Impossibility of interpersonal social identity diversification under binary preferences

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Abstract

Diverse identities, some socially shared, arise from a person's affiliation with multiple overlapping communities, which are non-disjoint subsets of persons in society. I prove that identification of each individual with binary preferences or their utility function representation, commonplace in economic theory, implies the impossibility of social-identity diversification of persons. Therefore, if the goal is to explain injustices based on social identity distinctions such as racial discrimination, the conceptual reach of economic theory needs extension. I propose a generalization by assigning non-binary preferences to each individual player to achieve endogenous social diversification, to potentially serve as a basis for explaining discrimination. (100 words)

Keywords: justice, social identity, discrimination, race, gender, non-binariness, maximization

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1. Introduction

In contemporary economic theory – in general equilibrium or game-theoretic form – each person in society or player in a game is characterized by a ranking relation of preference that satisfies the *property of binariness*.[1] In addition, Amartya Sen (1994a) points out that there is an implicit restriction in economic theory that for a decision maker, the *instrumental value of behavioral action norms is zero* in promoting or eroding individual wellbeing. Action norms, which exert their influence by deliberately restricting individual "instrumental possibilities" of persons who share that social identity, have the consequence of effectively restricting the set of actions considered socially acceptable in the community to which the person belongs, rendering acceptable actions a proper subset of the externally-feasible set, which is independent of community action norms. In this paper I argue that there is a compelling case for dropping both assumptions: binariness of preferences *and* zero instrumental value of restrictive action norms, at the individual level, so as to increase the scope and reach of economic theory. I do so, and explore some of the consequences of such a generalization for economic theory.[2] The rewards, as one would expect, are bountiful.

Alternative approaches to characterizing a person as a decision maker in economic theory can be subjected to informational analysis, by examining each approach in terms of the types of information that it admits and the types it excludes. Naturally, the *stronger* the restrictions placed on the characterization of a person, the greater is the information rendered *inadmissible* in describing a person. In turn, *narrower* will be the variability of the characteristics of any given person, and thus *greater* the extent of *homogeneity* across distinct persons. This binariness restriction on preference is so parasitic on information that it leaves very little wriggle room for exhibiting variety of traits in any given person. As expected, the entailed diversity of *distinctive traits* across persons is rendered severely limited.

In fact, I prove that if each person in society is identified with a ranking relation of preference that satisfies the property of binariness then social identity diversification of two or more individuals is impossible. This impossibility theorem renders binary-preference based economic theory, including every theory that identifies an individual with a utility function, quite

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[1] It could represent 'revealed choice behavior' or 'desire fulfillment' as payoff or wellbeing. I take up this matter later.

[2] I have nothing to add in this paper about "production units" in the economic theory contained in Arrow and Debreu (1954), nor about aggregate supply of each commodity, both of which I take on board exactly as in their work. My only innovation consists in characterizing their "consumption units" with a more general preference structure.

inadequate for examining many commonly observed forms of individual and group behavior in which interpersonal social-identity distinctions are material to the phenomena, including injustices such as racial, gender or ethnic discrimination, as discussed in Basu (2010), Akerlof and Kranton (2000) and Becker (1957), among others.[3] These analysts identify each person with a utility function, which entails binary individual preferences, and by the force of my impossibility result, it banishes social-identity diversification of persons, *inter alia* rendering discrimination on the basis of social identity impossible to occur in their conceptions of an economy.

If my claim is correct, it must be the case that in the aforementioned analyses, diversity of social identities is *exogenously imposed* on persons who, in fact, have *endogenously homogenous* identities. This is because the binariness restriction serves to purge the model of all information pertinent to the conceptual reach and terminological scope of social identity diversification. If everyone has a solitary, homogenous identity, it is not clear who exactly is at the receiving end of social-identity based injustice and who is accorded preferential treatment, except by an *ad hoc* assertion to that effect by the theorist.[4] In fact, in the context of sum-ranking utilitarianism, Sen and Williams (1982, p.4) claim that "persons do not count as individuals in this any more than individual petrol tanks do in an analysis of the national consumption of petroleum." Hence, if the object of investigation is social-identity diversification and injustices based thereon, escape from binariness of personal preferences is necessary.

Without endogenous differentiability of persons by social identity, all persons have a unique, solitary identity: if one is a single, white female in her thirties with two children in school, she lives in the United States and works full-time at a local diner at the prevailing minimum wage rate, then everybody else in society is a replica of her in terms of *all* these marks of social identity, except her income; not even two creatures with socially-distinct identities such as male and female can populate a society if everyone's preferences are binary. Discrimination

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[3] When I refer to a personal utility function, I refer strictly to the *utility indicator function* as part of rational choice theory in Arrow and Debreu (1954). This function $u = f(x)$ is a real-valued numerical representation of individual preferences, and it exists if personal preferences are represented by (*i*) a binary ranking relation R that stands for "at least as good as", it completely orders the person's feasible set S of mutually exclusive alternatives (is (*ii*) reflexive, (*iii*) transitive and (*iv*) complete), is (*v*) continuous, and if (*vi*) S is bounded from below, rendering $f(x)$ also continuous and bounded from below. Such a real valued, continuous function bounded from below qualifies to be called a utility function if and only if $\forall x, y \in S: ((xRy) \leftrightarrow [f(x) \geq f(y)]) \& ((xPy) \leftrightarrow [f(x) > f(y)])$, where P is the asymmetric part of R, and I is to be treated as the symmetric part of R.

[4] This critique applies to the work reported in Akerlof (1976), Stiglitz (1974), Phelps (1972), Spence (1974), and Basu (2005), among others, regardless of whether this is done in general equilibrium or game theoretic models.

by social identity is naturally impossible. Such a group of creatures may be many things, but a human society, as we know it, is not one of them. We are much more socially diverse.[5]

Some variation among persons is admissible: one person is distinct form another if, and only if, one person's ranking of at least a pair of alternatives is the negation of the other's ranking of the same pair of alternatives. No more diversity across distinct persons or players is admissible, however, based on preferences. That is the extent to which binariness purges information on *all other* criteria, such as social identities, on which persons or players could have differed in the model. There can thus be interpersonal differences among decision makers in terms of (i) distinct incomes or wealth holdings, or (ii) distinct environments of others' conjectures about specific persons,[6] or (iii) different information sets, all of which are external to preferences, or (iv) distinct preferences, with distinct rankings of at least a pair of alternatives (including due to differences in risk aversion). However, in terms of social identity, there cannot be any differences across persons.[7] Under the force of the binariness restriction (or of a utility function representation of binary preferences), as noted above, all persons are rendered completely homogenous in terms of having a solitary identity.[8]

Characterizing each individual by using greater information than is permitted by the binariness restriction is necessary to be able to distinguish between persons by their distinct, though possibly shared, social identities also. What is needed is an informational broadening of the conceptual foundations of behavioral economic theory to increase the reach of theory beyond the limits of behavioral phenomena it is at present capable of apprehending. One way of achieving this is to consider a society with a finite number of persons, as in Arrow and Debreu (1954), but by replacing their binariness-based preference structure by Kanger-Sen non-binary individual preferences, as in Sen (1994b).[9] The resultant model, with two additional restrictions, that preferences are reflexive and transitive, is a sufficiently rich framework – that has additional conceptual flexibility and vocabulary to go with it – to be able to *also* permit a coherent discussion of social identity differentiation and conflict or cooperation among individuals on that

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[5] In fact, it is not altogether clear that social norms constitute a well-defined concept for such a group of asocial creatures.

[6] Basu (2010) relies heavily on this feature of distinction among persons.

[7] In Basu's (2010) theory there can also be interpersonal differences in productivity, but more on that presently.

[8] The game sometimes called 'the battle of the sexes' is a misnomer; it is, in fact, a homogenous-sex battle.

[9] It is crucial to note that while binariness has a very precise definition, non-binariness can take a multitude of forms, and not all forms of non-binary individual preferences contain sufficient information to exhibit endogenous social identity diversification, on which more in Sections 3 and 5 presently.

basis. From this model, which is a generalization of the Arrow-Debreu model, new insights related to multiple identities of socially differentiated persons can be obtained *in addition* to the understanding already gained from the formal general equilibrium and game theory models with social-identity invariant individuals, including applications of these models to international trade, public economics, labor economics, industrial organizational structures, and so on.

The transition from binary preferences to Kanger-Sen non-binary preferences lets sufficient information back into the model to permit endogenous social identity diversification of individual persons or players, and more, including the possibility of multiplicity of motivations, although the existence of a utility function is a casualty of non-binariness.[10] This achieves a theoretical extension that characterizes an economy as a collection of individuals who have *distinct social identities*, who engage in volitional acts of choice mediated via *personal well-being maximization* with deliberate *restriction of individual instrumentalities* by social norms.[11]

Social norms are specific context-dependent rules of behavior (particular code of conduct) to which persons are obligated to conform, sometimes to some significant extent, because they are members of a community (subset of persons) in society. A person's affiliation with multiple, overlapping communities is precisely what gives that person a distinctive *set* of marks of social identification, which, taken collectively, define the person's social identity. Social identity gives a person a *social character* in terms of traits shared with others (e.g., ethnicity, gender), in addition to the *individual character* based on traits unique to a person (e.g., fingerprints, personal privacy). It is precisely in a society with such persons who have multiple, overlapping social identities that social norms are born, they evolve, and sometimes become fortified into rules of behavior considered acceptable *versus* contemptible conduct, which at times carries a punishment of excommunication or being ostracized from the community whose norms a person has flaunted.[12] Such shared values, which form one part of culture, vary from one society to another.

The economic theory I seek is grounded in firm choice-theoretic foundations, as in Arrow and Debreu. I shift attention to such foundations, to develop a more general economic

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[10] See Section 4 below.

[11] Sen (1994a, p. 385) alludes indirectly to a non-binary approach to preferences by giving the example of "Menu-dependent behavior, [which,]while socially important, would tend to violate" a set-theoretic property known as contraction consistency, and *inter alia*, "binariness" or "rationalizability" of choice, "as they are typically defined (i.e., over sets of outcomes only), would be violated too. With that would go real-valued representation of preference orderings over the outcomes."

[12] There can be a multitude of distinct reason why individual are affiliated with distinct communities.

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theory. In this context, Sen (1994a) makes an important distinction between the "choice-salience" and "well-being" *interpretations* of a person's preference, in "traditional economic theory, or standard game theory."[13] Sen (1994a, p. 386) explains, "A divergence between choice and well-being can easily arise when behavior is influenced by motivation other than the pursuit of one's own interest or welfare (e.g., through a sense of commitment, or respect for duty)."[14] Thus, in addition to the pursuit of one's own welfare, if another motivation also guides individual behavior, such as a self-imposed restraint of

> the Kantian requirements of action morality … "Act always on such a maxim as thou canst at the same time will it to be universal law", then "the people involved … could all end up having more fulfillment of their *unmodified* [personal well-being salient] preferences. … Obviously, confessing oneself is not such an act in the [unrepeated] prisoner's dilemma, since neither prisoner wants that behavior to be "universal." The Smithian-Kantian *self-imposed restraints* differ from constraints given from outside.

> It is, of course, possible to translate the impact of instrumental constraints – even self-imposed ones – into *reconstructed* "objective functions" … But then the interpretation of the "objectives" would have to be correspondingly different … The crucial question is not whether the "maximizing format" can continue to work in the Smithian-Kantian case (it certainly can), but whether there are reasonable arguments that support self-imposition of action norms, with a deliberate restriction of instrumental possibilities.[15]

Sen (1997) demonstrates two equivalent ways of modeling an individual who abides by social norms.[16] One is maximizing a person's *binary* wellbeing-salient preference subject to self-imposed restraints, and the other is maximizing a person's *non-binary* ranking relation of wellbeing-salient preference, which is formally a binary relation that is conditional on the person's background set, as in late Stig Kanger's departure from binariness.[17] Despite the equivalence that Sen establishes, if the object is to gain a better understanding of racial discrimination, tribal conflict, and the like, the binary preference approach gets trumped by the non-binary one, because the former cannot distinguish conceptually between any two persons on the basis of their distinct social identities.

I also prove that if each person is identified with a Kanger-Sen *non-binary* ranking relation (defined on a finite set) of weak preference in the well-being sense, and this relation is

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[13] Sen (1994a, p. 385).

[14] Sen (1994a) points out that multiplicity "of motivations was explicitly accepted" as significant influences driving individual behavior, "by the pioneers of utility theory in economics."

[15] Sen (1994a, p. 387), italics in original.

[16] Certainly, *one* way to proceed is to start with a person's *binary* preferences in the well-being sense, and maximize them subject to self-imposed constraints (e.g., Kant's categorical imperative), which restrict individual instrumental choices, in addition to non-instrumental constraints (such as those of the budget or availability), to arrive at a "reconstructed" personal binary preference that could be legitimately given a choice-salience interpretation because it reflects behavioral outcome.

[17] See Sen (1994b).

reflexive and transitive under all variations of a person's background set, then there exists a social state with endogenous diversification of persons by social identity, and it is Pareto optimal. This result follows immediately from Zorn's lemma. Despite Pareto optimality, however, this social state could be a manifestation of the vilest of social injustices; nasty remediable injustices do not disappear if the concept of a person is endowed with communal characteristics in addition to individualistic.

Further, since individual preferences are assumed to be wellbeing salient, there is no harm in enquiring if a social state, which I show exists, and in which every individual has maximized one's own non-binary preferences, if this state is Pareto optimal. This is because such optimality is itself interpreted in terms of the wellbeing achieved by each person.[18]

Characterizing a person as a maximizer of personal well-being salient Kanger-Sen non-binary ranking relation of weak preference has the added advantage that persons are *also* free to choose among alternative patterns of action-norm-restrictions of behavior, in addition to exhibiting a diversity of social identities. Thus in my model, the instrumental value of behavioral action norms can be zero, but it is also permitted to be positive.

At a practical level, the non-binary preference based model I propose is a framework with sufficient conceptual and informational richness that it can fruitfully be used for examining injustices based on social identity distinctions among individuals, and to investigate the nature of context-dependent implementable remedies.[19] It is not my purpose here to provide a theory of discrimination, but rather to provide a theoretical framework that can serve as a vehicle for explaining such phenomena, and for guidance on this specific matter, I rely mainly on Arrow (1998) and Basu (2010).[20]

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[18] By contrast, if the existence of a competitive general equilibrium is seen in terms of behavior-salient "revealed" preferences, then it is deeply problematic to juxtapose it with a social state called Pareto optimal, in which individual preferences are understood to have wellbeing-salient content.

[19] This paper makes no innovation on the production side of the model of an economy. Associated with command and accumulation of resources, production-sector association by social identity could be an enlightening issue to address. In India, for instance, members of certain caste tend to specialize more in business, statistically, while those of another persuasion adhere to more austere traditions such as teaching or preaching; the warlords would have land-holdings and dealings with labor.

[20] Arrow (1998) restrains himself from offering up a theory of discrimination, but asks instead, "What Has Economics to Say about Racial Discrimination?" Arrow (1998, p.93) writes,

> We have clear evidence that blacks were in the past excluded from a significant range of jobs and from purchase of housing and restaurant services. We have very strong evidence that these practices persist in some important measure. I … suggest … that market-based explanations will tend to predict that racial discrimination will be eliminated. Since they are not, we must seek elsewhere for non-market factors

Some analytical issues relating to the binariness of a choice-behavioral maximal function are covered in Section 2. The principal result on the impossibility of social identity diversification under binariness is proved in Section 3. Section 4 deals with some conceptual issues related to Kanger-Sen non-binary preference (in the well-being sense) relation of an individual, including the impossibility of existence of a utility function for non-binary preferences. Section 5 provides a brief, bare-bones approach to non-binary individual preferences in a finite society, and the possibility of social diversity of persons, and presents the main argument. Section 6 contains an existence and optimality result of a *social interaction outcome*, a generalization of the concept of general equilibrium, which may be fraught with injustices. Section 7 illustrates a preliminary examination of racial discrimination in the United States in a conjectural general equilibrium model, and deals with some methodological issues that such an exercise exposes. Section 8 contains some concluding remarks.

2. Binariness of Behavioral Maximal Function or of Wellbeing Ranking Relation

Let X be a universal set, and $\mathcal{K}$ a collection of non-empty subsets of X. In this paper I shall take X to be a finite set.[21] In a standard approach to the property of binariness, it is common to proceed by taking a maximal set as a primitive concept, in the following manner.

Definition 1: A *maximal set of* a set $\mathfrak{R}$ is defined as $\Theta(\mathfrak{R}) \leftrightarrow (\Theta(\mathfrak{R}) \subseteq \mathfrak{R} \ \& \ \exists \mathfrak{R} \in \mathcal{K})$.

Definition 2: A correspondence $\Theta^*: \mathcal{K} \longmapsto \mathcal{K}$ is defined as a *maximal function* over $\mathcal{K}$ if, and only if, $\Theta^*(\mathfrak{R}) = \{x \in \mathfrak{R} \mid \Theta(\mathfrak{R}) \subseteq \mathfrak{R} \ \& \ \Theta(\mathfrak{R}) \neq \emptyset \ \& \ \forall \mathfrak{R} \in \mathcal{K}\}$.

The following conditions are defined for a maximal function $\Theta^*(.)$ over $\mathcal{K}$.

Contraction Consistency (CC): $\forall S \in \mathcal{K} \ \& \ T \in \mathcal{K}: [x \in \Theta(S) \ \& \ x \in T \subseteq S] \longrightarrow x \in \Theta(T)$.[22]

Expansion Consistency (EC)[23]: $(\forall S_j \in \mathcal{K}_0 \subseteq \mathcal{K}): x \in W \longrightarrow x \in \Theta(V)$, where

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influencing economic behavior. The concepts of direct social interaction and networks seem to be good places to start.

Networks as communities with which persons are affiliated, and social interaction in the forms of interpersonal interaction among socially diverse creatures, is precisely what I attempt to achieve in this paper.

[21] This is not an essential assumption, but it is made only to obviate repeated additional qualifications that could detract from clarity in communicating the central points that I wish to drive home.

[22] Equivalently, $\forall S, T \in \mathcal{K}: \Theta(S \cup T) \subseteq [\Theta(S) \cup \Theta(T)]$ is the contraction consistency property of the maximal function.

[23] For *pairs* of sets $\forall S, T \in \mathcal{K}, [\Theta(S) \cap \Theta(T)] \subseteq \Theta(S \cup T)$ is Property γ^* in Sen (1971, p. 314), except that I have defined it for behavioral *maximal* sets, instead of his treatment in terms of *choice* sets. Property EC is the finite closure of Property γ^* in terms of maximal sets.

$$W = \bigcap_{S_j \in \mathcal{K}_0} \Theta(S_j) \ \& \ V = \bigcup_{S_j \in \mathcal{K}_0} S_j.$$

Venn diagram representations of CC and EC can be particularly illuminating.
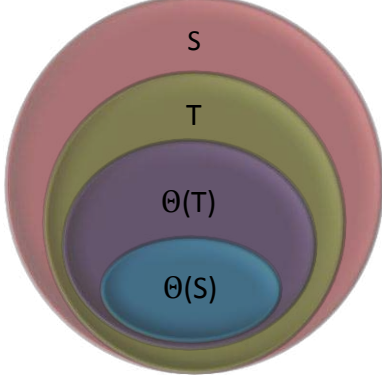

Figure 1
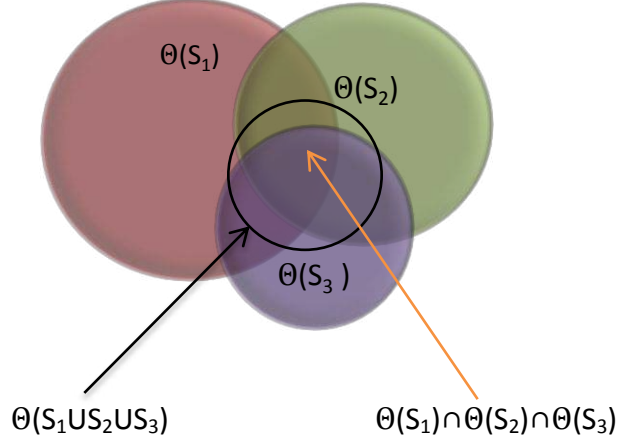Contraction Consistency of $\Theta^*(.)$ over $\mathcal{K}$

Figure 2
Expansion Consistency of $\Theta^*(.)$ over $\mathcal{K}$

Definition 3: A maximal function $\Theta^*(.)$ over $\mathcal{K}$ is *binary* if, and only if, $\Theta^*(\mathfrak{R}) = \Theta^B(\mathfrak{R}, \mathcal{F})$ &
$\Theta^B(\mathfrak{R}, \mathcal{F}) = \{x \in \mathfrak{R} \mid \exists \mathcal{F} \subseteq \mathfrak{R}^2 \ \&\sim[\exists y \in \mathfrak{R} \ \& \ y\mathcal{F}^*x] \& \forall \mathfrak{R} \in \mathcal{K}\} \neq \emptyset$,
where $\mathcal{F}^*$ is defined as $x\mathcal{F}^*y \leftrightarrow (x\mathcal{F}y \ \& \sim y\mathcal{F}x)$.

Theorem 2.1: $\Theta^*(.)$ over $\mathcal{K}$ is a *binary* maximal function if, and only if, it satisfies CC and EC.
That is, $\Theta^*(\mathfrak{R}) = \Theta^B(\mathfrak{R}, \mathcal{F})$ if, and only if, CC & EC are true.

Proof: (*Step 1*) Suppose $\Theta^*(.)$ over $\mathcal{K}$ is *binary*. Then $\Theta^*(\mathfrak{R}) = \Theta^B(\mathfrak{R}, \mathcal{F})$, as specified in Definition 3. Let $x \in \Theta^*(.)$ over $\mathcal{K}$. Then $x \in \Theta^B(\mathfrak{R}, \mathcal{F})$ by binariness, so that $\sim(\exists y \in \mathfrak{R}: y\mathcal{F}^*x)\&\forall \mathfrak{R} \in \mathcal{K}$. Also, if $x \in T \ \& \ \forall T \subseteq \mathfrak{R} \ \& \ \forall T \in \mathcal{K}$, then $\sim(\exists y \in T: y\mathcal{F}^*x)\& \forall T \in \mathcal{K}$, so that $x \in \Theta^B(T, \mathcal{F})$. But by binariness of the maximal function, $\Theta^B(T, \mathcal{F}) = \Theta^*(T)\&\forall T \in \mathcal{K}$. Therefore, $x \in \Theta^*(T)$. Thus binariness of a maximal function implies that it satisfies CC.

(*Step 2*) Suppose $(\forall S_j \in \mathcal{K}_0 \subseteq \mathcal{K})$, the antecedent of EC, $x \in W$, is true. This implies that $x \in \Theta^*(S_j) \ \& \ \forall S_j \in \mathcal{K}_0$. Suppose $\Theta^*(S_j)$ is binary $\forall S_j \in \mathcal{K}_0$. Then $\Theta^*(S_j) = \Theta^B(S_j, \mathcal{F}) \ \& \ \forall S_j \in \mathcal{K}_0$. Also, from Definition 3, $\sim(\exists y \in S_j: y\mathcal{F}^*x)\& \forall S_j \in \mathcal{K}_0$, which, in turn, implies that $\sim(\exists y \in V: y\mathcal{F}^*x)$, so that $x \in \Theta^B(V, \mathcal{F})$. But by binariness of the maximal function, $\Theta^B(V, \mathcal{F}) = \Theta^*(V)$. Therefore, $x \in \Theta^*(V)$, which is the consequent of EC. This proves that a binary maximal function $\Theta^*(.)$ over $\mathcal{K}_0$ satisfies EC.

(*Step 3*) To prove the converse, let $x \in \Theta^*(V)$. Notice that $S_j \subseteq V$. If $x \in S_j$, then CC implies that $x \in \Theta^*(S_j)$. Further, if x is in every set $\Theta^*(S_j) \ \& \forall j$, it must belong to their intersection. Thus, $x \in W$. So, under CC, $\Theta^*(V) \subseteq W$. Next, let $x \in W$. This and EC together imply that $x \in \Theta^*(V)$. So, under EC, $W \subseteq \Theta^*(V)$. Therefore, CC and EC imply that $\Theta^*(V) = W$. To complete the proof of the converse, we have to show that $W = \Theta^B(V, \mathcal{F})$. This follows from

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$\Theta^*(V) = W$ under CC and EC, together with definition 3, according to which $\Theta^*(V) = \Theta^B(V, \mathcal{F})$. Thus, $W(.)$ is a binary maximal function over $\mathcal{K}$, under CC and EC. ♦

Theorem 2.1 is a technical result that deals with purely logical relationships among sets, their subsets, collections of sets, and correspondences. In the triple $\langle X, \mathcal{K}, \Theta^B(., \mathcal{F}) \rangle$, it is only specified that X is a universal set, that $\mathcal{K}$ is a collection of non-empty subsets of X, that $\Theta^B(., \mathcal{F})$ over $\mathcal{K}$ is a binary maximal function, and that $\mathcal{F} \subseteq \mathfrak{R}^2$ for $\exists \mathfrak{R} \in \mathcal{K}$ is a *binary ranking relation*. That is all there is to it.[24]

Undoubtedly, $\Theta^*(.) = \Theta^B(., \mathcal{F})$ over $\mathcal{K}$ entails certain restrictions on the relationships among the elements of $\langle X, \mathcal{K}, \Theta^B(., \mathcal{F}) \rangle$, but these restrictions are of a context-independent character. If, however, a *contextual interpretation* is attributed to any single element of $\langle X, \mathcal{K}, \Theta^B(., \mathcal{F}) \rangle$, it permits as admissible some interpretations of the remaining elements, but renders inadmissible certain other interpretations of those elements. Inadequate appreciation of this issue can be, and has been the cause of considerable confusion in choice theory. I shall return to this issue later, after some additional conceptual issues are sorted out.

While the methodological approach described above takes maximal set and maximal function as primitive concepts, and derives a binary ranking relation from it, there is another approach to binariness that takes a binary ranking relation as a primitive concept, and derives a maximal function from it. This approach is also purely technical, devoid of any intrinsic contextual interpretation. I turn to this next. To this end, again let X be a universal set, and $\mathcal{K}$ a collection of non-empty subsets of X.

Definition 4: A *binary ranking relation* (e.g. $\geq$, or generically) $\mathcal{H}$ defined on a non-empty set $\mathfrak{R}$ is: $\mathcal{H} \subseteq \mathfrak{R}^2 \leftrightarrow (\exists x \in \mathfrak{R} \ \& \ \exists y \in \mathfrak{R} \ \& \exists \mathfrak{R} \in \mathcal{K} \ \& \ x\mathcal{H}y)$.

Definition 5: The set of all maximal elements of a given, non-empty set $\mathfrak{R}$ based on the binary ranking relation $\mathcal{H}$ is called the $\mathcal{H}$-*maximal set* over $\mathfrak{R}$ and is denoted:
$\mathcal{M}(\mathfrak{R}, \mathcal{H}) = \{x \in \mathfrak{R} \mid \sim[\exists y \in \mathfrak{R} \ \& \ y\mathcal{H}^*x] \& \exists \mathfrak{R} \in \mathcal{K}\}$, where $\mathcal{H}^*$ is $x\mathcal{H}^*y \leftrightarrow (x\mathcal{H}y \ \& \ \sim y\mathcal{H}x)$.

Definition 6: A correspondence $\mathcal{M}^*: \mathcal{K} \longmapsto \mathcal{K}$ defined as
$\mathcal{M}^*(\mathfrak{R}, \mathcal{H}) = \{x \in \mathfrak{R} \mid \sim[\exists y \in \mathfrak{R} \ \& \ y\mathcal{H}^*x] \ \& \forall \mathfrak{R} \in \mathcal{K}\} \neq \emptyset$ is called a $\mathcal{H}$-*maximal function* over $\mathcal{K}$.

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[24] For a treatment in terms of a choice function, instead of maximal function, see Sen (1971).

Remark 1:

(a) Both $\Theta(\mathfrak{R})$ and $\mathcal{M}(\mathfrak{R}, \mathcal{H})$ can be empty, but $\Theta^*(\mathfrak{R})$ and $\mathcal{M}^*(\mathfrak{R}, \mathcal{H})$ are non-empty.

(b) In general, $\Theta(\mathfrak{R}) \neq \mathcal{M}(\mathfrak{R}, \mathcal{H})$ and $\Theta^*(\mathfrak{R}) \neq \mathcal{M}^*(\mathfrak{R}, \mathcal{H})$, because $\Theta(\mathfrak{R})$ and $\Theta^*(.)$ over $\mathcal{K}$ are, in general, *not* binary, whereas both $\mathcal{M}(\mathfrak{R}, \mathcal{H})$ and $\mathcal{M}^*(\mathfrak{R}, \mathcal{H})$ are binary. However, in the special case that $\Theta^*(.) = \Theta^B(., \mathcal{F})$ over $\mathcal{K}$, we have the following result.

Lemma 2.1: $\forall \mathfrak{R} \in \mathcal{K}: \Theta^B(\mathfrak{R}, \mathcal{F}) = \mathcal{M}^*(\mathfrak{R}, \mathcal{H}) \leftrightarrow \mathcal{F} = \mathcal{H}$.

Proof: Follows from Definitions $1 - 3$ and $4 - 6$.

2.1 Interpretations of Binariness

     Lemma 2.1 is of crucial importance in clearing up some confusion in the part of choice theory that deals with the matter of revealed preference. Essentially this means that $\mathcal{F}$ and $\mathcal{H}$ are two binary ranking relations (as construed in relational logic) defined on the same collection of non-empty sets, and both rank elements in any given set exactly in the *same* order or partial order. That's all. And no interpretation associated with human beings, or associated with anything else, is ascribed to these two, indistinguishable binary relations, if the antecedent of Lemma 2.1 is true. This is because, seen purely as results in logic, $\Theta^B(., \mathcal{F})$ over $\mathcal{K}$ and $\mathcal{M}^*(., \mathcal{H})$ over $\mathcal{K}$ are equivalent in *form*, and form is the only essential feature of logical propositions; they are independent of contextual reference. Substantive issues of epistemic coherence arise however, once contextual interpretations of the concepts are entertained. This is because equivalence in form does not, by itself, imply equivalence in content.

     To see this, let X be interpreted as a set of alternatives of choice for a person. Then, definitions $1 - 3$ imply that an admissible interpretation is (*i*) that $\Theta^B(., \mathcal{F})$ over $\mathcal{K}$ is a binary maximal function that represents the outcome of choice conduct by a person, and (*ii-a*) that $\mathcal{F}$ is an *implicitly* defined binary ranking relation, which a person can be *hypothetically* seen as maximizing, to arrive at the behavioral outcome $\Theta^B(., \mathcal{F})$ over $\mathcal{K}$ – this, in Sen's (1994a) terminology, would be called the *choice-salience interpretation* of the binary ranking relation $\mathcal{F}$. However! Taken together with interpretation (*i*), it is *not* an admissible interpretation of $x\mathcal{F}y$ to regard it as (*ii-b*) "the person likes x at least as much as y". Nothing has been said about a person's yearnings, desires or wellbeing.

     Based on definitions $4 - 6$, instead of definitions $1 - 3$ above, it is perfectly legitimate to impose on $\mathcal{H}$ the interpretation that it (1) is a binary ranking relation of weak *preference* construed as the person "likes x at least as much as y", which, in Sen's (1994a) terminology,

would be called the *wellbeing interpretation* of the binary ranking relation $\mathcal{H}$. Also, $\mathcal{M}^*(.,\mathcal{H})$ over $\mathcal{K}$ can be legitimately regarded (2a) as a binary $\mathcal{H}$-*maximal function* over $\mathcal{K}$ of a person's choice based on maximizing preference, $\mathcal{H}$. But, taken together with (1), it is *not* an admissible interpretation of $\mathcal{M}^*(.,\mathcal{H})$ over $\mathcal{K}$ to consider it (2b) as representing the outcome of a person's choice *behavior*; instead, it merely represents a person's *hypothetical* choice behavior if it were driven solely by the impulse to maximize the personal wellbeing preference relation $\mathcal{H}$. Therefore, we have the following result.

Theorem 2.2: It is impossible, in general, to infer a binary ranking relation that represents personal wellbeing from a binary ranking relation that represents the outcome of personal behavior, and conversely.

Proof: Follows from Lemma 2.1.

A discussion of the underlying concepts is more illuminating that the shortest proof. Essentially, in terms of interpretation, $\Theta^B(.,\mathcal{F})$ over $\mathcal{K}$ represents choice behavioral outcome regardless of the *cause* of such conduct, thereby ascribing to $\mathcal{F}$ a behavior-salient interpretation. Instead, $\mathcal{H}$ refers to wellbeing-salient-preference relation, and $\mathcal{M}^*(.,\mathcal{H})$ over $\mathcal{K}$ refers to the outcome of $\mathcal{H}$ maximization as the cause, irrespective of the behavioral outcome. The distinction between choice-based-on-preference $\mathcal{M}^*(.,\mathcal{H})$ over $\mathcal{K}$ and behavioral outcome $\Theta^B(.,\mathcal{F})$ over $\mathcal{K}$ arises from additional, possibly divergent, drives that a person could have, such as commitment to adhere to social norms or to react so certain choice situations instinctively with fight-or-flight conduct. These additional human impulses could, in principle, intervene as *causal influences* to lead a person's choice based solely on preference away from the outcome of a person's behavior. Therefore, there is one interpretation, viz., that of *motivation*, that is imposed on $\mathcal{H}$ and a distinct interpretation, viz., that of *behavioral-outcome* explanation, entailed for $\mathcal{F}$.

Just because both $\mathcal{F}$ and $\mathcal{H}$ are binary ranking relations in form, it does not follow that they are equivalent in content in the contextual domain of conduct entailing personal well-being maximization. If the equivalence of $\mathcal{F}$ and $\mathcal{H}$ is sought, it would have to be asserted as an assumption, i.e., as an *imposed* condition, not one that is *entailed*.[25] Indeed, imposing the restriction of binariness on the outcome of maximizing behavior fails to reveal a person's motivation that underlies such behavior. The literature that regards $\Theta^B(.,F)$ over $\mathcal{K}$ as the
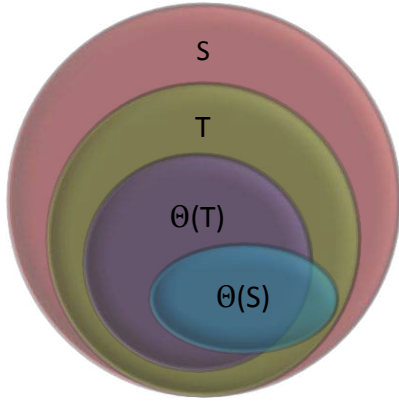
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[25] Sen (1993) has addressed this distinction with great clarity and force. See Footnote 28 below.

outcome of behavior, and simultaneously interprets the binary ranking relation $\mathcal{F}$ as a person's preference construed as wellbeing, which is allegedly "revealed" by the outcome of behavior, suffers from the problem of confounding equivalence in form with equivalence in content. In fact, $\mathcal{F}$ is completely bereft of any reference to a person's *motivation* that might possibly drive a person's acts or deeds. Individual motivations drive personal behavior: choice behavior refers to "*what* happened" but motivation refers to "*why* it happened".
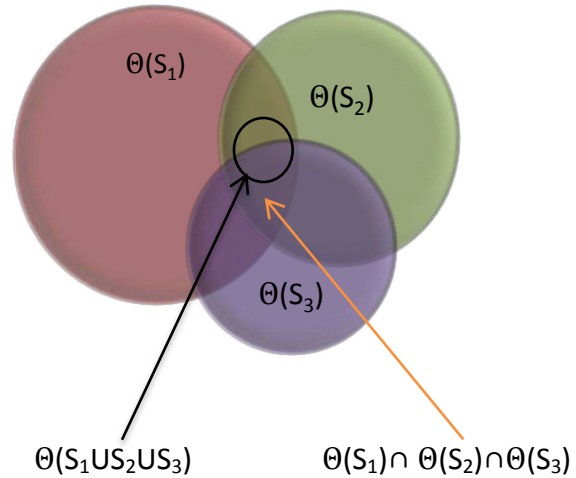
Moreover, as I have tried to argue, a very strong restriction – that the instrumental value of behavioral action norms is zero in promoting (or eroding) individual wellbeing, which is implicit in economic theory, defines away any distinction between the two interpretations. I permit the possibility of positive instrumental values to be attached to distinct sets of social norms – which deliberately restricts individual instrumental possibilities – sometimes to the benefit of human beings in society, but far too often by inflicting dreadful injustices on persons with specific social identities. I return to this issue later.

Notice also that in the context of the binary maximal function $\Theta^B(.\,,\mathcal{F})$ over $\mathcal{K}$, the issue is not why contraction consistency would be violated by human behavior. Rather, the pertinent question is: why must human behavior abide by contraction consistency? For example, $\{x\} = \Theta^*(\{x, y\})$ and $\{y\} = \Theta^*(\{x, y, z\})$ could be perfectly rational choice behavior, but violates contraction consistency.[26] After all, if the set T is a subset of S, why should the maximal set of T be a superset of S, even if x is in both T and S? If, however, due to contraction consistency the maximal set of T is a superset of S, this certainly constitutes a restriction, though, I would argue, an exceedingly demanding one, and *without* specifying the context there is absolutely no reason why it should be met by choice behavior. All that can be asserted is that $\Theta^*(S) \subseteq S \,\&\, \Theta^*(T) \subseteq T$, but even in the case that $T \subseteq S \,\&\, x \in T$, there is no *context-independent* reason why $\Theta^*(S) \subseteq \Theta^*(T)$ must be true. The violation of CC is depicted in Figure 3, and the violation of EC in Figure 4.

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[26] See Sen (1997).

Contraction Consistency violated

Figure 3

Expansion Consistency violated

Figure 4

Indeed it does not take a leap of faith to envision a circumstance illustrated by Figure 3, in which x belongs to S, T is a subset of S, x belongs to T, and x is in $\Theta(S)$, but it is not in $\Theta(T)$.[27] Behavioral maximal sets in Figure 3 violate contraction consistency of the associated maximal function, and thus pertain to a non-binary behavioral maximal function. Also, maximal sets displayed in Figure 4 violate expansion consistency, if $x \in ([\Theta(S_1) \cap \Theta(S_2)] - \Theta(S_3))$, and thus pertain to a non-binary maximal function of choice behavior.[28]

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[27] This would be the case if x is in $\Theta(S) - \Theta(T)$.

[28] Sen (1993, p. 495) writes,

Internal consistency of choice has been a central concept in demand theory, social choice theory, decision theory, behavioral economics, and related fields. … [T]his idea is essentially confused, and there is no way of determining whether a choice function is consistent or not *without* referring to something external to choice behavior (such as objectives, values or norms).'

In the same contribution, Sen (p. 496) goes on to say that he is

'not arguing, in general, against internal correspondences that may be *implied* by the substantive nature of the specific exercise … , but only against imposing such conditions in an *a priori* way as requirements of "internal consistency." There is a methodological difference between *imposed* internal correspondences … and *entailed* internal correspondences … The critique here is concerned solely with the former, not the latter.'

Sen (1993, p. 498-99) further argues that

'the basic difficulty arises from the implicit presumption … that *acts* of choice are, on their own, like *statements* which can contradict, or be consistent with, each other. That diagnosis is deeply problematic. Statements A and *not-A* are contradictory in a way that choosing x from $\{x, y\}$ and y from $\{x, y, z\}$ cannot be. … [Such] choices do not, *in themselves*, entail any such statements.'

Sen's (1993) critique of revealed preference theory, which to date has yet to be rebutted, is thoroughly devastating for the alleged claim that the observation of actual choice behavior, based on some imposed conditions of internal consistency of choice, can reveal the chooser's preferences. The chooser's preferences (or motivations) are, in fact, logically prior, and choice behavior, logically posterior.

2.2 Types of Non-binariness

Contrasted with the precise concept of binariness, non-binariness can take a *multitude* of different forms, and indeed has been accorded multiple conceptions and definitions in the literature.[29] Most discussions of non-binariness in economic theory have, however, tended to focus on a set function $\Theta^*: \mathcal{K} \longmapsto \mathcal{K}$ as a maximal function over $\mathcal{K}$ if, and only if, $\Theta^*(\mathfrak{R}) = \{x \in \mathfrak{R} \mid \Theta(\mathfrak{R}) \subseteq \mathfrak{R} \ \& \ \Theta(\mathfrak{R}) \neq \emptyset \ \& \ \forall \mathfrak{R} \in \mathcal{K}\}$, with such restrictions *imposed* on the function that they are either weaker than the necessary and sufficient conditions CC and EC taken together, or they violate at least one of them. This may be called the *maximal-function* approach to non-binariness.[30] However, as the foregoing discussion should make clear, there cannot be in this literature any genuine reference to wellbeing-salient preferences-based rankings or relations or sets. It is helpful to distinguish the *preference-relational* approach to non binariness, which is the only one I shall use henceforth, that regards R as a binary ranking relation of weak wellbeing-salient preference defined on a set S of alternative actions, and V is a background set on which R depends, so that $R(V)$ is a non-binary ranking relation of well-being-salient preference, and for person i it is given by $R_i(V_i) \ \forall i = 1, \dots n \geq 2$, defined on the set of alternative externally-feasible actions S_i. I return to this issue in Section 4. Interestingly, under non-binariness, even the *formal* correspondence between the two maximal functions breaks down, so that the question of content equivalence cannot arise.

On decision making, Arrow's (1998, p. 94) conception is that

> the individual actors act rationally (that is, by maximizing according to a complete ordering) within the constraints imposed by preferences, technology, and beliefs, and by the institutions which determine how individual actions interact to determine outcomes. Further, the beliefs are themselves formed by some kind of rational process. By economic theory, we mean that in some sense, markets are the central institution in which individual actions interact and that other institutions are of negligible importance.

It is important to note that in contrast with Arrow's conception, in Sen's (1997) formulation of rational choice, a person engages in a volitional act of choice by maximizing an *incomplete*

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[29] For instance, a binary ranking of a set of rankings of a set of alternatives, called a meta-ranking, is also a non-binary ranking of the set of alternatives. But non-binariness of a ranking relation in the strict Kanger-Sen sense is not at all like such a meta-ranking.

[30] See Deb (2012) for a very comprehensive non-binariness approach to social choice theory. The choice-behavioral maximal function approach to non-binariness is, for example, José Alcantud and Carlos Alós-Ferrer (2007), which is a perfectly legitimate exercise in mathematical logic, but this has nothing whatsoever to do with the preference-driven act of choice by a person. Deb also refers to the literature in which the collection of sets $\mathcal{K}$ is such that each set has at least h elements, with $h \geq 3$, as another way of defining non-binary choice-functional approach.

ranking by a *non-binary* personal preference relation that quasi-orders a feasible set. I return to the matter of "the beliefs are themselves formed by some kind of rational process" later.

I do not abandon maximization as a volitional act of choice. Nor do I redefine the set in Arrow and Debreu on which each person's ranking relation is defined. I also retain the reflexivity and transitivity of the ranking relations though I give up the completeness restriction. And, following Sen's (1997) formulation of choice theory, I assign a non-binary personal preference relation to each person in the economy. Sen (1994b and 1997) provides several contexts in which a person maximizes one's own preferences, while simultaneously violating binariness of preferences.[31] The Kanger-Sen non-binary preference ranking relation, in the well-being sense, can accommodate both *zero* instrumental value of action norms, as in traditional economic theory, and *positive* instrumental value in promoting or eroding unmodified personal wellbeing (pay-offs for players), in the economic theory developed in this paper.

3. Impossibility of Social Identity Diversification

An example and a simple theorem and proof follow. Suppose R_J and R_K are binary ranking relations of weak *preference* defined on a finite set S of alternatives with at least two elements. By definition, R_J and R_K are *identical* if, and only if, every pairwise ranking of elements of S is exactly the same under both relations. Conversely, R_J is *distinct* from R_K if, and only if, there exists a pair of elements (x, y) in S that is ranked in a different order under R_J than under R_K. Next,

Step 1: Suppose James is described exclusively by R_J and Kate exclusively by R_K both defined on the same set S.

Step 2: By definition, R_J is identical to R_K if every pair (x, y) in S is ranked in the same order by both R_J and R_K *and only if* it is ranked in the same order by both R_J and R_K.

Step 3: By negation, R_J is distinct from R_K if at least one pair (x, y) in S is ranked differently by R_J than by R_K *and only if* it is ranked differently by R_J than by R_K.

This implies that:

(a) James and Kate are indistinguishable *if* they rank *every* pair (x, y) in S in the same order, *and only if* they rank it in the same order.

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[31] Sen (1997, p. 477-478) mentions (i) reputation and indirect effects, (ii) social commitment and moral imperatives, (iii) direct welfare effects, and (iv) conventional rule following, as possible motivations for such behavior. And, in the same spirit, he also develops a Fruit Passing game with common knowledge of norms (p. 762).

(b) James and Kate are distinct *if* they rank at least one pair (x, y) in S differently, *and only if* they rank it differently.

From (a) and (b) the conclusion is:

(c) A pair-wise ranking difference over at least one pair (x, y) is the *exclusive* difference that is possible between James and Kate.

Suppose the two binary relations of preferences are distinct. Then, can James be interpreted as a dog and Kate as a cat? Or can they be seen as persons of different genders, races, religions, tribes, or as persons suffering from different diseases? No, they cannot. For, these would constitute inadmissible *interpretations* that are external to, and superimposed from outside of a preference-induced choice-theoretic exercise, and are thus prohibited by entailment of the binariness property of individual preferences. This is because the binariness of R_J and R_K renders James and Kate indistinguishable in every respect, as an entailment, *except* solely on the basis of a *pair-wise ranking difference* between them of at least one pair of alternatives. Different species, races or suffering from different diseases, are all *tertiary* considerations that go beyond the intrinsically admissible difference between two pair-wise rankings of at least one pair of alternatives in *S* by James and Kate.

Binariness of individual preference ranking relations completely precludes information regarding such tertiary considerations as social identity diversification of persons. To set the stage for an examination of social identity diversification of individuals, I first prove the following impossibility result.

Theorem 3.1 (Impossibility of Social Diversity): *Defined on the set of personal culmination outcomes or payoffs as the consequences of the volitional act of choice of actions by a person or strategies by a player, if a ranking relation represents individual preferences and it is binary then social identity diversification among individual persons or players is impossible.*

Proof: In a finite society of *n* persons, $|n| \geq 2$, consider two distinct persons, *J* and *K*. Denote the claim that 'Person *J* is identical to (indistinguishable from) Person *K*' $\leftrightarrow J = K$. Similarly, denote to the claim that 'Persons *J* and *K* are distinct' $\leftrightarrow J \neq K$. Since $(J \neq K) \leftrightarrow \sim(J = K)$, the claim that Person *J* is distinct from Person *K* is the *negation* of the claim that Persons *J* and *K* are indistinguishable.

Suppose next that Person *J* is identified exclusively with a binary preference relation R_J, and Person *K* is identified exclusively with a binary preference relation R_K, both defined on the same set *S*. Then, formally,

$$\left(R_J = R_K\right) \leftrightarrow \left([\forall x, y \in S] \& [\exists J, K = 1, \dots, n] : \left[(xR_Jy) \leftrightarrow (xR_Ky)\right] \& [\sim(xR_Jy) \leftrightarrow \sim(xR_Ky)]\right).$$

That is, every pair in S is ranked identically by Persons J and K if, and only if, Persons J and K are indistinguishable. From the negation of the consequent, it follows that the meaning of "Person G is distinct from Person K" is

$$\left(R_J \neq R_K \right) \leftrightarrow ([\exists x, y \in S]\&[\exists J, K = 1, \ldots, n, \ J \neq K]\colon \left[\left(xR_Jy\right) \rightarrow \sim(xR_Ky)\right]).$$

Notice that these are "if, and only if" claims, because they are definitions, and the "only if" part is *not* superfluous. Thus in a binary society, for two persons to be distinct it is *sufficient* that if one person has a specific ranking of a pair of alternatives, then a distinct person's ranking of at least that pair is different. This condition is *also* necessary. Hence in a binary society, any distinction between individuals J and K other than $R_J \neq R_K$ is excluded. ♦

Thus, a difference in the order of ranking of at least one pair of alternatives by two distinct persons is the one and only difference between them. This disallows all *other* sources of variation among persons. Distinct genders, races, tribal affiliations and all other forms of social identity diversification in society, by implication, cannot be a source of variation among persons if all persons are identified with binary preferences.

It is also important to note that the purpose of the work of Arrow and Debreu was to identify a set of conditions that are sufficient for the existence of a Walrasian competitive general equilibrium. This requires that there exist a finite vector of positive prices (in a unit simplex) at which the aggregate excess demand for every commodity is zero. For the aggregation of individual demands for any commodity across consumption units and of aggregate supply of that commodity across production units, the differentiation of persons by social identity is inconsequential – it matters not whether the persons demanding corn are male or female, black or white. Thus, in the interest or parsimony, Arrow and Debreu deliberately assume that each consumption unit is identified with a ranking relation (defined on the set of mutually exclusive alternatives) that satisfies the binariness property.

Given the purpose of Arrow and Debreu's contribution, it would *not* be a legitimate criticism of their work to ask that they should have modeled persons by means of a non-binary ranking relation so as to permit social differentiation in their model, because what they are after simply does not require such diversification. It also *cannot* be legitimate, by the same token, to employ binariness-salient models – whether of the general equilibrium nature or of the game-theoretic kind – to examine social-differentiation based injustices including racial and gender discrimination. Moreover, in economic theory, every person is concerned solely with parametrically given prices in the market, and thus personal decision-making expresses itself as a

person's direct interaction only with the market, not with other persons.[32] Direct social interaction is, therefore, completely absent in economic theory.[33]

4. The Concept of Kanger-Sen Non-binary Preference

My purpose is to develop an economic theory that Arrow seeks, both with regard to (a) possessing rational choice-theoretic foundations at the individual level and (b) in capturing "direct social interaction and networks" for including "non-market factors influencing economic behavior". To this end, I consider a finite society and embed the social identity of each person in a preference maximization framework. In the formulation I use, preferences are non-binary in the Stig Kanger sense: R_i is a binary ranking relation of weak preference defined on a finite set S_i of alternatives, and V_i is the background set of the ith person or player upon which R_i is dependent, so that $R_i(V_i)$ defined on S_i is that person's non-binary ranking relation of weak preference.

Note also that the personal feasible set S_i of mutually exclusive alternatives is very generally construed. However, in particular, it can also be taken to be a finite version of the same set as a consumption unit's feasible set in Arrow and Debreu (1954), *without redefining* the elements of a personal feasible set of culmination outcomes, in keeping with the typical case of the set of payoffs in choice theory, game theory and in general equilibrium theory.[34] Redefining

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[32] Arrow (1998) is clearly headed in this direction when he says (on p. 94),

> The theoretical picture of a market is one of *impersonal* exchange. … There is no particular relation between a supplier and a demander; that is, a supplier is indifferent about supplying one demander or another, and vice versa. … Certainly, employment of labor involves *direct personal relations* between employee and employer (or the latter's agents) as well as among employees. Similarly, credit relations … have typically required *direct personal interaction* between debtor and financial institution. … Let us ask whether a market-based model can broadly satisfy the[se] empirical constraints…. On the usual interpretation, it cannot. If the members of the two races, after adjusting for observable differences in human capital and the like, received different wages or were charged different prices in commodity or credit markets, an arbitrage possibility would be created which would be wiped out by competition.

[33] When the auctioneer announces a finite vector of prices, agents make offers of supplies and demands for goods. If there is positive (negative) excess demand for a commodity in the aggregate, the auctioneer raises (lowers) the price of that commodity. On this see Arrow and Hahn (1971). Once such an iterative process leads to zero excess demands for all commodities, a Walrasian equilibrium is achieved, and only then do agents engage in trades at such prices. Agents in such an economy never really have any form of direct interaction, social or otherwise.

[34] While I do not need it for the present exercise, for a richer formulation that would provide a formalization of culture, taken to be shared beliefs, I could take the liberty of assuming that S_i is, in some cases, a set of alternative, mutually exclusive *comprehensive outcomes* (action-process-consequence strings) that in individual preferences attach positive constitutive value *also* to the process by which the choice of action leads to the final consequence, rather than ascribing direct, constitutive value only to the *culmination outcomes*, which is the case with *game theory* in von Neumann and Morgenstern (1942) and in *general equilibrium theory* in Arrow and Debreu (1954) and Arrow and Hahn (1971), among others.

the elements of the feasible set also 'easily risks turning the "explanation" into a tautology' according to Arrow (1998, p.95), and is thus an approach to be avoided.[35]

In a *non-binary* relation of weak personal preference $R_i(V^i)$ defined on a finite set S_i, a variation in V^i (say, from V_1^i to V_2^i) can, in general, alter a pair-wise ranking R_i of a pair of alternative actions $x_1, x_2 \in S_i$, rendering $R_i(V^i)$ a non-binary relation. This is because R_i, which is by assumption a binary relation, is *not* invariant to *tertiary* considerations entailed by differences in the personal backgrounds of distinct persons, as in a distinction between V_1^i and V_2^i. For example, $R_1(V^1)$ could refer to the preference of a person who is an African American male, and $R_2(V^2)$ can represent a Caucasian male. While the gender of both persons is the same, a shared trait that they are both male, they are, in fact and in theory, persons of different races. Thus, both shared and distinctive social identities of persons can be accommodated in a model of a society with individual Kanger-Sen non-binary preferences.

In fact, a person's community affiliation begets obligation. A person's preference for an alternative, as more satisfaction or greater desire fulfillment from it, is a feeling, a state of mind, which could be *a* motivation, though not necessarily the *sole* motivation, in the causal shaping of entailed behavior. It has a component of yearning that the concept of behavior completely lacks. Undiluted personal preference for an alternative over another could well be intermediated via the obligations the person feels from having a social identity insofar as the person's community affiliation could keep the individual from exhibiting behavior that would have reflected personal preference in the absence of this sense of obligation. For example, a woman may actually have a strict preference for hot and spicy food over bland food. However, if she is also a lactating mother, she could feel obligated by her belonging to the community of breast-feeding women to exhibit behavior of eating bland food instead (possibly on the grounds – well-founded or not – that hot and spicy food may be bad for a suckling baby's health).[36]

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[35] However, this is only one possible interpretation. There are *other* useful and important interpretations of both the elements of the feasible set as well as of the background set of which the ranking relation is a function. They could be utilized to explain intra-family interactions, for example. See Sen (1989).

[36] Distinct social identities do, in fact, exist and these distinctions are embedded in the minds of persons in every country in the world. This social reality, in which we are "*diversely different*", shapes the way we think. Naturally, such distinctions in our minds also get injected into the informal parts of scripts of journal articles or monographs that we write in our capacity as economists, including in the contributions of Becker (1957) in the analysis of discrimination, and of Akerlof and Kranton (2000) in their examination of identity in economic theory, even though in their models all persons have the same identity.

To see that a non-binary preference ranking relation does not admit of a utility function representation, assume that a person's preferences are represented by a non-binary relation $R(V^j)$ defined on the set S of mutually exclusive alternatives, where R is a binary relation defined on S and V^j is the background set on which R is dependent. Suppose also that this person is identified with a utility function $f: S \longmapsto \mathcal{R}$, where S is the set of mutually exclusive alternatives and $\mathcal{R}$ is the set of real numbers. Then, by definition of a utility function, we have $\forall x, y \in S: [xR(V^j)y \leftrightarrow f(x) \geq f(y)]$. Consider $V^1 \neq V^2$. We know that since $R(V^j)$ is non-binary, $\exists x, y \in S: xR(V^1)y \,\&\, {\sim}xR(V^2)y$ are both admissible, so that $f(x) \geq f(y) \,\&\, {\sim}[f(x) \geq f(y)]$ are both true. Equivalently, $f(x) \geq f(y) \,\&\, f(x) < f(y)$, which is false. It follows that a non-binary preference relation cannot have a utility function representation.

5. Society with Non-Binary Personal Preference Relations

Next, consider a finite *n*-array of non-binary preference (desire or yearning) relations,

$$\{R_1(V_1), \dots, R_n(V_n)\}$$

each identified with a particular person in society. My purpose is to seek a non-empty set of social states at which all persons have achieved maximization of their respective personal non-binary preferences, which are required to be a quasi-ordering (reflexive and transitive, though not necessarily complete) of their respective personal feasible sets S_i.

5.1 Model

To develop the requisite theory, I make the following four assumptions

Axiom N (*Non-binariness Axiom*): Each individual is characterized by a personal non-binary ranking relation of weak preference that is defined on a finite set of alternatives. This preference relation is $R_i(V^i)$, which is defined on a finite set S_i of the person's feasible set of alternatives, and V^i is that person's *background set* upon which the *binary* ranking relation R_i is dependent, for all persons i from 1 to a finite n.

Notice that non-binariness of personal preferences is defined in a very specific sense.[37]

Axiom Q (*Quasi-Ordering Axiom*): Every person's non-binary relation of weak preference is reflexive and transitive but not necessarily complete.

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[37] The precise content of the concept of non-binariness of preferences taken here, in *Axiom N*, is exactly the one formulated by Stig Kanger on which additional work was done by Sen (1994b), to complete Kanger's unfinished work, to bring it to the domain of mainstream choice theory. This is precisely the formulation of personal preference in Sen's (1997) individual choice theory.

Axiom M (*Maximization Axiom*): Every person engages in a volitional act of choice by maximizing one's own personal preferences defined on the personal feasible set of actions.

Axiom I (*Social Interaction Axiom*): Social interaction takes the form of interpersonal interaction among individual social creatures.

These four axioms collectively entail the following three conditions:

Condition S (*Social Identity Diversification*): There exists of a social state with endogenous social differentiation of persons, with each person being a social creature with multiple social identities deriving from affiliation with distinct, though overlapping communities (or subsets) of persons in society.

Condition J (*Violations of Justice*): The social state identified in Condition *S* is consistent with flagrant violations of justice that take the form of conflicts between socially distinct groups.

Condition P (*Pareto optimality*): The social state identified in Condition *S* is Pareto optimal.

The groundwork for an examination of a non-binary society with social diversification is now laid. Some additional formalization is necessary for the demonstration of the principal conclusions of this paper in the next section.

5.2. Formalization

For a given set V_i^j, let $R_i(V_i^j)$ be person i's binary relation of weak preference that stands for "at least as good as", which is defined on a finite set S_i of alternatives social states, and V_i^j is a background set on which the binary relation R_i is dependent, with $i = 1, \dots, n$, and $j = 1, \dots, k_i$ specifying the possible parametric variations, V_i^j, of person i's background set. Here, $n \geq 2$ is finite, $k_i \geq 1$ is finite, and S_i also finite with at least three elements.

For $R_i(V_i^j)$, we can define the asymmetric part $P_i(V_i^j)$ that stands for "strict preference", and the symmetric part $I_i(V_i^j)$ that stands for "indifference" as follows.

Definition 9: $(\forall i, \forall j \, \& \forall x, y \in S_i): [[x R_i(V_i^j) y] \, \& \sim [y R_i(V_i^j) x]] \leftrightarrow [x P_i(V_i^j) y]$.

Definition 10: $(\forall i, \forall j \, \& \forall x, y \in S_i): [[x R_i(V_i^j) y] \, \& [y R_i(V_i^j) x]] \leftrightarrow [x I_i(V_i^j) y]$.

In this context, it is important to note that a variation in a *tertiary* consideration, viz., a parametric variation in the background set, can, in general, alter the order of personal preference

insofar as $(\forall i, \exists x, y \in S_i \ \& \ \exists l \neq m): \left[x R_i(\boldsymbol{V}_i^l) y \right] \ \& \sim [x R_i(\boldsymbol{V}_i^m) y]$, are both admissible, thereby rendering $R_i(\boldsymbol{V}_i^j)$ a non-binary relation.

My purpose here is to examine if a *social interaction outcome* based on *non-binary* personal preferences exists, and whether it is Pareto optimal. I prove that, in fact, there exists such an outcome, and that it is Pareto optimal, under some extremely mild conditions (*Axioms N*, *Q*, *M* and *I*), at least judging by the restrictions imposed in the literature to demonstrate the existence of alternative types of equilibria that are based on mainstream utility theory and choice theory.

Clearly, such a social outcome is a generalization that goes considerably beyond all of the alternative concepts of equilibrium in general equilibrium theory in economics that are predicated on binary personal preferences. It would also constitutes a generalization of some equilibrium concepts in game theory, and is sufficiently comprehensive in scope that it does not succumb to some crucial criticisms of the behavioral foundations of game theory discussed by Sen (1985), where the matter of restrictiveness of "privateness" of each person in economic theory versus "socialness" of personal identity in social reality is brought out most forcefully. I shall have more to say on this in Section 8.

The implications of the existence of a non-binary preference based Pareto optimal social interaction outcome are quite far-reaching in terms of providing formal explanations of many sociological, economic and political phenomena that binary-preference based conceptual structures fail to do.

6. Existence of a Pareto Optimal State under Non-Binariness

Importantly enough, once binariness is dropped as a feature of personal preferences in a society, the utility function as a numerical representation of personal preferences is also jettisoned from economic theory. Additionally, the non-binary personal preference theory is general enough to accommodate violations of the *Weak Axiom of Revealed Preference*, as would naturally occur if, in personal choice, preference were to change upon variation from one background set to another distinct one.

To achieve these objectives, I utilize three lemmas in Sen (1970), and with relatively minor generalizations, prove an existence theorem below.[38] First, however, some additional definitions are in order.

Definition 11: $R_i(V_i^j)$ is *reflexive* over S_i if and only if $(\forall i, \forall j \,\&\, \forall x \in S_i): [x R_i(V_i^j)x]$.

Definition 12: $R_i(V_i^j)$ is *transitive* over S_i if and only if the following holds:
$(\forall i, \forall j \,\&\, \exists x, y, z \in S_i): [[x R_i(V_i^j)y] \,\&\, [y R_i(V_i^j)z]] \rightarrow [x R_i(V_i^j)z]$.

Definition 13: A ranking relation that is reflexive and transitive is called a *quasi-ordering*.[39]

Let $J = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{k_i} V_i^j)$, and $S = \bigcap_{i=1}^{n} S_i \neq \emptyset$, and assume that S has at least three elements.

Definition 14: A social interaction outcome *rule* is a functional relation f that assigns exactly one social ranking $R(S, J)$ of S to an inter-personal non-binary preference profile, such that

$$R(S, J) = f\left(R_1(V_1^j), \ldots, R_n(V_n^j)\right), \text{ where } \forall i, j: R_i(V_i^j) \text{ is a quasi-ordering of } S_i.[40]$$

By $P(S, J)$ we denote the asymmetric part of $R(S, J)$. We next turn to *unanimity over a pair of alternatives* under all possible variations of the background set to define Pareto preference.

Definition 15A: $\forall j \,\&\, \forall x, y \in S: [\forall i: x\, R_i(V_i^j)\, y] \leftrightarrow x\, \hat{R}(S, J)\, y$.

Definition 15B: $\forall j \,\&\, \forall x, y \in S: [\forall i: x\, P_i(V_i^j)\, y] \leftrightarrow x\, \hat{P}(S, J)y$.

Remark: Definition 15A is a generalization of the Pareto 'preference or indifference' rule denoted by $\hat{R}(S, J)$ to non-binary personal preferences over the set S of alternative social states, and similarly, Definition 15B is a generalization of the Pareto 'strict preference' rule, denoted by $\hat{P}(S, J)$.[41]

Definition 16: A social state x in S is *Pareto optimal* if and only if it is not Pareto dominated by any state y in S in accordance with Definition 7B.

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[38] Lemma 6.2 below is Sen's (1997) own generalization to non-binary preferences. Lemma 6.1 below is my generalization to non-binary preferences, and Theorem 6.1 is our generalization of a lemma in Sen (1970), also to the case of non-binary preferences.

[39] Reflexivity of $R_i(V_i^j)$ is not a very demanding restriction. To see this, consider two persons, indexed 1 and 2, where 1 is male and 2 is female. Then $x R_1(V_1^j)x$ and $x R_2(V_2^j)x$ essentially means that both men and women regard an element of their feasible set at least as good as itself. Similarly for the transitivity property.

[40] A social interaction outcome rule is a generalization of what Sen (1970) calls a collective choice rule.

[41] The ethical principle in Definition 15B is also known as the weak Pareto rule.

Finally, using Definitions 14, 15A and 15B, and by requiring that $[x\,R(S,J)\,y] \leftrightarrow [x\,\hat{R}(S,J)y]$ and $[x\,P(S,J)\,y] \leftrightarrow [x\,\hat{P}(S,J)y]$, we can obtain a maximal social interaction outcome by using the following two lemmas.

LEMMA 6.1. $\hat{R}(S,J)$ *is a quasi-ordering of S.*

Proof: Following Sen (1970, Lemma 2*a, p.29)),

$\forall j$ & $\forall x \in S$, since by Definition 14, $\forall i: x\,R_i(V_i^j)\,x$, it follows that $\hat{R}(S,J)$ is reflexive. Also, $\forall x, y, z \in S: [x\,\hat{R}(S,J)y$ & $y\,\hat{R}(S,J)z] \rightarrow [\forall j$ & $\{\forall i: x\,R_i(V_i^j)\,y$ & $y\,R_i(V_i^j)\,z\}]$

$$\rightarrow \forall j\ [\forall i: x\,R_i(V_i^j)\,z]$$

$$\rightarrow x\,\hat{R}(S,J)z. \quad \blacklozenge$$

Next, consider

DEFINITION 17: $M(\hat{R}, S, J) = \{x | x \in S$ & $\sim[\exists y \in S: y\,\hat{P}(S,J)\,x]\}$.

Remark: The social interaction maximal set of socially undominated elements of S is fully captured by Definition 17 with respect to the weak Pareto rule $\hat{P}(S,J)$, which is the asymmetric part given in Definition 15B.

LEMMA 6.2. *The maximal set is non-empty for every finite set quasi-ordered by a non-binary preference relation.*

Proof: Again, following Sen (1970, Lemma 1*b, p.11, and Sen (1997)), let $S = \{x_1, \dots, x_m\}$. Assign a real number $a_1 = x_1$, and follow the recursive rule $x_{q+1}\,\hat{P}(S,J)x_q \rightarrow a_{q+1} = x_{q+1}$, and $a_{q+1} = a_q$ otherwise, so that by construction, x_m is a maximal element. $\quad \blacklozenge$

Remark: Note that since $(\forall i, \exists x, y \in S_i$ & $\exists l \neq m): [xR_i(V_i^l)y]$ & $\sim[xR_i(V_i^m)y]$ are both admissible, personal preferences are non-binary, but this poses no problem for obtaining a nonempty *social maximal* set since the personal non-comparability of a pair of alternatives in S_i is rendered irrelevant for defining the maximal set. This, of course, is not true of the *social optimal* set of best elements that is defined as $C(\hat{R}, S, J) = \{x | x \in S$ & $\forall y \in S: x\,\hat{R}(S,J)y\}$, which is rendered empty by $(\forall i, \exists x, y \in S_i$ & $\exists l \neq m): [xR_i(V_i^l)y]$ & $\sim[xR_i(V_i^m)y]$ being both admissible.

  Thus, requiring maximizing behavior as an act of volitional personal choice, instead of the more demanding optimization, does have an advantage in the case of non-comparability arising from non-binariness of personal preferences. In fact, it should not come as a surprise that once there is a social quasi-ordering which ranks at least one pair of alternatives, though not

necessarily all such pairs, if and only if these two alternatives are comparable over all individuals and over all background sets, there must exist an element which is Pareto undominated and thus Pareto optimal. This would also follow from Zorn's lemma.

In the case of personal choice theory, Sen (1997) exploits precisely this combination of non-binariness of preferences (and the entailed incomplete ranking), *and* maximizing behavior that precipitates the existence of a maximal element despite incompleteness. He thus obtains more general results than can be deduced from binary personal preferences constitute a complete ordering *and* optimizing behavior that necessitates completeness for the existence of an optimal element – this is the formulation of standard rational choice theory. Sen's (1997) formalization is of a decision maker who has non-binary preferences. To obtain a nonempty social interactional set, rather than a nonempty personal maximal set, a group of finite *n* persons needs to be considered. For this case, based on Lemma 6.1 and Lemma 6.2, the following result is immediate.

Theorem 6.1 (Existence of Unjust Pareto Optimal State): *Axioms N, Q, M and I are sufficient for the existence of a social state that is characterized by Conditions S, J and P.*

Proof:[42] It is to be proved that *for every set of non-binary personal preferences* $\left(R_1\left(V_1^j\right), \ldots, R_n\left(V_n^j\right)\right)$ *over a finite set S of alternative social states, where* $\forall i, j: R_i\left(V_i^j\right)$ *is a quasi-ordering, there exists a nonempty maximal social interaction set* $M(\hat{R}, S, J)$ *that contains at least one Pareto optimal state.*

By *Lemma 6.1*, the Pareto preference-or-indifference relation $\hat{R}(S, J)$ is a quasi-ordering of the set S of alternative social states. And the Pareto-optimal subset of S is identical to the social maximal set $M(\hat{R}, S, J)$. Further, since S is finite, and $\hat{R}(S, J)$ quasi-orders it, *by Lemma 6.2*, $M(\hat{R}, S, J)$ is nonempty. Hence a non-binary personal preferences based social interaction outcome exists, and it is Pareto optimal. ♦

Notice that, unlike the case of the existence of equilibrium in an Arrow-Debreu exchange economy, and unlike the case of existence of Nash equilibrium, for our existence result I do not impose the requirement of completeness; in fact, I cannot impose it because incompleteness is *entailed* by non-binariness, if it has any cutting power. Nor indeed do I require preferences to be

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[42] The proof of *Lemma 6.1*, and therefore of *Theorem 6.1*, goes through if transitivity is weakened to *Definition 12A*: *Acyclicity*: which holds if and only if $(\forall i, \forall j \ \& \ \forall x_1, \ x_2, \ldots, x_l \ \in S): [(x_1 P_i\left(V_i^j\right)x_2 \ \&$ $x_2 P_i\left(V_i^j\right)x_3, \ldots \& \ x_{l-1} P_1\left(V_i^j\right)x_l)] \to x_1 R\left(V_i^j\right)x_l$, provided "quasi-ordering" is replaced by "reflexive and acyclic ranking" after *Definition 12* is replaced by *Definition 12A*. These two may be called *Lemma 6.1A* and *Theorem 6.1A*, respectively. For further details, see Berdellima and Naqvi (2011).

binary, as in Kelsey (1995). Moreover, moving from *form* to alternative *interpretations* of the background set, and by considering parametric variations of this set, many of the inadequacies in explanations of social and economic phenomena entailed by binariness are entirely jettisoned, replaced instead by a much more comprehensive conceptual structure that still exhibits the unanimity property over a pair of alternatives. Also, the scope and reach of this non-binary model is so substantial that it also capable of formalizing the criticism of the game theory literature contained in Sen (1985).

7. Illustrations

In the conceptual structure of Sections 5 and 6, individuals can have distinct identities, they can identify with different groups of persons, they can share identities, escape, or not, from "the illusion of destiny,"[43] to diminish or exacerbate violence, exhibit, or not, the behavior of injustice in the form of discrimination, and so on. However, there are four issues in particular that arose in the development of the argument in previous sections, but were not addressed adequately, because the further development of the ideas would detract from the continuity of the argument. I shall deal with them in this section, before I sum up.

7.1 Racial Discrimination

The reason discrimination in one form or another has piqued the interest of some of the sharpest minds in our profession is that it is deemed profoundly unjust. We would rather have society exhibit greater harmony, less violence, have more say in their own individual lives – including political voice, people obeying tax laws, not violating criminal laws, and in general being civil to one another. It is this *civil* characteristic of a good society, when violated, often causes us to experience a feeling of revulsion at the unfairness or injustice of some actions, especially if we notice that these are ongoing actions that repeatedly harm the same segment of a society. Sometimes we may not notice such actions because they have become so commonplace in our social reality, unless, of course, we happen to be at the receiving end of the injustice. Discrimination is one such injustice. It is important to ask if it is remediable, and to the extent that it is, its mitigation can be taken to be one of the goals of public policy. Such an exercise would constitute an attempt to formalize the search for paths to peace and amity among people who are being uncivil to each other, as recently articulated by Sen (2006). Sen (2006) argues that

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[43] Sen (2006).

if different person's *multiple shared* identities are recognized and highlighted in reasoned public discourse, it would serve to diminish the influence of the fomenters of hatred which, as we shall see, have actual fatal consequences, sometimes in the form of carnage.

Fortuitously, Sen (2009) provides guidance in approaching this difficult subject of *economics and justice*. Sen (2009) argues that it can be a very constructive endeavor to define *The Idea of Justice* as the removal of remediable injustices. This is to be contrasted with all previous theories of justice that are *contractarian* in the sense of arriving at a single set of principles of justice, and seeking the characterization or attainment of a *perfectly just* arrangement in society, regardless of how this concept of perfectness is construed. In this regard, 'What moves us, reasonably enough,' writes Sen's (2009),

> is not that the world falls short of being completely just – which few of us expect – but that there are clearly remediable injustices around us which we want to eliminate. …. It is fair to assume that Parisians would not have stormed the Bastille, Gandhi would not have challenged the empire on which the sun used not to set, Martin Luther King would not have fought white supremacy in 'the land of the free and the home of the brave', without their sense of manifest injustices that could be overcome. They were not trying to achieve a perfectly just world (even if there were agreement on what that would look like), but they did want to remove clear injustices to the extent they could. (p. vii).

Sen's (2009) approach to justice is much more practical in nature, and can be easily and most fruitfully combined with a non-binary economic theory proposed here, to investigate many common pernicious social injustices and their remedies. A program worthy of pursuit would be an *Injustice Mitigation Program*: In a non-binary society, a public-policy induced manipulation of personal background sets, in favor of emphasizing in the public consciousness a wider domain of shared identities, especially aimed at groups in conflict, can serve to mitigate remediable injustices, including overcoming intolerance and discrimination, and thus violence by one group against another, especially if done via open and free reasoned public discourse, particularly in a democracy.

Such a program of investigation would take our understanding beyond the range of issues Sen (2009) covers in addition to Sen (2006). This, however, is the subject matter of another piece of work. What is important to remember is that a *contextual reference* is crucial to the investigation of any specific injustice. As an example of rampant discrimination, refer to Table 1.

Table 1

	L.E. Male	L.E. Female	M/F Ratio	Population Male(1000s)	Population Female(1000s)
Black	70.68	77.57	0.9169	19,996	21,808
White	76.19	81.21	0.9851	123,528	125,391
L.E. (WM-BM) =5.51	W(M/F)-B(M/F)=0.0682			1,488=BM Deficit	

Life Expectancy at birth (years) in U.S.A., 2007[44]

and

U.S. Resident Population (1000s) by Sex, Race, and Hispanic-Origin Status, 2009

According to the Census Bureau, in 2009 there were 917 African American males per thousand African American females in the United States. Per one thousand white females, there were 985 while males, however. Per 1000 African American females there are 68 fewer African American males than white males per 1000 white females. Since there were in 2009, in terms of thousands, 21,808 African American females, multiplying this by 68 gives almost 1.5 million missing African American males. That they are fewer than they should be, can also be confirmed by the life expectancy figures for 2007. African American are expected to live, on average, to the age of 71, white males to 76, black females to 78, and white females to 81. Why?

We know that pretty much all around the world, women outnumber men – except in India, China, Pakistan, and some other countries of north Africa, where females receive considerably adverse social, economic and political treatment than do males, as a consequence of powerful influence of their respective cultures on individual behavior in the two groups of countries.[45]

But, why do blacks live shorter lives than whites – whether male or female – in the United States?[46] Since pure races are almost extinct: certainly in the U.S. in 2007, almost all whites are a little black by heredity. Similarly, almost all blacks are a little white genetically. So, what is the cause of different life expectancies of "white males" and "black males," on the one

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[44] For Life expectancy figures, see source: Table 103. U.S. National Center for Health Statistics, National Vital Statistics Reports (NVSR), Deaths: Final Data for 2007 Vol. 58, No. 19, May 2010, at http://cdc.gov/NCHS/products/nvsr.htm#vol58/ . For population by race and gender, see http://www.census.gov/compendia/statab/2012/tables/12s0006.pdf.

[45] See Sen (1992) for some disturbing facts about women in parts of Asia and north Afica.

[46] Over generations, a higher life expectancy of females should, of course, imply females outnumbering males in any social grouping.

hand, and between whites' and blacks' life expectancies in general, on the other? Due to genetic mixing, the answer must be positioned not in the biological domain but in the field of social treatment received by blacks versus whites. This is an inescapable conclusion for this society at this time.

There is no need to belabor the point that social treatment of distinct socio-economic groups in society is invariably unequal. What is it that characterizes these groups of interest? And, what is the exact nature of the inequality of the social treatment they are receiving, and more importantly, accepting. Collective consciousness, imbued with beliefs (a) about reality, with values of (b) what is sacred and what profane, combined with (c) individual *free will* to make a reasoned (i) choice from amongst alternative sets of these very beliefs and values, and (ii) choice of a specific action out of a feasible set of alternatives, is what gets expressed as a *social interaction outcome* in a society, typically referred to as an equilibrium or a stationary state in economic theory. Barring some outliers, most persons tend to conform to the dominant form of beliefs and values of the time. More importantly, if multiple social interaction outcomes (corresponding to multiple equilibria) *can* occur under *mildly distinct* sets of collective beliefs, then it may be possible to identify the specific characteristics that differ, so that a policy-induced change in one set that supports a more *unjust* social outcome may be able to jolt the economy into switching to another social interaction outcome to achieve a little more justice in society.

Of course there are substantial issues of incentive compatibility here that have not been addressed, which are important in their own right. Also, requiring unanimity of ranking over a given pair of alternatives, embodied in the Pareto criterion, is a very exacting demand for beliefs and values to constitute part of culture. These however, are not the directions in which this paper purports to make a contribution.[47]

An economic theory of justice, which would permit the identification of injustice-curative policies, and which is of fundamental importance to individuals, society, national governments and world governance organizations, can, in fact, be developed in the framework of the model I have proposed in this paper. It can be argued that to understand the mechanisms by which at least the remediable injustices can be removed is a valuable goal of public policy and the case for

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[47] For Bayes-Nash implementation, or for *virtual* Bayes-Nash implementation, incentive compatibility would have to be formulated in terms of non-binary preferences of the above kind.

constructing an economic theory of such social injustices and their removal is unquestionably strong.

7.2 Arrow-Debreu Model with Social-cum-Economic Stratification

It would be helpful to have a formal theory of social interaction, which takes the form of interpersonal interaction among social beings with multiple community affiliations, *in addition to* each person's interaction with the Walrasian auctioneer, as in Arrow and Hahn (1971). Arrow and Debreu (1954) write, "we assume the existence of a number of consumption units, typically families or individuals but including also institutional consumers." Further, "The set of consumption vectors X_i available to individual i (=1, ... , m) is a closed convex subset of $\mathcal{R}^l$ which is bounded from below; i.e., there is a vector $\xi_i \leq x_i$ for all $x_i \in X_i$." (p. 278). Let every "consumption unit", which I shall call a person, be engaged in a volitional act of choice from amongst mutually exclusive actions, by maximization of one's own Kanger-Sen non-binary preference relation, which quasi-order each person's feasible set, *without redefining* the elements of the feasible set in Arrow and Debreu, except to take it as finite set. In this model, social differentiation of persons emerges in society, as I have demonstrated above. Each individual, in this *social economic theory* model is a social creature with multiple identities.

Further, in this model, let us retain the Arrow-Debreu (a) the production structure, (b) their exogenously given interpersonal distribution of commodity endowments, and (c) the exogenous distribution of property rights to shares in "production units" (or firms) in the economy.

The resultant model has the distinction that it is *also* possible to *exogenously assign* distinct commodity endowments and different property rights to fractional ownership of production units (corporate shares) to individuals who belong to socially distinct communities. Such an economy would tend to exhibit distinct endowment distributions and different corporate-share ownership that could vary systematically by race, gender, ethnicity, age group, and so on, including some unfortunate groups who would have no such rights to fractional ownership in firms. This is a model of an economy that has socially distinct communities that also differ by wealth holdings, and thus can exercise differential political power in influencing the social outcome.

Basu (2010) argues that if an entrepreneur's ability to make an investor and a customer better-off by setting up a business is dependent the customer's *and* the investor's *conjectures*

about the entrepreneur's ability to do so, then the content of the conjectures will determine whether entrepreneurial activity will or will not occur in equilibrium. Strategic reciprocity inherent in the supermodularity property of choice of actions by persons, by itself determines the entrepreneur's ability to create greater value for both the customer and the investor. Moreover, this can induce a real bifurcation among *ex ante* identical agents who are potential entrepreneurs, so that while *ex ante* identically productive, the subset who are believed by others (the customers and investors) to be more productive do, in fact, exhibit higher productivity *ex post*, and they start to wear a badge of higher productivity, while others who are *ex ante* identical do not. This productivity-based differentiation is precipitated precisely because the customer and the investor both *believe* in the higher productivity of a subgroup of entrepreneurs, so more clients are willing to sign contracts with the badge-wearing persons.[48] All this happens even without anybody having any personal prejudice, *individual discrimination*, at all, and the beliefs underlying the conjectures of the customer and the investor end up shaping the social reality of who gets to be an entrepreneur and who does not.

The productivity-based differentiation of persons in society is important because that is the sole concern of firms that demand labor, because they are simple profit maximizers, as in Arrow and Debreu. Behavior of a large enough number of persons based on beliefs that have no innate basis in social reality can actually produce a social reality that vindicates those beliefs. This is the upshot of conjectural general equilibrium theory, in which the Arrow-Debreu equilibrium arises if all agents have *competitive conjectures* (that market prices are parametrically external to their decisions).

More profound is Basu's recognition of multiple equilibria in these reciprocity-salient supermodular games, with an equilibrium here exhibiting stagnation but one there bursting with entrepreneurial excitement, energy and rapid development, and public action in such cases would be called for to bump the economy from the "bad" to the "good" equilibrium of the game. Unfortunately, the argument does not carry over to racial-discrimination equilibrium versus racial-harmony equilibrium as multiple equilibria of such a supermodular game, nor to gender, ethnicity or any other social-identity based distinctions or injustices. But, with a small extension, the argument would be applicable to social injustices also, and that is what I attempt to provide in this paper.

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[48] Higher productivity could be equivalent to fewer hours spent on producing the same additional income for a firm.

What Basu shows is the differentiation of a group of otherwise identical persons *ex ante*, due to the conjectures of other agents, into more productive and less productive subgroups *ex post*. However, the mapping of the set H of persons who are differentiated by productivity to the set Ψ of the social identity markers of the persons still remains to be established. And to construct this mapping, so that differentially productive persons can take on distinct, though overlapping social identities such as gender, race, ethnicity, and so on, requires a model that endogenously exhibits social identity diversification as an intrinsic feature of an attainable social state of an economy.

If this social identity diversification of person that I have proposed in this paper is combined with Basu's (2010, 89-95) model of endogenous productivity differentiation of otherwise productivity-wise identical individuals, it becomes a very rich and powerful model that is capable of providing formal explanations of a great many phenomena, especially in public choice theory, or new political economy, where social identity differentiation and differences in wealth holdings associated with socially distinct persons are material to the object of investigation, and such *wealth-cum-ethnicity outcomes* would arise *endogenously*. Such formally sound investigations have yet to be undertaken.[49]

The various marks of identification of individuals emerge from identifying with or associating with distinct, though overlapping communities – by the perception of others in society. And the collectivity of these *others* with whom we associate, due to one cause or another, with some of our social identities taking priority over others on different occasions, cannot but influence our own personal beliefs of what we believe to be reality, including, and especially *social reality*. Nor indeed can the expressed values by others in the varied communities to which we belong have no effect whosoever on our values of right and wrong, or of good and bad, or what is or is not one's duty or obligation to do in a given circumstance, solely because the person belongs to a community with some obligations. Association begets obligation. Obligation influences motivation. Motivation underlying behavior is, therefore, made up of preferences, community affiliation with obligations pertaining to one's social identity that deliberately restrict individual instrumentalities, and free will in both the form of maximizing

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[49] This would require, though, a reformulation of the mathematical concept of "supermodularity" in terms of non-binary preference relations that are reflexive and acyclic defined on a set of strategies, instead of reaction correspondences derived from utility functions.

behavior and in the form of selecting personal values from alternative sets. Interpersonal interaction of such social creatures is social interaction.

A model of a society of persons who engage in the act of maximizing their own personal non-binary preferences that quasi-order the mutually exclusive alternatives in their feasible (in some important cases, budget) sets brings to life injustices suffered by one group of persons in a finite society in what *appears* to be at the hands of another socially distinct group, although it is *actually* collective prejudice, distinguished from individualistic prejudice, that is contained in collective beliefs. Such a model can also help identify redress of each of these collective, rather than individualistic, injustices that spring from social identity diversification of persons in society.

7.3. A Methodological Issue

An equilibrium exists in this model in the sense of a non-empty social maximal set defined on a finite set of alternatives that are quasi-ordered by the social non-binary ranking relation $R(V)$. This model endogenously exhibits social identity differentiation, and there can be multiple equilibria supported by distinct rational conjectures in the manner of Basu, thereby implying the existence of equilibria in which, *ex post*, African American males exhibit a lower productivity, a lower life expectancy and lower peak median income than white males, not to mention a disgracefully higher incarceration rate that African Americans males suffer.

At a very fundamental level, Arrow (1998) addresses an issue of some intricacy that gets to the heart of the issue I am addressing in this paper, at the level of economic theory:

> Most analysts, following Becker (1957), add to the usual list of commodities some special disutility which whites attach to contact with blacks, taste-based discrimination. … The trouble with these explanations is that they contradict in a direct way the usual view of employers as simple profit-maximizers. While they do not contradict rational choice theory, they undermine it by introducing an additional variable.
> There are at least two objections to this line of analysis. One is that introducing new variables easily risks turning the "explanation" into a tautology. …and it certainly would be a parody of economics to multiply entities in this anti-Occamian fashion. Perhaps more serious is the neglect of Darwinian principles. (pp. 94-95)

So, Arrow considers (i) contradict[ing] in a direct way the usual view of employers as simple profit-maximizers, (b) violating the principle of survival of the fitter, and (c) 'introducing new variables [, which] easily risks turning the "explanation" into a tautology', objectionable.

In fact, taken together with the Basu (2010) twist, the non-binary personal preferences based model I have proposed does not violate any of the minimal requirements Arrow (1998) has suggested for an adequate theory of racial discrimination.

In my explanation of social identity diversification of individuals, I have the same feasible sets of individuals as the "consumption units" have in Arrow and Debreu. Each person is still a maximizer of one's own individual quasi-ordering of feasible alternatives. Choice theory has, therefore, not been compromised, especially since everybody is a personal non-binary preference maximizer of a quasi-ordered set of alternatives. Further, "the producer" is also a "simple profit maximizer", who hires or does not hire black or white employees in specific occupations. However, the producer-entrepreneur also imbibes values from the collective beliefs that embody prejudice against blacks. Unfortunately however, upon examination of the imbibed beliefs, the simple-profit maximizing producer would find that, due to the force of such pernicious beliefs in society, his prejudicial beliefs are vindicated, because by reducing the incentives for African Americans to engage in education, training and self-improvement, they have *ex post* rendered vast numbers, though, of course, not all, of the African American community to actually exhibit lower productivity, thereby reinforcing the stereotype. After all, racial discrimination does manifest itself in terms of just shy of *one-and-a-half million missing African American males* in early 21st Century. It also expresses itself in terms of a 5 ½-year shorter life expectancy for African American males compared to white males during the same period. Why would it not express itself also in terms of endogenous productivity differentials in the manner Basu (2010) has suggested in a different context?

Since the model I am proposing here is actually diversified by persons with distinct social-identity dependent capabilities and social-identity dependent wellbeing-achievements, and employers are, as in Arrow and Debreu (1954), simple profit maximizers, the force of Darwinian principles is somewhat muted. The methodological issue of introducing new variables does need to be addressed, however. I deal with this next.

Were it not for utter existential intolerance of any forms of social heterogeneity under the preference structure ascribed to consumption units in Arrow and Debreu (1954), there would be no need to introduce any new variable. But the binariness property of preferences in their model is so inimical to social identity diversification that, in principle, for racial discrimination, or of any other kind, to exhibit itself, it is necessary to introduce a new variable. I use $R_i(V_i)$, as a

non-binary relation of personal preference dependent on that person's background set, instead of only R_i in Arrow-Debreu, without the background set V_i that I include.

The operational issue turns not on whether a new variable should be introduced (it certainly does have to be for social differentiation), but on what exactly should be the properties of the newly introduced set, V_i called a person's background set in $R_i(V_i)$. In Arrow and Debreu (1954) and in von Neumann and Morgenstern (1942), $V_i = \overline{V}$. In the context of racial discrimination in the United States, as expressed in terms of males, $V_1 = \{African\ male\}$ and $V_2 = \{Caucasian\ male\}$. This characterization of background sets would not apply at all if the interest were in examining gender discrimination, whether in the U.S., or elsewhere.[50] Individual conjectures would also have to be correspondingly different, in terms of substantive content, pertaining to genders, rather than relating to racial differences. Notice, however, that the consequences for life expectancy differences may not be the same, though biased in the same direction, in the distinct forms of racial and gender discrimination, caused by adverse social treatment of the aggrieved. Of course, neither specification of background sets would be applicable if the object were to understand caste discrimination in India.

The crucial point is that the 'new variable', which is a person's background set, is dependent on the context of the issue under investigation. We ought not to look for some mathematical properties such as compactness or convexity, but rather treat background sets as context-dependent. The ingenuity of the economist would then reside in the proximity-in-predictive (or explanatory) power of the specification of the background set in explaining a specific social interaction outcome.

I am actually advocating a move in the direction back from generality to particularity of modeling the specific phenomenon under investigation. In ascertaining aggregate demand for a commodity at any given price, it is irrelevant whether the demander is black or white. Not so, however, if the object of investigation is racial discrimination. The characterization of a person's background set is necessarily context dependent.

This context-dependent background set in an analysis of racial discrimination in the United States is by no means the same as in the matter of gender discrimination in the world. The form it takes, the groups involved, the proportions of the population with distinct social

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[50] The ratio of women to men in Sub-Saharan Africa is 1.01, USA, 1.03 whereas that in India is 0.92, for instance, in 2011.

identities, all vary greatly. Caste based conflict in India is another context completely distinct from both gender and racial discrimination, although all three, far too often, lead to the same disagreeable outcome, of making the lives of far too many persons considerably more morbid and shorter. In the pursuit of more and more general results in economic theory, we appear to have gone too far in the direction of homogenizing persons by our assumptions to make our models incapable of addressing specific social injustices. Some back-tracking in the direction of context-dependent particularity seems to be necessary for addressing issues of specific social injustices and their remedies.

8. Concluding Remarks

Existing collective beliefs that effectively mediate personal wellbeing maximization as a volitional act of choice by each of a finite number of players, by deliberately limiting individual instrumentalities, taken together with representation of the maximand by a Kanger-Sen non-binary ranking relation that quasi-orders a belief-independent feasible set of alternatives of choice, is the formal framework I present in this paper, whether taken as an exchange model or with production as in Arrow and Debreu (1954). This model is considerably richer in terms of the information that it deems admissible in the characterization of an individual member of a group. The replacement of a binary by a non-binary ranking relation precipitates the possibility of social identity diversification of individuals in the group with overlapping community affiliations, which is disallowed by binariness.

Since each community affiliation obligates an individual to not violate specific types of action norms that the community deems unacceptable, individual instrumental possibilities become further restricted. By explicitly admitting the possibility that the instrumental value (over and above the constitutive value) of action norms is *not* zero – unlike the implicit assumption in contemporary economic theory that it is zero – collective beliefs begin to have some cutting power in translating personal preference maximization into exhibited behavior. In some instances, the role of collective beliefs may be to engender cooperation and thereby enhance every player's own unmodified wellbeing, as in an escape from the prisoners' dilemma.[51] In

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[51] There are some additional substantive issues that still need to be addressed, but with which I have not dealt in this paper. For instance, in a *binary* ranking relation of a person's preference, $xRy \leftrightarrow$ "I like x as much, if not more, than I like y," careful attention needs to be paid to the two concepts in this statement; "like" and "I". First, for me, *liking* means that my preference, seen as desire or yearning to attain, is such that it *simultaneously* satisfies (i) goal-completeness, (ii) goal-self-regarding-ness, (iii) goal-priority, and (iv) mutual knowledge, as in Sen (1985, p. 342-

other instances, collective beliefs can support conflicts that are abhorrently unjust social outcomes, such as discrimination, shorter life spans of some members of a group for reasons that are extra-biological and higher incarceration rates, to name a few.

Sen (2006) identifies *the illusion of destiny* of a person's unique identity that can breed hatred and violence, when fomenters of communal discord emphasize excessively such a solitary, divisive social identity of persons, *inter alia* devaluing a great many other social identities that the same persons actually share. Sen (2006) also argues that bloodshed need not, in fact, be the outcome if the spuriousness of this illusion of destined divisive identity is exposed through public discourse. He champions transparency and uninhibited public discourse in also serving to remove injustices of discrimination and killings inevitably inflicted by propagators of intolerance on vulnerable segments of society. I have tried in this paper to provide the outlines of a formal economic-theoretic framework in which these weighty matters can be discussed coherently.

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43). Second, preference are characterized by *my* 'privateness' or autonomy, as being completely independent of others in terms of my immunity to influences that are external to me. Thus my preference does *not* intrinsically incorporate any positive-feeling (such as sympathy) or negative-feeling (such as envy) or neutral-feeling (such as commitment) towards any other person. Thus, the concept of a person (or player) whose preference is being discussed is a very private person, and this *privateness* is characterized *simultaneously* by (a) Self-centered welfare, (b) Self-welfare goal, and (c) Self-goal choice, again as in Sen (1985, p. 347).

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