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Abstract: This short paper is the encyclopedia entry on Divisia Monetary Indexes to appear in the second edition of the *International Encyclopedia of the Social Sciences*. The encyclopedia is edited by William A. Darity and forthcoming from Macmillan Reference USA (Thomson Gale).

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JEL Classifications: E4, E5, C43, G12

Aggregation theory and index-number theory have been used to generate official governmental data since the 1920s. One exception still exists. The monetary quantity aggregates and interest rate aggregates supplied by many central banks are not based on index-number or aggregation theory, but rather are the simple unweighted sums of the component quantities and averages of interest rates. The predictable consequence has been induced instability of money demand and supply functions, and a series of "puzzles" in the resulting applied literature. Without coherence between data aggregation formulas and the models within which aggregates are embedded, stable structure can appear to be unstable. This phenomenon has been called the "Barnett critique" by Alec Chrystal and Ronald MacDonald (1994). In contrast, the Divisia monetary aggregates, originated by William A. Barnett (1980), are derived directly from economic index-number theory and are now available from some central banks.

Statistical index-number theory provides indexes which are computable directly from quantity and price data, without estimation of unknown parameters. For decades, the link between statistical index number theory and microeconomic aggregation theory was weaker for aggregating over monetary assets than for aggregating over other goods and asset quantities. Once monetary assets began yielding interest, monetary assets became imperfect substitutes for each other, and the "price" of monetary-asset services was no longer clearly defined. That problem was solved by Barnett (1980), who derived the formula for the user cost of demanded monetary services. Barnett's results on that user cost set the stage for introducing index number theory into monetary economics.

Let $\mathbf{m}_t = (m_{1t}, m_{2t}, \ldots, m_{nt})'$ be the vector of real balances of monetary assets during period t, let $\mathbf{r}_t = (r_{1t}, r_{2t}, \ldots, r_{nt})'$ be the vector of nominal holding-period yields for monetary assets during period t, and let R_t be the yield on the benchmark asset during period t. The benchmark asset is defined to be a pure investment that provides no services other than its yield, so that the asset is held solely to accumulate wealth. Let $\boldsymbol{\pi}_t = (\pi_{1t}, \pi_{2t}, \ldots, \pi_{nt})'$ be the vector of monetary-asset real user costs, with

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t}.$$

The user cost formula measures the foregone interest, which is the opportunity cost, of holding a unit of monetary asset i.

In economic aggregation theory, there exists an exact aggregator function over quantities. Let that aggregator function over monetary assets be u, so that the exact monetary aggregate, M_t, is $M_t = u(\mathbf{m}_t^*)$. Statistical index-number theory enables us to track M_t exactly, without estimating the unknown function, u.

In continuous time, the exact monetary aggregate, $M_t = u(\mathbf{m}_t^*)$, can be tracked exactly by the Divisia index, which solves the differential equation

$$\frac{d \log M_t}{dt} = \sum_{\mathrm{i}} s_{it} \frac{d \log m_{it}^*}{dt},$$

where

$$s_{it} = \frac{\pi_{it} m_{it}^*}{y_t}$$

is the i'th asset's share in expenditure on the total portfolio's service flow. The dual user cost price aggregate, $\Pi_t = \Pi(\boldsymbol{\pi}_t)$, can be tracked exactly by the Divisia price index, which solves the differential equation

$$\frac{d \log \Pi_t}{dt} = \sum_{\mathrm{i}} s_{it} \frac{d \log \pi_{it}}{dt} .$$

In continuous time, the Divisia index, under conventional neoclassical assumptions, is exact. In discrete time, the Törnqvist approximation is:

$$\log M_t - \log M_{t-1} = \sum_{\mathrm{i}} \overline{s}_{it} (\log m_{it}^* - \log m_{i,t-1}^*) ,$$

where

$$\overline{s}_{it} = \frac{1}{2} (s_{it} + s_{i,t-1}) .$$

The user cost aggregate, Π_t, then can be computed directly from Fisher's factor reversal formula, $\Pi_t M_t = \boldsymbol{\pi}_t{}' \mathbf{m}_t$.

The formula for Divisia monetary aggregation has been extended to risk aversion and to multilateral (international) aggregation within a common currency area, with particular reference to the concerns of the European Central Bank. Many of those extensions have been collected together in Barnett and Apostolos Serletis (2000).

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