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Abstract

In this study, we develop a monetary Schumpeterian growth model with endogenous market structure (EMS) to explore the effects of monetary policy on the number of firms, firm size, economic growth and social welfare. EMS leads to richer implications and different results from previous studies in which market structure is exogenous. In the short run, a higher nominal interest rate leads to lower growth rates of innovation, output and consumption and also smaller firm size due to a reduction in labor supply. In the long run, an increase in the nominal interest rate reduces the equilibrium number of firms but has no effect on economic growth and firm size because of a scale-invariant property of the model as a result of entry and exit of firms. Although monetary policy has no long-run effect on economic growth, an increase in the nominal interest rate permanently reduces the levels of output, consumption and employment. Taking into account transition dynamics, we find that social welfare is decreasing in the nominal interest rate. Given that a zero nominal interest rate maximizes welfare, Friedman rule is optimal in this economy.

JEL classification: O30, O40, E41
Keywords: monetary policy, economic growth, R&D, endogenous market structure

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1 Introduction

In this study, we develop a monetary Schumpeterian growth model to explore the effects of monetary policy on economic growth, social welfare and endogenous market structure (EMS). In contrast to previous studies with exogenous market structure, we find that monetary policy has only transitory, not permanent, effects on the rate of economic growth. The reason for this difference is that the economy's market structure responds endogenously to changes in labor supply induced by monetary policy. In other words, market structure, measured by the number of firms and the size of each firm, is endogenously determined through the entry and exit of firms in response to macroeconomic conditions. More importantly, each firm's incentives to invest in R&D depend on the size of its market, which is determined by market structure but not aggregate market size.

To capture EMS and R&D in a dynamic framework, we use a variant of the second-generation R&D-based growth model, pioneered by Peretto (1998), Young (1998), Howitt (1999) and Segerstrom (2000). To our knowledge, this is the first analysis of monetary policy in the second-generation R&D-based growth model that is free of scale effects.[1] The model features two dimensions of technical progress: variety expansion (i.e., horizontal innovation) and quality improvement (i.e., vertical innovation). In the horizontal dimension, entrepreneurs create new firms by introducing new products, and the number of firms in equilibrium determines two important elements of market structure: market concentration and firm size. In the vertical dimension, each incumbent firm performs in-house R&D to improve the quality of its products, and the return to in-house R&D is determined by the size of the firm. In this economy, technological progress and market structure are jointly determined in equilibrium: market structure is measured by the number of firms, whereas technological progress is determined by the growth rate of vertical innovation. One advantage of this second-generation R&D-based growth model is that it is consistent with stylized facts in the industrial organization (IO) literature. For example, the return to R&D depends on firm size rather than aggregate market size; see Cohen and Klepper (1996a,b). Furthermore, theoretical implications of the second-generation R&D-based growth model with EMS are supported by empirical studies, such as Laincz and Peretto (2006) and Ha and Howitt (2007).

In this growth-theoretic framework, an increase in the nominal interest rate reduces labor supply via a cash-in-advance (CIA) constraint on consumption. In the short run, this reduction in labor supply caused by the higher nominal interest rate reduces average firm size and leads to lower growth rates of innovation, output and consumption.[2] Intuitively, when the nominal interest rate increases,[3] households decrease consumption and increase leisure due to an extra cost of consumption imposed by the CIA constraint. As a result, the reduced supply of labor causes lower employment per firm in the short run, which in turn reduces economic growth temporarily. In the long run, an increase in the nominal interest rate reduces the equilibrium number of firms but has no effect on economic growth and firm size because of a scale-invariant property of the model. Intuitively, in the long run, some

[1] See Jones (1999) and Laincz and Peretto (2006) for a discussion of scale effects in R&D growth models.

[2] For example, Evers *et al.* (2007) provide empirical evidence for a negative effect of inflation and the nominal interest rate on total factor productivity growth.

[3] In this study, the nominal interest rate is a policy instrument chosen by the monetary authority.

firms exit the market as a result of the smaller aggregate market size measured by the supply of labor, and the number of firms adjusts such that employment per firm returns to the initial level. Therefore, long-run economic growth is independent of the nominal interest rate. Although monetary policy has no long-run effect on economic growth, an increase in the nominal interest rate permanently reduces the levels of output, consumption and employment. Furthermore, taking into account transition dynamics, we find that social welfare is decreasing in the nominal interest rate. Intuitively, the supply of labor is suboptimally low in equilibrium, so that a positive nominal interest rate that reduces labor supply is suboptimal. Given that a zero nominal interest rate maximizes social welfare, Friedman rule is optimal in this economy.[4] To our knowledge, this is the first analytical derivation of optimal monetary policy that takes into account transition dynamics in the R&D-based growth model.

This study relates to the literature on inflation and economic growth;[5] see Tobin (1965) and Stockman (1981) for seminal studies and Wang and Yip (1992) for a discussion on different approaches of modelling money demand. A common approach of modelling money demand in this literature is through a CIA constraint on consumption; see for example Gomme (1993), Dotsey and Ireland (1996) and Mino (1997). In this study, we follow this approach to model money demand. Studies in this literature often analyze the growth and welfare effects of monetary policy in variants of the overlapping generations model or the Neoclassical growth model. For example, Wu and Zhang (2001) also analyze the effects of inflation on the number of firms and firm size in a Neoclassical growth model; however, they do not consider R&D-driven economic growth and transition dynamics. Our study takes into consideration these elements and relates to a more recent subbranch of the literature that analyzes the growth and welfare effects of monetary policy in R&D-based growth models; see for example, Marquis and Reffett (1994), Funk and Kromen (2010), Chu and Lai (2012), Chu *et al.* (2012) and Chu and Cozzi (2012). These studies consider either the variety-expanding model or the quality-ladder model. The present study differs from them by analyzing the effects of monetary policy in a more recent vintage of R&D-driven growth models based on Peretto (2007) in which both the number of firms and the growth rate of vertical innovation are endogenous. In other words, we consider a scale-invariant Schumpeterian growth model with EMS; see Peretto (1996, 1999) for seminal studies in R&D-based growth models with EMS and Etro (2012) for an excellent textbook treatment. This study contributes to the literature with a novel analysis of monetary policy on EMS in an R&D-based growth model and also provides a novel result that the long-run effects of monetary policy in an R&D-based growth model with EMS are reflected in the economy's market structure measured by the number of firms rather than the rate of economic growth.

The rest of this study is organized as follows. Section 2 presents the monetary Schumpeterian growth model. Section 3 analyzes the effects of monetary policy on economic growth and social welfare. The final section concludes.

[4]See Mulligan and Sala-i-Martin (1997) for a discussion of Friedman rule.

[5]Gillman and Kejak (2005) provide a survey of this literature.

2 A monetary Schumpeterian growth model with EMS

Our growth-theoretic framework is based on the Schumpeterian model with in-house R&D and EMS in Peretto (2007). We introduce money demand into the model via a CIA constraint on consumption. As in standard CIA models, monetary policy affects the economy by distorting households' tradeoff between consumption and leisure. In our analysis, we provide a complete closed-form solution for the economy's transition dynamics as well as its balanced growth path.

2.1 Households

There is a representative household, who has the following lifetime utility function

$$U = \int_0^\infty e^{-\rho t} \ln u_t dt = \int_0^\infty e^{-\rho t} [\ln c_t + \gamma \ln(L - l_t)] dt, \tag{1}$$

where c_t denotes consumption of final goods (numeraire) at time t and l_t denotes labor supply. The parameters $\rho > 0$ and $\gamma > 0$ determine respectively subjective discounting and leisure preference. Each household maximizes (1) subject to the following asset-accumulation equation

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t + \tau_t - c_t - \pi_t m_t. \tag{2}$$

a_t is the real value of assets (i.e., ownership of monopolistic firms) owned by each household, and r_t is the real interest rate. Each household has a labor endowment of L units and elastically supplies l_t units to earn a real wage rate w_t. The household also faces a lump-sum transfer (or tax) τ_t from the government. The household carries real balances m_t to facilitate purchases of consumption goods.[6] The cost of holding money is the inflation rate π_t. The CIA constraint is given by $\xi c_t \leq m_t$, where the parameter $\xi \in (0, 1]$ determines the importance of the CIA constraint. In the limiting case $\xi \to 0$, monetary policy would have no effect on the real economy.

The optimality condition for consumption is

$$\frac{1}{c_t} = \eta_t (1 + \xi i_t), \tag{3}$$

where $i_t = r_t + \pi_t$ is the nominal interest rate and η_t is the Hamiltonian co-state variable on (2).[7] The optimality condition for labor supply is

$$w_t (L - l_t) = \gamma c_t (1 + \xi i_t). \tag{4}$$

The intertemporal optimality condition is

$$-\frac{\dot{\eta}_t}{\eta_t} = r_t - \rho. \tag{5}$$

[6] In this study, we focus on a single type of money, namely currency. See for example Santomero and Seater (1996) for an analysis of an economy with several types of money.

[7] There is also a co-state variable on the CIA constraint, and we have substituted out this co-state variable using the first-order conditions in order to derive (3).

In the case of a constant nominal interest rate i,[8] combining (3) and (5) yields the familiar Euler equation $\dot{c}_t/c_t = r_t - \rho$.

2.2 Final goods

Following Aghion and Howitt (2005) and Peretto (2007), we assume that final goods Y_t are produced by competitive firms using the following production function

$$Y_t = \int_0^{N_t} X_t^\theta(j)[Z_t^\alpha(j)Z_t^{1-\alpha}l_{y,t}(j)]^{1-\theta}dj, \tag{6}$$

where $\theta, \alpha \in (0,1)$ and $X_t(j)$ denotes intermediate goods $j \in [0, N_t]$. The productivity of $l_{y,t}(j)$ workers using intermediate good $X_t(j)$ depends on the quality $Z_t(j)$ of that good and also on the average quality $Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_t(j)dj$ of all intermediate goods. From profit maximization, the conditional demand function for $l_{y,t}(j)$ is

$$l_{y,t}(j) = \left(\frac{1-\theta}{w_t}\right)^{1/\theta} X_t(j)[Z_t^\alpha(j)Z_t^{1-\alpha}]^{(1-\theta)/\theta}, \tag{7}$$

and the conditional demand function for $X_t(j)$ is

$$X_t(j) = \left(\frac{\theta}{p_t(j)}\right)^{1/(1-\theta)} Z_t^\alpha(j)Z_t^{1-\alpha}l_{y,t}(j), \tag{8}$$

where $p_t(j)$ denotes the price of $X_t(j)$ denominated in units of Y_t. The demand for type-j intermediate goods depends on firm size measured by $l_{y,t}(j)$ but *not* on aggregate market size $l_{y,t} \equiv \int_0^{N_t} l_{y,t}(j)dj$. This dependence on firm size rather than aggregate market size is the key difference between this model and previous studies of monetary policy and R&D-driven economic growth. Both N_t and $l_{y,t}(j)$ are endogenously determined in equilibrium. Perfect competition implies that final goods producers pay $\theta Y_t = \int_0^{N_t} p_t(j)X_t(j)dj$ to intermediate goods firms and pay $(1-\theta)Y_t = \int_0^{N_t} w_t l_{y,t}(j)dj$ to workers.[9]

2.3 Intermediate goods

There is a continuum of industries producing differentiated intermediate goods $X_t(j)$ for $j \in [0, N_t]$. Each type of intermediate goods is produced by a single monopolistic firm that has price-setting power. Thus, the number of intermediate goods N_t is the same as the number of firms that produce them. There are two types of R&D, vertical and horizontal. Vertical R&D is quality improvement, carried out by incumbent firms in an attempt to increase the demand for their products. Horizontal R&D is the invention of new products, carried out by entrepreneurs who enter the market as new firms producing the newly invented goods. Through the entry of firms, the number of firms and the size of each firm are endogenously determined in equilibrium.

[8] Given that the nominal interest rate is exogenously chosen by the monetary authority, the inflation rate endogenously responds to changes in the real interest rate.

[9] Free movement of workers across firms implies that wages must be equal across firms.

2.3.1 Incumbents

Existing intermediate goods firms produce differentiated goods with a technology that requires one unit of final goods to produce one unit of intermediate goods. Following Peretto (2007), we assume that the firm in industry j incurs $\phi Z_t^{\alpha}(j) Z_t^{1-\alpha}$ units of final goods as a fixed operating cost. This specification implies that managing facilities are more expensive to operate in a technologically more advanced environment. To improve the quality of its products, the firm invests $R_t(j)$ units of final goods in R&D. The innovation process is

$$\dot{Z}_t(j) = R_t(j). \tag{9}$$

The cash flow of firm j is

$$F_t(j) = [p_t(j) - 1] X_t(j) - \phi Z_t^{\alpha}(j) Z_t^{1-\alpha}, \tag{10}$$

and the profit flow is

$$\Pi_t(j) = F_t(j) - R_t(j). \tag{11}$$

The value of the monopolistic firm in industry j is

$$V_t(j) = \int_t^{\infty} \exp\left(-\int_t^u r_s ds\right) \Pi_u(j) du. \tag{12}$$

Taking the conditional demand function (8) as given, the firm sets its own price and devotes resources to in-house R&D to maximize $V_t(j)$. The current-value Hamiltonian for this optimization problem is[10]

$$H_t(j) = \Pi_t(j) + q_t(j)\dot{Z}_t(j). \tag{13}$$

Following the standard approach in this class of models, we consider a symmetric equilibrium in which $Z_t(j) = Z_t$ for $j \in [0, N_t]$.[11] The return to in-house R&D is increasing in firm size, where size is measured as employment per firm l_t/N_t. This property is consistent with the stylized facts in the IO literature discussed in the introduction.

Lemma 1 *The return to in-house R&D is given by*

$$r_t^I = \alpha \left[\theta^{(1+\theta)/(1-\theta)}(1-\theta)\frac{l_t}{N_t} - \phi\right]. \tag{14}$$

Proof. See the Appendix. ■

[10] See the Appendix for the solution of this optimization problem.

[11] See Peretto (1998, 1999, 2007) for a discussion of the symmetric equilibrium being a reasonable equilibrium concept in this class of models.

2.3.2 Entrants

A firm that is active at time t must have been born at some earlier date. A new firm pays a setup cost $\beta X_t(j)$ at time t to set up its operation and introduce a new variety of product.[12] Following the standard treatment in the literature, we assume that the new product comes into existence with the average level of quality as existing products. We refer to this process as entry. Suppose entry is positive (i.e., $\dot{N}_t > 0$). Then, the no-arbitrage condition is

$$V_t(j) = \beta X_t(j). \tag{15}$$

Under symmetry, $V_t(j) = V_t$, and the familiar Bellman equation implies that the return to entry is

$$r_t^E = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{16}$$

2.4 Monetary authority

The nominal money supply is denoted by M_t, and its growth rate is $\mu_t \equiv \dot{M}_t/M_t$. The real money balance is $m_t = M_t/P_t$, where P_t is the price of final goods. The monetary policy instrument that we consider is i_t.[13] Given a nominal interest rate i_t exogenously chosen by the monetary authority, the inflation rate is endogenously determined according to $\pi_t = i_t - r_t$. Then, given π_t, the growth rate of the nominal money supply is endogenously determined according to $\mu_t = \pi_t + \dot{m}_t/m_t$. To balance the budget, the monetary authority provides a lump-sum transfer (or tax) to households, and this transfer has a real value of $\tau_t = \dot{M}_t/P_t = \dot{m}_t + \pi_t m_t$.

2.5 Aggregation

Under symmetry, the labor market clearing condition is

$$l_t = N_t l_{y,t}. \tag{17}$$

The resource constraint on final goods is

$$Y_t = c_t + N_t(X_t + \phi Z_t + R_t) + \beta X_t \dot{N}_t. \tag{18}$$

[12] The setup cost is proportional to the new firm's initial volume of output. This assumption captures the idea that the setup cost depends on the amount of productive assets required to start production. See Peretto (2007) for a discussion.

[13] On the balanced growth path, $i = r + \pi = \rho + \mu$. Therefore, it is the rate of change in money supply that affects the real economy in this model. A one-time change in the level of money supply affects the price level and has no effect on the real economy. This is the well-known distinction between the neutrality and superneutrality of money. The evidence generally favors neutrality and rejects superneutrality, consistent with our model. See for example Fisher and Seater (1993) for a discussion on the neutrality and superneutrality of money.

Substituting (8) into (6) and imposing symmetry yield the aggregate production function

$$Y_t = \left(\frac{\theta}{p_t(j)}\right)^{\theta/(1-\theta)} Z_t N_t l_{y,t} = \theta^{2\theta/(1-\theta)} Z_t l_t, \tag{19}$$

where the second equality uses (17) and markup pricing $p_t(j) = 1/\theta$.

2.6 Equilibrium

The equilibrium is a time path of allocations $\{m_t, a_t, c_t, Y_t, l_t, l_{y,t}(j), X_t(j), R_t(j)\}$, prices $\{r_t, w_t, p_t(j), V_t\}$ and policy $\{i_t\}$. Also, at each instant of time, the following holds:

- Households choose $\{m_t, a_t, c_t, l_t\}$ to maximize utility taking $\{r_t, w_t, \pi_t\}$ as given;
- Competitive final goods firms choose $\{l_{y,t}(j), X_t(j)\}$ to maximize profits taking $\{w_t, p_t(j)\}$ as given;
- Incumbents in the intermediate goods sector choose $\{p_t(j), R_t(j)\}$ to maximize the present value of profits taking $\{r_t\}$ as given;
- Entrants make entry decisions taking $\{V_t\}$ as given;
- The monetary authority balances the budget such that $\tau_t = \dot{m}_t + \pi_t m_t$;
- The value of all existing monopolistic firms adds up to the value of households' assets such that $a_t = N_t V_t$;
- The market-clearing condition of labor holds;
- Finally, the market-clearing condition of final goods holds.

3 Growth and welfare effects of monetary policy

In this section, we analyze the effects of monetary policy on the number of firms, firm size, economic growth and social welfare. Specifically, we consider the effects of the nominal interest rate i. In Section 3.1, we analyze the effects of monetary policy on economic growth. In Section 3.2, we analyze the effects of monetary policy on social welfare.

3.1 Effects of monetary policy on economic growth

In the Appendix, we show that the consumption-output ratio c_t/Y_t jumps to a unique and stable steady-state value, a property that greatly simplifies the analysis of the transition dynamics.

Lemma 2 *The consumption-output ratio jumps to a unique and stable steady-state value*

$$(c/Y)^* = 1 - \theta + \rho\beta\theta^2. \tag{20}$$

Proof. See the Appendix. ■

Given a constant nominal interest rate i and a stationary consumption-output ratio, one can use (4) to show that the supply of labor l_t also jumps to its steady-state value given by

$$l^* = \left[1 + \gamma(1 + \xi i)\left(1 + \frac{\rho\beta\theta^2}{1-\theta}\right)\right]^{-1} L. \tag{21}$$

Equation (21) shows that the equilibrium supply of labor is decreasing in the nominal interest rate i. Intuitively, an increase in the nominal interest rate increases the cost of consumption relative to leisure because of the CIA constraint on consumption, and as a result, households reduce consumption and increase leisure. Given that labor supply is stationary for any given nominal interest rate i, (19) and (20) imply that

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \tag{22}$$

where the last equality uses the Euler equation. Setting $r_t^I = r_t$, one can then use (14) and (22) to derive the equilibrium growth rate given by

$$g_t \equiv \frac{\dot{Z}_t}{Z_t} = \max\left\{\alpha\left[\theta^{(1+\theta)/(1-\theta)}(1-\theta)\frac{l^*}{N_t} - \phi\right] - \rho, 0\right\}, \tag{23}$$

which is increasing in firm size measured by employment per firm l^*/N_t.[14] The growth rate g_t is strictly positive if and only if

$$N_t < \overline{N} \equiv \frac{\theta^{(1+\theta)/(1-\theta)}(1-\theta)}{\phi + \rho/\alpha} l^*.$$

This inequality means that if the number of firms is below a critical level $\overline{N}$, each firm's market size is large enough to make it profitable for firms to do in-house R&D. Otherwise, there are too many firms diluting the return to R&D; as a result, firms do not invest in R&D, and the growth rate of vertical innovation is zero. In the Appendix, we provide the derivations of the dynamics of N_t.

[14] Laincz and Peretto (2006) provide empirical evidence that is consistent with the theoretical prediction from this class of models that economic growth is positively related to the average firm size.

Lemma 3 *The growth rate of N_t is given by*

$$\frac{\dot{N}_t}{N_t} = \left\{ \begin{array}{ll} \frac{1-\theta}{\beta\theta} - \left(\phi + \frac{\dot{Z}_t}{Z_t}\right) \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} - \rho & \text{if } N_t < \overline{N} \\ \frac{1-\theta}{\beta\theta} - \phi \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} - \rho & \text{if } N_t > \overline{N} \end{array} \right\}. \tag{24}$$

Proof. See the Appendix. ■

The following Lemma provides the steady-state values of $N_t = N^*$ and $g_t = g^*$ as well as the parameter restrictions that ensure $N^* \in (0, \overline{N})$ and $g^* > 0$.[15]

Lemma 4 *Under the parameter restrictions that $\frac{1-\theta}{\beta\theta} - \alpha\phi < \rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}$,[16] the economy is stable and has a positive and unique steady-state value of N_t as well as a positive and unique steady-state growth rate given by*

$$N^* = \left[\frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho\right] \frac{\beta\theta^{2/(1-\theta)} l^*}{\phi(1-\alpha) - \rho} > 0, \tag{25}$$

$$g^* = \alpha \left[\theta^{(1+\theta)/(1-\theta)}(1-\theta)\frac{l^*}{N^*} - \phi\right] - \rho = \frac{(\rho + \alpha\phi)\beta\theta - (1-\theta)}{(1-\alpha)(1-\theta)/\rho - \beta\theta} > 0. \tag{26}$$

Proof. See the Appendix. ■

In the following proposition, we provide our first main result: an increase in i reduces the steady-state equilibrium number of firms but does not affect the steady-state equilibrium growth rate. Intuitively, an increase in i reduces the supply of labor l^* in (21), which in turn leads to a decrease in the steady-state equilibrium number of firms N^*. A reduction in labor supply decreases the aggregate market size, which in turn induces some firms to exit the market such that the average firm size remains constant in the long run. Because of this scale-invariant property of the model, steady-state employment per firm l^*/N^* remains unchanged; as a result, the steady-state equilibrium growth rate in (26) is independent of the nominal interest rate.

Proposition 1 *The steady-state equilibrium number of firms is decreasing in the nominal interest rate, but the steady-state equilibrium growth rate is independent of the nominal interest rate.*

[15] In this model, we have assumed zero population growth, so that N_t converges to a steady state. If we assume positive population growth, it would be the number of firms per capita that converges to a steady state instead, and our main results would be unchanged.

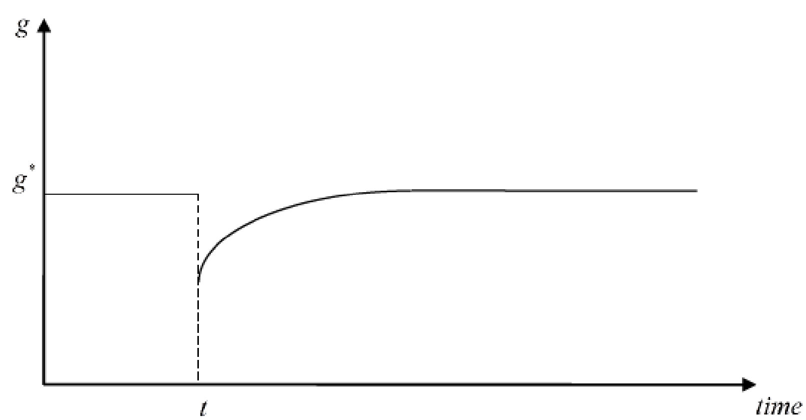
[16] This parameter restriction would depend on a larger set of parameters if we parameterize R&D productivity in (9) and the productivity in producing intermediate goods from final goods. For simplicity, we have implicitly normalized these productivity parameters to unity.

Proof. Use (21), (25) and (26). ■

The above result differs from previous studies, such as Chu and Lai (2012) and Chu *et al.* (2012), who find that an increase in i reduces the steady-state equilibrium growth rate. The difference is due to the fact that the earlier literature uses a monetary R&D-based growth model that exhibits an aggregate scale effect. The scale effect is the dependence of the growth rate on the size of the economy and is a consequence of the exogenous market structure assumed by earlier endogenous growth models. Empirical evidence strongly rejects the aggregate scale effect. In contrast, our model is scale-invariant. The reason is endogenous market structure. Entry and exit of firms in response to profit opportunities imply that the number of firms increases or decreases with aggregate market size and so eliminate the scale effect. The same mechanism implies that the number of firms changes in response to endogenous changes in labor supply leading to our result that changes in labor supply have no effect on economic growth in the long run.

In the rest of this subsection, we analyze the effects of monetary policy on economic growth along the transition path. The model features transition dynamics because N_t is a state variable that gradually converges to its state-state value N^*. When the monetary authority increases the nominal interest rate, the equilibrium supply of labor l^* adjusts instantly, but the equilibrium number of firms adjusts slowly. Given that the equilibrium growth rate is determined by firm size l^*/N_t, monetary policy can have an effect on economic growth during the transition to the steady state. Indeed, in the following proposition, we show that an increase in the nominal interest rate reduces the growth rates of vertical innovation, output and consumption on the transition path. Figure 1 illustrates the transitional effects of an increase in the nominal interest rate at time t.

Figure 1: Transitional effects of the nominal interest rate on economic growth



Proposition 2 *An increase in the nominal interest rate reduces the growth rates of vertical innovation, output and consumption on the transition path.*

Proof. Use (21), (22) and (23). Also, recall that N_t is a state variable. ■

Intuitively, an increase in the nominal interest rate reduces labor supply, which adjusts instantly and leads to a temporary decrease in the average firm size l^*/N_t. The smaller firm size reduces the returns to R&D in (14) and the equilibrium growth rate in (23). Over time, the smaller aggregate market size determined by l^* induces some firms to leave the market. As a result, the average firm size l^*/N_t gradually increases and returns to the initial level at which point, the equilibrium growth rate also returns to the initial level as shown in Figure 1. This transitional dynamic analysis of the effects of monetary policy is novel relative to previous studies, such as Marquis and Reffett (1994), Funk and Kromen (2010), Chu and Lai (2012), Chu *et al.* (2012) and Chu and Cozzi (2012), which focus on the steady-state equilibrium growth rate.

3.2 Effects of monetary policy on social welfare

In this subsection, we analyze the welfare effects of monetary policy. Specifically, we consider the effects of a permanent change in the nominal interest rate at time 0 on flow utility $\ln u_t$ at any arbitrary time $t \geq 0$. We show that $\partial \ln u_t/\partial i < 0$, which is sufficient for $\partial U/\partial i < 0$ because $U = \int_0^\infty e^{-\rho t} \ln u_t dt$. Taking the log of (19), we obtain

$$\ln Y_t = \frac{2\theta}{1-\theta}\ln\theta + \ln Z_t + \ln l_t = \frac{2\theta}{1-\theta}\ln\theta + \int_0^t g_s ds + \ln l^*, \tag{27}$$

where we have normalized $Z_0 = 1$. Taking the log of (20), we obtain

$$\ln c_t = \ln(1-\theta+\rho\beta\theta^2) + \ln Y_t. \tag{28}$$

Therefore, an increase in the nominal interest rate at time 0 decreases the levels of output and consumption at any arbitrary time $t > 0$ through two channels. First, it reduces the supply of labor l^*. Second, it temporarily reduces the growth rate of technology, which decreases the level of technology in the future.

Proposition 3 *An increase in the nominal interest rate at time 0 decreases the levels of output and consumption at any arbitrary time $t > 0$.*

Proof. Use Proposition 2 and (21) in (27) and (28). ■

Substituting (27) and (28) into flow utility $\ln u_t$ in (1) and then differentiating it with respect to i yield

$$\frac{\partial \ln u_t}{\partial i} = \underbrace{\int_0^t \frac{\partial g_s}{\partial i} ds}_{-} + \underbrace{\frac{\partial \ln l^*}{\partial i}}_{-} + \gamma \underbrace{\frac{\partial \ln(L-l^*)}{\partial i}}_{+}. \tag{29}$$

An increase in the nominal interest rate i thus has three effects on social welfare. First, it reduces welfare by temporarily decreasing the growth rates of vertical innovation, output and consumption. Second, it reduces welfare by decreasing the levels of output and consumption through a decrease in labor supply l^*. Third, it improves welfare by increasing leisure $L-l^*$. Although the overall welfare effects seem ambiguous, we find that $\partial \ln u_t/\partial i < 0$ because the loss of consumption dominates the gain in leisure. Intuitively, the supply of labor is suboptimally low in equilibrium partly because the CIA constraint imposes an extra cost on consumption relative to leisure. To see this result,

$$\frac{\partial \ln l^*}{\partial l^*} + \gamma \frac{\partial \ln(L-l^*)}{\partial l^*} = \frac{L-(1+\gamma)l^*}{l^*(L-l^*)} > 0 \tag{30}$$

because $L/(1+\gamma) > l^*$ in (21). As a result, a positive nominal interest rate that reduces labor supply is suboptimal. We summarize these welfare implications in the following proposition.

Proposition 4 *Social welfare is decreasing in the nominal interest rate; therefore, Friedman rule (i.e., a zero nominal interest rate) is socially optimal in this economy.*

Proof. Use (29) and (30). Also, recall from (21) that $\partial l^*/\partial i < 0$. ■

Previous studies, such as Marquis and Reffett (1994), Chu and Lai (2012) and Chu *et al.* (2012), also find that Friedman rule is optimal in the R&D-based growth model;[17] however, these studies mostly focus on steady-state welfare. To our knowledge, our result is the first analytical derivation of optimal monetary policy that takes into account transition dynamics in the equilibrium growth rate of an R&D-based growth model.

4 Conclusion

In this study, we have analyzed the effects of monetary policy on economic growth, social welfare and endogenous market structure in a scale-invariant Schumpeterian growth model. Unlike previous studies that analyze the effects of monetary policy on economic growth either in an AK-type growth model or the first-generation R&D-based growth model, this study analyzes the effects of monetary policy in a second-generation R&D-based growth model with both vertical and horizontal innovation, and we have obtained some novel results and richer implications. A novel result is that monetary policy has a negative effect on economic growth only in the short run; in the long run, monetary policy has no effect on the equilibrium growth rate because of the endogenous response of the economy's market structure to changes in labor supply induced by monetary policy. This result differs from previous studies that analyze the effects of monetary policy in R&D-based growth models with either horizontal or vertical innovation, but not both. Furthermore, we analyze optimal monetary policy by analytically deriving the complete changes in welfare along the transition path and find that Friedman rule is socially optimal in this economy.

[17] See Chu and Cozzi (2012) for an analysis of the suboptimality of Friedman rule in the Schumpeterian growth model.

A potential direction for future research is investigation of the effects of monetary policy on economic growth and social welfare in a growth-theoretic framework in which R&D endogenously alters the importance of labor as a factor of production. The behavior of labor is central to our results, so a model in which the importance of labor changes as a result of R&D might deliver interesting new insights into the relation between money and real economic activity. See Peretto and Seater (2011) for the recent development of such a model without money.

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Appendix

Proof of Lemma 1. Substituting (8), (10) and (11) into (13) yields

$$H_t(j) = \theta \left[Z_t^\alpha(j) Z_t^{1-\alpha} l_{y,t}(j)\right]^{1-\theta} [X_t(j)]^\theta - X_t(j) - \phi Z_t^\alpha(j) Z_t^{1-\alpha} - R_t(j) + q_t(j) R_t(j). \quad \text{(A1)}$$

The first-order conditions include

$$\frac{\partial H_t(j)}{\partial X_t(j)} = 0 \Leftrightarrow p_t(j) = \theta \left[\frac{Z_t^\alpha(j) Z_t^{1-\alpha} l_{y,t}(j)}{X_t(j)}\right]^{1-\theta} = \frac{1}{\theta}, \quad \text{(A2)}$$

$$\frac{\partial H_t(j)}{\partial R_t(j)} = 0 \Leftrightarrow q_t(j) = 1, \quad \text{(A3)}$$

$$\frac{\partial H_t(j)}{\partial Z_t(j)} = \alpha(1-\theta) \frac{\theta \left[Z_t^\alpha(j) Z_t^{1-\alpha} l_{y,t}(j)\right]^{1-\theta} [X_t(j)]^\theta}{Z_t(j)} - \alpha\phi Z_t^{\alpha-1}(j) Z_t^{1-\alpha} = r_t^I q_t(j) - \dot{q}_t(j). \quad \text{(A4)}$$

Substituting (A2) and (A3) into (A4) yields

$$r_t^I = \alpha \left[(1-\theta)\theta^{(1+\theta)/(1-\theta)} l_{y,t} - \phi\right], \quad \text{(A5)}$$

where we have applied $Z_t(j) = Z_t$. Finally, substituting (17) into (A5) yields (14). ■

Proof of Lemma 2. Substituting $\tau_t = \dot{m}_t + \pi_t m_t$ into (2) yields

$$\dot{a}_t = r_t a_t + w_t l_t - c_t. \quad \text{(A6)}$$

Then, substituting (15) into $a_t = V_t N_t$ yields

$$a_t = \beta X_t N_t = \beta \frac{p_t X_t N_t}{p_t} = \beta\theta^2 Y_t, \quad \text{(A7)}$$

where the last equality uses (A2) and $p_t X_t N_t = \theta Y_t$. Substituting (A7) into (A6) yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{a}_t}{a_t} = r_t + \frac{w_t l_t - c_t}{\beta\theta^2 Y_t}. \quad \text{(A8)}$$

Substituting the Euler equation and $w_t l_t = (1-\theta) Y_t$ into (A8) yields

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{Y}_t}{Y_t} = \frac{c_t/Y_t}{\beta\theta^2} - \left(\frac{1-\theta}{\beta\theta^2} + \rho\right). \quad \text{(A9)}$$

Therefore, the dynamics of c_t/Y_t is characterized by saddle-point stability such that c_t/Y_t must jump to its steady-state value in (20). ■

Proof of Lemma 3. Substituting (10), (11), (15) and (A2) into (16) yields

$$r_t^E = \frac{1-\theta}{\beta\theta} - \frac{\phi Z_t + R_t}{\beta X_t} + \frac{\dot{X}_t}{X_t}, \quad \text{(A10)}$$

where we have applied $Z_t(j) = Z_t$ and $\dot{V}_t/V_t = \dot{X}_t/X_t$. Substituting (17) and (A2) into (8) yields

$$X_t = \theta^{2/(1-\theta)} Z_t \frac{l^*}{N_t}. \tag{A11}$$

Substituting (9) and (A11) into (A10) yields

$$r_t^E = \frac{1-\theta}{\beta\theta} - \left(\phi + \frac{\dot{Z}_t}{Z_t}\right) \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} + \frac{\dot{Z}_t}{Z_t} - \frac{\dot{N}_t}{N_t}, \tag{A12}$$

where we have used $\dot{X}_t/X_t = \dot{Z}_t/Z_t - \dot{N}_t/N_t$. Setting $r_t^E = r_t$ and substituting (22) into (A12) yield the dynamics of N_t given by[18]

$$\frac{\dot{N}_t}{N_t} = \frac{1-\theta}{\beta\theta} - \left(\phi + \frac{\dot{Z}_t}{Z_t}\right) \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} - \rho. \tag{A13}$$

Equation (A13) describes the dynamics of N_t when $N_t < \overline{N} \equiv \frac{\theta^{(1+\theta)/(1-\theta)}(1-\theta)}{\phi+\rho/\alpha} l^*$. When $N_t > \overline{N}$, $\dot{Z}_t/Z_t = 0$ as shown in (23). ■

Proof of Lemma 4. This proof proceeds as follows. First, we prove that under $\rho < \min\left\{\phi(1-\alpha), \frac{(1-\alpha)(1-\theta)}{\beta\theta}\right\}$, there exists a stable, unique and positive steady-state value of N_t. Then, we prove that under $\rho > \frac{1-\theta}{\beta\theta} - \alpha\phi$, the growth rate of vertical innovation is strictly positive. Finally, the above parameter conditions can be merged into $\frac{1-\theta}{\beta\theta} - \alpha\phi < \rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}$, which ensures $\frac{(1-\alpha)(1-\theta)}{\beta\theta} < \phi(1-\alpha)$. We consider the equilibrium under which there is positive in-house R&D. Substituting (23) into the first equation of (24) yields

$$\frac{\dot{N}_t}{N_t} = \frac{\rho - \phi(1-\alpha)}{\beta\theta^{2/(1-\theta)} l^*} N_t + \frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho. \tag{A14}$$

Because N_t is a state variable, the dynamics of N_t is stable if and only if $\rho < \phi(1-\alpha)$. Solving $\dot{N}_t = 0$, we obtain the steady-state value of N_t in an economy with positive in-house R&D.

$$N^* = \left[\frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho\right] \frac{\beta\theta^{2/(1-\theta)} l^*}{\phi(1-\alpha) - \rho}. \tag{A15}$$

Given $\rho < \phi(1-\alpha)$, (A15) shows that $N^* > 0$ if and only if

$$\rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}. \tag{A16}$$

Combining $\rho < \phi(1-\alpha)$ and (A16) yields

$$\rho < \min\left\{\phi(1-\alpha), \frac{(1-\alpha)(1-\theta)}{\beta\theta}\right\}. \tag{A17}$$

Substituting (A15) into (23) yields (26). Given (A16), (26) shows that $g^* > 0$ if and only if $\rho > \frac{1-\theta}{\beta\theta} - \alpha\phi$. ■

[18] It is useful to note that we have followed the standard approach in this class of models to treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also βX_t); therefore, $V_t = \beta X_t$ always holds. Otherwise, there would be an infinite number of either entries or exits.