the analysis that follows we may use firstly a known model reported by Benchekroun and Long (2006) (hereafter BL model). Following the BL model, and taking the oligopolistic strategies as fixed, a taxation scheme which is independent of the state (i.e. the remainder stock) obviously will transfer the overall tax liability onto the value functions, i.e. on the discounted revenues. Moreover a tax (or subsidy) on quantity, offered by the outset to each firm in order to compensate the tax burden, will reduce each firm's extraction feedback strategy. In the same way, taking extraction strategies as given, a tax scheme depending on the state is expected to be inducing efficient in the sense of Karp and Livernois (1994) or even better in the sense of Benchekroun and Long (1998).

Moreover as the implications of an additional stock endowed to each firm from the outset are well-known (and in a static model are not surprising) we study the implications of such a taxation scheme. Starting from the Cournot equilibrium it is familiar that a marginal reduction of all firms' production will be beneficial to the firms and will move them closer to the cooperative equilibrium. Conversely, increasing the output of all firms is likely to move them further from the cooperative outcome and will reduce their profits.

In a dynamic framework with free time horizon, this reasoning is not necessarily valid. The typical extraction path under non-cooperation is monotonically decreasing over time with production level below the production level of cooperative exploitation for at least some interval of time, which we refer to as a scarcity phase. As it is well known, when a firm receives an additional stock it splits its extra-exploitation between the scarcity phase and the phase where production is above the cooperative level (Benchekroun and Long, 2002, 2006). Increasing exploitation during the phase where production is above the cooperative level decreases instantaneous profits but increasing exploitation in the former phase raises instantaneous profits, resulting in an unclear conclusion for the overall impact in firms profits.

The rest of the paper is organized as follows. Section 2 describes the model of resource extraction with an isoelastic demand function. Section 3 provides the Markov perfect Nash equilibrium strategies that are time consistent and the resulting value function for the strategies. Section 4 proposes some policy instruments based on changes of the allowed resource stock, while the last section concludes the paper.

2. The Benchekroun – Long (BL) model

Let us assume that there are N firms in an oligopoly market. Firm i is endowed with a stock of a resource $S_i(t)$ at time t, with $S_i(0)=S_i^0$. Let $S(t)$ denote the sum of all stocks at time t, that is

$$S(t)=\sum_{i=1}^{N} S_i(t)$$

We define $S_{-i}(t)=S(t)-S_i(t)$. We then also assume that the rate of change of firm's i resource stock is[1]

$$\frac{dS_i(t)}{dt}=\dot{S}_i(t)=-h_i(t)$$

where $h_i(t)$ is firm's i extraction rate at time t.

The inverse demand function is given by

$$D(h(t))=(h(t))^{a}$$

with $f(a)=-a$, $a\in(0,1]$ and $h(t)=\sum_{i=1}^{N} h_i(t)$ denotes the overall extracted quantity. The function $[f(a)]^{(-1)}=1/a\geq 1$ determines, in absolute value, the instantaneous elasticity of demand, i.e. the inverse demand function is always elastic and takes the hyperbolic shape if $a=1$ (i.e. a constant), but is always convex.

Here in order to form the dynamic problem it is assumed utility is derived from revenues, so firms in industry are rather revenues maximizers. Moreover, the resource stock is not restrictive for the firms' decisions (i.e. extraction rate) but the regulator is the decision maker of the state variable, i.e. the remainder resource stock as you will

[1] A similar adoption in the resource reduction equation is made by Batabyal (1995, 1996).

see below. One of the results[2] of the model is that the control trajectory is strictly dependent on the extraction trajectory as well as on the instantaneous elasticity. So, the state variable in the problem, as affected from the control, is an optimal control for every involved firm.

Having these assumptions the dynamic can be presented as follows. Firm's i revenues are given by the expression:

$$R_i\left(h_i, h_{-i}\right) = h_i\left(h_i + h_{-i}\right)^{-a}$$

where $$h_{-i} = h - h_i$$

The objective function of firm i is to maximize the present value of the stream of cash flow subject to the system dynamics, that is the problem[3]

$$\max \int_0^\infty h_i\left(h_i + h_{-i}\right)^{-a} e^{-\rho t} dt \qquad (1)$$

subject to $$\dot{S}_i(t) = -h_i(t) \qquad (2)$$

with $$S_i(0) = S_i^0$$

The control variable of firm i is its quantity h_i, while the state variable is its remainder resource S_i.

The goal is to find a strategy and the value function of the dynamic problem under the closed loop[4] or Markovian Nash informational structure equilibrium which is by definition the concept of equilibrium in which the choice of player's i current action is conditioned on current time t and on state vector too.

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[2] In another perspective a second result could be the fact that a tightening of the regulation on total allowed resource stock can lead to an increase in firms' NPV of discounted revenues.

[3] In this setting, where the state variable does not enter into the objective function, the induced game seems to be a trivial one.

[4] For more details about the informational structures of the dynamics games, see Basar and Olsder (1995).

Under the closed–loop informational structure and stationarity of the game the player's i strategy space[5] is this of mappings

$$\phi_i : \mathbb{R}^n_+ \to \mathbb{R}$$

which associates to a vector of resource stock $(S_1, S_2, ..., S_N) \in \mathbb{R}^n_+$ the quantity $\phi_i(S_1, S_2, ..., S_N)$ to extract. Each player i of the game has to choose a quantity $h_i(t) \in \mathbb{R}$ of the resource, and the price of that resource is then set according to

$$D(h_1, h_2, ..., h_N) = \left(\sum_{i=1}^{N} h_i(t) \right)^{-a}$$

The utility (total revenues) enjoyed by firm i is then given by

$$U_i : (\phi_1, \phi_2, ..., \phi_N) \to \int_0^\infty D(\phi_1(S), \phi_2(S), ..., \phi_i(S), ..., \phi_N(S)) e^{-\rho t} dt$$

where $(S_k)_{k=1,...,N}$ evolve according to the differential equation determined by (2). Equilibrium should then be defined as a set of strategies for which no player has a profitable deviation.

Imposing this assumption on informational structure of the game, clearly the history of the game is important and is reflected in the current value of the state vector. Consequently, player's i optimal time paths take into account at any point of time the control variables (quantity extracted) of the other players. This type of equilibrium affects the state variables, requiring a revision of the player's i controls at any time instant.

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[5] By strategy spaces, we mean the information available to each player together with a set of functions with this information as domain. These functions are actually the permissible ways in which the players are allowed to use that information. Open-loop strategies, where at each instant of time t the players have knowledge of the present time instant t and the initial condition S(0) of the state, result in different equilibrium from the strategies where at each instant of time t the players have knowledge of the time t, the initial state S(0) and the current state S(t).

Here the Hamilton – Jacobi – Bellman (HJB) equation is applied to prove that the conjectured strategy is a Markovian strategy and consequently a strongly time consistent one. In contrast to the open-loop informational structure the closed-loop is a strongly time consistent one, but the open-loop is not. Here the time consistent property is in the sense of sub–game perfectness (for more details see Dockner *et al.* 2000). Then the Markov Perfect Nash Equilibrium (MPNE) is characterized as follows.

Let ϕ_i be the strategy that specifies firm's i extraction rate as a function of time t and the vector of remainder resource stock at the same time. This is the strategy

$$h_i(t) = \phi_i(S(t))$$

Each firm takes competitors strategies as given and determines its optimal strategy that solves problem (1) subject to the constraint expressed by (2).

Proposition 1 (Benchekroun and Long, 2006)

Markov Perfect Nash Equilibrium exists, where the equilibrium strategy of firm i has the property that its extraction level depends on its own resource stock and on elasticity of demand. That is

$$h_i(t) = \frac{\rho}{a} S_i(t) \qquad\qquad i = 1, \dots, N$$

The discounted sum of firm's i revenues $V_i(S)$, when the total resource stock is S, are given by

$$V_i(S) = \left(\frac{a}{\rho}\right)^a \frac{S_i}{S^a} \tag{3}$$

Given the discounted revenues expression by (3), it is easy to see the impact from a change in elasticity of demand in the discounted revenues. Therefore taking the derivative of the value function expression given by (3) with respect to the demand curvature a, assuming that the initial resource stock of firm's i and the overall resource stock S remains unchanged, i.e.:

$$\frac{dV_i}{da} = \left(\frac{\rho S}{a}\right)^{-a}\left(-\ln\left(\frac{\rho S}{a}\right)+1\right)S_i = V_i(S)\left(1-\ln\left(\frac{\rho S}{a}\right)\right)$$

which is a negative or positive quantity, meaning that the discounted revenues' change will be negative or positive, depending on the sign of the quantity $1-\ln\left(\frac{\rho S}{a}\right)$. The latter means that it remains profitable for each oligopolistic firm to extract, with respect to the demand changes, only if the total resource stock fulfills the inequality $S < e(a/\rho)$

2.1. The unary elasticity demand (A special case)

Consider for a moment that elasticity of demand equals to one independent of time, i.e. $a = 1$. As it is simply clear in this case the market demand function collapses to a hyperbolic shape, this being a special case of a more general class of models based on isoelastic demand curves. An isoelastic demand function was used to study the stability for a general Cournot oligopoly (Chiarella and Szidarovsky, 2000) and in many variations (Puu, 1991, 1996; Puu and Norin 2003; Puu and Marin, 2006).

Furthermore isoelastic demand functions is a result in the case the consumers maximize utility functions of the Cobb–Douglas type in a static environment. The static problem for the i consumer is to $\max\left(D_1^i\right)^{a_1^i}\left(D_2^i\right)^{a_2^i}\ldots$, subject to the budget constraint $y^i = p_1 D_1^i + p_2 D_2^i + \ldots$ with p_k to denote the prices of the commodities and D_k^i denote the quantities demanded. The well-known outcome of this static constrained maximization is $p_k D_k^i = a_k^i y^i$ where a_k^i is the fixed spending share of the i's consumer income y^i on the k-th good.

From the above problem's solution the resulting demand for each consumer is reciprocal to price charged that is $D_k^i = \frac{a_k^i y^i}{p_k}$, so dropping commodities indices, the aggregate demand is obtained as (the sum of all consumers) $D = \sum_i D^i = \frac{\sum_i a^i y^i}{p} = \frac{R}{p}$. Puu (2008) also uses the following price specification $p = \frac{R}{\sum_{i=1}^{N} q_i}$ where p denotes market price, $\sum_{i=1}^{N} q_i$ is the total quantity produced, while R is the sum of the total budget shares that all consumers spend in the particular good. It is well-known from the literature[6] in such a case the maximum problem of a firm choosing the output level is indeterminate if marginal cost is zero, since the revenues generated by a hyperbolic demand are constant, thus economically unacceptable.

But even in this special case the model under closed-loop informational structure yields linear strategies and value function as well. More precise setting demand elasticity to one, $a = 1$, the model solution yields the following results for strategies and value function respectively:

$$h_i = \rho S_i \qquad i = 1, \ldots, N \tag{4}$$

$$V_i(S) = \frac{S_i}{\rho S} \tag{5}$$

The latter reasoning leads us to conclude the following corollary.

[6] For an exposition of a differential oligopoly model where firms face implicit menu costs of adjusting output over time due to sticky market price, see Lambertini (2007).

Corollary 1

The BL model of an exhaustible resource extraction even in the special case of isoelastic demand, so for constant consumers' budget share, yields deterministic Markovian linear strategies and value functions given by (4), (5) *respectively.*

3. Simple taxing schemes in the oligopoly

In this section we investigate the implications of a tax factor imposed on prices charged by the extracting oligopolists. Thus we assume that the tax factor τ it is proportional to the price. In this way the discounted revenues for the i firm is the following integral

$$PV = \int_0^\infty e^{-\rho t}\left[h_i\left[(1-\tau)(h_i + h_{-i})\right]^{-a}\right]dt \tag{6}$$

subject to the constraint expressed by (2).

The simple taxation scheme leaves the extraction strategy unaffected and the only impact is on the value functions of each oligopolist. The above result is recorded in the next proposition.

Proposition 2.

In the BL model, there is a taxation scheme which leaves the extraction strategy unaffected, also reducing each firm's discounted revenues, as described by equation (6). The tax scheme is depending on elasticity and the reduction on discounted revenues is given by the binomial expression $f(\tau) = (1-\tau)^{-a}$. The oligopolistic firms follows the same extraction strategies $h_i(t) = (\rho/a)S_i(t)$ and the reduced discounted revenues are given by the expression $V_i(S) = (a/\rho)^a S_i \big/ \left(S(1-\tau)\right)^a$.

Proof (In the appendix).

The tax factor τ is easily approximated using the linear approximation of the binomial expression as[7] $f(a,\tau)=(1-\tau)^{-a}=1+a\tau$ since the tax factor τ is a real number close to zero and a is a real number.

In what follows we will try with the same tax scheme on price charged but now a subsidy is given to all firms from the outset in order to compensate the tax burden. The subsidy is proportional to the quantity supplied in the market, therefore the discounted revenues for the i firm are given by the following present value

$$PV_1=\int_0^\infty e^{-\rho t}\left[(1+\delta)h_i\left[(1-\tau)(h_i+h_{-i})\right]^{-a}\right]dt \qquad (7)$$

Supposing that every firm adheres to the same extraction rate, i.e. the equation of motion (2) remains valid, then the following proposition holds true.

Proposition 3.

In the case the firms receives a constant subsidy, as a fraction of quantity supplied in the market, then in the feedback equilibrium every firm change their strategy, reducing the extraction rates equal to tax factor imposed. Every firm raise the discounted revenues with the quantity equal to the subsidy fraction, as it is expected. In the model terms the feedback equilibrium strategy is defined as $h_i(t)=(\rho/a)(1-\tau)S_i(t)$ and the discounted revenues are given by the expression $V_i(S)=(a/\rho)^a(1+\delta)S_i\big/\big(S(1-\tau)\big)^a$.

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[7] Let $f(\tau)=(1-\tau)^{\beta}$ then $f'(\tau)=\beta(1-\tau)^{\beta-1}$ and setting $\tau=0$, $f'(0)=\beta$. Using linear approximation $f(\tau)\approx f(\beta)+f'(\beta)(x-\beta)$, $f(\tau)\approx f(\beta)+f'(\beta)(\tau-\beta)$ for which $(1-\tau)^{\beta}\approx 1+\beta\tau$

Proof (is the same as in proposition 2).

4. A tax scheme dependent on the resource stock

Let now consider a taxation scheme for which the tax function is dependent on the remainder stock own in every extracting oligopolistic firm. The regulator takes the extraction strategy as it is given and imposes the tax function $\delta\left(S_1\right)$ regardless for the extraction rate, since that strategy is one of a family of feedback strategies for every firm at equilibrium. As it is intuitively known these taxation schemes are more fair and therefore efficient, called "*efficient inducing taxation schemes*" (Benchekroun and Long, 1998). Hence, in the same model the stream of the discounted revenues are now given by the following expression:

$$PV_i = \int_0^{\infty} e^{-\rho t}\left[\left[1-\delta\left(S_i\right)\right]h_i\left[\left(h_i+h_{-i}\right)\right]^{-a}\right]dt \qquad (8)$$

subject to the equation of the own stock motion (2)

Proposition 4.

Given the model of equations (8) and (2) there exists a tax function dependent on the stock, thus an efficient tax, for which the analytical expression is given by

$$\delta\left(S_i\right) = \frac{S_i^{-1/a}S}{S_i^{-1/a}S-1}$$

Proof (In Appendix)

5. Concluding remarks

In this paper we set up a very simple model of taxation for extracting oligopolists, where the demand is not linear and the resulting game is not a linear quadratic one. Due to the model's demand function the resulting feedback strategies are linear. As the analytic expressions of strategies and value function are known, it is easy for the policy maker to construct tax schemes that may be efficient.

First, we consider a tax function which is based on the price charged and as it is expected the above (intuitive) tax is not efficient in the sense of Benchekroun and Long. Since this first tax function is not dependent on the state variable we try with an improved taxation scheme dependent on the stock, which is may be more efficient. As a result we calculate the analytical expression of the second tax scheme. The results, in our opinion, are useful for a policy maker to make distributed extraction policies on the industry in total as well as partially on a firm.

As it is known, from the BL model, a new technology that reduces the total amount of the extraction stock is not necessarily approved by all firms in the industry. The question raised and left for future research relates to the taxation scheme proposed here and if it is an efficient one in the case of a windfall gain for every oligopolistic firm.

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Appendix

Proofs of Propositions

Proof of Proposition 2

First we check that if firm's j strategy is $h_j = \frac{\rho}{a} S_j$, then firm's i best response will be $h_i = \frac{\rho}{a} S_i$. The Hamilton-Jacobi-Bellman (hereafter HJB) equation for firm's i maximization problem is the following

$$\rho V_i = h_i \left(h_i + (1-\tau) \frac{\rho}{a} S_{-i} \right)^{-a} + \frac{\partial V_i}{\partial S_i}(-h_i) + \sum_{j \neq i, j=1}^{N} \frac{\partial V_i}{\partial S_j} \left(-\frac{\rho}{a} S_j \right)$$

Maximization of the RHS of the HJB equation with respect to h_i gives

$$\frac{\partial V_i}{\partial S_i} = (1-\tau)^{-a} \left(h_i + \frac{\rho}{a} S_{-i} \right)^{-a} \left[1 - a \frac{h_i}{\left(h_i + \frac{\rho}{a} S_{-i} \right)} \right] \qquad (A.1)$$

Where S_{-i} represents the sum of all resource stocks except firm's i stock, that is $S_{-i} = S - S_i$ and $S = \sum_{j=1}^{N} S_j$

We try with the value function

$$V_i = \left(\frac{a}{\rho} \right)^a S_i \left((1-\tau) \sum_{j=1}^{N} S_j \right)^{-a}$$

Differentiation of the value function with respect to S_i yields

$$\frac{\partial V_i}{\partial S_i} = \left(\frac{\rho}{a} S \right)^{-a} (1-\tau)^{-a} \left(1 - a \frac{S_i}{S} \right) \qquad (A.2)$$

with $S = \sum_{j=1}^{N} S_j$ the same as above.

Equating the terms with the same power of $(A.1)$ and $(A.2)$ we have the resulting system of equations.

$$1-a\frac{h_i}{\left(h_i+\frac{\rho}{a}S_{-i}\right)}=1-a\frac{S_i}{S} \qquad (A.3)$$

and

$$h_i+\frac{\rho}{a}S_{-i}=\frac{\rho}{a}S \qquad (A.4)$$

Both equations $(A.3)$ and $(A.4)$ have the same solution

$$h_i=\frac{\rho}{a}S_i.$$

Now we prove that substituting the above strategies into the RHS of the HJB function we have equality with the LHS of the same equation. The partial derivative of the value function V_i with respect to S_j is

$$\frac{\partial V_i}{\partial S_j}=-a\left(\frac{a}{\rho}\right)^a S_i S^{-(a+1)}<0 \qquad (A.5)$$

so the RHS of the HJB becomes

$$\text{RHS(HJB)}=\frac{\rho}{a}S_i\left(\frac{\rho}{a}S\right)^{-a}-\frac{\rho}{a}S_i\left(1-a\frac{S_i}{S}\right)\left(\frac{\rho}{a}S\right)^{-a}+\sum_{j\neq i,j=1}^{N}-a\left(\frac{a}{\rho}\right)^a S_i S^{-(a+1)}\left(-\frac{\rho}{a}S_j\right)=$$
$$=\frac{\rho}{a}S_i\left(\frac{\rho}{a}S\right)^{-a}\left(1-1+a\frac{S_i}{S}+a\frac{S_{-i}}{S}\right)=\rho S_i\left(\frac{\rho}{a}S\right)^{-a}=\rho V_i(S)=\text{LHS(HJB)}$$

Where as above we have set $S=\sum_{j=1}^{N}S_j$ and $S_{-i}=\sum_{j=1,j\neq i}^{N}S_j$

Proof of Proposition 4

Maximization of the RHS of the HJB function yields

$$\left(h_i + \frac{\rho}{a} S_{-i}\right)^{-a} \left[\left(1-\delta\left(S_i\right)\right)\left(1 - \frac{aS_i}{h_i + \frac{\rho}{a} S_{-i}}\right)\right] = \frac{\partial V_i}{\partial S_i} \qquad (\text{B.1})$$

The partial derivative of the value function w.r.t. the state becomes

$$\frac{\partial V_i}{\partial S_i} = \left(\frac{\rho}{a}\right)^{-a} \left(S_i + S_{-i}\right)^{-a} \left(1-\delta\left(S_i\right)\right)^{-a} \left[1 - \frac{aS_i}{S_i + S_{-i}} - \frac{aS_i}{1-\delta\left(S_i\right)} \frac{\mathrm{d}}{\mathrm{d}S_i} \delta\left(S_i\right)\right] (\text{B.2})$$

Equating the terms inside the brackets for both equations yields the differential equation

$$\delta\left(S_i\right) \frac{\left(1-\delta\left(S_i\right)\right)}{aS_i} \left(\frac{aS_i}{S_i + S_{-i}} - 1\right) = \frac{\mathrm{d}}{\mathrm{d}S_i} \delta\left(S_i\right)$$

For which the solution is:

$$\delta\left(S_i\right) = \frac{S_i^{-1/a} S}{S_i^{-1/a} S - 1}$$