The Multiple Discrete-Continuous Extreme Value Model (MDCEV) with fixed costs

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Abstract

In this paper, we present a model that can be viewed as an extension of the traditional Tobit model. As opposed to that specific model, ours also accounts for the the fixed costs of car ownership. That extension is needed since being carless is an option for many households in societies that have a good system of public transportation, the main reason being that carless households wish to save the fixed costs of car ownership. So far, no existing model can adequately map the impact of these fixed costs on car ownership. The Multiple Discrete-Continuous Extreme Value Model (MDCEV) with fixed costs fills this gap. In fact, this model can evaluate the effect of policies intended to influence household behaviour with respect to car ownership, which can be of great interest to policy makers. Our model makes it possible to compute the effect of policies such as taxes on fuel or on car ownership on both the share of carless households and the average driving distance.

We calibrated the model using data on Swiss private households in order to be able to forecast responses to policies. One result of particular interest that cannot be produced by other models is the evaluation of the impact of a tax on car ownership. Our results show that a tax on car ownership has a much lower impact on aggregate driving demand – per unit of tax revenues – than a tax on fuel.

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Introduction

Being carless is an option for many households in economies having good system of public transportation – as it is the case in Switzerland. Thus, a good model should be able to map this option. In particular, it should also be able to map how the fixed costs of holding a car affects car ownership. So far, no model can be found in the literature that adequately maps this option. This paper presents the theoretical model that fills this gap.

The drawbacks of the existing modelling techniques can be summarized as follows: The OLS fails to map carless households. The Tobit model is unable to map the impact of fixed costs. The sample selection model fails due to the lack of an instrumental variable: there is no variable that influences only the choice of whether or not to own a car whilst not influencing the demand for driving at the same time. An interesting candidate for solving this problem is the Discrete-Continuous Choice model introduced by Dubin and McFadden (1984). This model can be used to explore the ownership of certain car types and their use. Unfortunately, the model only allows the choice of being carless to be captured if the annual mileage travelled using public transport is given in the dataset. Since this information is not available in most micro-census datasets, this model cannot be applied.

The Multiple Discrete-Continuous Extreme Value Model (MDCEV) with fixed costs overcomes the drawbacks of these models. As mentioned above, the proposed model can measure the impact of changes in the fixed costs of cars on driving demand and on the probability of households being carless. This ability to map the impact of income, fuel price and the fixed costs of car ownership on both car ownership and car use could not be found in the literature.[1] The MDCEV model makes it possible to compute the effects of policies such as taxes on fuel or car ownership on both the share of carless households and the average driving distance.

The MDCEV model was introduced by Bhat (2005).[2] This model consists of a direct utility function and a budget restriction. It is assumed that it maps the utility maximisation process of a household and is based on

1 One exception is the model of De Jong (1990), used later by Ramjerdi and Rand (1992) and Bjorner (1999). In contrast to our model, it is based on an indirect utility function instead of a direct function. Unfortunately, De Jong's (1990) model has an assumption that violates its compatibility with a microeconomic utility maximisation framework. In addition, it yields rather unrealistic results, particularly with respect to the impact of changes in fixed costs on car ownership. We believe that the MDCEV model with fixed costs maps reality much more effectively and lead to realistic results.

2 The first application of Bhat's model was to explain the time tourists spend for different activities. The model reflects that each activity can be chosen or not and how many hours are spent for the activities, subject to the time restriction of 24 hours a day, Bhat (2005). Later, Bhat applied this modeling framework to the case where households can choose to own none, one or several cars of different car types and decide of the driving distances the different cars are used for, Bhat (2006). In this model, Bhat ignores the fact that holding cars causes fixed costs and thus according to the model it would not be irrational to hold a number of cars even when the preference for car driving is low. Thus, we want to overcome this drawback by introducing fixed cost in our MDCEV model.

the assumption that a household chooses certain amounts of goods from a set of goods including the possibility of a household choosing not to consume any good at all. This means that a household may choose not to consume any goods at all. In order to adapt the model for examining car ownership and car use, we modified this model in two ways: first, we restricted it to the case with only two goods. This means that households may only choose whether or not to own and use a car and spend the remaining income for a consumption basket containing any other good. Secondly, we extended this model to the case where driving a car requires car ownership, incurring fixed costs, which is our contribution to the theory.

Assumption on household behaviour

The basic idea behind the model is described in the following. We assume that all decisions are taken at the household level. In the case of non-single households, we do not make any assumptions on who might have the most influence on the driving decisions. We also assume that each household compares the utility yielded from the following two options: first, it establishes the utility level it would gain if it owned a car. In this case, the household income would be reduced by the fixed costs of car ownership. Given that the household would then decide what annual distance x_2 it would drive in order to yield maximal utility. Note that the household spends its remaining income entirely on good one x_1, which we consider to be a consumer basket containing all goods apart from car driving, e.g. housing, food, medical care, holidays, and so on. We assume that utility is driven exclusively by the kilometres driven and not by the car ownership. Second, we assume that the household establishes the utility in the case that it decides not to own a car. In this case, it would save the fixed costs of car ownership and would spend all its income on good one x_1. The household then decides which option would give it the highest utility. This behaviour can be mapped using a standard microeconomic utility maximisation approach where the utility level can be computed by the direct utility function. The calculation of households' utility maximisation as described above can be illustrated as follows:

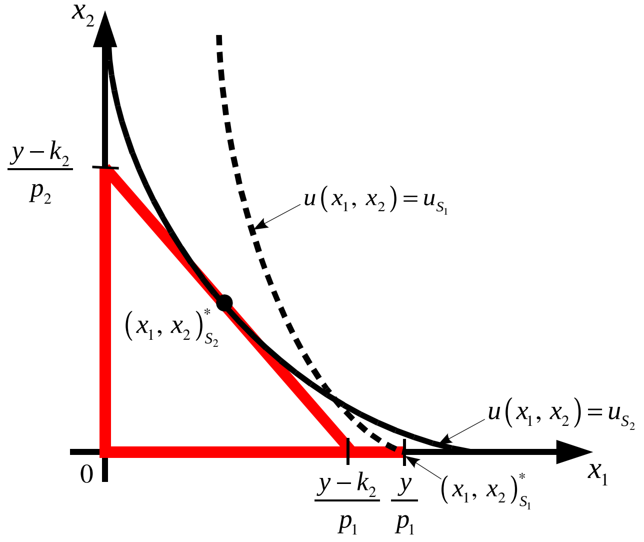

Figure 1: Optimum decisions of two households with different preferences

This figure illustrates the optimal consumption plan of two households with identical income but different car driving preferences. The solid lined iso-utility curve $u(x_1, x_2) = u_{S_2}$ represents a household with a high preference for car driving. It decides to own a car – this choice is denoted by index S_2 – and chooses x^*_{2,S_2} as its optimal annual driving distance given its income *y*, the fixed costs of car ownership k_2 and marginal driving costs p_2. We set the price of good one p_1 as numeraire, so the utility of x_1 can also be interpreted as the utility of income remaining after having paid all expenses incurred by the car. With this household, the consumption vector $(x_1, x_2)^*_{S_1} = (y/p_1, 0)$ is below the iso-utility curve and therefore yields a lower utility. In contrast, the dashed lined iso-utility curve $u(x_1, x_2) = u_{S_1}$ represents the household with a low preference for car driving. Since any point on the budget line defined by points $(0, (y-k_2)/p_2)$ and $((y-k_2)/p_1, 0)$ yield a lower utility than spending the total income on good one $(x_1, x_2)^*_{S_1} = (y/p_1, 0)$, the household decides not to own a car. This choice is denoted by index S_1.

Derivation of the MDCEV Model and its Maximum Likelihood function

Our choice of the utility function corresponds to the one in Bhat (2005:686). Since in our model, a household can only choose between the good "annual car driving distance" x_2 and consumption basket x_1 containing all other goods, the utility function is then written as:

$$U = (X_1 + a_1)^{d_1} + \exp(m + \beta \cdot \varsigma) \cdot (X_2 + a_2)^{d_2}, \quad (1)^3$$

with $m = \gamma \cdot s$, (2)

where ς is a logistically distributed stochastic parameter

$$\varsigma \sim F_\varsigma(x) = \frac{1}{1 + e^{-x}}. \quad (3)$$

We assume a positive marginal utility that is decreasing in all arguments. Thus d_1 and d_2 are bounded to lie between zero and one:[4] $0 < d_j < 1,\ j = 1,2$. The smaller d_j is, the faster the marginal utility of good *j* decreases when X_j increases. Parameters a_1 and a_2 can be considered as shifting parameters, since they can move the indifference curves of the utility function along the *x*- and in the *y*-axis, respectively. Note that the marginal utility of X_1 is infinite if X_1 approaches $-a_1$, which is also true if X_2 approaches $-a_2$. The values $-a_1$ and $-a_2$ therefore define the lower limits of optimal solutions for X_1 and X_2 respectively. Since consumption basket X_1 contains essential goods such as food and housing, it must always be consumed. Therefore, a_1 is non-positive in order to ensure that the solution for X_1 is always positive. Following Bhat (2008)[5], we choose $a_1 = 0$.[6] Expression $\exp(m + \beta \cdot \varsigma)$ is a weight on $(X_2 + a_2)^{d_2}$. The higher $\exp(m + \beta \cdot \varsigma)$ is, the stronger is the preference for driving. This weight is determined by socio-demographic variables in *s* that influence the preference for driving, $m = \gamma \cdot s$. This means, for instance, that households in rural areas usually have a greater preference for driving than households in urban areas. If a household

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3 This utility function is based on the utility function proposed by Bhat (2005:686):

$$U = \sum_i \exp(m_i + \beta \cdot \xi_i) \cdot (X_i + a_i)^{d_i},$$

where the random terms are assumed to be iid Gumbel distributed: $\xi_j \sim iid\ gu(0,1)$, $f_\xi(x) = e^{-x} \cdot \exp(-e^{-x})$.

Transforming the utility function by multiplying by $\exp(m_i + \beta \cdot \xi_i)^{-1}$ yields Equation (1). Note that the stochastic component ς in (1) corresponds to $\varsigma = \xi_2 - \xi_1$ and is therefore logistically distributed (for a proof see Appendix A1). Note that we use capital letters for X_1 and X_2, because these variables are also stochastic since their solution in optimality will depend on the stochastic parameter ς.

4 This is to ensure decreasing marginal utility in both goods and the concavity of the utility function, see Appendix A2.

5 "Note that there is no translation parameter γ_k for the first good, because the first good is always consumed" Bhat (2008: 290). Note that γ_k, which Bhat uses, corresponds to α_k, which we use.

6 In this case, the so-called INADA-condition $\lim_{x_1 \to 0} \partial u(x_1, x_2, .., x_J) / \partial x_1 = \infty$ is fulfilled for x_1. It ensures that x_1 is greater than zero when solving the maximisation problem.

moves from an urban to a rural area, therefore, *m* is expected to increase in line with an increase in the household's preference for car driving. The random term ς represents socio-demographic variables $\tilde{s}$ that cannot be observed by the researcher and that can be interpreted as an unobserved preference for car driving. Following Bhat (2005), we assume this random term to be logistically distributed. Note that the preference for driving, *m*, does not contain any car-specific component, since the model captures only one car type, which is assumed to be the same for each household. To allow for a substantial simplification and to avoid identification problems, we choose to set: [7]

$$d = d_1 = d_2 \cdot \tag{4}$$

We assume that the household maximises its utility by selecting optimal values for X_1 and X_2, subject to its budget constraint:

$$y = p_1 \cdot X_1 + p_2 \cdot X_2 + I\left(X_2 > 0\right) \cdot k_2, \tag{5}$$

where k_2 stands for the fixed costs of car ownership, $I\left(X_2 > 0\right)$ is an indicator function that takes the value one if $X_2 > 0$ and zero otherwise, and the non-negativity constraint $X_2 \geq 0$.[8]

The household's utility for the case S_1, where only good one is consumed, is therefore

$$u_{S_1} = \left(\frac{y}{p_1} + a_1\right)^d + \exp\left(m + \beta \cdot \varsigma\right) \cdot \left(a_2\right)^d \text{, with } a_1 = 0\,. \tag{6}$$

The household's demand for car-km for the case S_2, where the households owns a car, is as follows:

$$x_2\left(y - k_2,\, p_1,\, p_2,\, A,\, a_2\right) = \frac{A \cdot \dfrac{y - k_2}{p_1} - a_2}{1 + A \cdot \dfrac{p_2}{p_1}} \text{ and } A = \left(\frac{p_1}{p_2} \cdot \exp\left(m + \beta \cdot \varsigma\right)\right)^{\frac{1}{1-d}}. \qquad (7)^9$$

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7 Bhat (2008) even proposes that some parameter values are fixed: "Alternatively, the analyst can stick with one functional form a priori, but experiment with various fixed values of a_k for the γ_k -profile [...]"; Bhat (2008: 282), footnote 9. The term "functional form" refers to the three utility functions (32) in Bhat (2008: 290). The so-called " γ_k -profile" corresponds to the model based on the third utility function of (32) in Bhat (2008: 290). The utility function (1) we use is a positively transformed function of that third utility function; we fix its parameter value $d = d_1 = d_2$ and estimate all other parameters.

8 Since $X_1 > 0$ is ensured by the choice of utility function, condition $X_2 \geq 0$ does not need to be stated.

9 This Marshallian demand function is obtained by solving the corresponding Lagrangian function. For details, see appendix A3.

Using this Marshallian demand function, we can now compute the maximum level of utility the household can achieve:

$$u_{S_2} = \left(\frac{y-k_2}{p_1} - \frac{p_2}{p_1} \cdot \frac{A \cdot \frac{y-k_2}{p_1} - a_2}{1 + A \cdot \frac{p_2}{p_1}} \right)^d + \exp\left(m + \beta \cdot \varsigma\right) \cdot \left(\frac{A \cdot \frac{y-k_2}{p_1} - a_2}{1 + A \cdot \frac{p_2}{p_1}} + a_2 \right)^d . \qquad (8)$$

By use of the utility functions (6) and (8), the value of the probability of a household choosing to own a car can be computed:

$$P\left(X_2 = 0 \mid \theta, p_1, p_2, y, k_2\right) = F_\varsigma\left(\varsigma_c\right), \qquad (9)$$

where $F_\varsigma(x)$ denotes the density function of the logistic distribution and $\varsigma_c = \varsigma_c\left(\theta, p_1, p_2, y, k_2\right)$ corresponds to the so-called "critical" unobserved preference given all parameters and economic variables at which the household would switch from owning a car to being carless, $u_{S_2} - u_{S_1} \big|_{\varsigma \geq \varsigma_c} \geq 0$ and $u_{S_2} - u_{S_1} \big|_{\varsigma < \varsigma_c} < 0$.[10]

The density of the Marshallian demand can be computed using the first-order conditions of the Lagrangian associated with the utility maximisation problem:[11]

$$f_{X_2 \wedge (X_2 > 0)}\left(z \mid \theta,\, p_1,\, p_2,\, y - k_2, s\right) = \frac{1}{\beta} \cdot f_\varsigma\left(\frac{V_1 - V_2}{\beta}\right) \cdot \left(\frac{1-d}{\frac{y - k_2 - p_2 z}{p_1} + a_1} \cdot \frac{p_2}{p_1} + \frac{1-d}{z + a_2} \right), \text{with } a_1 = 0 \quad (10)$$

where

$$V_1 = \ln(d) - \ln(p_1) - (1-d) \cdot \ln\left(\frac{y - k_2 - p_2 z}{p_1}\right), \qquad (10a)$$

$V_2 = \ln(d) - \ln(p_2) - m - (1-d) \cdot \ln(z + a_2)$, with $m = \gamma \cdot s$, and (10b)

$$f_\varsigma(x) = \frac{e^{-x}}{\left(1 + e^{-x}\right)^2} \qquad (10c)$$

is the density of the logistically distributed random term ς.

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10 Note that the Marshallian demand (7) at ς_c is always greater than zero and that ς_c is always unique. For proof, see Appendix A5.

11 For details, see Appendix A3.

Since we assume that the random terms ς are independent across households, the Maximum Likelihood function is thus:

$$L_{MLE}\left(\left(X_2 = x_{2n}\right)_{n=1,2,..,N} \mid \theta,\ p_1,\ p_{2n=1,2,...,N}, y_{n=1,2,...,N}, k_2, s_{n=1,2,..,N}\right)$$

$$= \prod_{n=1}^{N} P\left(x_{2n} = 0 \mid \theta,\ p_1,\ p_{2n}, y_n, k_2, s_n\right)^{I(x_{2n}=0)} \cdot \prod_{i=n}^{N} f_{X_2 \wedge (X_2>0)}\left(x_{2n} \mid \theta,\ p_1,\ p_{2n}, y_n - k_2, s_n\right)^{I(x_{2n}>0)}, \quad (11)$$

where $I(z>0)$ and $I(z=0)$ are indicator functions, being one when the argument is true and zero otherwise. Vector θ contains all parameters, $\theta = \{d, a_2, \gamma, \beta\}$. Probability $P(\bullet)$ is defined in (9) and density $f_{X_2 \wedge (X_2>0)}(\bullet)$ in (10). Index *n* corresponds to the *n*-th observation in the dataset.

It is important to note that this likelihood function is only defined for values $x_{2n} = 0$ and $x_{2n} \geq x_2\left(\varsigma_{c,n}\right)$, since for values x_{2n} in interval $0 < x_{2n} < x_2\left(\varsigma_{c,n}\right)$ the probability of observation is zero. Thus as soon as any observation belongs to the interval $0 < x_{2n} < x_2\left(\varsigma_{c,n}\right)$, the likelihood function (11) equals zero, which makes it impossible to compute optimal parameters θ using the Maximum Likelihood Estimation routine. We thus propose an estimation routine where all observations $0 < x_{2n} < x_2\left(\varsigma_{c,n}\right)$ are removed from the dataset before we apply the Maximum Likelihood Estimation routine. Since the value $x_2\left(\varsigma_{c,n}\right)$ only depends on parameters a_2 and *d*,[12] parameters γ and β can then be computed by Maximum Likelihood estimation using the modified dataset and given parameters a_2 and *d*. Since both a_2 and *d* also influence the shape of the density function (10) as well as the probability that a household is carless (9), we cannot set them arbitrarily. For this reason, we propose minimising the following "penalty function" for choosing optimal values for parameters a_2 and *d*:

$$Q = \left(\frac{P_{sim}(X) - \text{mean}(x=0)}{\text{mean}(x=0)}\right)^2 + c_1 \cdot \left(\frac{E_{sim}(X) - \text{mean}(x)}{\text{mean}(x)}\right)^2 + c_2 \cdot \left(\frac{\#\text{ elim. observations}}{\text{size of initial observations}}\right)^2, \quad (12)$$

where c_1 and c_2 weigh the corresponding error components and $P_{sim}\left(X_2 = 0\right)$ and $E_{sim}\left(X_2\right)$ are defined as follows:

$$P_{sim}\left(X_2 = 0\right) = \frac{1}{N} \cdot \sum_{n=1}^{N} P\left(X_2 = 0 \mid \theta, p_{1n}, p_{2n}, y_n, k_2, s_n\right), \quad (13)$$

$$E_{sim}\left(X_2\right) = \frac{1}{N} \cdot \sum_{n=1}^{N} \int_{z=x_2\left(\varsigma_{c,n}\right)}^{z=y_n-k_2} z \cdot f_{X_2 \wedge (X_2>0)}\left(z \mid \theta,\ p_{1n},\ p_{2n},\ y_n - k_2, s_n\right) dz\ . \quad (14)$$

12 For proof, see Appendix A6.

Therefore, we propose the following estimation routine:

1. Choose values for d and a_2 .
2. Eliminate all observations with $0 < x_{2n} < x_2\left(\varsigma_{c,n}\right)$ from the dataset.
3. Estimate parameters γ by MLE conditional on d and a_2 using (11).
4. Compute the penalty function (12).
5. Repeat steps 1 to 4 for a number of different values for d and a_2 (grid search).
6. Choose values d and a_2 so that the lowest value of the penalty function is yielded ("optimal values" d and a_2).

Note that functions (13) and (14) are also used to compute aggregate impacts on driving demand and the probability of being carless when the economic variables p_2 , k_2 and y change, e.g. these functions will be used to compute the corresponding elasticities.

Empirical Results

We obtained results using the micro-census data about travel behaviour in Switzerland collected in 2005 by the Swiss Federal Statistical Office SFSO.[13] We chose this data because it contains a large number of observations, namely 33,000, and a number of useful socio-demographic variables concerning the households. Since our model captures only one car type, it is considered to be an "average car". The fixed costs of maintaining a car and the marginal costs of driving are thus assumed to be equal to those of an average car owned by a Swiss household. The values we retain for k_2 and p_2 were taken from the Swiss touring club TCS (2007) and comprise:[14]

$$k_2 = 7033 \text{ and } p_2 = 0.1601 + 0.0778 \cdot p_{fuel} . \tag{15}$$

Note that the annual fixed costs k_2 mainly consist of depreciation, which is unrelated to the car's use, such as rusting, and loss in value due to the technical progress of new cars, capital costs, taxes on car ownership and parking costs. Since we neglect such costs as evaluation and registration costs, we assume that owing a car is similar to renting a car and that households can switch from owning a car to being carless without any cost. The costs dependent on the number of kilometres driven consist of fuel costs $0.0778 \cdot p_{fuel}$ and non-fuel-related costs such as the wear of tyres and mechanical components, which account for CHF 0.1601 per kilometre. The fuel price p_{fuel} is the average fuel price from the last twelve months prior to interviewing the household to which the information on annual driving distance refers.[15] To explain the deterministic component of the preference for driving *m*, we used a dummy "rural" standing for the type of the households' location and a the number of people living in the individual households.

Table 1 below shows the results for two cases. In the case denoted as "∞", expectation value $E_{sim}(X_2)$ is computed according to (14); in the case of "60,000 km" the upper limit "$(y_n - k_2)/p_{2n}$" is replaced by "$\min((y_n - k_2)/p_{2n}, 60{,}000\text{km})$". We believe that the latter produces more realistic results since the theoretical density (10) has some tail above this value, while as the empirical distribution does not, since households simply have not time to drive such long distances. Thus, integrating to an upper limit above 60,000 km when computing (14) would simply result in too high and therefore unrealistic values.[16] Hence,

13 For details see SFSO (2006a) and SFSO (2006b).

14 According to TCS (2007), the total annual costs of an average car amounted to CHF 11,600 when the annual distance driven was 15,000 kilometres (km). 17.4% of these costs, namely CHF 2,018.4, were fuel costs. Based on the average fuel price paid for petrol 98 octane of CHF 1.729/litre in 2007 (SFSO 2009), it can be computed that the TCS (2007) based this fuel cost on a fuel consumption of 7.7825 litres/100 km: (CHF 2,018.4/15,000 km) / (CHF 1.729/litre) = 7.7825 litres/100 km. The fuel costs of an average car per kilometre are therefore 7.7825 litres/100 km/100 multiplied by the fuel price per litre paid by households. Non-fuel-related marginal costs of a car were calculated to be 20.7% of the total costs, 0.207 · CHF 11,600 = CHF 3,312, amounting to CHF 3,312/15,000 km = CHF 0.1601/km, see TCS (2007).

15 The computation of p_{fuel} is based on the monthly average price of petrol 98 octane, as published by the SFSO (2009a).

16 This argument is discussed more in detail in Appendix A7.

we believe it is justifiable to restrict the upper limit of the integral to this value. For the penalty function (12), we choose arbitrarily $c_1 = 1$ and $c_2 = 0.5$.[17] However, it is important to note that changing parameters c_1 and c_2 only has a limited impact on the measures of interest, namely elasticities.[18] The results processed for the aforementioned datasets are as follows:

Upper limit of integrating, $E(X_2)$	∞	**60,000 km**
$\varepsilon_{E(X_2),p_2}$	− 1.07 (0.0069)	− 0.67 (0.0075)
$\varepsilon_{E(X_2),p_{fuel}}$	− 0.49 (0.0031)	− 0.28 (0.0031)
$\varepsilon_{E(X_2),y}$	1.17 (0.0031)	0.75 (0.0069)
$\varepsilon_{E(X_2),k_2}$	− 0.16 (0.0030)	− 0.16 (0.0029)
$\varepsilon_{P(X_2=0),p_2}$	0.24 (0.0052)	0.24 (0.0049)
$\varepsilon_{P(X_2=0),p_{fuel}}$	0.14 (0.00005)	0.11 (0.0028)
$\varepsilon_{P(X_2=0),y}$	− 1.42 (0.0087)	− 1.41 (0.0081)
$\varepsilon_{P(X_2=0),k_2}$	1.27 (0.0102)	1.33 (0.0111)

The values in parentheses "(...)" represent standard deviations computed using the bootstrapping method with 10 random samples of 200 obs. each.

Table 1: Simulated elasticities when using a modified density function to compute the expectation value.[19]

The results yielded by the model for the fuel price elasticities of travelling demand $\varepsilon_{E(X_2),p_{fuel}}$ are of major interest. Since our model assumes no costs when switching from owning a car to being carless and vice versa, our elasticities can be interpreted as long-term fuel price elasticities. These correspond approximately to average values determined in international studies (-0.31), such as in Graham and Claister (2004). The income elasticity of aggregate driving we obtained (0.77) is also very close to the average values established in international studies (0.73) by both Graham and Claister (2005) and Goodwin et. al. (2004). In contrast, both values $\varepsilon_{P(X_2>0),p_{fuel}} = 0.026$ and $\varepsilon_{P(X_2>0),y} = 0.33$ that can be computed from $\varepsilon_{P(X_2=0),p_{fuel}}$ and $\varepsilon_{P(X_2=0),k_2}$ are quite smaller in absolute value than the elasticities of the car stock determined in international studies.[20] We explain this difference by the fact that our elasticities refer to the case of "at least one car" and the

17 Note, that choosing arbitrarily $c_1 = 1$ and $c_2 = 0.5$ yields that in the optimum the number of "irrational" observations that are removed account for about 9% of the total observations. We propose not to choose values lower than 0.5 for c_2, since this would lead to a "dropout-rate" of observations of more than 9%, which we would consider a too high. Note, that removing these observations should not induce a significant change in the elasticities of driving demand we compute, since these households drive a very low annual mileage and thus do not contribute much to the aggregate driving distance.

18 We applied values of 0.5, 1.0 and 2.0 in various combinations on both parameters c_1 and c_2. Despite this quite dramatic change in the parameters of the penalty function, the resulting values of $\varepsilon_{E(X_2),p_2}$ remained in the region of about 20% of its absolute value, meaning $\left(\max\left(\varepsilon_{E(X_2),p_2}\right) - \min\left(\varepsilon_{E(X_2),p_2}\right)\right) \Big/ \left(\left(\max\left(\varepsilon_{E(X_2),p_2}\right) + \min\left(\varepsilon_{E(X_2),p_2}\right)\right) \cdot 0.5\right) = 0.12$, while the same measure for $\varepsilon_{P(X_2),p_2}$ amounts to 0.2 and for both $\varepsilon_{P(X_2=0),p_2}$ and $\varepsilon_{P(X_2=0),y}$ to 0.02.

19 The point estimates are based are based on the complete dataset.

income elasticity for buying a second or even a third car can be assumed to be greater since the latter can be considered a luxury good. In contrast, $\varepsilon_{P(X_2>0),k_2} = 0.31$ is quite similar to the values determined by Dargay (2001) and Johansson and Shipper (1997) for the elasticity of the car stock with respect to the car's fixed costs.[21] However, it is also important to note that the results found in international studies for the elasticity of car ownership vary greatly and thus it is hard to judge whether the values a model yields are plausible.

Furthermore, our model also yields that both elasticities $\varepsilon_{E(X_2),p_{fuel}}$ and $\varepsilon_{E(X_2),y}$ are only weakly driven by households that switch to being carless, despite them switching from an annual mileage of about 5,000 km to zero – according to the model.[22]

Finally, an important result of our model is also that the effect of a tax on car ownership on aggregate driving distance is – per unit of tax revenue – more than ten times weaker than the effect of a tax on car ownership.[23]

One criticism of calibrating the model and producing these results by using the micro-census dataset of the SFSO 2005 is that the fuel price does not vary enough across households. For this reason,we also calibrate the model by using stated preference datasets with a large variation in fuel price.[24] It is important to note that all elasticities with respect to the aggregate driving demand produced by using this dataset differ at most by 13% in absolute terms from the results produced by the micro-census dataset of the SFSO 2005 as presented in table 1.

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20 Note, that $\varepsilon_{P(X_2>0),p_{fuel}} = \dfrac{\partial P(X_2>0)}{\partial p_{fuel}} \cdot \dfrac{p_{fuel}}{P(X_2>0)} = -\dfrac{\partial P(X_2=0)}{\partial p_{fuel}} \cdot \dfrac{p_{fuel}}{P(X_2=0)} \cdot \dfrac{P(X_2=0)}{P(X_2>0)} = -\varepsilon_{P(X_2=0),p_{fuel}} \cdot \dfrac{P(X_2=0)}{P(X_2>0)} =$ $= -1.33 \cdot \dfrac{0.19}{81} = 0.31$, where for $P(X_2=0)$ we use the value of the dataset from which the observations $0 < x_2 < X_2(\varsigma_c)$ and $x_2 > 60{,}000\,\text{km}$ were removed.

21 The only study in which we could find a model where the effect of a tax on car ownership was examined was in Johansson and Shipper (1997). In their model, this tax was imposed by a tax on car purchase. Annualising one unit of this tax yields an increase in the fixed costs of car ownership of about 2%, yielding a 0.6% decrease in car stock. Thus, a 1% increase in fixed costs would reduce the vehicle stock by 0.3%.

22 This effect contributes only about 2.5% to the total effect on aggregate demand in the case of $\varepsilon_{E(X_2),p_{fuel}}$ and 11.5% in the case of $\varepsilon_{E(X_2),y}$.

$$\frac{dx_2}{dp_2/p_2} = \varepsilon_{E(X_2),p_{fuel}} \cdot \text{mean}(x_2) = 0.28 \cdot 13{,}890\,\text{km} = 3889.2\,\text{km}, \quad \frac{dx_{2,P(X_2=0)effect}}{dp_2/p_2} = \frac{dP(X_2=0)}{dp_2/p_2} \cdot x_2(\varsigma_c) = \frac{dx_2}{dp_2/p_2} =$$
$$= \varepsilon_{P(X_2=0),p_{fuel}} \cdot P(X_2=0) \cdot x_2(\varsigma_c) = 0.10 \cdot 0.1890 \cdot 5000 = 94.5\,\text{km}. \;\; 94.5/3889.2 = 2.4\%.$$
$$\frac{dx_2}{dy/y} = \varepsilon_{E(X_2),y} \cdot \text{mean}(x_2) = 0.80 \cdot 13{,}890\,\text{km} = 11{,}112\,\text{km}, \quad \frac{dx_{2,P(X_2=0)effect}}{dy/y} = \frac{dP(X_2=0)}{dy/y} \cdot x_2(\varsigma_c) = \frac{dx_2}{dy/y} =$$
$$= \varepsilon_{P(X_2=0),y} \cdot P(X_2=0) \cdot x_2(\varsigma_c) = 0.80 \cdot 0.1890 \cdot 5000 = 1275.8\,\text{km}. \;\; 1275.8/11{,}112 = 11.5\%.$$

23 The relative effect can be computed as follows:
$\varepsilon_{E(X_2),p_2} / \varepsilon_{E(X_2),k_2} \cdot k_2 / \left(\text{mean}(x_2) \cdot \text{mean}(p_2 - 0.1601)\right) = \approx 0.26 / \approx 0.11 \cdot 7{,}033\,\text{km} / \left(13{,}890\,\text{km} \cdot (0.2745 - 0.1601)\right) = 10.5.$

24 We used the same dataset as Axhausen and Erath (2010). We gratefully thank Prof. Kay Axhausen and Dr. Alexander Erath for providing their dataset.

Conclusion

In contrast to currently existing models, ours is able to quantify the effects of a tax on fuel *and/or* a tax on car ownership on both the car ownership *and* the cars' use. Our model made it also possible to measure the effects of two mechanisms leading to a decrease in aggregate driving distance when the fuel price is increased, namely: The first one is determined by households with a rather high preference for car driving that will keep the car, but they will reduce their annual mileage. The second mechanism is determined by households with a rather low preference for car driving will switch form owning a car to become carless and therefore reducing their annual mileage form about at least 5,000km per year to zero. Our model shows, that the effect of the first mechanism dominates the one of the second, since only a few households will sell their car, if fuel prices increase.

Furthermore, the model made it possible to show that a tax on car ownership is – per unit of tax revenue – much less effective as a tax on fuel. It is noteworthy that the model adapts the data very well, even though we only estimate four parameters.[25]

The fact that the model contains a utility function opens the way for more applications such as computing the Hicksian compensating variation when fuel prices increase for each household or the household's willingness to pay for car ownership.

25 For details see Appendix A7.

References

Axhausen, Kay W. and Alexander Erath, 2010, "Long term fuel price elasticity: Effects on mobility tool ownership and residential location choice", Bundesamt für Energie, 2010, Publikation 29015. http://www.bfe.admin.ch/php/modules/enet/streamfile.php?file=000000010334.pdf&name=000000290158

Bhat, Chandra R., 2005, "A multiple discrete-continuous extreme value model: formulation and application to discretionary time-use decisions", Transportation Research Part B 39 (2005), 679-707.

Bhat, Chandra R. and Sudeshna Sen, 2006, "Household vehicle type holdings and usage: an application of the multiple discrete-continuous extreme value (MDCEV) model", Transportation Research Part B 40 (2006) 35–53.

Bhat, Chandra R., 2008. "The multiple discrete-continuous extreme value (MDCEV) model: role of utility function parameters, identification considerations, and model extensions", Transportation Research Part B 42 (2008) 274–303.

Bjørner, Thomas Bue, 1999, "Demand for Car Ownership and Car Use in Denmark: A Micro Econometric Model", International Journal of Transport Economics, Vol. XXVI, No. 3, pp. 377-395.

De Jong, G. C., 1990, "An indirect utility model of car ownership and private car use", European Economic Review, Elsevier, Vol. 34(5), 971-985, July.

Dubin, Jeffrey A. and Daniel L. McFadden, 1984, "An Econometric Analysis of Residential Electric Appliance Holdings and Consumption", Econometrica, Vol. 52, No. 2 (Mar., 1984), 345-362.

Goodwin, P and Dargay, J and Hanly, M., 2004, "Elasticities of road traffic and fuel consumption with respect to price and income: a review", Transport Reviews, 24 (3) , 275-292.

Graham, Daniel and Stephen Glaister, 2001, "Review of income and price elasticities of demand for road traffic", Final Report. Centre for Transport Studies, Imperial College of Science, Technology and Medicine.

Graham, D.J. and Glaister, S., 2004, "A Review of Road Traffic Demand Elasticity Estimates, Transport Reviews", 24 (3), 261-276.

Johansson, Olof and Lee Schipper, 1997, "Measuring the Long-Run Fuel Demand of Cars", Journal of Transport Economics and Policy, September 1997.

Ramjerdi, F. and L. Rand, 1992, "The National Model System for Private Travel", Institute of Transport Economics. TØI rapport 150/1992. Oslo.

Swiss Federal Statistical Office SFSO, 2009a, "Konsumentenpreisindex, LIK, Durchschnittspreise für Benzin und Diesel, Monatswerte", Swiss Federal Statistical Office SFSO, Neuenburg (Switzerland) 2009. http://www.bfs.admin.ch/bfs/portal/de/index/themen/05/02/blank/key/durchschnittspreise.Document.88015.xls

Swiss Federal Statistical Office SFSO, 2006a, "Mikrozensus 2005 zum Verkehrsverhalten", Swiss Federal Statistical Office SFSO, Neuenburg (Switzerland) 2006.

Swiss Federal Statistical Office SFSO, 2006b, "Mikrozensus 2005 zum Verkehrsverhalten, Kurzversion Fragebogen (Hauptbefragung)", Swiss Federal Statistical Office SFSO, Neuenburg (Switzerland) 2006. http://www.portal-stat.admin.ch/mz05/files/de/00.xml

Touring Club der Schweiz (TCS), 2007, "Kosten eines Musterautos". http://www.tcs.ch/main/de/home/auto_moto/kosten/kilometer/musterauto.html

Appendix

A1: The distribution of the random term ς of the utility function

As mentioned in footnote 1, the random term is equal to the difference of the two iid gumbel distributed random variables ξ_1 and ξ_2, $\xi_1, \xi_2 \sim iid\ f_\xi(x) = e^{-x} \cdot \exp\left(-e^{-x}\right)$, $\varsigma = \xi_1 - \xi_2$.

The cumulative density function (cdf) of ς can be computed as follows:

First, given that the cumulated density function (cdf) of $F_\varsigma(y)$ is equivalent to $F_\varsigma(y) = P(X_1 - X_2 < y)$, thus $F_\varsigma(y)$ can then be computed as follows:

$$F_\varsigma(y) = P(X_1 - X_2 < y) = P(X_1 < y + X_2) = \int_{x_2=-\infty}^{x_2=\infty} \int_{x_1=-\infty}^{x_1=y+x_2} f(x_1, x_2)\, dx_1 dx_2 = \int_{x_2=-\infty}^{x_2=\infty} \int_{x_1=-\infty}^{x_1=y+x_2} f_\xi(x_1) \cdot f_\xi(x_2)\, dx_1 dx_2 =$$

$$= \int_{x_2=-\infty}^{x_2=\infty} f_\xi(x_2) \cdot \int_{x_1=-\infty}^{x_1=y+x_2} f_\xi(x_1)\, dx_1 dx_2 = \int_{x_2=-\infty}^{x_2=\infty} f_\xi(x_2) \cdot F_\xi(y + x_2)\, dx_2 = \int_{x_2=-\infty}^{x_2=\infty} e^{-x_2} \cdot \exp\left(-e^{-x_2}\right) \cdot \exp\left(-e^{-(x_2+y)}\right) dx_2 =$$

$$= \int_{x_2=-\infty}^{x_2=\infty} e^{-x_2} \cdot \exp\left(-e^{-(x_2+y)} - e^{-x_2}\right) dx_2 = \int_{x_2=-\infty}^{x_2=\infty} e^{-x_2} \cdot \exp\left(-e^{-x_2} \cdot \left(e^{-y} + 1\right)\right) dx_2 = \int_{x_2=-\infty}^{x_2=\infty} e^{-x_2} \cdot \exp\left(-e^{-x_2} \cdot e^{\ln\left(e^{-y}+1\right)}\right) dx_2 =$$

$$= \int_{x_2=-\infty}^{x_2=\infty} e^{-x_2} \cdot \exp\left(-e^{-\left(x_2 - \ln\left(e^{-y}+1\right)\right)}\right) dx_2.$$

This expression can be reformulated by substituting $q = x_2 - \ln\left(e^{-y} + 1\right)$, $dq = dx_2$, $x_2 = q + \ln\left(e^{-y} + 1\right)$:

$$F_\varsigma(y) = \int_{q=-\infty}^{q=\infty} e^{-q-\ln\left(e^{-y}+1\right)} \cdot \exp\left(-e^{-q}\right) dq = e^{-\ln\left(e^{-y}+1\right)} \cdot \int_{q=-\infty}^{q=\infty} e^{-q} \cdot \exp\left(-e^{-q}\right) dq = \frac{1}{e^{-y}+1} \cdot \int_{q=-\infty}^{q=\infty} f_\xi(q)\, dq = \frac{1}{1+e^{-y}}.$$

A2: Concavity of the utility function

In the following we show that the utility function we use is concave. To do so we show that the Hessian matrix is negative (semi-)definite. We first compute its diagonal elements:

$$\frac{\partial^2 U}{\partial X_j^{\,2}} = d_j \cdot \left(d_j - 1\right) \cdot \exp\left(m_j + \xi_j\right) \cdot \left(X_j + a_j\right)^{d_j - 2} < 0, \text{ if and only if } 0 < d_j < 1.$$

Since the non-diagonal elements are zero, the Hessian matrix is negative (semi-)definite and therefore the utility function is concave:

$$\begin{vmatrix} \dfrac{\partial^2 U}{\partial X_1^{\,2}} & \dfrac{\partial^2 U}{\partial X_1 \partial X_2} \\ \dfrac{\partial^2 U}{\partial X_1 \partial X_2} & \dfrac{\partial^2 U}{\partial X_2^{\,2}} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial^2 U}{\partial X_1^{\,2}} & 0 \\ 0 & \dfrac{\partial^2 U}{\partial X_2^{\,2}} \end{vmatrix} = \frac{\partial^2 U}{\partial X_1^{\,2}} \cdot \frac{\partial^2 U}{\partial X_2^{\,2}} > 0 \text{ and } \frac{\partial^2 U}{\partial X_j^{\,2}} < 0, \text{ if and only if } 0 < d_j < 1,\ j = 1,2 \text{ and}$$

$X_1 > -a_1$ and $X_2 > -a_2$.

The term $\dfrac{\partial^2 U}{\partial X_1 \partial X_2}$ is equal to zero because the utility function is of the additive separable type.

A3: Derivation of the Marshallian demand function and its probability density function

The derivation of the Marshallian demand function as well as of its probability density function is based on solving the Lagrangian function representing the case, where the households owns a car.

$$L = \left(X_1 + a_1\right)^d + \exp\left(m + \varsigma\right) \cdot \left(X_2 + a_2\right)^d + \lambda\left(y - k_2 - p_1 X_1 - p_2 X_2\right), \text{ with } a_1 = 0 . \qquad \text{(A3.1)}$$

The corresponding first-order conditions are as follows:

$$d \cdot \frac{1}{\left(X_1 + a_1\right)^{1-d}} - \lambda \cdot p_1 = 0 , \qquad \text{(A3.2)}$$

$$d \cdot \exp\left(m + \varsigma\right) \cdot \frac{1}{\left(X_2 + a_2\right)^{1-d}} - \lambda \cdot p_2 = 0 , \text{ with } m = \gamma \cdot s . \qquad \text{(A3.3)}$$

We first derive the Marshallian demand function. To do so, we solve (A3.2) for λ, insert the result in (A3.3) and reformulate in order to get the resulting expression:

$$\frac{X_2+a_2}{X_1+a_1}=\exp\left(\frac{m+\varsigma}{1-d}\right)\cdot\left(\frac{p_1}{p_2}\right)^{\frac{1}{1-d}}. \tag{A3.4}$$

From the budget restriction follows that

$$X_1=\left(y-k_2-p_2\cdot X_2\right)/p_1. \tag{A3.5}$$

Including this expression in (A3.4) and solving for X_2 yields the Marshallian demand function:

$$X_2=x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)=\frac{A\cdot\frac{y-k_2}{p_1}-a_2+A\cdot a_1}{1+A\cdot\frac{p_2}{p_1}}, \qquad \text{(A3.6)[1]}$$

with $A=\left(\frac{p_1}{p_2}\cdot\exp\left(m+\beta\cdot\varsigma\right)\right)^{\frac{1}{1-d}}$.

Note that $x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)$ depends on the random term ς and there exists a value $\varsigma=\varsigma_0$ such that

$$x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)\big|_{\varsigma\le\varsigma_0}\le 0 \text{ and } x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)\big|_{\varsigma>\varsigma_0}>0. \qquad \text{(A3.7)[2]}$$

Secondly, we derive the probability density function of the Marshallian demand function. To do so, we start by solving each of the first order conditions (A3.2) and (A3.3) for λ and then taking the logs:

$$V_1=\ln(\lambda)\text{, with: } V_1=\ln(d)-\ln(p_1)-(1-d)\cdot\ln(X_1+a_1) \text{ and } X_1=\left(y-k_2-p_2\cdot X_2\right)/p_1 \tag{A3.8}$$

$$V_2+\varsigma=\ln(\lambda)\text{, with: } V_2=\ln(d)-\ln(p_2)+m-(1-d)\cdot\ln(X_2+a_2). \tag{A3.9}$$

Plugging (A3.8) in (A3.9) and solving for ς yields:

$$\varsigma=V_1-V_2. \tag{A3.10}$$

From this follows

$$P\left(\varsigma<V_1-V_2\right)=F_\varsigma\left(V_1-V_2\right). \tag{A3.11}$$

1 Note that in the case of $d_1\neq d_2$ the expression resulting when plugging the expression (A3.5) in (A3.4) could not been solved explicitly for X_2.

2 For a proof, see Appendix A4.

We can now compute the density of driving demand at a given driving distance x_2 by deriving (A3.11) with respect to x_2 :[3]

$$f_{X_2 \wedge (X_2>0)}\left(z \mid \theta, p_1, p_2, y, s\right) = f_\varsigma\left(V_1 - V_2\right) \cdot \left(\frac{1-d}{\frac{y-p_2 z}{p_1} + a_1} \cdot \frac{p_2}{p_1} + \frac{1-d}{z+a_2} \right), \tag{A3.12}$$

where V_1 and V_2 are given by (A3.8) and (A3.9), $\theta = \{d, a_2, m, \beta\}$ and $f_\varsigma(x)$ is the probability density function (pdf) of the logistic distribution, $f_\varsigma(x) = \frac{\exp\left(e^{-x}\right)}{\left(1+\exp\left(e^{-x}\right)\right)^2}$.

A4: Boundary solution in the case of the model with fixed costs

In this section, we shall show that there exists a so called critical relative preference expressed by parameters $m+\varsigma$. If the relative preference is below this level, the demand for car driving yields a boundary solution that means the driving distance is zero. If the relative preference is above this level, the driving demand will be positive. This means, that – given a fixed value for m – a value $\varsigma = \varsigma_0$ exists such that

$$x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)\big|_{\varsigma \le \varsigma_0} \le 0 \text{ and } x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)\big|_{\varsigma > \varsigma_0} > 0\,, \tag{A4.1}$$

where

$$x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right) = \frac{A \cdot \frac{y-k_2}{p_1} - a_2}{1 + A \cdot \frac{p_2}{p_1}} \text{ and } A = \left(\frac{p_1}{p_2} \cdot \exp\left(m + \beta \cdot \varsigma\right) \right)^{\frac{1}{1-d}}.$$

Since this is not obvious when looking only at the utility function and the budget restriction, we prove (A4.1). Note that the value $\varsigma = \varsigma_0$ also plays a role when we show in appendix A5 that ς_0 is always smaller than the critical preference at which the household switch from owning a car to being carless in the case where owning a car is connected with fixed costs.

3 For this case, the result can also be computed as follows. From $P\left(\varsigma \le V_1 - V_2\right) = F_\varsigma\left(V_1 - V_2\right)$ it follows that:
$f_{X_2 \wedge (X_2>0)}(z) = \frac{\partial F_\varsigma\left(V_1-V_2\right)}{\partial\left(V_1-V_2\right)} \cdot \frac{\partial\left(V_1-V_2\right)}{\partial z} = f_\varsigma\left(V_1-V_2\right) \cdot \left(\frac{\partial V_1}{\partial X_2} \cdot \frac{\partial X_2}{\partial z} - \frac{\partial V_2}{\partial z} \right)$, where $\frac{\partial V_1}{\partial X_1} \cdot \frac{\partial X_1}{\partial z} = -\frac{1-d}{x_1+a_1} \cdot \frac{-p_1}{p_2} > 0$, $\frac{\partial V_2}{\partial z} = -\frac{1-d}{z+a_2} < 0$ and $x_1 = \frac{y - p_2 \cdot z}{p_1}$. Note that the expression $\frac{\partial V_1}{\partial X_2} \cdot \frac{\partial X_2}{\partial z} - \frac{\partial V_2}{\partial z}$ is positive for any value z that is in the feasible range $0 \le z < y/p_2$. Note that this is a necessary condition for the validity of the theorem of densities of transformed variables.

The proof that (A4.1) is correct follows from these conditions:

i. $\lim\limits_{\varsigma\to-\infty} x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)\le 0$,

ii. $\lim\limits_{\varsigma\to\infty} x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)>0$,

iii. $\dfrac{\partial x_2\left(y-k_2,\ p_1,\ p_2,\ A,\ a_1,\ a_2\right)}{\partial A}\cdot\dfrac{\partial A}{\partial\varsigma}>0$.

Thus, only conditions i., ii. and iii. need to be verified.
The proof of condition i. follows from

$\lim\limits_{\varsigma\to-\infty} A=\lim\limits_{\varsigma\to-\infty}\left(\dfrac{p_1}{p_2}\cdot\exp\left(m+\beta\cdot\varsigma\right)\right)^{\frac{1}{1-d}}=0$. This implies that $\lim\limits_{\varsigma\to-\infty} x_2\left(\bullet\right)=\lim\limits_{\varsigma\to-\infty}\dfrac{A\cdot\dfrac{y-k_2}{p_1}-a_2}{1+A\cdot\dfrac{p_2}{p_1}}=-a_2<0$.

The proof of condition ii. follows from

$\lim\limits_{\varsigma\to\infty} A=\lim\limits_{\varsigma\to\infty}\left(\dfrac{p_1}{p_2}\cdot\exp\left(m+\beta\cdot\varsigma\right)\right)^{\frac{1}{1-d}}=\infty$.

This implies that $\lim\limits_{\varsigma\to-\infty} x_2\left(\bullet\right)=\lim\limits_{\varsigma\to-\infty}\dfrac{A\cdot\dfrac{y-k_2}{p_1}-a_2}{1+A\cdot\dfrac{p_2}{p_1}}=\dfrac{A\cdot\dfrac{y-k_2}{p_1}}{A\cdot\dfrac{p_2}{p_1}}=\dfrac{y-k_2}{p_2}<0$.

This result is rather intuitive. If $\varsigma\to\infty$, this means that the household has a very strong preference for car driving, and it is therefore plausible that it spends all income $y-k_2$ on car driving.
In order to prove condition iii., the derivative simply has to be computed. This yields:

$$\frac{\partial x_2\left(\bullet\right)}{\partial\varsigma}=\frac{\partial\left(\left(A\cdot\dfrac{y-k_2}{p_1}-a_2\right)\cdot\left(1+A\cdot\dfrac{p_2}{p_1}\right)^{-1}\right)}{\partial\varsigma}=\frac{A'\cdot\dfrac{y-k_2}{p_1}\cdot\left(1+A\cdot\dfrac{p_2}{p_1}\right)-\left(A\cdot\dfrac{y-k_2}{p_1}-a_2\right)\cdot A'\cdot\dfrac{p_2}{p_1}}{\left(1+A\cdot\dfrac{p_2}{p_1}\right)^2}=$$

$$=A'\cdot\frac{\dfrac{y-k_2}{p_1}+A\cdot\dfrac{y-k_2}{p_1}\cdot\dfrac{p_2}{p_1}-A\cdot\dfrac{y-k_2}{p_1}\cdot\dfrac{p_2}{p_1}+a_2\cdot\dfrac{p_2}{p_1}}{\left(1+A\cdot\dfrac{p_2}{p_1}\right)^2}=A'\cdot\frac{\dfrac{y-k_2}{p_1}+a_2\cdot\dfrac{p_2}{p_1}}{\left(1+A\cdot\dfrac{p_2}{p_1}\right)^2}>0\text{, with}$$

$$A'=\frac{1}{1-d}\cdot\frac{p_1}{p_2}\cdot\left(\frac{p_1}{p_2}\cdot\exp\left(m+\beta\cdot\varsigma\right)\right)^{\frac{1}{1-d}-1}=\frac{1}{1-d}\cdot\frac{p_1}{p_2}\cdot\left(\frac{p_1}{p_2}\cdot\exp\left(m+\beta\cdot\varsigma\right)\right)^{\frac{d}{1-d}}=\frac{\beta}{1-d}\cdot\frac{p_1}{p_2}\cdot A^d>0.$$

A5: Minimal driving distance

As shown in this paper, there is a minimal driving distance x_{2c} associated with relative preference ς_c, so that $x_{2c} = x_2(\varsigma_c)$. We now prove that $\varsigma = \varsigma_c$ exists such that[4]

$$u_{S_2} - u_{S_1}\,|_{\varsigma \geq \varsigma_c} \geq 0 \text{ and } u_{S_2} - u_{S_1}\,|_{\varsigma < \varsigma_c} < 0. \tag{A5.1}$$

The proof that (A5.1) is correct follows from these conditions:

i. There exists a $u_{S_2} - u_{S_1}\,|_{\varsigma = \varsigma_c} < 0$,

ii. $\lim\limits_{\varsigma \to \infty} u_{S_2} - u_{S_1} > 0$,

iii. $\dfrac{\partial\left(u_{S_2} - u_{S_1}\right)}{\partial \varsigma} > 0,\ \varsigma \in (\varsigma_c, .., \infty]\cdot$

Therefore, only conditions i., ii., and iii. need to be verified.

We shall start by proving iii.

To compute $\dfrac{\partial\left(u_{S_2} - u_{S_1}\right)}{\partial \varsigma}$, we use formula (8):

$$\frac{\partial u_{S_2} - u_{S_1}}{\partial \varsigma} = \frac{\partial u_{S_2} - u_{S_1}}{\partial X_2} \cdot \frac{\partial X_2}{\partial \varsigma} + \frac{\partial u_{S_2} - u_{S_1}}{\partial \varsigma} = \ldots$$

$$\ldots = \left(d \cdot \left(\frac{y - k_2}{p_1} - \frac{p_2}{p_1} \cdot X_2 \right)^{d-1} \cdot \left(-\frac{p_2}{p_1} \right) + d \cdot \exp\left(m + \beta \cdot \varsigma\right) \cdot \left(X_2 + a_2\right)^{d-1} \right) \cdot \frac{\partial X_2}{\partial \varsigma} + \ldots$$

$$\ldots + \exp\left(m + \beta \cdot \varsigma\right) \cdot \left(X_2 + a_2\right)^d - \exp\left(m + \beta \cdot \varsigma\right) \cdot \left(a_2\right)^d. \tag{A5.1}$$

We then choose $\varsigma = \varsigma_0$, which corresponds to $x_2(\bullet) = 0$.[5] It follows from this that

$$\frac{\partial u_{S_2} - u_{S_1}}{\partial \varsigma}\,|_{\varsigma = \varsigma_0} = \left(d \cdot \left(\frac{y - k_2}{p_1} \right)^{d-1} \cdot \left(-\frac{p_2}{p_1} \right) + d \cdot \exp\left(m + \beta \cdot \varsigma\right) \cdot \left(a_2\right)^{d-1} \right) \cdot \frac{\partial X_2}{\partial \varsigma}. \tag{A5.2}$$

It also follows from the first-order conditions (A3.3) that

$$\frac{X_2 + a_2}{X_1} = \left(\frac{p_1}{p_2} \cdot \exp\left(m + \beta \cdot \varsigma\right) \right)^{\frac{1}{1-d}}, \tag{A5.3}$$

---

4 This statement is equivalent to (14).

5 For details see Appendix A4.

which we denote as A; see (A3.6).

Since we chose $X_2 = 0$, it follows that

$$\frac{p_1 \cdot a_2}{y-k_2} = \left(\frac{p_1}{p_2} \cdot \exp(m + \beta \cdot \varsigma) \right)^{\frac{1}{1-d}}. \qquad \text{(A5.4)}$$

Plugging this into (A5.2) yields

$$\frac{\partial u_{S_2} - u_{S_1}}{\partial \varsigma}\Big|_{\varsigma=\varsigma_0} = \ldots$$

$$\ldots = \left(d \cdot \left(a_2 \cdot \left(\frac{p_1}{p_2} \cdot \exp(m+\beta\cdot\varsigma) \right)^{-\frac{1}{1-d}} \right)^{d-1} \cdot \left(-\frac{p_2}{p_1} \right) + d \cdot \exp(m+\beta\cdot\varsigma)\cdot(a_2)^{d-1} \right) \cdot \frac{\partial X_2}{\partial \varsigma} = 0. \qquad \text{(A5.5)}$$

If any value $\varsigma > \varsigma_0$ that corresponds to $x_2(\bullet) > 0$ is plugged into (A5.5), derivative $\frac{\partial\left(u_{S_2} - u_{S_1}\right)}{\partial \varsigma}$ becomes greater than zero.

Proof:

If X_2 increases, then also both expressions $-\frac{p_2}{p_1} \cdot d \cdot \left(\frac{y-k_2}{p_1} - \frac{p_2}{p_1} \cdot X_2 \right)^{d-1} + d \cdot \exp(m+\beta\cdot\varsigma)\cdot(X_2 + a_2)^{d-1}$ and $\exp(m+\beta\cdot\varsigma)\cdot(X_2+a_2)^d - \exp(m+\beta\cdot\varsigma)\cdot(a_2)^d$ increase.

Since $\partial X_2 / \partial \varsigma > 0$ – as shown in Appendix A4 – and from (A3.5), it follows that $\frac{\partial\left(u_{S_2} - u_{S_1}\right)}{\partial \varsigma} > 0$ for all $\varsigma > \varsigma_0$.

We shall now prove i. The proof is straightforward: plugging $\varsigma = \varsigma_0$ that corresponds to $x_2(\bullet) = 0$ into (A5.2) yields[6]

$$u_{S_2} - u_{S_1} = \left(\frac{y-k_2}{p_1} \right)^d - \left(\frac{y}{p_1} \right)^d < 0. \qquad \text{(A5.6)}$$

6 Note that $u_{S_2} - u_{S_1} = \left(\frac{y-k_2}{p_1} \right)^d + \exp(m+\beta\cdot\varsigma)\cdot(a_2)^d - \left(\frac{y}{p_1} \right)^d - \exp(m+\beta\cdot\varsigma)\cdot(a_2)^d = \left(\frac{y-k_2}{p_1} \right)^d - \left(\frac{y}{p_1} \right)^d < 0$.

The proof of ii. is also straightforward: Plugging $\varsigma = \infty$ that corresponds to $x_2(\bullet) = \frac{y-k_2}{p_2}$ – as shown in Appendix A4 – in (A5.2) yields[7]

$$\lim_{\varsigma\to\infty} u_{S_2} - u_{S_1} = \lim_{\varsigma\to\infty} \exp(m+\varsigma)\cdot\left(\left(\frac{y-k_2}{p_2}+a_2\right)^d - (a_2)^d\right) - \left(\frac{y}{p_1}\right)^d = \infty\,. \quad \text{(A5.7)}$$

Note that it also follows from i., ii., and iii. that

$$\varsigma_c > \varsigma_0\,. \quad \text{(A5.8)}$$

The proof we have just presented can also be illustrated:

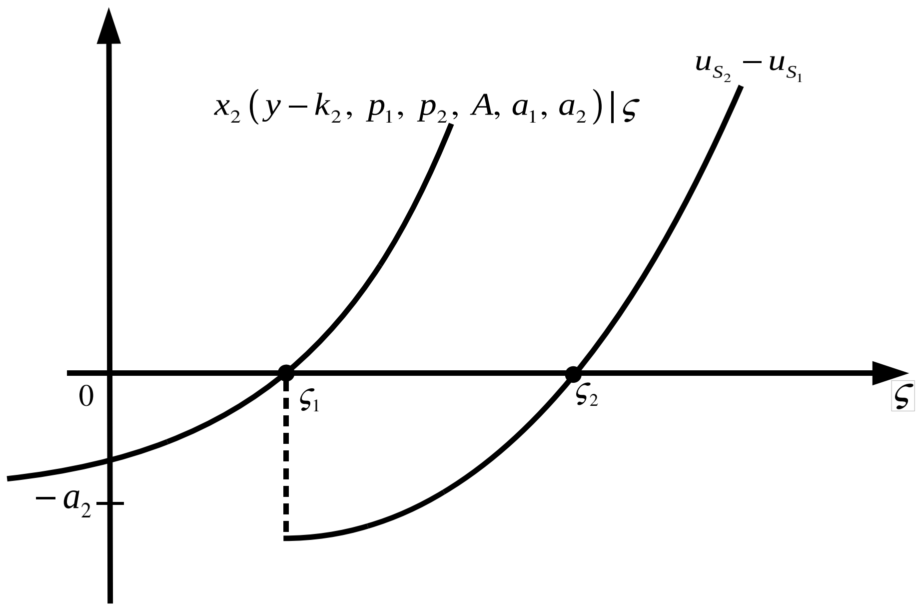

Figure A5.1: An illustration of the effect of the relative preference on choice.

7 Recall that parameter a_2 is always greater than zero.

$$\lim_{\varsigma\to\infty} u_{S_2} - u_{S_1} = \lim_{\varsigma\to\infty}\left(\frac{y-k_2}{p_1} - \frac{p_2}{p_1}\cdot\frac{y-k_2}{p_2}\right)^d + \exp(m+\beta\cdot\varsigma)\cdot\left(\frac{y-k_2}{p_2}+a_2\right)^d - \left(\frac{y}{p_1}\right)^d - \exp(m+\beta\cdot\varsigma)\cdot(a_2)^d = \ldots$$

$$\ldots = \lim_{\varsigma\to\infty}\exp(m+\beta\cdot\varsigma)\cdot\left(\left(\frac{y-k_2}{p_2}+a_2\right)^d - (a_2)^d\right) - \left(\frac{y}{p_1}\right)^d = \infty.$$

A 6: The impact of model parameters on the minimal driving distance

In this section, we shall illustrate the impact of changes in model parameters a_2 and d on the minimum driving distance $X_2(\varsigma_c)$, since this is one of the key points of this model. These impacts can be illustrated by looking at the iso-utility curves corresponding to the critical value ς_c in a x_1/x_2-diagram.

We start with illustrating the case of changes in the parameter a_2.

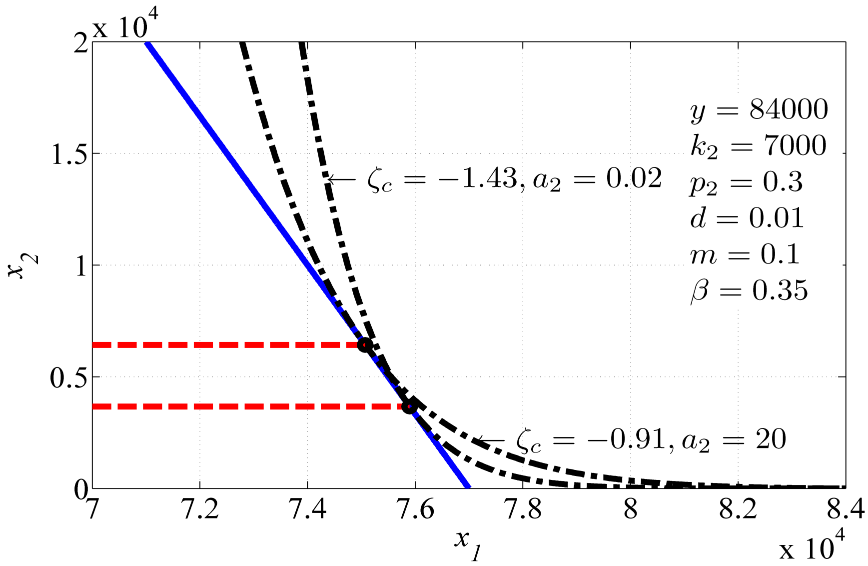

Figure A6.1: Indifference curves and minimum consumption for different parameters a_2.

This diagram shows that the minimum consumption levels indicated by the dashed lines increase if parameter a_2 increases. It also shows that for $a_2 = 0.02$ the preference where households would be indifferent between owing and not owning a car ζ_c is smaller than in the case $a_2 = 20$. This also explains why probability $P(X_2 = 0 | \theta, p_1, p_2, y, k_2, s)$ becomes smaller if a_2 decreases. Note that if the indifference curves of the utility function were not restricted to $x_2 \geq 0$, they would approach the horizontal line at value $-a_2$.

We then illustrate the case of changes in the parameter d. The following diagram shows that the minimum consumption levels indicated by the dashed lines increase if parameter d increases. It also shows that for $d = 0.02$ the preference where households would be indifferent between owning and not owning a car ζ_c is smaller than in the case $d = 0.3$. This also explains why probability $P(X_2 = 0 | \theta, p_1, p_2, y, k_2, s)$ becomes smaller if d decreases. Note also that the lower the value of d, the more cornered the indifference curve is. Note that if the indifference curves of the utility function were not restricted to $x_2 \geq 0$, they would approach the horizontal line at value $-a_2$.

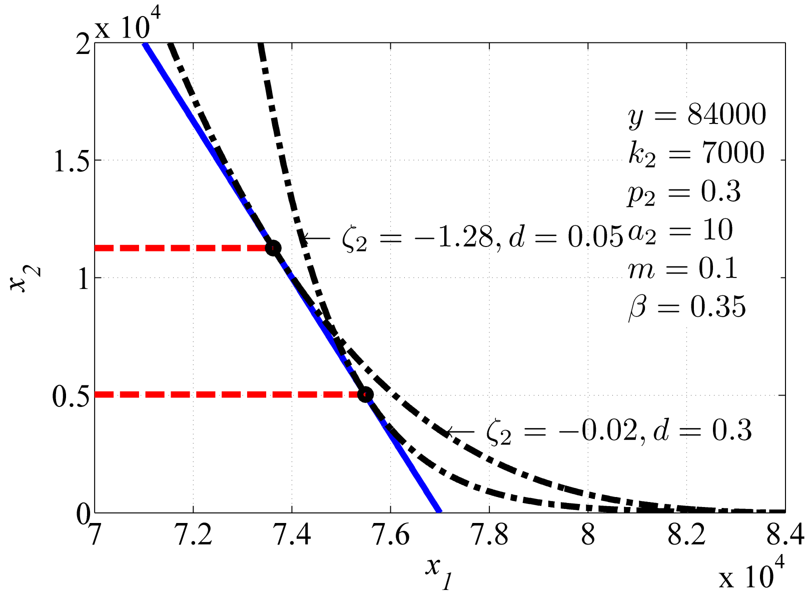

Figure A6.2: Indifference curves and minimum consumption for different parameters *d.*

This diagram shows that the minimum consumption levels indicated by the dashed lines increase if parameter a_2 increases. It also shows that for $a_2 = 0.02$ the preference where households would be indifferent between owing and not owning a car ζ_c is smaller than in the case $a_2 = 20$. This also explains why probability $P(X_2 = 0 | \theta, p_1, p_2, y, k_2, s)$ becomes smaller if a_2 decreases. Note that if the indifference curves of the utility function were not restricted to $x_2 \geq 0$, they would approach the horizontal line at value $-a_2$.

The following diagram shows that the minimum consumption levels indicated by the dashed lines increase if parameter d increases. It also shows that for $d = 0.02$ the preference where households would be indifferent between owning and not owning a car ζ_c is smaller than in the case $d = 0.3$. This also explains why probability $P(X_2 = 0 | \theta, p_1, p_2, y, k_2, s)$ becomes smaller if d decreases. Note also that the lower the value of d, the more cornered the indifference curve is. Note that if the indifference curves of the utility function were not restricted to $x_2 \geq 0$, they would approach the horizontal line at value $-a_2$.

A 7: Adaptation of the densities to the empirical values

The following first two diagrams figure A7.1 and figure A7.2 show that even though the probability density function (10) adapts the empirical distribution quite well, computing the expectation value by the formula (14) leads to too high results, particularly when the income level increases. Since this difference increases even over-proportionally with the income, we assume that simulated elasticities with respect to the income are too high if the upper limit of the integral in (14) is not bound to 60,000 km.

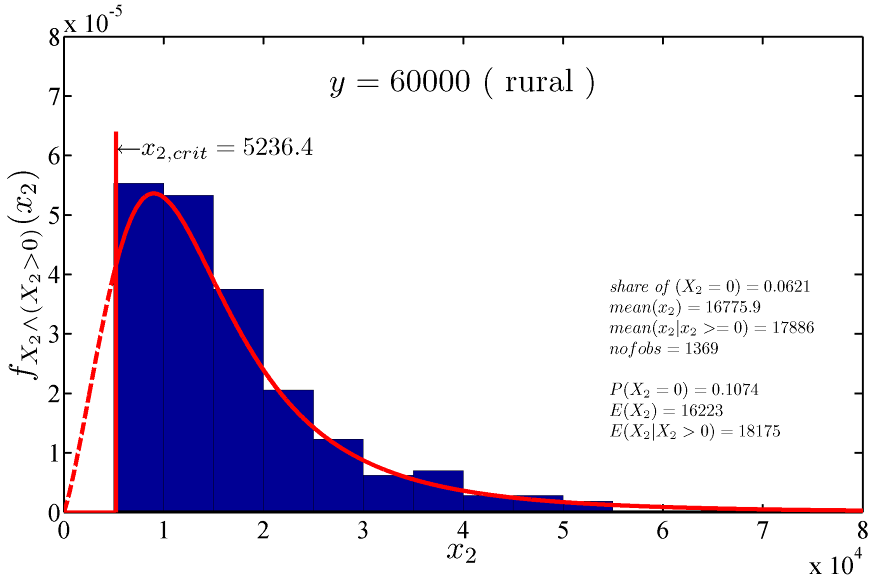

Figure A7.1: Histogram and density function of households in rural areas with an income of 60,000km

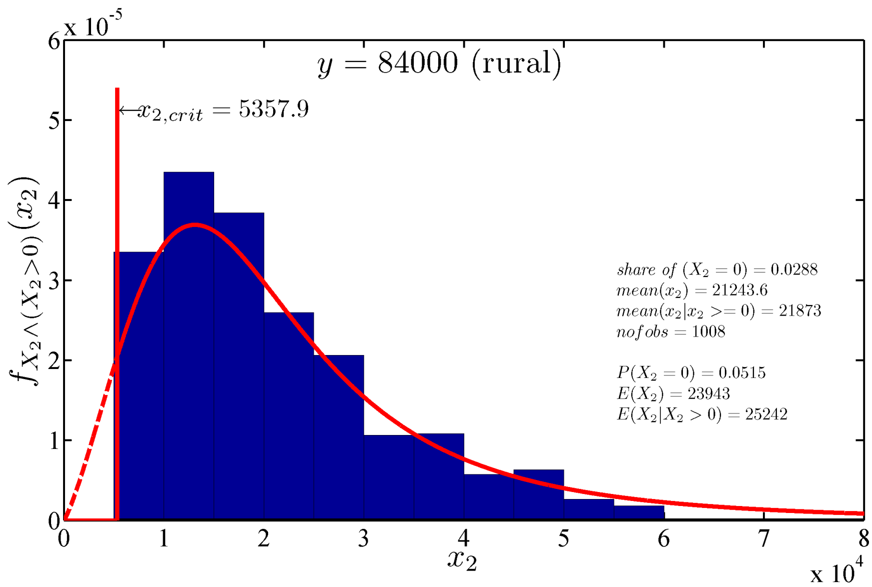

Figure A7.2: Histogram and density function of households in rural areas with an income of 84,000km

The following figures show that when the model is based on a modified function (14) with the upper boundary of the integral bound to 60,000km, the difference between the expectation value to the empirical value does not increase with the income. Therefore, we conclude that the model based on the modified function (14) yields unbiased values when computing simulated elasticities.

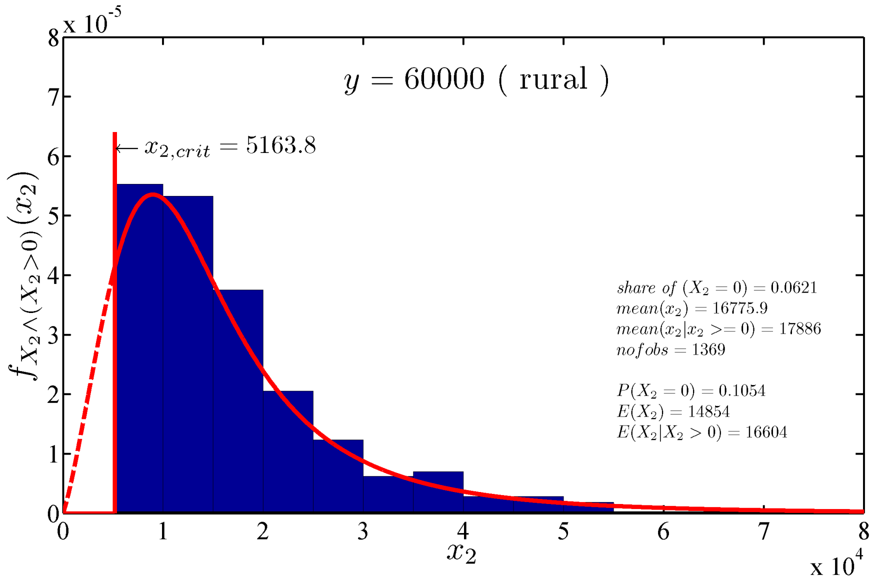

Figure A7.3: Histogram and density function of households in rural areas with an income of 60,000 CHF using the model based on the modified density function

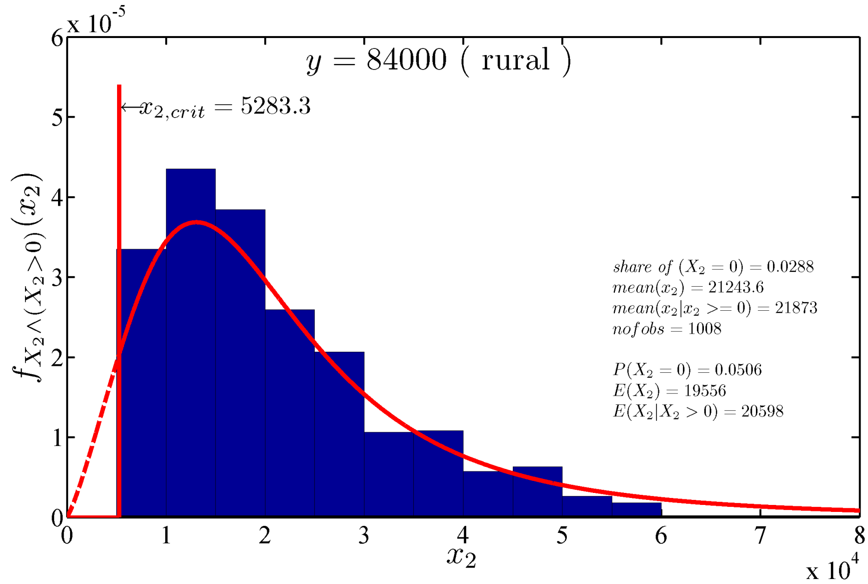

Figure A7.4: Histogram and density function of households in rural areas with an income of 84,000 CHF using the model based on the modified density function

The fact that in the case of the model with the modified computation of the expectation value, the difference between the computed expectation value and the empirical value does not increase with the income. This is shown by the following figure.

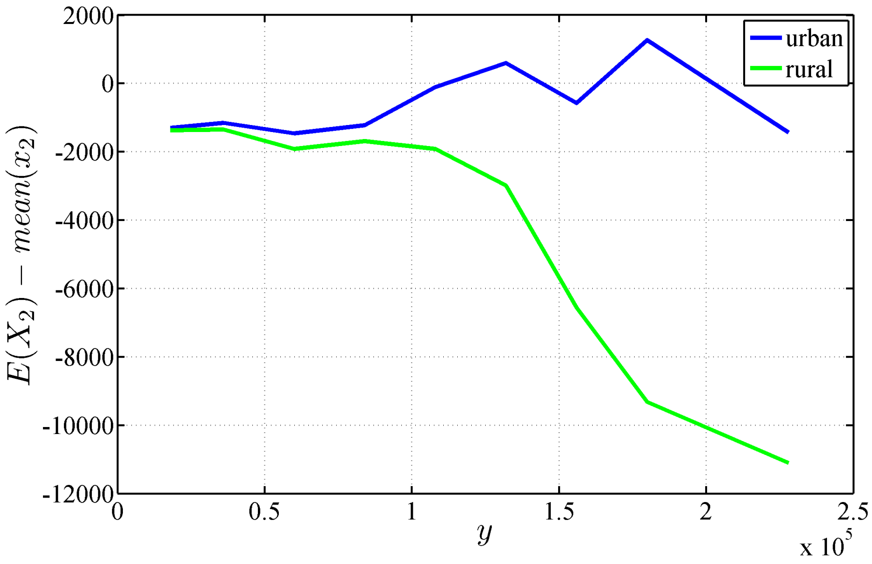

Figure A7.5: Difference between the computed expectation value and the empirical mean when using the model with the modified computation of the expectation value

The figure above shows that the model's expectation value is quite close to the empirical value and the difference between the simulated and the empirical values increases only weakly with the income of urban households. In contrast, the difference between the simulated and the empirical values decreases quite strongly in the case of the rural households. But since only less than one forth are rural households and the fact that the difference between the simulated and the empirical values of urban households is slightly increasing in income leads to our conclusion, that the income elasticity of the aggregate driving demand computed by use of this model is unbiased.

In contrast, the model based on the non-modified expectation function (12) produces too high elasticities, since in this case the difference between the simulated and the empirical values increases quite strongly when the households' income increase, see figure A7.6.

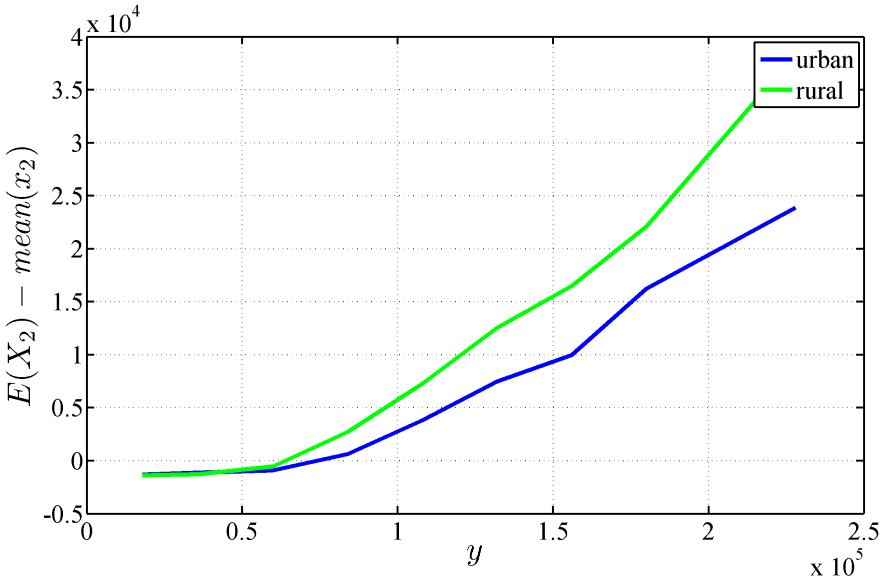

Figure A7.6: Difference between computed expectation value using the model based on the non-modified density function and the empirical mean value

In the case, a researcher is interested in computing only the elasticity of driving demand of certain households, e.g. rural households, we assume that it is better to run a separate model, in this case for the rural households. Note that all the diagrams in this appendix A7 are based on a model that includes only the dummy “rural”.