To be or not to be informal?: A Structural Simulation

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Fundacion ARU

18 May 2012

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May 18, 2012

Abstract

The paper presents estimations of the informal economy size in Bolivia from an application of a Dynamic General Equilibrium Model. The parameter estimation is performed using maximum likelihood method to obtain, as an intermediate result, a latent variable estimation of the informal economy size. This procedure is new, as the estimate of the size of the informal economy using a dynamic structural model represents an alternative study area to latent variable models which assume relationships without a strong support in theory (MIMIC models).

The results suggest that the size of the informal economy represents 60% of Bolivian GDP in 2010 and that the trend has been decreasing in the last decade. In addition, we simulated four alternative policies to reduce the size of the underground economy. Some of them allow to identify surprising response mechanisms which allows to analyze the flow of workers from the informal sector into the formal sector and vice versa.

The research, besides quantifying the informal economy size, tries to provide a tool and methodology for evaluating alternative policy scenarios related to fiscal policy and labor mobility in a framework of an economy with a large informal sector and evasion.

Key Words: Informal Economy, Kalman filter, Structural Model.

JEL Classification: C61, E26, E62, O43

*Comments are welcome to `jpvargas@fen.uchile.cl`. I thank Cecilia Salas and Carola Tito for their research assistance. All errors are my responsibility. The views expressed in this document do not necessarily represent those of the ARU Foundation. I would like to thank Gustavo Machicado, and the assistants to the Third Bolivian Conference on Development and the Applied Research Workshops (Universidad Católica Boliviana).

1 Introduction

Measuring informal economy size has attracted considerable interest in recent decades[1], specially in developing countries [2]. However, measuring size of informal sector in the economy is not an easy task. One of the main challenges is the lack of a clear definition for informal economy. A wide range of similar terms are used in the literature, as illegal, underground, unreported economy and so on. In addition, designation is just the first issue on informality. There are also large differences in determining which should be considered as informal sector activities, estimation processes and usage of results for making economic analysis and taking public policy decissions. Because of these difficulties and despite its importance *"...we are still far from the time when the results of studies of the underground economy can have immediate consequences for policy or for the adjustment of various macroeconomic variables"* (Tanzi, 1999). This study seeks to promote new research in order to overcome these problems.

Growing informal activity around the world is an underlying concern (Schneider and Enste, 2000), because -so far- it has not been possible to obtain a consensus about how to measure it. Thus any attempt to measure informal economy is complicated, mainly because activities that make up the informal economy seek to avoid official registration.

In this context, and given the large magnitude that informal economy size represents in Bolivia[3] it is necessary to promote the economic study of this phenomenon in order to suggest alternative economic policies. Following this objective, and focusing on quantifying and proposing policy alternatives for the size of the underground economy in Bolivia, this document is organized as follows: the second section reviews the definitions and estimations methods of informal economy size identified in the literature, third section briefly describes behavior of informal economy in Bolivia, section 4 details the strategy proposed to obtain an alternative measure of informal economy. In section 5 the results related to three policy simulations are presented. Finally, section 6 reviews the major findings and conclusions.

[1] The literature about informal economy received an initial boost in Tanzi (1982).

[2] Schneider (2005) estimates suggest that the informal economy size in developing countries is almost twice (as a proportion of GDP) compared to developed countries.

[3] According to Schneider (2005) the size of the informal sector in the Bolivian economy would represent almost 70% of GDP. A parallel approach (Morales, 2008), considering the proportion of workers in the informal sector as informality measure, found that 70% of workers are working in the underground economy.

2 Definition and Estimation Methods of Informal Economy Size

According to Feige (1996) underground economy includes the following types of activities[4]:

- *Illegal* economy, which consists of the outcomes generated by economic activities that involve a violation of legal statutes that define the scope of legitimate forms of commerce.
- *Undeclared* economy, which consists in those legal and illegal economic activities that evade tax rules as stated in the tax laws.
- *Informal* economy, including economic activities that avoid costs and are excluded from benefits and rights incorporated in the administrative laws and rules that regulate the relations of property, commercial licensing, labor contracts, liability, credit financial and social systems.

Alternatively, Portes et al. (1989) define informal economy as the income earning activities that are not regulated by institutions of society, in a legal and social framework in which similar activities are regulated. This document, for measurement purposes, will contain three alternative and not exclusive definitions:

- Informal economy size is the share of total output produced within productive sector of unregistered economy.
- Unreported economy size is the proportion of existing evasion, defined as the share of total output that represents the undeclared production of productive registered enterprises.
- The employment size in informal sector is the share of all workers who decide to work in underground sector of economy (enterprises not registered or officially recognized).

The following subsections will give an overview of the state of art about existing procedures for estimating informal economy size. These procedures can be categorized into three types of methods[5]:

[4] An alternative definition of underground economy can be found in Cowell (1990). This document will take informal economy as synonymous of informal underground economy.

[5] This classification and descriptions closely follow Klinglmair and Schneider (2004). For more details you should refer to that document.

2.1 Direct Approaches

These are mainly composed by microeconomic approaches that use surveys based on voluntary participation and data collection methods based on fiscal audits.

The main disadvantage of using surveys is that they present the common failures of this type of processes: accuracy of findings and results will depend on response and veracity of participants. The main advantage of using surveys is that they provide detailed information about informal economy structure, allowing to disaggregate the information almost as much as required.

On the other hand, the size of the informal economy might be estimated as the difference between declared incomes for fiscal purposes and those measured by audits. The problems facing this methodology is that the use of random audits is equivalent to applying a biased sample of the population. The advantage is that objective data are available (tax returns and financial statements).

2.2 Indirect Approaches

The indirect methods try to estimate the informal economy size from the behavior of variables that should reflect the existence of underground activities. Next, we identify five variants of this type of calculation: i) discrepancies between the measurement of expenditure and income; ii) discrepancies between official and actual size of the labor force, iii) approximation of transactions; iv) the cash demand approach, and v) the physical input method (approximation of the production by means of electrical consumption behavior).

2.3 Structural Approaches

All methods described so far are designed to estimate the size and development of the informal economy based on a single indicator that would capture its effects.

On the other hand, the structural estimates explicitly consider the use of multiple causes that lead to the existence and growth of informal economy, as well as its multiple effects. The empirical method used is based on the statistical theory of unobserved variables, which considers multiple causes and multiple indicators of the phenomenon to be measured. For estimation, it uses a factor-analytic approach to measure the informal economy as an unobserved variable over time (latent variable). The unknown coefficients are estimated on a set of structural relationships represented by equations in which the variable can not be measured directly[6].

According to the literature, some causes of the underground economy, usually included in MIMIC models, are: i) the burden of direct and indirect taxes both real

[6] This kind of models, Multiple Indicators and Multiple Causes are known as MIMIC models (for its acronym in English). An extension of these models which considers intertemporal effects is known as DYMIMIC model (Dynamic Multiple Indicators and Multiple Causes model). Both are special cases of SEM (Structural Equation Models).

as perceived, ii) the burden of regulation as a proxy for other State activities, since it is assumed that an increase in the burden of regulation creates a strong incentive to enter the informal economy (Loayza et al., 2005), and iii) the 'tax morale' as the attitudes of citizens towards the State that describes layout of the people to leave their official occupations and enter informal economy (Torgler and Schneider, 2009; Frey and Torgler, 2007; Güth and Sausgruber, 2004). Among the indicators or effects of the underground economy, researchers typically include: i) the performance of monetary indicators: if activities in the informal economy increases, it requires a further increase in monetary transactions, ii) labor market development, as an increase in the share of workers in the informal sector implies a decrease in their participation in the formal economy, and iii) the production market performance: an increase in the informal economy means that inputs (specially labor) will move from the official economy (at least in part), causing a shift that could have a depressive effect on the official growth rate of the economy.

The estimation method we propose and which is detailed in Section 4, is a logical extension of structural models that seeks to add a theoretical background to estimate the size of the informal economy as a latent variable. But previously, the following section contextualizes the behavior of the informal economy in Bolivia, based on information from previous studies.

3 The Informal Economy in Bolivia

The size of the informal economy in Bolivia has been estimated by different techniques, by different authors, and under different definitions of informality. There are three commonly applied empirical definitions: one that defines the informal economy as the proportion of all workers who carry out their activities without complying with employment records and existing legal obligations; as the proportion of the official product, which is generated by informal activities, and; as the proportion of tax evasion. Table 1 summarizes the results of some estimations, where the common factor is that the informal economy size in Bolivia is not an isolated element, but rather represents a significant proportion of economic activity, both in the proportion of workers involved, and the proportion of output generated. In general, most estimations suggest that the informal economy size would be in a range between 60% and 70%, measured as both, the share of total workers, and the share of official output. See Table 1.

Country	Author	Methodology	Years	Results
31 Latin America and the Caribbean countries.	Loayza A. Norman (1997)	Approximation of informal sector participation on the basis of a statistical model that considers the relative size of informal sector (the ratio between the level of informal sector production and total GDP) as a variable "latent" that potentially has multiple causes and so that can be explained by multiple factors.	Data base used 1980-1992	Estimations are considered valid to quantify the informal economy in the early 90's. The results showed that Bolivia is the country that has a greater degree of informality and the contribution to the economy is 65.6% of GDP.
Bolivia	Humerez Quiroz Julio (2005)	The methodology used is the cash demand, that is to make inferences from the information contained in monetary aggregates, on the assumption that agents use cash in order to make informal activities.	1994-2003	The size of the shadow economy (informal sector) is estimated in average 47%.
21 Latin America and the Caribbean countries.	Gasparini and Tornatolli (2007)	The authors use two definitions of informality: the legalistic and productivity to define the size of the informal economy through household surveys.	1989-2005 (Between 1993 and 2002 for Bolivia)	Results suggest a informal economy size in Bolivia of 77%.
Bolivia	Landa and Yañez (2007)	Proportion of workers in the informal sector information. Based on surveys of households, which are representative at the urban level.	1996-2006	The informal economy would be about 60% with decreasing trend.
Bolivia	Morales (2008)	Informal Sector definition according to ILO.	1999-2005	The informal economy size (in proportion of workers) would be about 70% with decreasing trend.
Bolivia	Mártinez and Chumacero (2009)	The methodology used considers those occupational categories in survey and are assumed to jobs of low productivity due to be labor intensive and not capital intesive.	1995, 2000 and 2005.	The degree of informality that was found was 65.3% in 1995, a percentage that dropped to 62.4% in 2000 and 2005. It means that almost two thirds of those employed in urban areas are informal, with a low productivity employment and low income taking them close to poverty.
120 Countries	Schneider and Buehn (2007)	Calculations based on the combination of a MIMIC process (latent variable method for multiple indicators and multiple causes)	1996-2006	The size of the informal economy as a proportion of GDP, would be about 67.3% in average for the period of analysis

Table 1. Informal Economy Size in Bolivia.

Figure 1 compares the size of the informal economy in Bolivia with averages from other world regions. One element that makes the analysis of informality in Bolivian economy interesting is its magnitude. According to Schneider and Buehn (2007) estimates for the 1999-2006 period, 67.3% of Bolivia's GDP would be generated by informal activities, which means that among 120 countries, Bolivia occupies the second place (only behind Georgia) among countries with larger informal economies. According to these data, the informal economy, measured as a share of official GDP, has a slight upward trend in Bolivia in the recent years.

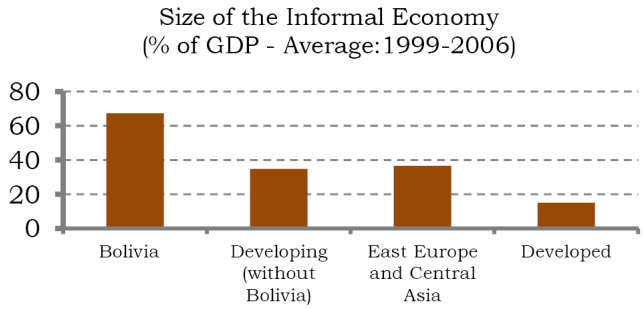


Figure 1. Estimated size of Economy in Bolivia and other groups of countries. (Source: Schneider and Buehn, 2007)

An alternative definition of informality based on employment (Landa and Yañez, 2007) show three approaches to the proportion of workers who perform their work in underground sector of the economy. The first identifies the informal workers as those who do not receive social protection (legal definition), the second measures by the size of the companies (where firms with less than 5 workers are considered informal), and the third bases on company registration in the national tax records. Figure 2 summarizes these results, and suggest that the informal economy size, under any of the three employment measurement definitions, is relevant, although it shows a slight decrease in the 1996-2006 period of study. In any case, it is necessary to clarify that the data of Landa and Yañez (2007) only represent the behavior of the urban area, suggesting that the size of the underground economy at national level might be higher, considering that rural sector activities are typically performed without any official registration.

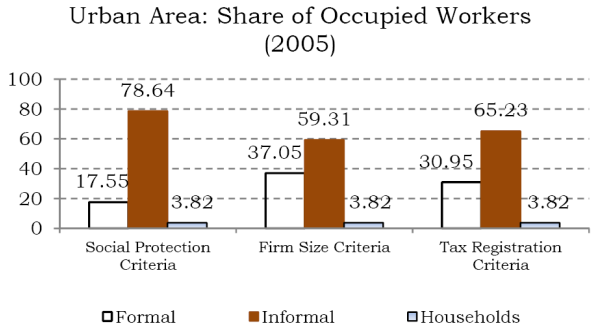


Figure 2. Estimated Size of the Urban Informal Economy in Bolivia (Source: Landa and Yañez, 2007)

The results of the estimations compiled in this section coincide that the size of the informal economy is non-negligible. Furthermore, a common factor in the mentioned researches, is that they primarily focus on estimating or characterizing the informal economy size; the estimation method we propose in next section, has the virtue that, in addition to providing a new series of the size of the informal economy, seeks to promote the study of the underlying mechanisms from a structural scope.

4 The Estimation Method

As mentioned above, the method we propose to quantify the size of the informal economy is a natural extension of DYMIMIC models. The main difference resides in the equations included in the model, which are derived from the linearization of a Dynamic General Equilibrium Model. This means that relations are based on optimizer behavior of model agents. In this sense, we present a model that can be understood as an extension of the dynamic general equilibrium models of Chen (2003) and Busato et al. (2006). To the best of our knowledge, this type of model has not been used to quantify the informal economy size by applying the Kalman filter to estimate a latent variable. However, there are some approaches in that direction: Karanfil and Ozkaya (2007) used the Kalman filter to estimate the size of the unrecorded economy in Turkey, using records of energy consumption as an indicator of it. The difference with our approach is that, while based on a structural model, this model does not come explicitly from optimizing behavior of agents. On the other hand, Arango et al. (2006) also apply the Kalman filter to measure the size of the underground economy in Colombia between 1976-2003, and presents a critique of DYMIMIC and MIMIC models. Their estimates are based on the estimation of a cash demand function, combined with a linear approximation of the implications

of the structural model Loayza (1997). This approach is methodologically similar to the one we propose; however, it arises from a non-unified structural model.

Within the model we propose it is possible to consider the size of the informal economy for at least three definitions or components: i) the proportion of unreported income by officially registered businesses (evasion), ii) the proportion of output that is generated by companies that are not registered by the government, and iii) the proportion of workers who choose to work in the informal sector.

The model is composed by three actors: government, households and firms[7]. It includes various events that promote and - under some conditions - ensure the co-existence of an informal sector with the formal sector. Thus, agents are able to evade taxes, meaning evasion for underreporting of income, but can also avoid the full payment of such taxes by producing from the informal sector. The two possibilities are not mutually exclusive, in fact the model allows both: i) a firm which chooses to produce some share in the informal sector, and ii) evade taxes related with its formal sector production.

Firms face the likelihood to be detected by the government, both in the case of tax evasion as in the case of producing in the informal sector. In the case of detection, they face fines (penalties) that apply on the amount of the omitted tax. Probabilities and fines may be the same or different for the two types of infringement. Besides, we include a mechanism where detection probabilities are endogenous, and depend on a regulatory quality indicator of the activities of government, holding the line of findings of Loayza et al. (2005). Endogeneity is supported because regulatory quality is affected by a proportion of government spending used to improve it.

The model depends on a set of parameters that need to be defined so that it is a good representation of the Bolivian economy. There is much literature regarding the better strategy to define these parameters, which highlights an important distinction between calibration and estimation processes (Cooley 1997, Hansen and Heckman 1996; Gomme and Rupert 2005; and Dejong and Dave 2007). In this paper we choose the parameter estimation by the method of maximum likelihood.

The maximum likelihood estimation is based on the resolution of the model - by a first order approximation - according to the perturbation method proposed in Schmitt-Grohe and Uribe (2004). Once the model solution is found, it can be expressed as a set of difference equations. Then the system of difference equations is assigned to a vector of observable variables known as measurement equations. The measurement equations system is then used in the Kalman filter to construct likelihood functions under the assumption of normality of error terms. An important intermediate output of the Kalman filter is observed and unobserved smoothed variables generated in the estimation process, which are, in the case of unobserved variables, latent variable estimations[8], including the variable that identifies the informal

[7] Model details are in Apendix A.

[8] This exercise is analogous to that performed in DeJong et al. (2000).

economy size[9]. The application of the proposed method, whose results are described in the next section, was performed using Dynare v.4.2.1 (a Matlab Toolbox).

5 Results

In the following we describe the main findings and policy exercises supported by the method of the preceding section. As mentioned above, the first step is to estimate the model parameters so that it could be considered a good approximation of the Bolivian economy. The model includes 32 parameters to determine, since this large number of parameters leads to difficulties for joint estimation, we used a block estimation strategy, in such a way to find areas where the log-likelihood function is well behaved. We included five observable variables on the estimation process: the official GDP, government spending, investment, consumption, and the index of regulatory quality. The former four were obtained from national accounts in Bolivia, while the latter corresponds to the Governance Matters project (World Bank). The estimated parameters are summarized in Table 2.

Parameter	Value	Parameter	Value	Parameter	Value
A	0.98	s	1.27	ω	0.1
B	1.12	t	1.4	$\bar{q}$	-0.24
α	0.36	T_1	-17	ρ_1	0.99
δ	0.07	T_2	45	ρ_2	0.99
τ_F	0.24	T_3	-5.1	ρ_q	0.99
τ_L	0.16	T_4	1	σ_{e_1}	0.04
η	2.28	W_1	0.01	σ_{e_2}	0.03
σ	2	W_2	1	σ_{e_3}	0.39
h	0.7	φ	0.29	σ_{e_4}	0.03
β	0.99	υ	0.83	σ_{e_5}	0.02
θ	0.57	H	2		

Table 2. Estimated Parameters for the model

It is important to notice that a subset of variables (σ, H) were not estimated, but taken from the literature and/or fixed to reasonable values because it was not possible to build a well-behaved likelihood function for them. In general, the other estimated parameters are within the ranges suggested by the theory and highlight elements that deserve further analysis: the underground sector technology would be higher than the official sector $(B > A)$; the penalty for evading in the formal sector is lower than the penalty for being found producing in the underground sector

[9]Further details about method of solution can be found in Dejong and Dave (2007), Canova (2007), and Adjemian et al. (2011)

($s < t$); the probability of being detected evading in the formal sector is highly sensitive to regulatory quality, while the probability of detection when producing in the underground sector, is not so sensitive to regulatory quality ($T_2 > W_2$); the share of government spending representing a positive externality for the formal sector is higher than the share which represents a positive externality for the underground sector ($\upsilon > \varphi$). Except for the first one, these results are intuitive according to the evidence and current tax laws.

The parameters in Table 2 are consistent with the following values of the variables in steady-state:

Variable	Steady-State Value
y	1
c/y	0.46
g/y	0.28
i/y	0.25
y^O/y	0.34
y^U/y	0.66
μ	0.33
ξ	0.75
p	0.48
o	0.61
q	0.46

Table 3. Steady-State of the Model

Above results are quite consistent with other authors' estimations described in the previous section. At steady-state, the informal economy size represent 66% of the official product (y^U/y), about 67% of workers who belong or have activities in the informal sector ($1 - \mu$); the officially registered companies would declare only 75% of their income (ξ) which implies a 25% output tax evasion ($1 - \xi$); and regulatory quality would be around the average of the countries in the sample ($q = 0.46$). Furthermore, as mentioned in the previous section, it is possible to have an estimation of informal economy time evolution. This estimation shows a decreasing behavior of the informal economy size which, as theory suggests, is accompanied by an increase in the size of the formal economy (see Figure 3 and Table 4).

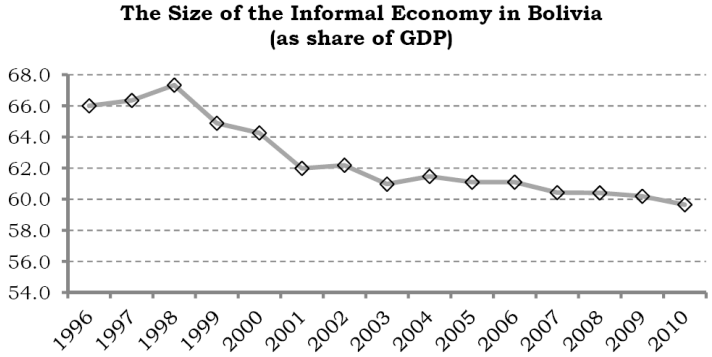


Figure 3. Estimated Size of the Informal Economy in Bolivia

Year	y^U/y
1996	66.0
1997	66.3
1998	67.3
1999	64.9
2000	64.3
2001	62.0
2002	62.2
2003	61.0
2004	61.5
2005	61.1
2006	61.1
2007	60.4
2008	60.4
2009	60.2
2010	59.6

Table 4. Size of the Informal Economy

Previous calculation of the informal economy size follows the same procedure as in Loayza (1997). To transform a time series from a latent variable (with relative scale) to a time series with absolute scale we must fix two points: the first is the estimation of informal employment of Landa and Yañez (2007) for 1996, and the second - to adjust the distance between ranks - is the data of informality for 2006 from the same source. Since Landa and Yañez (2007) use an employment approach to measure the size of informal economy, we performed a scale adjustment in order to get a measure of informality as the share of GDP.

According to model estimations, the main reason for the decreasing trend of informal economy should be attributed to positive technology shocks in formal sector,

which is reinforced by negative shocks in technology in the underground economy[10], and a decline in regulatory quality. The mechanism under which a reduction in regulatory quality can increase the size of the official sector, according to the model, is as follows: a diminution in the regulatory quality decreases the detection probability of evasion, so that agents find out that tax burden within the official sector is lower, and therefore reduce their incentives to move to underground economy.

In the following subsection we introduce four alternative scenarios of economic policy experiments; they aim to reduce the informal economy size, without increasing formal sector productivity.

5.1 Four Scenarios for Policy Making

We raise four alternative strategies that an economic policy maker might propose to reduce the informal economy size, namely: i) increase the penalties to evade capture in the formal sector, ii) increase the penalties to be caught working in the underground sector, iii) increase government regulatory quality (or reduce corruption), and iv) policies ii) and iii), jointly.

Table 5 summarizes the main findings:

Variable	Baseline Scenario	Scenarios of Policy			
		i) $\Delta^+ s$	ii) $\Delta^+ t$	iii) $\Delta^+ q$	iv) $\Delta^+ t \wedge \Delta^+ q$
y^U/y	0.66	$+$	$-$	$+$	$-$
y^O/y	0.34	$-$	$+$	$-$	$+$
$1-\xi$	0.24	$-$	$-+$	$-$	$-$
μ	0.33	$-$	$+$	$-$	$+$

Table 5. Effects of four alternative policies on the size of the informal economy

With regard to the first policy option, it seems surprising that increasing the penalty to formal sector workers who evade, decreases the size of the formal sector boosting the agents to work in the informal sector. The transmission mechanism is as follows: evasion becomes more expensive due to increased penalties, so the agents must face a higher tax burden (evading less), but higher tax burden boost to leave the formal sector and move to underground sector.

On the other hand It seems clear that, from the point of view of effectiveness, increasing penalties to agents working in the informal sector (second policy alternative) might be a better choice. Thus, the second policy scenario in Table 5, which suggests increasing the penalty in the underground sector, appears to reach desirable results in order to reduce the size of the informal economy. In fact, the increment in t increases expected costs of producing in the underground sector, so the option to operate in the formal sector marginally becomes more attractive.

[10] Productivity shocks were analyzed through the smoothed series generated by the Kalman filter.

The third policy experiment, focused on increasing regulatory quality provided by government (variable highly correlated with control of corruption and government effectiveness index[11]) presents similar results to those of the first experiment. The increase in regulatory quality, per se, is not an effective policy, since it leads to increase capture probability in the formal sector (more than in the informal sector), this implies that it becomes more costly to evade (in fact the policy reduces the levels of evasion $1-\xi$), so agents move to the informal sector because they face a heavier tax burden in the formal sector.

Finally, we simulate a combined policy, under which the increase in regulatory quality can be combined with an increase in the penalties imposed when an agents are caught producing in the informal sector. This fourth scenario, is highly effective in reducing the informal economy size ($^{-}\Delta(y^U/y)$), displacing workers and capital into the formal sector ($\Delta^{+}\mu$) and decreasing the evasion levels ($^{-}\Delta(1-\xi)$).

The sensitivity of the main variables to the first three policy proposals outlined can be inferred from the following graphs[12].

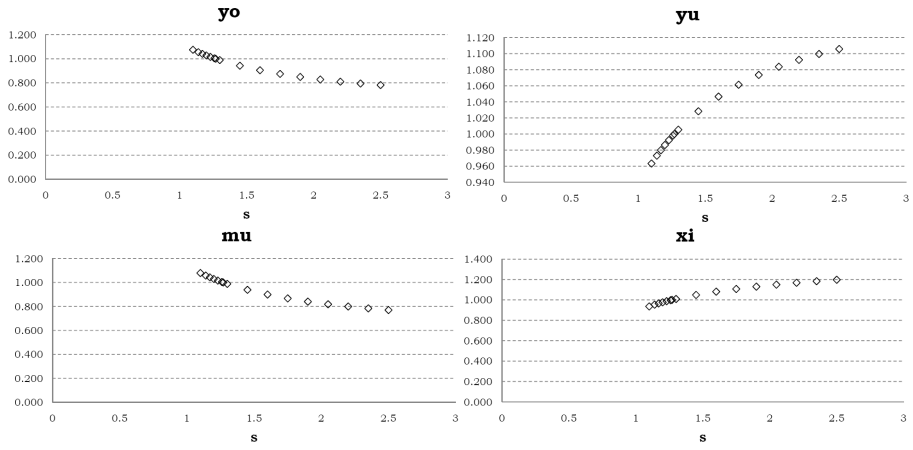


Figure 4. Changes in the penalties for evasion in the formal sector

Figure 4 corroborates results of case i) (Table 5). That is to say, an increase in penalties for evading implies, at first glance counterintuitively, an increase in the informal sector of the economy. When increasing the size of the penalties in the formal sector, from 1.27 (27% of effective fine) to about 2 (100% effective penalty), the size of the formal sector (y^O) and the proportion of its workers ("*mu*"*or* μ) are reduced by almost 20%, the informal sector expands in 8% and the proportion of income reported by companies to the Tax Agency ("*xi*"*or* ξ) are increased by 15%, therefore decreasing evasion $(1-\xi)$ in the same amount.

[11] The correlation between regulatory quality and government effectiveness is 0.93 (using World Bank Governance Matters series).

[12] In Figures 4, 5 and 6, the vertical axis is adjusted so that the value of 1 corresponds to steady-state value in Table 3.

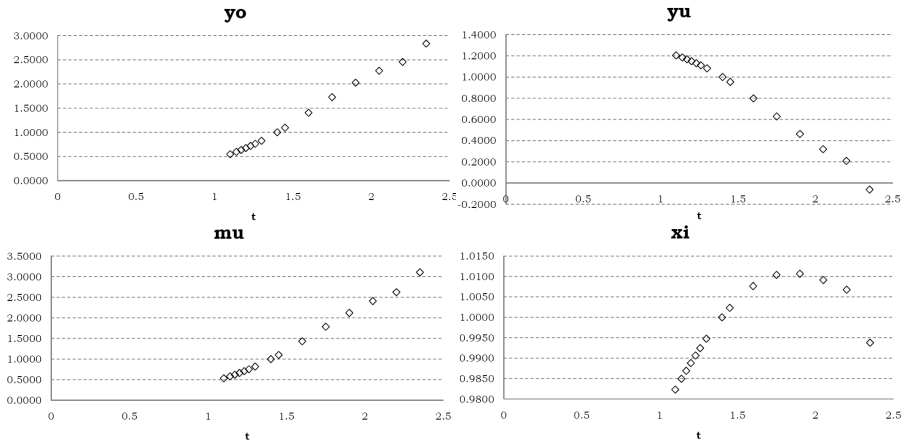
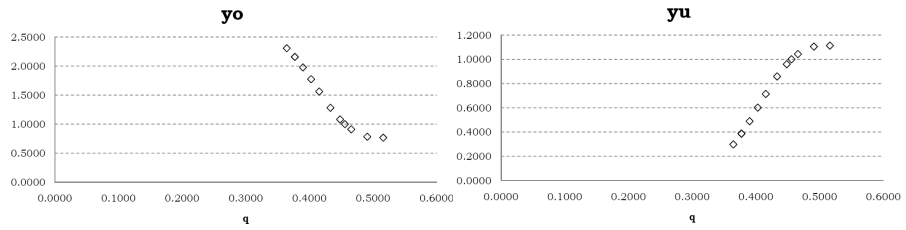


Figure 5. Changes in the penalties for producing in informal sector.

The sensitivity analysis of the second policy (case ii) in Table 3) is shown in Figure 5. The estimated value of effective penalties in the baseline scenario is $t = 1.4$ (i.e. 40% of fines over unrecorded production value). If, as in the previous case, the penalty increases to reach 100% of fines, that is to say $t = 2$, the model suggests that the size of the formal economy, and the proportion of workers in the formal sector would increase just over 120%, the size of the informal economy (y^U) would be reduced by 70%, and evasion ($1-\xi$) would be reduced to a certain threshold, after that it begins to increase again. This nonlinear behavior deserves some explanation: increasing penalties to informal sector producers decreases informal sector production and increases formal sector production; at some stage formal sector production could not compensate the decline in the informal sector production, so the revenues from the formal sector could not compensate the fall in revenues from the informal sector, creating a kind of Laffer curve on government revenues, this nonlinearity is transmitted to regulatory quality levels which depend on the level of government revenues, and finally, the probabilities of being caught in both sectors depend on the levels of regulatory quality, and evasion levels depends on the probability of detection. Thus, the initial increase and subsequent decline of total output, generate an initial reduction of evasion followed by an increase of it, nevertheless this behavior is marginal in its magnitude. Also, this result suggest that increasing indiscriminately the penalties in the underground sector does not necessarily result in a maximizing production policy.



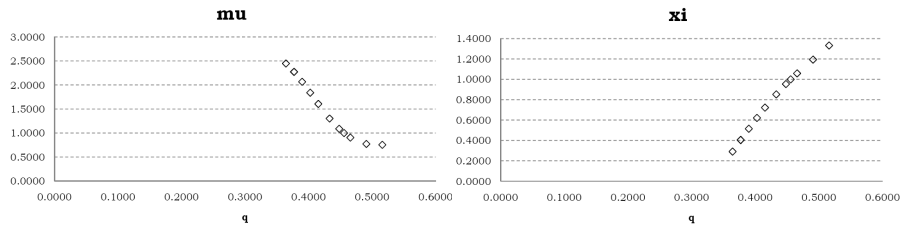


Figure 6. Changes in regulatory quality

Finally, Figure 6 shows the sensitivity analysis of a policy aimed at increasing regulatory quality. To simulate this policy, we assume a positive shock on the autonomous regulatory quality level (variable $\bar{q}$ in equation A.12 in Annex A). The increase in $\bar{q}$ would be reflected in an increase in regulatory quality index q, whose value is represented in the horizontal axis of Figure 6 charts. Thus an increase in regulatory quality does not necessarily imply a smaller size of the informal economy, in fact, the increase in regulatory quality implies an increase of the informal economy size and a decrease in the size of the formal economy. This happens because the increase in q promotes better control (i.e. increase in probability of detection) in both the formal sector through the detection probability p as in the informal sector through o. Furthermore, the results of the estimation of the model imply that the probability of detection within the formal sector is much more sensitive to regulatory quality that the probability of detection within the informal sector. Therefore, the increase in regulatory quality discourages evasion more than the disincentive to produce in the informal economy. Finally, the decline of evasion increases the effective tax and therefore the tax burden in the formal sector, leading to a transfer of formal sector workers into the informal sector.

To conclude this section it is important to review the results in column iv) of Table 3. In the fourth policy option we propose to increase penalties for informal sector and improve regulatory quality, both at the same time. This policy mix would allow: to promote formal sector size, decreasing the levels of evasion as well, and decrease the size of the informal sector by increasing the relative cost of staying in it. Obviously, this is not the only possible combination, and a thorough analysis of public policy should consider other factors besides the mentioned here.

The aim of the performed exercises is to establish that a policy which tries to reduce informal economy might not necessarily be made considering isolated elements of the evidence, and instead it should take into account indirect effects that often can be counterintuitive. In this context, it is important to emphasize that one of advantages of this type of methodology and estimations of the size of the underground economy, is that it can be extensible for considering the effects of different policies on other desirable objectives such as welfare, economic growth or revenues, to name a few.

6 Concluding Remarks

This paper aims to quantify the informal economy size in Bolivia and propose policy alternatives based on the results of a dynamic general equilibrium model. The method used to calculated the size of the informal economy is novel because allows to take advantage of the parameter estimations by maximum likelihood to calculate a latent variable (unobserved) which represents the proportion of output that is generated in the informal sector of the economy.

The model presented in the paper allows to quantify the size of the informal economy in Bolivia from different definitions: as the total output share; as the existing tax evasion in the formal (registered) sector, and; as the proportion of employees working in the informal (unregistered) sector. The results suggest that the size of the informal economy in Bolivia - measured as the share of total output of the economy - declined in recent years, which would potentially mean 60% of the total economy in 2010. This decreasing trend follows closely the results of Morales (2008), and Landa and Yañez (2007).

Furthermore, we proposed four policy simulation exercises aimed to reduce the size of the informal economy in Bolivia: i) increasing penalties for tax evaders in the formal sector of the economy ii) increasing the penalties for agents who are producing in the informal sector of economy, iii) increasing government regulatory quality, and iv) combining policies ii) and iii). According to the estimated model parameters, results indicate that not all these policies are desirable and in some cases the results are counterintuitive and would imply increasing the size of the informal economy. In fact, an increase of penalties in the formal sector of economy, and an increase of regulatory quality levels imply an increase in the size of the underground economy. On the other hand, we find that a combined policy, where the penalties for those who produce in the informal economy are increased, and regulatory quality is improved, could have desirable effects when trying to reducethe size of the informal sector, and evasion levels.

However, it is necessary to make the analysis of these policies within alternative definitions of what *desirable* means, understanding that there might be superior objectives besides those considered in this paper proposal, for instance: increasing well-being, increasing consumption, reducing tax revenues volatility, increasing the share of workers in the formal sector, etc. In this sense, we believe that the proposed methodology is a useful and extensible tool to lead public policy debate related to the size of the underground economy from different goals and an integral approach to the problem.

References

[1] Adjemian, S., Bastani, H., Juillard, M., Mihoubi, F., Perendia, G., Ratto, M., and Villemot, S. Dynare: Reference manual, version 4. Dynare Working Papers 1, CEPREMAP, 2011.

[2] Arango, C., Misas, M., and López, E. Economía subterránea en colombia 1976-2003: Una medición a partir de la demanda de efectivo. Borradores de Economia 335, Banco de la Republica de Colombia, 2006.

[3] Busato, F., Chiarini, B., and Rey, G. Equilibrium implications of fiscal policy with tax evasion. *SSRN eLibrary* (2006).

[4] Canova, F. *Methods for applied macroeconomic research*. No. v. 13 in Methods for applied macroeconomic research. Princeton University Press, 2007.

[5] Chen, B.-L. Tax evasion in a model of endogenous growth. *Review of Economic Dynamics 6*, 2 (April 2003), 381–403.

[6] Cooley, T. F. Calibrated models. *Oxford Review of Economic Policy 13*, 3 (Autumn 1997), 55–69.

[7] Cowell, F. *Cheating the Government: The Economics of Evasion*. MIT Press, 1990.

[8] DeJong, D., and Dave, C. *Structural macroeconometrics*. Princeton University Press, 2007.

[9] DeJong, D. N., Ingram, B. F., and Whiteman, C. H. Keynesian impulses versus solow residuals: identifying sources of business cycle fluctuations. *Journal of Applied Econometrics 15*, 3 (2000), 311–329.

[10] Dell'Anno, R. Stimare l'economia sommersa con un approccio ad equazioni strutturali. un'applicazione all'economia italiana (1962-2000). Dipartimento di Scienze Economiche e Statistiche, Università degli Studi di Salerno, 2001.

[11] Feige, E. L. Overseas holdings of u.s. currency and the underground economy. In *Exploring the Underground Economy*. Kalamazoo, Michigan, pp. 5-62, 1996.

[12] Frey, B. S., and Torgler, B. Tax morale and conditional cooperation. *Journal of Comparative Economics 35*, 1 (March 2007), 136–159.

[13] Gasparini, L., and Tornarolli, L. Labor informality in latin america and the caribbean: Patterns and trends from household survey microdata, Feb. 2007.

[14] Giles, D. Modeling the hidden economy and the tax-gap in new zealand. Tech. rep., Working Paper, Department of Economics, University of Victoria, Canada, 1999.

[15] Giles, D. Measuring the hidden economy: Implications for econometric modelling. The Economic Journal , 109/456, pp.370-380, 1999a.

[16] Gomme, P., and Rupert, P. Theory, measurement, and calibration of macroeconomic models. Working Paper 0505, Federal Reserve Bank of Cleveland, 2005.

[17] Güth, W., and Sausgruber, R. Tax morale and optimal taxation. Tech. rep., 2004.

[18] Hansen, L. P., and Heckman, J. J. The empirical foundations of calibration. *Journal of Economic Perspectives 10*, 1 (Winter 1996), 87–104.

[19] Henley, A., Arabsheibani, G. R., and Carneiro, F. G. On defining and measuring the informal sector. Institute for the Study of Labor (IZA), Nov. 2006.

[20] Hernández, M., and de la Roca, J. Evasión tributaria e informalidad en el perú. Economía y Sociedad 62, CIES, 2006.

[21] Humerez, J. Una estimación del tamaño de la economía informal, un ejercicio para bolivia: 1990-2003, Abril 2005.

[22] Johnson, S., Kaufmann, D., and Zoido-Lobatón, P. Regulatory discretion and the unofficial economy. American Economic Review, Vol. 88, pp. 387Ű92, 1981.

[23] Karanfil, F., and Ozkaya, A. Estimation of real gdp and unrecorded economy in turkey based on environmental data. *Energy Policy 35*, 10 (October 2007), 4902–4908.

[24] Landa, F., and Yañez, P. Informe especial: La informalidad en el mercado laboral urbano 1996-2006. UDAPE, 2007.

[25] Loayza, N. The causes and consequences of informality in peru. Banco Central de Reserva del Perú, 2007.

[26] Loayza, N. A. The economics of the informal sector : a simple model and some empirical evidence from latin america. The World Bank, Policy Research Working Paper Series 1727, Feb. 1997.

[27] Loayza, N. V., Oviedo, A. M., and Serven, L. The impact of regulation on growth and informality - cross-country evidence, May 2005.

[28] Macias, J. B. The dynamics of parallel economies. measuring the informal sector in méxico. CaŠ Foscari University, 2008.

[29] Martínez, D. y. C. M. El sector informal urbano en bolivia, 1995-2005, Octubre 2009.

[30] Morales, R. El sector informal en bolivia. reflexiones teóricas y realidad estadística. Superintendencia de Empresas - Bolivia, 2008.

[31] OIT. Panorama laboral. Organización Internacionald el Trabajo, 2004.

[32] Portes, A., Castells, M., and Benton, L. World underneath: The origins, dynamics, and effects of the informal economy. In *The Informal Economy: Studies in Advanced and Less Developed Countries*. Johns Hopkins University Press, Baltimore, 1989.

[33] Schmitt-Grohe, S., and Uribe, M. Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control 28*, 4 (January 2004), 755–775.

[34] Schneider, F. Measuring the size and development of the informal economy. can the causes be found and the obstacles be overcome? In *Essays on Economic Psychology*. Berlin, Heidelberg, Springer Publishing Company, pp. 193-212., 1994a.

[35] Schneider, F. Can the informal economy be reduced through major tax reforms? an empirical investigation for austria. Supplement to Public Finance/ Finances Publiques, 49, pp. 137-152., 1994b.

[36] Schneider, F. The shadow economies of western europe. Journal of the Institute of Economic Affairs, Vol. 17, pp. 42Ű48, 1997.

[37] Schneider, F. Further empirical results of the size of the informal economy of 17 oecd-countries over time. Paper presented at the 54. Congress of the IIPF Cordoba, Argentina and discussion paper, Department of Economics, University of Linz, Linz, Austria, 1998.

[38] Schneider, F. The size and development of the shadow economies of 22 transition and 21 oecd countries. Institute for the Study of Labor, 2002.

[39] Schneider, F. Size and measurement of the informal economy in 110 countries around the world. Workshop of Australian National Tax Centre, ANU, Canberra, Australia., 2002.

[40] Schneider, F. The shadow economy and shadow economy labor force: What do we (not) know? IZA Discussion Papers 5769, Institute for the Study of Labor (IZA), June 2011.

[41] Schneider, F., and Enste, D. H. Shadow economies: Size, causes, and consequences. Journal of Economic Literature, Vol. XXXVIII, 2000.

[42] Schneider, F. G., and Buehn, A. Shadow economies and corruption all over the world: revised estimates for 120 countries. *Economics - The Open-Access, Open-Assessment E-Journal 1*, 9 (Version) (2007), 1–53.

[43] Tanzi, V. The underground economy in the united states and abroad. Lexington (Mass.), Lexington, 1982.

[44] Tanzi, V. The underground economy in the united states, reply to comments by feige, thomas, and zilberfarb. IMF - Staff Papers, 33/ 4, pp. 799-811, 1986.

[45] Tanzi, V. Uses and abuses of estimates of the underground economy, June 1999.

[46] Tedds, L. The underground economy in canada. McMaster University, 2005.

[47] Torgler, B., and Schneider, F. The impact of tax morale and institutional quality on the shadow economy. *Journal of Economic Psychology 30*, 2 (April 2009), 228–245.

[48] Vuletin, G. Measuring the informal economy in latin america and the caribbean. International Monetary Fund, 2008.

A Model

The model is a slightly modified version of Vargas (2009), which considers a representative agent model, both for consumption as for production in which taxes are applied on labor and output. The production technology has two sectors:

- A legal, formal or registered sector in which it is possible to declare - for fiscal purposes - income levels lower than the real. In this sector there might be tax evasion.
- An unregistered sector, which evades all taxes on labor and product factor. This sector uses a proportion of all capital and labor available for the firm.

Logically, the specification includes the government sector, which provides public goods and services and determines (in this case exogenously) taxes on labor and production factor. Another important feature of the model is that it includes the effect of regulatory quality / control of corruption on the decision of the agents, by integrating them through endogenous coefficients of detection probability that firms and workers perceive. Regard to consumers, they face a utility function that depends on consumption and leisure.

In the remainder of this section we show model conditions.

A.1 The firms problem

We assume the following specifications to represent the technology of firms in both sectors, official and underground[13]:

$$y^O \equiv y^O\left(A, k^O, l^O, g\right) \quad \text{and} \quad y^U \equiv y^U\left(A, k^U, l^U, g\right) \tag{A.1}$$

The above equations consider a positive externality in the production functions of the official and underground sector, they are generated by government spending as in Barro (1992). Constant returns to scale in the three productive factors are assumed, that is capital, labor and government spending. It is assumed that production functions have positive and decreasing returns in each factor separately, and also fulfill the Inada conditions.

The firm's income for product sales are equal to $y = y^O + y^U$ (normalizing prices to 1); however, taxes must be deducted and/or penalties to which the firm is subject plus costs for the retributions to productive factors. In this way we calculate profit of the firm.

The government charges a tax on formal output y^O according to a percentage aliquot τ_F. Since the government ignores the actual output of firms, it is possible

[13] The above indices O and U represent the official (formal) and underground (informal) sector, respectively.

that they declare only a proportion ξ from its total output, where $0 < \xi < 1$. The government finds, with a probability p, firms declared $\xi < 1$, and applies in this case a proportional penalty on the tax evaded as well (with $s > 1$).

The firm's income also comes from its underground operations y^U. We assume that the company - in underground sector - faces a probability of detection (o) and penalties (t), different from those within the formal sector. In addition, firm decides the share of labor that goes to the official sector and implicitly the share of labor it is assigned to the underground sector. Given these elements, the benefit function of firms is:

$$\pi = (1 - \tau_E)\, y^O + (1 - o \cdot t \cdot \tau_F)\, y^U - w \cdot l - (r + \delta) \cdot k \tag{A.2}$$

where $\tau_E \equiv \tau_F(\xi + p \cdot s(1-\xi))$ and $\rho \cdot k = k^O, (1-\rho) \cdot k = k^U, \mu \cdot l = l^O, (1-\mu) \cdot l = l^U, k^O + k^U = k$ y $l^O + l^U = l$.

Under the assumption of competitive markets, a firm takes the wage w, r capital income and job level l as given, and maximizes its profit by equating the marginal productivity of capital to its rent. That is:

$$r^* = y_k^O\,(1 - \tau_E) + y_k^U\,(1 - o \cdot t \cdot \tau_F) - \delta \tag{A.3}$$

Given that the assumption of competitive markets should ensure the zero profit condition. The salary must equal the marginal product of labor to capital level of equation (A.3). That is:

$$w^* = \frac{(1 - \tau_E)\left(y^0 - k \cdot y_k^O\right) + (1 - o \cdot t \cdot \tau_F)\left(y^U - k \cdot y_k^U\right)}{l} \tag{A.4}$$

Where y_k^j represents the marginal productivity of capital in sector j. In addition, firms choose to declare a proportion of their income ξ^* from the condition:

$$\frac{\partial \pi}{\partial \xi} = 0 \tag{A.5}$$

Finally, firms determine the proportion of labor and capital will use with each production technology. This means that the parameters ρ and μ are determined endogenously. For this, the firm solves the following conditions:

$$\frac{\partial \pi}{\partial \rho} = 0 \qquad \frac{\partial \pi}{\partial \mu} = 0 \tag{A.6}$$

In particular, the following functional forms are defined for production functions:

$$y^O = A\,(k\rho)^{\alpha}\,(\mu l)^{\theta}\,(\upsilon g)^{1-\alpha-\theta} \tag{A.7}$$

$$y^U = M\,((1-\rho) \cdot k)^{\alpha}\,((1-\mu) \cdot l)^{\theta}\,(\varphi g)^{1-\alpha-\theta} \tag{A.8}$$

Furthermore, we must take into account the following constraints:$0 < \rho < 1, 0 < \mu < 1, 0 < \theta < 1, 0 < \alpha < 1, \alpha + \theta < 1, or < \nu, \varphi < 1$ and $\mu < \varphi$. The last inequality reflects the intensity of the positive externality of government spending on the production function is lower in the underground sector than in the formal sector.

Finally, the relevant probabilities of capturing to the firm p and o can be considered in terms of regulatory quality q. These assumptions are consistent with the empirical evidence that suggests that the detection probability and statutory penalties are not significant to explain the size of the informal economy.

$$p = \frac{1}{1 + e^{-(T_1 + T_2 \cdot q + T_3 \cdot \xi + T_4 \cdot \tau_F)}} \tag{A.9}$$

$$o = \frac{1}{1 + e^{-(W_1 + W_2 \cdot q)}} \tag{A.10}$$

In addition, the regulatory quality index is bounded in the interval (0,1), where higher values represent better regulatory quality. Besides, regulatory quality would positively depend on a proportion ω of government spending g. The last couple of features are reflected in the following specifications[14]:

$$q_t = \frac{1}{1 + e^{-(\widetilde{q}_t)}} \tag{A.11}$$

$$\widetilde{q}_t = \overline{q}_t + H\left(\omega \cdot g_t\right) \tag{A.12}$$

The variable $\bar{q}_t$ can be understood as an autonomous level of regulatory quality, and encompass all the elements of regulatory quality which are not directly affected by government spending. It can be understood that this variable is associated with cultural elements such as morality or tax consciousness and acceptance of corruption, for example. Empirically, we consider it as a standardized variable.

A.2 The Problem of households, workers and investors.

Consumers seek to maximize their expect utility (U) which in this case depends on consumption (c) and the valuation of leisure ($1 - l$) discounted by a factor β to an infinite period of time:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, l_t\right) \tag{A.13}$$

Households use the income (after taxes) they do not consume to accumulate assets (a), according to the following rule of accumulation that takes into account a tax aliquot τ_L proportional to the wage.

[14] The relative weight of the effect of level of self-regulatory quality and government spending on regulatory quality is reflected in the parameter H

$$a_{t+1} - a_t = (1 - \tau_L) w_t \mu_t l_t + (1 - o \cdot t \cdot \tau_L) w_t (1 - \mu_t) l_t + r_t \cdot a_t - c_t \tag{A.14}$$

Households seek to maximize their utility derived from discounted flow of consumption and labor. According to Bellman Optimality Principle (1957), the problem can be expressed as:

$$V(a_t) = \max \{U(c_t, l_t) + \beta \cdot E[V(a_{t+1})]\} \tag{A.15}$$

Solving the problem we can find the first order conditions that maximize the utility from the choice of optimal consumption and employment. These conditions are presented in equations (A.16) and (A.17):

$$1 = E_t \left[\beta \frac{U_{c,t+1}}{U_{c,t}} (1 + r_{t+1}) \right] \tag{A.16}$$

$$1 = E_t \left[\frac{U_{l,t}}{U_{c,t} (1 - \tau_L) w_t \cdot \mu_t + (1 - o \cdot t \cdot \tau_L) w_t (1 - \mu_t)} \right] \tag{A.17}$$

Since families are the owners of assets, it is possible to even $a_t = k_t$ in (A.14), then, including relations in equations (A.3) and (A.4) and the identity $y_t = y_t^O + y_t^U$ we can reach the following expression, which is simply the resource constraint of the economy:

$$k_{t+1} - k_t = y_t - g_t - k_t \cdot \delta - c_t \tag{A.18}$$

In particular, we define a utility function with a coefficient σ of constant relative risk aversion (CRRA) separable in consumption and leisure $(1 - l)$:

$$U(c_t, l_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + h \frac{(1 - l_t)^{1-\eta}}{1 - \eta} \tag{A.19}$$

Where $h \geq 0$ and $1/\eta$ represents the elasticity of intertemporal substitution of leisure.

A.3 Government Restriction.

The government collects taxes based on output and labor. Furthermore, there are revenues related to the penalties charged when tax evaders or informal producers are caught.

From the behavior of firms is straightforward to define the revenue from activities within the formal sector as:

$$R^O = Y^O \cdot \tau_F (\xi + p \cdot s \cdot (1 - \xi)) + w \cdot L \cdot \mu \cdot \tau_L \tag{A.20}$$

Using the identity $\tau_E \equiv \tau_F(\xi + p \cdot s(1-\xi))$ we can express (A.20) as:

$$R^O = Y^O \cdot \tau_E + w \cdot L \cdot \mu \cdot \tau_L \tag{A.21}$$

While, when the informal sector is discovered the revenue generated equals to:

$$R^U = o \cdot t \cdot Y^U \cdot \tau_F + o \cdot t \cdot L \cdot (1-\mu) \cdot w \cdot \tau_L \tag{A.22}$$

With these definitions we can specify a combined budget constraint of the government as follows:

$$Y^O \tau_E + w \cdot L \cdot \mu \cdot \tau_L + Y^U \cdot o \cdot t \cdot \tau_F + o \cdot t \cdot L \cdot (1-\mu) \cdot w \cdot \tau_L \tag{A.23}$$

$$\max(\upsilon, \varphi) \cdot G + \omega \cdot G + (1 - \max(\upsilon, \varphi) - \omega)\, G = G = R^O + R^U \tag{A.24}$$

The left side of the last restriction represents that government spending might be distributed between: i) goods that are useful in the production of goods, ii) in resources that improves quality regulatory and revenue, and iii) government expenses that not belong to the group i) or ii), this last one term considers public spending which does not generate benefits for society. When simulations are performed we assume that $(1 - max(\upsilon, \varphi) - \omega) > 0$ that means there is no a binding constraint between allocating resources that benefit the productive sector and allocate resources which benefit the level of regulatory quality.

B Related Studies

The following table is a summary of studies measuring the informal economy using different methodologies, applied to different countries and different periods of time.

Country	Author	Methodology	Años	Results
Africa, Asia, LA, Central Europe countries, ex URSS and OECD members	Schneider and Enste (2000)	Calculations based on physical inputs, cash demand and modeling approaches.	Averaga from 1990 to 1993	Measuring informal economy in developing, transition and developed countries.
Portugal	Dell'Anno, Roberto (2006)	Multiple Indicators and Multiple Causes Model (MIMIC)	1977 - 2004	Measuring Portugal informal economy and policy recommendations to adopt.
Canadá	Tedds, Lindsay (2005)	Multiple Indicators and Multiple Causes Model (MIMIC)	1976 - 2001	MIMIC model application to measure the informal economy in Canada over time.
Perú	Hernández, Manuel and De la Roca, Jorge (2006)	Discrepancies in level of consumption among groups - from underreporting.	2000	Right and proper measurement of informal economy and the share of informal PEA in Peru.
Ex URSS, Central and Eastern Europe	Schneider, Friedrich (2002)	Cash Demand and Model MIMIC approach.	1998 - 1999 and 2001 - 2002	Factors that cause an increase in the informal economy.
Italia	Dell'Anno, Roberto (2003)	Approach using structural equations and MIMIC model.	1962 - 2000	Utilization of two methodologies to measure informal economy in Italy. Compare and contrast.
145 countries	Schneider, Friedrich	Direct, indirect and Modelistic methods - depending on the characteristics of each country.	1999 - 2003	The increase in tax rates and social security contributions, combined with the growing labor market regulations, boost growth of the underground economy.
LA and the Caribbean countries	Vuletin, Guillermo (2008)	Utilization of Modelistic approach.	Early 2000	Taxes system Strengthening and regulation, along with high inflation and a predominance of agriculture in production, are key factors in measuring informal economy size.
México	Brambila Macias, Jose (2008)	Error Correction Model	1970 - 2006	It is found a positive correlation bewteen informal sector and economic growth in the long-run.
Brasil	Reza Arabsheibani, G and Carneiro, Francisco (2006)	Three approaches are used: employment contract registers, social security registers and employee caracterization in Brasil using survey households.	1992 - 2001	The conditional impact of particular factors changes according to the methodology used to measure informality. For this reason, definitions of informality based on occupation and employment size seem to be the most arbitrary measures but conceptually substantiated.