



Munich Personal RePEc Archive

## **Firms' entry, monetary policy and the international business cycle**

Cavallari, Lilia

University of Rome III (Italy)

July 2012

Online at <https://mpra.ub.uni-muenchen.de/41876/>

MPRA Paper No. 41876, posted 12 Oct 2012 20:11 UTC

# Firms' entry, monetary policy and the international business cycle

Lilia Cavallari\*

University of Rome III

## Abstract

This paper provides a theory of the international business cycle grounded on firms' entry and sticky prices. It shows that under simple monetary rules pro-cyclical entry and counter-cyclical markups can generate fluctuations in macroeconomic aggregates and trade variables as large as those observed in the data while at the same time providing positive international comovements. Both firms' entry and sticky prices are essential for reproducing the synchronization of the business cycles found in the data.

Keywords: firm entry, international business cycle, international comovements, variable markup, Taylor rule, exchange rate regimes.

JEL codes: E31; E32; E52

---

\*Lilia Cavallari, University of Rome III, DIPES, Via Chiabrera, 199, 00145 Rome, Italy, email: cavallar@uniroma3.it. I wish to thank two anonymous referees and the co-Editor Giancarlo Corsetti for useful suggestions on previous drafts. I also thank participants in seminars at Cesifo, Luiss, University of Pavia, University of Tor Vergata (International Seminars 2010), University of Istanbul (EcoMod 2010), University of Crete (ICMAIF 2010) and Birmingham Business School. Remaining errors are mine.

# 1 Introduction

This paper provides a theory of the international business cycle grounded on firms' entry and sticky prices. It shows that under simple monetary rules entry can generate fluctuations in output, investments, employment and trade flows close to those in the data while at the same time providing positive international comovements. The capacity to capture established facts of the international business cycle overcomes the well-known difficulties of standard models in this regard. As first shown by Backus, Kehoe and Kydland (1992), international business cycle models typically generate very low or even negative cross-correlations (the comovement puzzle), with output less correlated than any other macroeconomic variable across countries (the quantity anomaly). In addition, they fail to match the counter-cyclical behavior of net exports found in the data. These empirical incongruities arise as a consequence of a strong incentive at work in artificial economies to use inputs where they are most productive. This needs not be the case as long as a business cycle expansion in one country leads to additional entry in the trading partner's market. This paper shows that firms' entry indeed provides a channel for positive comovements.

I propose a two-country dynamic stochastic general equilibrium model with monopolistic competition where producers are subject to a sunk entry cost, a one-period production lag and to an exogenous exit shock. Investments are represented by entry of new firms. Prior to entry, investors must acquire a basket of materials comprising domestic and foreign goods. Specifying entry costs in terms of goods allows investment demand to have a non-negligible component of imports as in the data. Later in the paper, I discuss the implications of modelling entry costs in terms of (non-tradable) labor. The economy has complete financial markets and a fully specialized structure of production. Risk sharing together with the assumption of costless trade imply that agents are in principle able to move resources in the most productive economy. I will argue that they have only a moderate incentive to do so as long as prices are sticky and entry costs comprise foreign goods. In the model, nominal rigidity is captured by a price-setting à la Calvo (1983). Monetary policy is represented in the standard form of a feedback rule as in the Neo-Wicksellian framework (Woodford (2003)) and the global nature of the monetary regime is captured by the interaction of interest rules followed by the monetary authorities in the two countries. I consider floating regimes under symmetric Taylor rules, with or without interest smoothing, and a regime where the exchange rate is fixed at all dates. In order to assess the extent to which sticky prices play an essential role in reproducing the international business cycle, I also consider a flexible price equilibrium.

Simulation results show that both outward-looking entry costs and sticky prices are essential for matching the volatility and synchronization of business cycles observed in the data. With flexible

prices or with labor entry costs, comovements. are negative as in Backus et al. (1992) and essentially for the same reasons. In the benchmark model, on the contrary, a business cycle expansion in one country stimulates entry in the trading partner's market, thereby generating positive comovements. The reason is threefold. First, changes in the terms of trade, by affecting entry costs, help to bridge the productivity gap between countries. Consider for instance a positive productivity shock in the home country. In the face of the shock, the price of home-produced goods falls relative to the price of foreign-produced goods, deteriorating the home terms of trade. This in turn reduces entry costs in the partner country (the low productivity economy), stimulating investments in new firms. Clearly, this channel is obscured in a setup with labor entry costs. Second, the failure to set prices in an optimal way reduces the prospective profits from investing in a new firm and discourages entry compared to a flexible price economy. A lower volatility of investments in the benchmark model reflects a limited ability of agents to move resources towards the more productive economy. Last but not least, a noticeable difference with the alternative specifications concerns the behavior of markups. These are counter-cyclical in the benchmark model, pro-cyclical with labor entry costs and constant with flexible prices. Counter-cyclical markups weaken the incentive to move investments towards the more productive economy.

The paper contributes to a recent line of research stressing the role of firms' dynamics in the business cycle.<sup>1</sup> In a pioneering paper, Cook (2002) shows that counter-cyclical markups can generate positive comovements. in a model with sequential entry, time-varying capital utilization and incomplete financial markets. Other early attempts to model entry in open economy have mainly focused on explaining trade margins and foreign investments. In all of these contributions, monetary policy is either overlooked (as in Ghironi and Mélitz (2005) and Helpman, Mélitz and Yeaple (2007), among others) or considered as an exogenous source of business cycle variability (as in Russ (2007) and Cavallari (2007, 2010)). Yet, there are good reasons for studying monetary policy in a model with firms' entry. Recent evidence documents that a monetary easing, i.e. a drop in the nominal interest rate, has a positive impact on the number of firms entering the market, suggesting that monetary policy may play a relevant role in a firm's decision whether to start-up a new production unit.<sup>2</sup>

There are surprisingly few open economy models that combine endogenous monetary policy

---

<sup>1</sup>See, among others, Ghironi and Mélitz (2005), Corsetti, Martin and Pesenti (2007, 2008) and Bilbiie, Ghironi and Mélitz (2007, 2011). The introduction of realistic assumptions on firms' dynamics ameliorates the capacity of artificial economies to match the business cycle properties of the data. In a closed economy, Bilbiie, Ghironi and Mélitz (2007, 2011) show that the moments generated by their models with entry come very close to the data, outperforming a typical fixed-entry real business cycle model.

<sup>2</sup>See Bergin and Corsetti (2008), Lewis (2009) and Uusküla (2010) among others.

and firm entry. The contributions closer to this paper are Auray, Eyquem and Poutineau (2012) and Auray and Eyquem (2011). The former focuses on a transmission mechanism that may be considered complementary to the one analyzed here. It emphasizes the role of asset prices in a context of complete financial integration and labor entry costs. The free entry condition in the model (equalizing the value of the firm to labor marginal costs) provides a direct link between asset prices and inflation that is absent in my setup. This, however, has the unappealing consequence of implying a positive relation between entry and interest rate innovations that is at odds with the data. The latter paper considers incomplete financial markets with given asset prices. In the model, firm value is tied to an exogenous entry cost and shocks are transmitted through changes in the real return on assets. An increase in the real return on equity (as after a positive productivity shock) is brought about by an increase in expected dividends. Since the real returns are equalized across countries by arbitrage, the increase in expected dividends in the low productivity economy requires a fall in the number of entrants. The model fails to match the positive cross-correlation of investments observed in the data.

The remainder of the paper is organized as follows. Section 2 describes the benchmark two-country model. Section 3 explains the log-linear solution of the model. Section 4 compares simulation results of the benchmark model and alternative specifications for entry costs and monetary rules. Section 5 concludes. The appendix contains the steady state of the model and the log-linearized equations.

## 2 The model

The world economy comprises two countries labelled Home, H, and Foreign, F, each specialized in the production of one type of good as in Corsetti and Pesenti (2002). It is populated by a continuum of agents of unit mass, where the segment  $[0, \gamma)$  belongs to country H and the segment  $(1 - \gamma]$  to country F. A typical agent in the economy is both a consumer and a worker: he supplies labor services in a competitive labor market and consumes all the goods produced in the world economy. In the Home country, there is a continuum of monopolistically competitive firms, each producing a different variety of the Home good  $h \in (0, N^H)$ , where  $N^H$  is the number of Home firms. Similarly, in the foreign country there is a continuum of firms  $f \in (0, N^F)$ . The stocks of Home and Foreign producers are determined endogenously in the model.

## 2.1 Preferences

In each period  $t$ , a typical agent  $i$  in country  $J = H, F$  derives utility from consuming a basket  $C$  containing all the goods produced in the world economy while suffering disutility from labor effort,  $L$ . Agents maximize the expected discounted value of flow utility  $U$  over their life horizon. Flow utility is assumed additive-separable:

$$U_{it}^J = \frac{(C_{it}^J)^{1-\rho}}{1-\rho} - \frac{\varphi\chi}{1+\varphi} (L_{it}^J)^{\frac{1+\varphi}{\varphi}} \quad (1)$$

where  $\rho > 0$  is the inter-temporal elasticity and  $\varphi > 0$  is the Frisch elasticity of labour supply.

The consumption basket  $C$  comprises home,  $C_H$ , and foreign goods,  $C_F$ , :

$$C^J = \frac{(C_H^J)^\gamma (C_F^J)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \quad (2)$$

where  $C_H$ ,  $C_F$  are given by:

$$\begin{aligned} C_H^J &= \left[ \int_0^{N^H} C^J(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}} \\ C_F^J &= \left[ \int_0^{N^F} C^J(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}} \end{aligned} \quad (3)$$

and  $\theta > 1$  denotes the elasticity of substitution across varieties. Consumer price indexes are given by<sup>3</sup>:

$$P^J = (P_H^J)^\gamma (P_F^J)^{1-\gamma} \quad (4)$$

while producer prices are:

$$\begin{aligned} P_H^J &= \left[ \int_0^{N^H} P^J(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}} \\ P_F^J &= \left[ \int_0^{N^F} P^J(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}} \end{aligned} \quad (5)$$

I assume that the law of one price holds, i.e.  $P^H(h) = \varepsilon P^F(h)$  and  $P^H(f) = \varepsilon P^F(f)$ , where the nominal exchange rate  $\varepsilon$  is the price of currency F in terms of currency H. Given identical preferences, purchasing power parity also holds. In my setup with entry, the assumption is less restrictive than it

---

<sup>3</sup>For nominal variables a superscript denotes the currency of denomination. So,  $P_H^F$  for instance denotes the Home producer price index in Foreign currency.

might appear at first. Firms can in principle insulate the final price of their products from exchange rate changes by letting their markups vary according to local market conditions. Simulation results will show that this is indeed the case as long as prices are sticky. Clearly, the presence of trade frictions would play a role in the decision whether to access foreign markets in the first place and eventually whether to serve them with exports or by engaging in investments overseas. The analysis of endogenous changes in trade openness or in the mode of accessing foreign markets is beyond the scope of the present paper. <sup>4</sup>

Finally, I define the terms of trade of country F,  $T$ , as the price of goods produced in country F relative to the price of goods produced in country H:

$$T = \frac{P_F^H}{P_H^H} = \frac{P_F^F}{P_H^F} \quad (6)$$

## 2.2 Firms

Producers in the world economy face an identical linear technology with labor as the sole factor. Output supplied by a firm  $j = h, f$  in country  $J = H, F$  is given by:

$$y_t^J(j) = Z_t^J L_t^J(j) \quad (7)$$

where  $Z^J$  is a country-specific shock to labor productivity.

Prior to entry, firms face an exogenous sunk entry cost (as in Grossman and Helpman (1991) and Romer (1990), among others).<sup>5</sup> In order to start the production in period  $t + 1$ , at time  $t$  a firm needs to purchase  $f_e^J$  units of the following combination of Home and Foreign varieties  $f_e^J = (C_H^J)^\sigma (C_F^J)^{1-\sigma}$ , at the price  $P_{f,t}^J = (P_{H,t}^J)^\sigma (P_{F,t}^J)^{1-\sigma}$  with  $\sigma \in (0, 1)$ . The cost of entry in units of the consumption basket is therefore  $f_e^J \frac{P_{f,t}^J}{P_t^J}$ . In this specification, entry requires installing a bundle of materials that may have a different composition than that of the consumption basket. Others, as Bilbiie et al. (2007) and Cavallari (2007), specify entry in effective labor units. Entry costs in this case coincide with labor marginal costs.

How to model entry costs is an open question. It has implications for aggregate accounting: labor costs imply a wedge between output of the consumption sector and GDP that is absent with entry costs in terms of goods. More importantly, it may affect the mechanism of monetary transmission. A monetary easing can in principle lead to an increase or a drop in the cost of entry and therefore

---

<sup>4</sup>In a setup with exporters and multinational firms, Cavallari (2010) shows that the currency of denomination of international trade affects both dimensions of the decision to serve foreign markets.

<sup>5</sup>For a model with endogenous entry costs see, among others, Bergin and Corsetti (2008) and Arespa (2012).

to opposing responses on the part of entrants. In a closed economy with labor entry costs, Uusküla (2010) shows that sticky price models predict a positive relation between firms' entry and interest rate innovations in contrast to what found in the data.<sup>6</sup> The reason is that an increase in the interest rate, by restraining labor demand, reduces real wages and entry costs. A similar mechanism extends to open economies as long as entry costs are entirely non-tradable (as labor costs). It is, however, conceivable that setup costs comprise a non-negligible component of tradable goods. This is certainly so for investment goods in general and one does not see why first-time investments should be different. Clearly, changes in the terms of trade will affect entry costs in this case. As will be clear soon, a monetary expansion has a positive effect on entry with tradable investment goods.

As in Ghironi and Méltitz (2005), all firms entered in a given period are able to produce in all subsequent periods until they are hit by a death shock, which occurs with a constant probability  $\delta \in (0, 1)$ . Therefore, a firm entered in period  $t$  will only start producing at time  $t + 1$ , introducing a one-period time-to-build lag into the model. She will eventually exit with an exogenous probability  $\delta$ . In each period, in addition to incumbent firms there is a finite mass of entrants,  $N_e^J$ . Entrants are forward looking and decide to start a new firm whenever its real value,  $\nu^J$ , given by the present discounted value of the expected stream of profits  $\{d_s^J\}_{s=t+1}^\infty$ , covers entry costs:

$$\nu_t^J = E_t \left[ \sum_{s=t+1}^{\infty} \beta (1 - \delta) \left( \frac{C_{s+1}^J}{C_s^J} \right)^{-\rho} d_s^J \right] = f_e^J \frac{P_{f,t}^J}{P_t^J} \quad (8)$$

The free entry condition above holds as long as the mass of entrants is positive. Macroeconomic shocks are assumed to be small enough for this condition to hold in every period. Note that upon entry, firms' profits vary and can even turn negative for a while (although the firm value remains positive). This is a key difference relative to models of frictionless entry. In the absence of sunk costs, in fact, the free entry condition requires zero profits in every period.

Finally, the timing of entry and the one-period production lag imply the following law of motion for producers:

$$N_t^J = (1 - \delta) (N_{t-1}^J + N_{e,t-1}^J) \quad (9)$$

## 2.3 Consumers' choices

I assume complete financial markets within and between countries. Agents can invest their wealth in a set of nominal state-contingent bonds,  $B$ , denominated in the currency of country  $H$  that span all

---

<sup>6</sup>Uusküla (2010) shows that a 1% increase in the Federal Funds rate rate leads to a 0.6% fall in the entry rate.



the states of nature.<sup>7</sup> In addition to state-contingent bonds, they hold a share  $s$  of a well-diversified portfolio of domestic firms. The budget constraint of a typical home agent is given by:

$$\sum_{\Omega} q_t^H(\Omega_{t+1}) \frac{B_{it}^H}{P_t^H} + s_t^H (N_t^H + N_{e,t}^H) v_t^H \leq \frac{B_{it-1}^H}{P_t^H} + s_{t-1}^H (v_t^H + d_t^H) + \frac{W_t^H L_{it}^H}{P_t^H} - C_{it}^H \quad (10)$$

where  $W$  is the nominal wage. A similar constraint holds for the foreign economy.

Agents choose consumption, labor effort, share and bond holdings in period  $t$  so as to maximize utility (1) over their whole life horizon subject to a budget constraint as (10) or its foreign analogue. Consumers' optimization requires the following first order conditions:

$$\frac{q_t^J(s_{t+1})}{P_t^J} (C_t^J)^{-\rho} = \beta E_t \left( \frac{C_{t+1}^J}{P_{t+1}^J} \right)^{-\rho} \quad (11)$$

$$(C_t^J)^{-\rho} = \beta (1 - \delta) E_t \left[ \frac{d_{t+1}^J + v_{t+1}^J}{v_t^J} (C_{t+1}^J)^{-\rho} \right] \quad (12)$$

$$\begin{aligned} C_t^J(h) &= \left( \frac{P_t^J(h)}{P_{Ht}^J} \right)^{-\theta} C_{Ht}^J \\ C_t^J(f) &= \left( \frac{P_t^J(f)}{P_{Ft}^J} \right)^{-\theta} C_{Ft}^J \end{aligned} \quad (13)$$

$$\frac{W_t^J}{P_t^J} = \chi (L_t^J)^{\frac{1}{\varphi}} (C_t^J)^{\rho} \quad (14)$$

Note that the Euler equation for bonds (11) together with the no arbitrage condition in international asset markets,  $q_t^H(s_{t+1}) = \varepsilon_t q_t^F(s_{t+1})$ , yields the uncovered interest rate parity, UIP:

$$E_t \left( \frac{P_t^H C_t^{H\rho}}{P_{t+1}^H C_{t+1}^{H\rho}} \right) = E_t \left( \frac{P_t^F C_t^{F\rho}}{P_{t+1}^F C_{t+1}^{F\rho}} \right) \frac{1 + i_{t+1}^F}{1 + i_{t+1}^H} \quad (15)$$

The assumption of complete markets together with the law of one price and the fact that consumption bundles are identical across countries imply that consumption is equalized worldwide, i.e.  $C^H = C^F$ .

---

<sup>7</sup>Each bond pays one unit of the Home currency if state  $\Omega_{t+1} \in \Psi$  occurs at time  $t+1$ , where  $\Psi$  is the set of finitely states that can occur in each period. The price of such a bond at date  $t$  is  $q^J(\Omega_{t+1})$ .

## 2.4 Pricing

Goods markets are monopolistically competitive. Each producer sets the price for its own variety facing the following market demand:

$$\begin{aligned} y_t(h) &= \left( \frac{P_t^J(h)}{P_{Ht}^J} \right)^{-\theta} \{ T_t^{1-\gamma} C_t + T_t^{1-\sigma} (f_e^H N_{e,t}^H + f_e^F N_{e,t}^F) \} \\ y_t(f) &= \left( \frac{P_t^J(f)}{P_{Ft}^J} \right)^{-\theta} \{ T_t^{-\gamma} C_t + T_t^{-\sigma} (f_e^H N_{e,t}^H + f_e^F N_{e,t}^F) \} \end{aligned} \quad (16)$$

where  $C \equiv \int_0^1 C_i di$  indicates world consumption.

I introduce nominal rigidities through a Calvo-type contract. In each period a firm can set a new price with a fixed probability  $1 - \alpha$  which is the same for all firms, both incumbent firms and new entrants, and is independent of the time elapsed since the last price change. In every period there will therefore be a share  $\alpha$  of firms, comprising incumbents and entrants, whose prices are pre-determined. In a symmetric equilibrium, pre-determined prices at a given point in time coincide with the average price chosen by firms active in the previous period.<sup>8</sup>

The simplifying assumption that new entrants behave like incumbent firms is without loss of generality. Allowing entrants to make their first price-setting decision in an optimal way would have only second order effects in my setup with Calvo pricing.<sup>9</sup> It might have major consequences in a setting where firms face costs of price adjustment as it would introduce heterogeneity in price levels across cohorts of firms entered at different points in time (see Bilbiee et al. (2007)). As the number of price-setters that face no cost of adjusting to a past pricing decision moves over the cycle, the aggregate degree of price stickiness becomes endogenous. The analysis of endogenous changes in price stickiness is beyond the scope of this paper.

Each firm  $j = h, f$  sets the price for its own variety so as to maximize the present discounted

---

<sup>8</sup>The average pre-determined price for home goods  $P_H^H$  will be:

$$(P_{Ht}^H)^{1-\theta} = \frac{(P_{H,t-1}^H)^{1-\theta}}{N_{t-1}^H}$$

and similarly for  $P_F^F$ . These properties are used in deriving the Calvo state equations below.

<sup>9</sup>The pre-determined price in each period would coincide with the average price chosen by firms entered in all previous periods and survived to the death shock:

$$(P_{Ht}^H)^{1-\theta} = \frac{\sum_{s=2}^{\infty} (1-\delta)^{s-1} N_{t-s}^E (P_t^H(h))^{1-\theta}}{N_{t-1}}$$

value of future profits, taking into account market demand (16) and the probability that she might not be able to change the price in the future, yielding:

$$P_t^J(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} (\alpha\beta(1-\delta))^k \frac{W_{t+k}^J y_{t+k}^J(j)}{Z_{t+k}^J P_{t+k}^J C_{t+k}^{-\rho}}}{E_t \sum_{k=0}^{\infty} (\alpha\beta(1-\delta))^k \frac{y_{t+k}^J(j)}{P_{t+k}^J C_{t+k}^{-\rho}}} \quad (17)$$

with  $J = H, F$ . The above expression can be re-arranged in a more familiar form as:

$$P_t^J(j) = \frac{\theta}{\theta - 1} (1 - \alpha\beta(1-\delta)) \frac{W_t^J}{Z_t^J} + \alpha\beta(1-\delta) E_t P_{t+1}^J(j) \quad (18)$$

where the optimal price in each period  $P^J(j)$  depends on current nominal marginal costs and expected future prices. Clearly, when  $\alpha = 0$  optimal pricing implies a constant markup  $\frac{\theta}{\theta-1}$  on marginal costs at all dates. With  $\alpha > 0$ , prices respond less than proportionally to a marginal cost shock, implying variable markups. Changes in markups over the cycle generate a real rigidity at the firm level.

Aggregating across firms the expressions above and recognizing that the pre-set price level coincides with the average market price in the previous period, yields the following state equations:

$$(P_{Jt}^J)^{1-\theta} = \alpha \frac{N_t^J}{N_{t-1}^J} (P_{Jt-1}^J)^{1-\theta} + (1-\alpha) N_t^J (P_t^J(j))^{1-\theta} \quad (19)$$

Producer prices in each period depend on current and previous stocks of active firms. An increase in the number of producers over time will reduce the aggregate price level everything else given. This is a direct consequence of love for variety. A higher number of, say, home varieties raises the value of consumption per unit of expenditure in home goods. Home producer prices therefore fall.

## 2.5 Aggregate accounting

Define real GDP in country  $J = H, F$  as  $Y^J \equiv \int_0^{N^J} \frac{P^J(j)}{P^J} y(j) dj$  where  $y(j)$  is given by (16). Goods market clearing requires:

$$\begin{aligned} Y_t^H &= T_t^{2(1-\gamma)} C_t + T_t^{2-\sigma-\gamma} (N_{et}^H f_e^H + N_{et}^F f_e^F) \\ Y_t^F &= T_t^{-2\gamma} C_t + T_t^{-\sigma-\gamma} (N_{et}^H f_e^H + N_{et}^F f_e^F) \end{aligned} \quad (20)$$

Labor market clearing implies:

$$L_t^H \equiv \int_0^\gamma L_{it}^H di \geq \int_0^{N_t^H} \frac{y_t(h)}{Z_t^H} dh \quad L_t^F \equiv \int_\gamma^1 L_{it}^F di \geq \int_0^{N_t^F} \frac{y_t(f)}{Z_t^F} df \quad (21)$$

Aggregating across agents the budget constraint (10) and its foreign equivalent, using the equilibrium conditions  $s_{t+1}^J = s_t^J = 1$  and assuming that initial financial wealth is zero, yields the accounting equations:

$$\begin{aligned} Y_t^H - \gamma C_t - N_{et}^H v_t^H &= \frac{B_{Ht}^H}{P_t^H} \\ Y_t^F - (1 - \gamma)C_t - N_{et}^F v_t^F &= \frac{B_{Ft}^H}{\varepsilon_t P_t^F} \end{aligned} \quad (22)$$

where  $B_H^H \equiv \int_0^\gamma B_i^H di$  and  $B_F^H \equiv \int_\gamma^1 B_i^H di$ .<sup>10</sup> In open economies, a discrepancy between output and domestic absorption reflects into a change in net foreign assets (here represented by bonds denominated in Home currency). Finally, asset market equilibrium requires  $B_H^H = -B_F^H$ .

## 2.6 Interest rules

The model is closed by specifying the monetary policy rules in place in the world economy. The monetary instrument is the one-period risk-free nominal interest rate,  $i_t^J$ , and monetary policy belongs to the class of feedback rules:

$$1 + i_t^J = f_t^J(\Theta_t) \quad (23)$$

where  $f$  is a generic function and  $\Theta$  is the information set at time  $t$ .

## 3 The log-linear model

The model does not provide a closed-form solution. It is log-linearized around a symmetric steady state with zero inflation. In the steady state, stochastic shocks are muted at all dates,  $Z^J = 1$ . This section discusses the main linearized equations while the Appendix contains the steady state and the full log-linearization.

---

<sup>10</sup>I used  $Y_t^J = \frac{W_t^J L_t^J}{P_t^J} + N_t^J d_t^J$  in deriving the current account equations.

### 3.1 Demand block

The aggregate demand block is derived from the log-linear approximation to the first order conditions of consumers in the Home and Foreign countries. Consumers allocate their wealth among consumption, nominal risk-free securities and shares. Inter-temporal optimization requires that the marginal rate of substitution between current and one-period ahead consumption equalizes the real return on nominal assets, both the risk-free bonds and shares. A first set of Euler equations, one for each country, will therefore describe the dynamic link between current and expected one-period ahead consumption and relate it to the risk-free return in units of consumption. A second set of Euler equations, again one for each country, will relate the inter-temporal profile of consumption to the real return on shares. The real value of the firm, which coincides with the entry cost in equilibrium, is the forward solution to the Euler equations for shares.

Recognizing that consumption risks are perfectly insured in the world economy, the bond Euler equations in the Home and Foreign countries can be combined, yielding:

$$E_t \widehat{C}_{t+1} = \widehat{C}_t + \frac{\gamma}{\rho} \left( \widehat{i}_t^H - E_t \pi_{t+1}^H \right) + \frac{1-\gamma}{\rho} \left( \widehat{i}_t^F - E_t \pi_{t+1}^F \right) \quad (24)$$

where a hat over a variable denotes the logdeviation from the steady state,  $\pi_{t+1}^J = \ln P_{Jt+1}^J / P_{Jt}^J$  is producer inflation in country  $J = H, F$  and  $E$  is the expectation operator. The above expression says that an increase in the world real interest rate, wherever it is originated, raises the return on bonds, therefore making it more attractive to postpone consumption in the future.

The definition of the terms of trade (6) yields the following state equation:

$$\widehat{T}_t = \widehat{T}_{t-1} + \Delta \widehat{\varepsilon}_t + \pi_t^F - \pi_t^H \quad (25)$$

Movements in the terms of trade around the steady state are driven by changes in the nominal exchange rate and by cross-country inflation differentials. Monetary policy can directly affect the terms of trade through uncovered interest parity:

$$E_t \Delta \widehat{\varepsilon}_{t+1} = \widehat{i}_t^H - \widehat{i}_t^F \quad (26)$$

### 3.2 Supply block

The supply block is derived from a log-linear approximation to the pricing and entry decisions of firms in the Home and Foreign countries together with a log-linearization of agents' labor supply. First, consider the optimal price (17) for, say, a home variety. Using market demand (16) and labor

supply (14), re-arranging and linearizing gives:

$$E_t \sum_{k=0}^{\infty} \alpha \beta (1 - \delta)^k \left[ \widehat{P}_{t,t+k}^H + \left(1 + \frac{1}{\varphi}\right) (\gamma - 1) \widehat{T}_{t+k} - \left(\rho + \frac{1}{\varphi}\right) \widehat{C}_{t+k} + \left(1 + \frac{1}{\varphi}\right) \widehat{Z}_{t+k}^H - \frac{1}{\varphi} \widehat{N}_{t+k}^H + \frac{\theta}{\varphi} \widehat{P}_{t,t}^H \right] = 0$$

where  $\widehat{P}_{t,t+k}^H = \ln P_t^H(h)/P_{H,t+k}^H$ . Note that by definition  $\widehat{P}_{t,t+k}^H = \widehat{P}_{t,t}^H - \sum_{s=1}^k \pi_{t+s}^H$ , namely changes in the real price of a home variety between  $t$  and  $t+k$  coincide with the variety effect, the first addend, less producer inflation over the period. Using the Calvo state equation (19), the variety effect is:

$$\widehat{P}_{t,t}^H = \frac{\alpha}{1 - \alpha} \pi_t^H + \frac{1}{(1 - \alpha)(\theta - 1)} \widehat{N}_t^H - \frac{\alpha}{(1 - \alpha)(\theta - 1)} \widehat{N}_{t-1}^H$$

With  $\alpha = 0$ , an increase in the number of Home producers raises the real price of Home varieties and the more so the lower the elasticity of substitution  $\theta$ . This effect is dampened with  $\alpha > 0$ .

Combining the two equations above and re-arranging gives:

$$\begin{aligned} \pi_t^H &= \zeta \left[ \frac{(1 - \gamma)(1 + \varphi)}{\varphi} \widehat{T}_t + \left(\rho + \frac{1}{\varphi}\right) \widehat{C}_t - \frac{1}{(1 - \alpha)(\theta - 1)} \widehat{N}_t^H - \frac{(1 + \varphi)}{\varphi} Z_t^H + \frac{\alpha}{(1 - \alpha)(\theta - 1)} \widehat{N}_{t-1}^H \right] \\ &\quad + \beta (1 - \delta) E_t \pi_{t+1}^H \end{aligned} \quad (27)$$

where  $\zeta = \frac{(1 - \alpha \beta (1 - \delta))(1 - \alpha)}{\alpha(\varphi + \theta)}$ .

Producer inflation in the foreign country is obtained in a similar way:

$$\begin{aligned} \pi_t^F &= \zeta \left[ \frac{-\gamma(1 + \varphi)}{\varphi} \widehat{T}_t + \left(\rho + \frac{1}{\varphi}\right) \widehat{C}_t - \frac{1}{(1 - \alpha)(\theta - 1)} \widehat{N}_t^F - \frac{(1 + \varphi)}{\varphi} Z_t^F + \frac{\alpha}{(1 - \alpha)(\theta - 1)} \widehat{N}_{t-1}^F \right] \\ &\quad + \beta (1 - \delta) E_t \pi_{t+1}^F \end{aligned}$$

The country-specific inflation rates depend on next period expected inflation as well as on deviations of the terms of trade, consumption, the number of producers and productivity from their steady state values. These deviations are correlated with current marginal costs. To begin with, consider an increase in  $T$ , i.e. a deterioration in the home terms of trade. The rise in  $T$  switches world demand in favor of home products, pushing up labor demand in the home economy. Real wages and marginal costs therefore increase, fuelling inflation. A similar mechanism explains why a rise in world consumption leads to higher inflation. A rise in home productivity, on the contrary, directly reduces marginal costs. The number of producers is related to inflation via the variety effect. An increase

in the current stock of producers makes a larger range of home varieties available for consumption. Because of love for variety, the value of consumption per unit of expenditure in home varieties raises. Hence producer prices must fall. The opposing effect is true for a rise in  $\widehat{N}_{t-1}$ . In this case, previous period's prices drop and result into higher inflation in the current period.

Second, a log-linear approximation to the number of entrants can be obtained from the current account equations (22) as a function of output minus absorption and net foreign assets:

$$\begin{aligned}\widehat{N}_{et}^H &= \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\widehat{Y}_t^H + \left(1 - \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\right)\widehat{C}_t - \widehat{v}_t^H - \frac{(1-\delta)}{\delta}\widehat{nfa}_t \\ \widehat{N}_{et}^F &= \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\widehat{Y}_t^F + \left(1 - \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\right)\widehat{C}_t - \widehat{v}_t^F + \frac{(1-\delta)}{\delta}\widehat{nfa}_t\end{aligned}\quad (28)$$

where  $\widehat{nfa}_t = \widehat{B}_t^H - \frac{1}{\beta}\widehat{B}_{t-1}^H$  and  $B_t^H = \frac{B_{Ht}^H}{Y^H P_t^H}$ . Note that the resource constraint implies a trade-off between investments in new varieties and consumption of existing goods (the coefficient on C is negative). The law of motion of firms is:

$$\widehat{N}_t^J = (1-\delta)\widehat{N}_{t-1}^J + \delta\widehat{N}_{et-1}^J \quad (29)$$

Finally, using the property that the aggregate price markup  $\mu^J \equiv \int_0^{N^J} \frac{P^J(j)}{W^J/Z^J} dj$  coincides with the inverse of the labor share,  $\frac{Y^J P^J}{W^J L^J}$ , one can substitute away the real wage in labor supply (14) and together with the GDP definition obtain an expression for aggregate labor. In log-linear terms, this gives :

$$\begin{aligned}\widehat{L}_t^H &= -\rho\varphi\widehat{C}_t + \varphi\left(\widehat{Z}_t^H - \widehat{\mu}_t^H + (\gamma-1)\widehat{T}_t + \widehat{P}_{t,t}^H\right) \\ \widehat{L}_t^F &= -\rho\varphi\widehat{C}_t + \varphi\left(\widehat{Z}_t^F - \widehat{\mu}_t^F - \gamma\widehat{T}_t + \widehat{P}_{t,t}^F\right)\end{aligned}\quad (30)$$

where it is clear that employment decreases with consumption, because of inter-temporal substitution between leisure and labor, while raising with the real wage (the term in brackets).

### 3.3 Interest rate rules

I consider one regime with fixed exchange rates and two floating regimes. The fixed regime is a unilateral peg to the Foreign currency featuring a fixed exchange rate at all dates. It is implemented by the interest rule  $\widehat{i}_t^H = \widehat{i}_t^F - \varsigma\widehat{\varepsilon}_t$  with  $\varsigma > 0$ . In the rule, the presence of an exchange target (normalized to zero) ensures determinacy.

In the floating regime, monetary policy follows a symmetric Taylor rule  $\widehat{i}_t^J = \phi_\pi^J \pi_t^J$ . The Taylor

principle, requiring that policy-makers react more than proportionally to inflation, i.e.  $\phi_\pi^J > 1$ , ensures determinacy. Taylor rules have been extensively analyzed since the seminal paper by Taylor (1993). They are empirically plausible, especially in the last few decades when the objective of price stability has gained a major role in monetary policy-making. In order to account for the need to reduce swings in interest rates in an environment characterized by long and variable lags in monetary transmission, I also consider a variant with interest rate smoothing, i.e.  $\hat{i}_t^J = \phi \hat{i}_{t-1}^J + \phi_\pi^J \pi_t^J$ .

For ease of comparisons with flexible price models, I finally consider a Wicksellian regime in which the nominal interest rate is set so as to reproduce a flexible price equilibrium with zero inflation. The Wicksellian interest rates  $\tilde{i}^J$  are given by:

$$\begin{aligned}\tilde{i}_t^H &= \rho \left( E_t \hat{C}_{t+1} - \hat{C}_t \right) + (1 - \gamma) \left( E_t \hat{T}_{t+1} - \hat{T}_t \right) \\ \tilde{i}_t^F &= \rho \left( E_t \hat{C}_{t+1} - \hat{C}_t \right) - \gamma \left( E_t \hat{T}_{t+1} - \hat{T}_t \right)\end{aligned}\tag{31}$$

With flexible prices, nominal interest rates mimic changes in the world natural (real) interest rate. As is well-known, the Wicksellian policy can be implemented recurring to a credible threat to deviate from a zero inflation target, i.e.  $i_t^J = \tilde{i}_t^J + \vartheta \pi_t^J$  with  $\vartheta > 1$ .

## 4 Simulations

The model is simulated using first-order perturbation methods. To begin with, I consider productivity shocks as the main source of business cycle volatility, abstracting from interest rate innovations. This facilitates comparisons with real business cycle models. Next, I focus on a one-time innovation in nominal interest rates for the purpose of illustrating the mechanism of monetary transmission.

### 4.1 Calibration

I calibrate a US-EMU model in which country H represents the United States and country F the EMU 12. The relative dimension of these two economies is captured by  $\gamma = 0.6$ . This parameter also measures the share of domestic goods in US consumption in the model, while the analogous share in European consumption is  $1 - \gamma = 0.4$ . These values are consistent with a higher home bias in US consumption relative to that in Europe. As regards the share of domestic goods in US investment, I set  $\sigma = 0.4$ , in accord with the fact that the share of imports in investments is higher than that in consumption. For robustness, I let  $\sigma$  vary between 0.2 and 0.8.

In the simulations, periods are interpreted as quarters and  $\beta = 0.99$  as usual in quarterly models of the business cycle. The size of the exogenous exit shock  $\delta$  is 0.025 as suggested by Bilbiie et



al. (2007). The rate of firm disappearance is consistent with a 10 percent rate of job destruction per year as found in the US. Moreover, such a moderate rate does not overstate the capacity of the model to generate persistence. The elasticity of substitution across varieties  $\theta$  is equal to 7.88 as in Rotemberg and Woodford (1999a). This yields a reasonable average markup of approximately 18 percent. Studies based on disaggregated data usually find a much lower  $\theta$ , roughly around 4. Simulations with a lower elasticity deliver qualitatively identical results and will not be reported.<sup>11</sup> Other preference parameters are  $\varphi = 2.13$  as in Rotemberg and Woodford (1999a) and  $\rho = 1$  as in Bilbiie et al. (2007). I also experiment with a value of the Frisch elasticity as high as 6.

The degree of nominal rigidity is estimated by Galí, Gertler and Lopez-Salido (2001). In their study, the value of  $\alpha$  ranges between 0.407 and 0.66 in the US and between 0.67 and 0.771 in Europe. As suggested by Benigno and Benigno (2008), I take the middle points from these intervals and set  $\alpha = 0.49$  in the US and  $\alpha = 0.72$  in Europe. These values imply an average duration of nominal contracts of, respectively 2.3 and 3.65 quarters. I also experiment with a common value of 0.66 as Rotemberg and Woodford (1999a), obtaining qualitatively identical results. Initial conditions for productivity shocks, the terms of trade and the nominal exchange rate do not affect the dynamics of the model and are set at unity without loss of generality.

Finally, the vector of productivity shocks  $Z_t = (Z_t^H, Z_t^F)$  follows a bivariate autoregressive process,  $Z_t = AZ_{t-1} + \epsilon_t$  where  $\epsilon_t = (\epsilon_t^H, \epsilon_t^F)$  is distributed normally and independently over time with variance  $V$ . The correlation between the technology shocks  $Z_t^H$  and  $Z_t^F$  is determined by the off-diagonal elements of  $A$  and  $V$ .

## 4.2 Technology shocks

### 4.2.1 Moments

This subsection illustrates the performance of the model in replicating stylized facts of the international business cycle. Tables 1 and 2 report statistics of the model's artificial time series together with statistics in the data. As with the data, statistics refer to Hodrey-Prescott filtered variables with smoothing parameter of 1600. The reported statistics are averages across 100 simulations.

In comparing the model to properties of the data, the treatment of variety effects deserves a particular attention. As argued in Ghironi and Méltitz (2005), empirical relevant variables - as opposed to welfare-consistent variables - net out the effects of changes in the range of available products. The reason is the inability of statistical measures of CPI inflation to adjust for availability of new products as in the welfare-consistent price index. Hence, the data are closer to the producer

---

<sup>11</sup>These results are available upon request.

than to the consumer price index. In what follows, any variable that in the model is measured in units of consumption will be deflated by producer inflation and converted into units of output (for any variable  $X$  in consumption units the corrected measure will be  $X^R = XP^H/P_H^H$ ). A similar correction applies to the real value of household investments in new firms,  $I = v^H \frac{P_H^H}{P(h)} N_e^H$ .

The benchmark model features sticky prices and entry cost in terms of goods. In this model, monetary policy follows a Taylor rule with interest smoothing  $i_t^J = 0.8i_{t-1}^J + 0.3\pi_t^J$  as in Bilbiee et al. (2007). In the flexible price economy, monetary policy is given by the Wicksellian rule  $i_t^J = \tilde{i}_t^J + 1.2\pi_t^J$ . For comparison with other models of firms' entry, I also consider a variant with entry costs in labor units (see the Appendix for linearized equations).

First, consider international comovements. In the data, comovements of output, consumption, investment and employment are strikingly positive across a large number of countries, although cross-correlations are not too strong especially in more recent times (see Ambler et al. (2004)). In addition, international synchronization is higher for output than for any other variable, while the synchronization of investments is larger than that of employment. As first shown by Backus, Kehoe and Kydland (1992, 1995), there are important discrepancies between these facts and what standard models predict. Real business cycle models typically generate negative cross-correlations (the comovement puzzle) and a lower correlation of output than that of any other macroeconomic aggregate (the quantity anomaly). Many candidates have been suggested to propose a solution to these puzzles, yet no agreement has been reached on what is the best way to solve them.<sup>12</sup> In general, they have been relatively unsuccessful in finding a solution to all the anomalies simultaneously.

Panel A of Table 1 contains correlations of Home and Foreign variables in the benchmark model, in a calibration with  $\sigma = 0.8$  or with  $\sigma = 0.2$ , in the flexible price economy, in the model with labor entry costs and in the data for the United States and Europe from Ambler et al. (2004). Panel B contains statistics of Home variables in the three models above together with US data from King and Rebelo (1999). To facilitate comparisons, I focus on country-specific productivity shocks with symmetric standard deviation equal to 0.0852, correlation 0.258 and persistence 0.906 as in Backus et al. (1992).

---

<sup>12</sup>Successful strategies comprise, among others, the introduction of non-tradable goods, investment and consumption of durable goods, distribution services, capital market frictions, adjustment costs to investments as well as government spending and taste shocks.

Table 1:

## A: International Comovement

	Benchmark model	Flex price model	Labor entry costs	High $\sigma$	Low $\sigma$	EU-US data
	correlation of domestic and foreign variables					
$Y^R$	0.58	-0.78	-0.89	0.50	0.47	0.66
$L$	0.14	-0.98	-0.98	0.04	0.00	0.33
$I$	0.28	-0.99	-0.98	0.26	0.21	0.53

## B. Business Cycle Properties

	Benchmark model			Flex price model			Labor entry costs			US data, KR (1999)		
	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\rho_X$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\rho_X$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\rho_X$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\rho_X$
$Y^R$	1	1	0.86	1	1	0.74	1	1	0.61	1	1	0.84
$L$	1.02	0.95	0.85	0.61	0.91	0.71	1.03	0.99	0.96	0.99	0.88	0.88
$I$	3.83	0.57	0.81	11.9	0.79	0.56	11.2	0.99	0.63	2.93	0.80	0.87

$\sigma_X$  is the standard deviation of variable X,  $\sigma_{XY}$  is the correlation of variable X with output, Y, and  $\rho_X$  is the auto-correlation of variable X.

The benchmark model (with sticky prices and entry costs in terms of goods) reproduces the positive comovements of output, investments and employment found in the data.<sup>13</sup> In addition, it matches the ranking in the data, with output more correlated than investments and investments more correlated than employment across countries. However, the cross-correlation of investments and employment is low compared to the data. Correlations remain positive although far below those in the data when the share of imports in entry costs is either very low or very high.

In Backus et al. (1992), negative correlations arise as a consequence of a strong incentive to use inputs where they are most productive. In their model, agents are able to shift substitutable goods costlessly between countries and to trade in complete markets for state contingent claims. They are therefore induced to move production effort to the country with a high technology shock. A similar incentive is at work in a setup with entry whenever prices are flexible or entry requires hiring workers. Not surprisingly, the performance of these economies in reproducing the international business cycle is very close indeed to that of Backus et al. (1992). A near perfect correlation of investments between countries reflects a strong incentive to establish new firms where productivity is high (the Home country in the simulation). A negative cross-correlation of employment captures the effect of risk sharing: higher income in the low productivity economy (the Foreign country in the simulation) induces agents to reduce labor supply. The combination of these effects leads to a negative correlation of output between countries.

<sup>13</sup>Others have matched the positive comovements observed in the data, for example Kose and Yi (2003) and Corsetti, Dedola and Leduc (2008). But the cross-correlation of output in these models still falls short of the empirical findings.

In the benchmark model, three factors work in the direction of reducing the incentive to move resources towards the high productivity economy. First, a gradual drop in the costs of acquiring materials for setting up a new firm favors investments in the low productivity economy. This is a key difference relative to a setup with labor entry costs. Second, the failure to set prices in an optimal way reduces the prospective profits from investing in a new firm and discourages entry relative to a flexible price economy. Last but not least, investors realize that profits and markups will move upon entry. As will become apparent soon, counter-cyclical markups reduce the incentive to move investments towards a more productive economy. Pro-cyclical profits, on the other hand, mitigate the negative impact of risk sharing on labor supply, generating a positive comovement of employment.

The ability to match comovements in the data does not come at the cost of implausible business cycle properties. Panel B in Table 1 shows that the benchmark model generates volatilities of investment and employment in terms of output close to those in the data. The flexible price economy and the model with labor entry costs, on the contrary, display excessive volatility of investments. The intuition is that a lower volatility of investments reflects a limited ability of agents to move production where it is more productive.

Consider now trade variables. Despite ample heterogeneity across countries, a number of stylized facts emerge with clarity. Exports and imports are more volatile than output, positively correlated with each other and pro-cyclical, while net exports are less volatile than output and counter-cyclical (Engel and Wang (2011)). Table 2 reports statistics of Home real imports, real exports and the ratio of net exports to GDP in the benchmark model (column a), in a calibration with  $\sigma = 0.8$  or with  $\sigma = 0.2$ , in the flexible price economy (column b), in the model with labor entry costs (column c) and in US data from Engel and Wang (2011). The parametrization of the productivity shock is as before.

Table 2: Trade statistics

	(a)		(b)		(c)		High $\sigma$		Low $\sigma$		US data	
	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$	$\frac{\sigma_X}{\sigma_Y}$	$\sigma_{XY}$
Real Imports	3.12	0.78	3.18	0.74	3.59	0.89	2.48	0.69	2.07	0.68	3.34	0.83
Real Exports	2.67	0.73	1.79	-0.95	1.72	-0.98	1.50	0.95	1.65	0.96	2.63	0.41
Net Exports/GDP	0.22	-0.23	1.72	0.85	2.32	0.98	0.99	-0.64	0.50	-0.56	0.25	-0.47

$\sigma_X$  is the standard deviation of variable X and  $\sigma_{XY}$  is the correlation of variable X with output, Y. US data are from Engel and Wang (2011).

Column a refers to the benchmark model, column b to the flexible price economy and column c to the model with labor entry costs.

The benchmark model matches the properties of trade variables fairly well. It captures the volatility and the pro-cyclical behavior of trade flows as well as the counter-cyclical behavior of net exports (in the model as in the data, counter-cyclical net exports derive from a higher correlation of imports with GDP than that of exports). In this respect, it performs better than standard international real business cycle models, as Backus et al. (1992) and Heathcote and Perri (2002), and in line with recent models as Corsetti, Dedola and Leduc (2008) and Engel and Wang (2011). The finding is robust to varying the composition of investments goods. In my setup, the volatility of trade variables depends on the volatility of demand for consumption and investments goods as well as on the substitution between Home and Foreign goods. In spite of a low elasticity of substitution (equal to 1 in the model) and a low volatility of consumption (the standard deviation of C relative to output is 0.75 in the benchmark model), investment demand can generate very volatile imports and exports in the benchmark as well as in the other specifications. A noticeable difference is that exports are counter-cyclical and net exports are pro-cyclical in the models with flexible prices and with labor entry costs. This result reflects once again a strong incentive to move resources towards the most productive economy. Falling investments in the low productivity economy, in fact, imply a drop in the demand for Home exports.

#### 4.2.2 Impulse responses

Figure 1 shows impulse response functions of selected Home and Foreign variables for a one-standard-deviation shock to Home productivity. The vertical axis shows percentage deviations from the steady state (a value of, say, 0.01 denotes a 1 percent deviation) and the horizontal axis shows the number of periods after the shock. For consistency with second moments, the shock has a persistence of 0.906. The impulse responses are displayed under a Wicksellian policy (dotted line),  $i_t^J = \tilde{i}_t^J + 1.2\pi_t^J$ , and a Taylor smoothing rule (solid line),  $i_t^J = 0.8i_{t-1}^J + 0.3\pi_t^J$ .

Focus on the responses under flexible prices (i.e. with the Wicksellian policy). The positive shock to Home productivity creates a more favorable business environment, attracting entrants into the home market and leading to a gradual increase in the number of producers over time.<sup>14</sup> A larger number of producers, in turn, rises the relative price of Home varieties (the variety effect) together with marginal costs while markups remain unchanged. Because the shock is persistent, there is also a significant wealth effect that pushes up the demand for both Home and Foreign goods. As a result of all these effects, aggregate consumption and GDP increase in the home country.

---

<sup>14</sup>The pro-cyclical response of entry is consistent with an ample evidence. In the US, the cyclical properties of firms' entry have been documented by, among others, Dunne, Roberts and Samuelson (1988), Chatterjee and Cooper (1993), Campbell (1998), Bilbiee et al. (2007) and Lewis (2009).

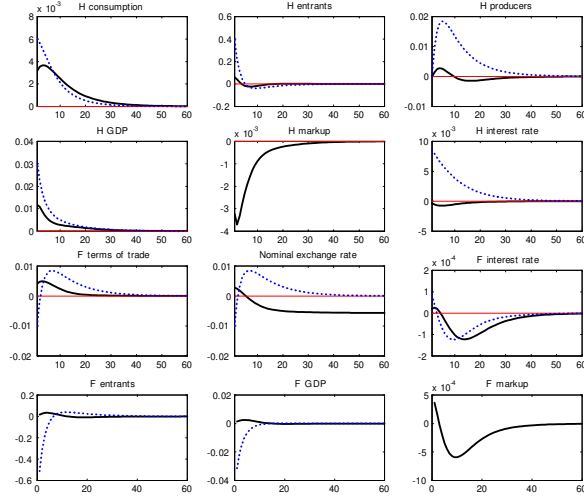


Figure 1: impulse response functions for a one-standard-deviation shock to H productivity with sticky prices (solid line) and with flexible prices (dotted line)

Trade in goods and assets spreads the effects of the productivity shock worldwide. In the face of a home positive shock, the price of Home-produced goods falls relative to Foreign-produced goods, deteriorating the home terms of trade ( $T$  is above the steady state for most of the transition). Consequently, world expenditure switches in favor of home goods. An analogous shift materializes in the portfolio of investors as a consequence of arbitrage in financial markets. In the wake of the productivity shock, the real return on assets (bonds and shares) increases in the world economy. In the high productivity economy (the Home country), the increase in the real return on shares is brought about by a decrease in today's price of equity (the value of the firm) relative to tomorrow's while the opposite is true in the low productivity economy.<sup>15</sup> Therefore firms' entry is above the steady state in the Home country and below the steady state in the Foreign country. As will be clear soon, this needs not be the case when markups (and returns) move counter-cyclically. Note that the response of Foreign entrants is the mirror image of that of Home entrants. This is a consequence of the fact that a productivity shock favors production of existing goods relative to the creation of new varieties. In the foreign country, the drop in the number of entrants and producers reduces GDP for most of the transition. As already noted, a strong incentive to move resources in the most

<sup>15</sup>With flexible prices, profits are a constant share of revenues,  $d_t^J = \frac{Y_t^J}{\theta N_t^J}$ . Therefore, firm's value (the price of equity) is given by:

$$\nu_t^J = \frac{\beta(1-\delta)}{\theta} E_t \left[ \sum_{s=t+1}^{\infty} \left( \frac{C_{s+1}^J}{C_s^J} \right)^{-\rho} \left( \frac{Y_{t+1}^J}{N_{t+1}^J} \right) \right]$$

The real return on shares is  $r_{t+1}^J \equiv \frac{(v_{t+1}^J + d_{t+1}^J)}{\nu_{tt}^J}$ .

productive economy generates negative comovements between countries.

Comparing the macroeconomic dynamics with sticky and flexible prices reveals a number of interesting features. Consistently with the statistics reported in the preceding subsection and in line with New-Keynesian models, sticky prices imply significant departures from a flexible price equilibrium. First, the response of entrants is subdued, translating into a moderate change in the stock of producers over time. Second, Home markups stay below the steady state for the whole transition. In the foreign country, the response of markups is positive on impact then it gradually declines and eventually turns negative before converging to the steady state. Markups are therefore counter-cyclical as observed in the data.<sup>16</sup> <sup>17</sup> The behavior of markups is a consequence of firms' entry moving prices and marginal costs in opposite directions. On the one side, an increase in the number of producers, by pushing on labor demand, raises wages and marginal costs. On the other side, it fosters price competition. These effects, virtually present in any model with entry, may be obscured by offsetting changes in markups or in other sources of marginal costs. With costs of price adjustments, for instance, (as in Bilbiee et al.(2007), Auray and Eyquem (2011) and Auray et al. (2012)), the pressure on wages implied by entry is typically accommodated by a drop in firms' markups.

Third, output and investment spillovers are positive as in the data. In order to see why, consider entry behavior in the foreign country. Markups above the steady state rise the expected return on investments in new firms. Moreover, the Home currency depreciation reduces the costs of importing materials for setting up a new firm. The combination of these effects reduces today's price of equity relative to tomorrow's, attracting new entrants.

My findings differ from previous models of endogenous entry, as Bilbiee et al. (2007) and Auray et al. (2012), where the performance of the sticky price model is virtually indistinguishable from that of the flexible price economy. In models with labor entry costs there is a direct link between asset prices and inflation that is absent in my setup. Consider, for instance, a temporary interest rate cut that reduces the real return on bonds and shares. The fall in the return on shares is brought about by an increase in today's price of equity relative to tomorrow's. The price of equity is related

---

<sup>16</sup>Studies based on different methodological approaches converge on the view that markups are counter-cyclical in major economies. In the US, this is indeed the case for studies using mostly aggregate data as Rotemberg and Woodford (1999b) as well as two digit industry level data as in Bils (1987). In a study with 14 OECD countries and industry level data, Martins, Scarpetta and Pilat (1996) find counter-cyclical markups in 52 out of the 56 cases they consider.

<sup>17</sup>In New-Keynesian models, variable margins of profits are typically powered by exogenous price stickiness. For models that combine variable markups and menu costs see, among others, Gopinath and Iskhoki (2010) and Bhattaraj and Schoenle (2010). Nominal rigidity is by no means essential for generating variable markups. In a setup with flexible prices, Atkeson and Burstein (2008) show that firms can discriminate prices across markets by letting their markups vary based on local market conditions. Alessandria (2009) shows that consumers' search can also deliver variable markups.

to marginal costs by the free entry condition in these models, therefore marginal costs rise, markups fall and, through the Phillips curve, inflation rises. Sticky prices will affect entry only marginally whenever simple monetary rules manage to control inflation, as is the case with Taylor rules. In my setup, instead, the price of equity is directly related to the cost of acquiring home and foreign investment goods.

They differ also from models with exogenous asset prices, as Auray and Eyquem (2011). In these models, shocks are transmitted through a change in the expected return on assets. In the face of a positive productivity shock, the increase in the real return on shares is brought about by an increase in expected dividends. In the country hit by the productivity shock, markups rise and more firms enter the market while the opposite occurs in the partner country (as with flexible prices). In my setup, on the contrary, the behavior of markups is counter-cyclical. Declining markups in high productivity economies tend to amplify deflationary pressures while rising markups in low productivity economies have the opposite effect. Monetary authorities have therefore an incentive to move interest rates in a pro-cyclical way in the attempt to stabilize prices (see Figure 1). Pro-cyclical interest rates, in turn, exacerbate exchange rate volatility via the interest parity. This accords with a large evidence documenting that nominal exchange rates among major currencies revert to their mean value with very long lags.<sup>18</sup> In the model, non-stationarity derives from the state equation of the terms of trade (25), which splits a given change in the terms of trade into changes in the nominal exchange rate and inflation differentials between countries. Although the terms of trade are stationary and revert to the initial value after a shock, there is nothing in the floating regimes considered that forces the exchange rate towards the initial steady state, unless inflation rates are zero at all dates (as with the Wicksellian policy). Mechanically, firms' entry contributes to this non-stationarity by generating inflation differentials between countries.

Entry behavior might in principle be affected by the exchange rate regime. Simulating the benchmark model under a unilateral peg in the Foreign country,  $i_t^F = i_t^H - 0.2\varepsilon_t$ , I find that this is indeed the case. Investments and output are negatively correlated between countries as with flexible prices (not shown in Figure 1). The reason is easy to grasp. Fixed exchange rates (combined with sticky prices) limit the extent to which foreign agents can take advantage of the Home productivity shock, especially in the early part of the transition. By contrast, arbitrage in financial markets requires the real return on assets to increase *immediately* in both economies. In floating regimes, a sharp depreciation of the Home currency helps to bridge the gap between high and low productivity economies. With fixed exchange rates, on the contrary, adjustment is brought about mainly by

---

<sup>18</sup>In a famous paper, Meese and Rogoff (1983) show that for major nominal exchange rates against the dollar a random walk model outperforms any of the structural models within a one-year forecasting horizon.



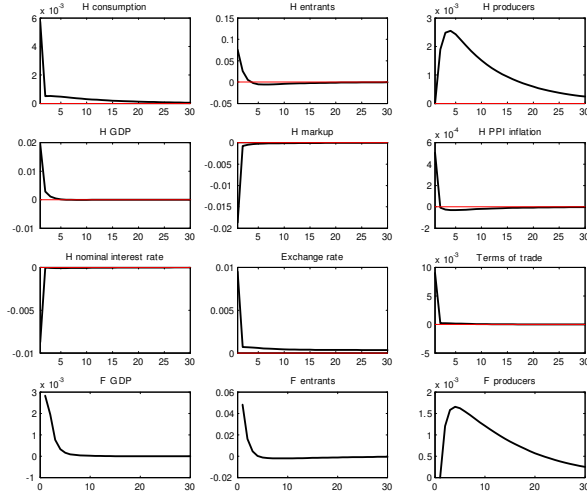


Figure 2: impulse responses for a one percent transitory fall in the Home interest rate

the price of equity. In the low productivity economy, today's price of equity increases relative to tomorrow's, deterring investments in new firms.

### 4.3 Monetary policy shocks

In order to give some insight on monetary transmission in the model, this subsection considers monetary policy shocks. Figure 2 displays impulse response functions of Home and Foreign variables for a one percent transitory cut in the Home nominal interest rate. The impulse responses are calculated under a symmetric Taylor rule  $i_t^J = 1.5i_{t-1}^J$  and with the baseline calibration.

The monetary expansion boosts world demand as long as prices are sticky, leading to a spike in world consumption. Over time, as prices slowly return to their natural levels, consumption converges to the steady state. The rise in consumption reflects a drop in the world real interest rate, i.e. a drop in the return on bonds. Arbitrage in financial markets requires the real return on shares to fall as well. The decrease in the real return on shares is brought about by a fall in the return ( $v_{t+1} + d_{t+1}$ ) relative to today's price of equity  $v_t$ . The price of equity is tied to the cost of acquiring investments goods by the free entry condition in the model (8). On impact, this cost falls, favoring investments in new firms.

Changes in the terms of trade spread the effects of the monetary easing worldwide. The drop in the Home nominal interest rate depreciates the Home currency, deteriorating the Home terms of trade.<sup>19</sup> Consequently, world demand shifts in favor of home goods. In my setup, foreign investors

<sup>19</sup>Given the assumption of costless trade, the terms of trade are independent of changes in the relative price of non-

take advantage of cheaper home goods (and lower entry costs) by expanding the range of available products. Fuelled by investment demand, foreign GDP stays above the steady state during the whole transition.

Firms might decide to accommodate market demand by increasing their size (the internal margin) as opposed to producing a wider range of varieties (the external margin). Simulations show that they have a strong incentive to do so (and a weak incentive to vary the external margin) whenever markups are relatively stable over the cycle. In the model, the volatility of markups crucially depends on the elasticity of labor supply. A high  $\varphi$ , by reducing the pro-cyclical movements of wages implied by firms' entry, generates less volatile markups. In the baseline calibration, firms' markups sharply drop on impact, thereby reducing the inflationary consequences of the monetary expansion. In simulations with  $\varphi$  as high as 6 (not shown in Figure 2), markups are fairly stable, entry is less volatile, inflation is higher and firms' size is larger compared to the baseline case.

## 5 Conclusions

This paper developed a two-country model with firms' entry and feedback monetary rules. The benchmark model, featuring a price-setting à la Calvo (1983) and entry subject to a sunk entry cost in units of investments goods, replicates the properties of the international business cycle fairly well. It generates positive comovements between output, investments and employment as those observed in the data while at the same time providing a solution to the quantity anomaly (i.e., the fact that the cross-correlation of output is higher than that of any other variable). Moreover, the theoretical moments of trade variables closely match those in the data. Simulations under alternative specifications show that both sticky prices and outward-looking entry costs are essential for replicating these facts.

## References

- [1] Alessandria G., 2009. Consumer search, price dispersion and international relative price fluctuations, *International Economic Review*, vol. 50, 3, 803-829
- [2] Ambler S., Cardia E. and C. Zimmerman, 2004. International business cycles: what are the facts?, *Journal of Monetary Economics* vol. 51, 257-276

---

tradable goods in this model. The role of trade frictions in firm entry and their implications for monetary transmission is ground for future research.

- [3] Arespa Castellò M., 2012. A new open economy macroeconomic (NOEM) model with endogenous portfolio diversification and firms entry, *Mimeo* Universitat Autònoma de Barcelona
- [4] Atkeson A. and A. Burstein, 2008. Pricing to market, trade costs and international relative prices, *American Economic Review* vol. 98, 5, 1998-2031
- [5] Auray S. and A. Eyquem, 2011. Do changes in product varieties matter for fluctuations and monetary policy in open economies?, *International Finance* vol.14, 3, 507-539
- [6] Auray S., Eyquem A. and J.C. Poutineau, 2012. The effect of a common currency on the volatility of the extensive margin of trade, forthcoming in *Annales d'Economie et Statistique*
- [7] Backus, D., Kehoe, P. and F. Kydland, 1992. International real business cycles, *Journal of Political Economy* vol. 100, 4, 745-775
- [8] Backus, D., Kehoe, P. and F. Kydland, 1995. International business cycles: theory and evidence. In: Cooley, T.F. (Ed.), *Frontiers of Business Cycle Research*. Princeton University Press, Princeton, NJ, pp. 331–356.
- [9] Bhattaraj S. and Schoenle R., 2010. Multi-product firms and price setting: theory and evidence from US producer prices, *Mimeo* Brandeis University
- [10] Benigno G. and P. Benigno, 2008. Exchange rate determination under interest rate rules, *Journal of International Money and Finance* vol. 27, 971-993
- [11] Bergin P. and G. Corsetti, 2008. The external margin and monetary policy, *Journal of Monetary Economics* vol. 55, 7, 1222-1237
- [12] Bilbiie F., Ghironi F. and M. Méltiz, 2007. Monetary policy and business cycles with endogenous entry and product variety, in Acemoglu D., Rogoff K. and M. Woodford, eds., *NBER Macroeconomic Annual 2007*, 22, pp. 299-353, University of Chicago Press, Chicago (IL)
- [13] Bilbiie F., Ghironi F. and M. Méltiz, 2011. Endogenous entry, product variety and business cycles, *CEPR discussion paper* no.8586
- [14] Bils M., 1987. The Cyclical behavior of marginal cost and price, *American Economic Review* vol.77, 5, 838-855.
- [15] Calvo G., 1983. Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* vol.12, 983-998

- [16] Campbell, J.,1998. Entry, exit, embodied technology and business cycles, *Review of Economic Dynamics* vol. 1, 371-408
- [17] Cavallari L., 2007, A Macroeconomic model of entry with exporters and multinationals, *The B.E. Journal of Macroeconomics* vol. 7, issue 1 (Contributions), Article 32
- [18] Cavallari L., 2010. Exports and FDI in an endogenous-entry model with nominal and real uncertainty, *Journal of Macroeconomics* vol.32, 300-313
- [19] Chatterjee S. and R. Cooper,1993. Entry and exit, product variety and the business cycle. *NBER Working Paper* 4562
- [20] Cook D., 2002. Market entry and international propagation of business cycles, *Journal of International Economics* vol. 56, 155–175
- [21] Corsetti G., Dedola L. and S. Leduc, 2008. International risk sharing and the transmission of productivity shocks, *Review of Economic Studies* 75, 443-473
- [22] Corsetti G., Martin P. and P. Pesenti, 2007. Productivity spillovers, terms of trade and the “home market effect”, *Journal of International Economics* vol. 73, 1, 99-127
- [23] Corsetti G., Martin P. and P. Pesenti, 2008. Variety, the transfer problem and the costs of current account adjustment, *CEPR discussion paper* no. 6660
- [24] Corsetti G. and P. Pesenti, 2002. Welfare and macroeconomic interdependence, *Quarterly Journal of Economics* vol. 116, 2, 421-446
- [25] Dunne T., Roberts M. J. and L. Samelson, 1988. Patterns of firm entry and exit in U.S. manufacturing industries, *The RAND Journal of Economics* vol. 19, 4, 495-515
- [26] Engel C. and J. Wang, 2011. International trade in durable goods: understanding volatility, cyclicalities and elasticities, *Journal of International Economics* vol. 83, 1, 37-52
- [27] Galí J., Gertler S. and D. Lopez-Salido, 2001. European inflation dynamics, *European Economic Review* vol. 45, 7, 1237-1270
- [28] Ghironi F. and M. Melitz, 2005. International trade and macroeconomic dynamics with heterogeneous firms, *Quarterly Journal of Economics* vol. 120, 865-915
- [29] Gopinath G. and O. Itskhoki, 2010. In search of real rigidities, in Acemoglu D. and M. Woodford, eds., *NBER Macroeconomic Annual* 2010, 25, pp.261-309 , University of Chicago Press, Chicago (IL)

- [30] Grossman and E. Helpman, 1991. *Innovation and Growth in the Global Economy*, MIT Press, Cambridge, MA
- [31] Heathcote and Perri, 2002. Financial autarky and international business cycles, *Journal of Monetary Economics* 49, 601-627
- [32] Helpman E., Melitz M. and S. Yeaple, 2004. Export versus FDI with Heterogeneous Firms, *American Economic Review* vol. 94, 1, 300-316
- [33] King R. and S. Rebelo, 1999. Resuscitating business cycles, in Taylor J. and M. Woodford eds., *Handbook of Macroeconomics* vol.1B, 927-1007, Elsevier, Amsterdam
- [34] Kose, A. and K. Yi, 2003. Can the standard international business cycle model explain the relation between trade and comovement?, *Journal of International Economics*, vol. , 267-295.
- [35] Lewis V., 2009. Business cycle evidence on firm entry, *Macroeconomic Dynamics* vol.13, 605-624
- [36] Martins J.O., Scarpetta S. and D. Pilat, 1996. Mark-Up ratios in manufacturing industries: estimates for 14 OECD countries, *OECD Economics Department Working Papers* no. 162
- [37] Meese R. and K. Rogoff, 1983. Empirical exchange rate models of the seventies: do they fit out of sample?, *Journal of International Economics* vol. 14, 3-24
- [38] Romer, P. 1990. Endogenous technological change, *Journal of Political Economy* 98, 71-102
- [39] Rotemberg J. and M. Woodford, 1999a. Interest-rate rules in an estimated sticky-price model, in: Taylor J.B. ed., *Monetary policy rules*, University of Chicago Press, Chicago, IL
- [40] Rotemberg J. and M. Woodford, 1999b. The cyclical behaviour of prices and costs, in Taylor J. B. and M. Woodford ed., *The Handbook of Macroeconomics*, vol. 1B. North-Holland, Amsterdam pp.1051–1135.
- [41] Russ K., 2007. The endogeneity of the exchange rate as a determinant of FDI: a model of money, entry and multinational firms”, *Journal of International Economics* vol. 71, 344-372
- [42] Taylor J. B., 1993. Discretion versus policy rules in practice, *Carnegie-Rochester Series on Public Policy* vol. 39, 195-214
- [43] Uusküla L., 2010. Limited participation or sticky prices? New evidence on firm entry and failures, *Mimeo*, European University Institute

- [44] Woodford M., 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Cambridge, MA

## 6 Appendix

### 6.1 Steady state

The model is solved in log-deviation from a symmetric steady state equilibrium without inflation where  $C^H = C^F = C$ ,  $Y^H = Y^F = Y$ ,  $N^H = N^F = N$ ,  $N_e^H = N_e^F = N_e$ ,  $L^H = L^F = L$  and  $\varepsilon = T = 1$ . Assuming  $Z^H = Z^F = f_e^H = f_e^F = 1$ , the steady state of the economy is such that:

$$N = \left( \frac{\theta(1 - \beta(1 - \delta)) - \delta\beta}{\beta(1 - \delta)} \right)^{\frac{\theta-1}{2-\theta}}$$

Other variables are given by:

$$\begin{aligned} i &= \frac{1 - \beta}{\beta}, \quad v = 1, \quad d = \frac{(1 - \beta(1 - \delta))}{\beta(1 - \delta)}, \quad \mu = \frac{\theta}{(\theta - 1)}, \quad \frac{P(h)}{P_H^H} = \frac{P(f)}{P_F^F} = N^{\frac{1}{\theta-1}} \\ C &= \theta N \left[ \frac{1 - \beta(1 - \delta)}{\beta(1 - \delta)} - \frac{\delta}{\theta(1 - \delta)} \right], \quad L = \theta d N^{\frac{2-\theta}{1-\theta}}, \quad Y = \theta d N, \quad N_e = \frac{\delta}{(1 - \delta)} N \end{aligned}$$

### 6.2 Loglinear model

Loglinearized conditions for households are:

$$\begin{aligned} E_t \widehat{C}_{t+1}^J &= \widehat{C}_t^J + \frac{1}{\rho} \left( \widehat{i}_t^J - E_t \pi_{t+1}^J \right) \\ E_t \widehat{C}_{t+1}^J &= \widehat{C}_t^J + \widehat{v}_t^J + \frac{1}{\rho} E_t \left( \frac{i + \delta}{1 + i} d_{t+1}^J + \frac{1 - \delta}{1 + i} \widehat{v}_{t+1}^J \right) \\ \widehat{L}_t^H &= -\rho \varphi \widehat{C}_t^J + \varphi \left( \widehat{Z}_t^H - \widehat{\mu}_t^H + (\gamma - 1) \widehat{T}_t + \widehat{P}_{t,t}^H \right) \\ \widehat{L}_t^F &= -\rho \varphi \widehat{C}_t^J + \varphi \left( \widehat{Z}_t^F - \widehat{\mu}_t^F - \gamma \widehat{T}_t + \widehat{P}_{t,t}^F \right) \end{aligned}$$

Loglinearized conditions for firms are:

$$\begin{aligned} \widehat{N}_t^J &= (1 - \delta) \widehat{N}_{t-1}^J + \delta \widehat{N}_{et-1}^J \\ \widehat{\mu}_t^J &= \alpha \beta (1 - \delta) \left( \widehat{P}_{t,t+1}^J - \widehat{P}_{t,t}^J + E_t \pi_{t+1}^J \right) \\ \pi_t^J &= \zeta m c_t^J + \beta (1 - \delta) E_t \pi_{t+1}^J \end{aligned}$$

where  $mc$  denotes an index of current marginal costs defined by the term in squared brackets in equation (27) in the main text.

Other log-linear equilibrium conditions are:

$$\begin{aligned}
\widehat{P}_{t,t}^J &= \frac{\alpha}{1-\alpha}\pi_t^J + \frac{1}{(1-\alpha)(\theta-1)}\widehat{N}_t^J - \frac{\alpha}{(1-\alpha)(\theta-1)}\widehat{N}_{t-1}^J \\
\widehat{Y}_t^J &= \widehat{Z}_t^J + \widehat{L}_t^J \\
\widehat{Y}_t^H &= \frac{(4-3\gamma-\sigma)\beta(1-\delta)}{\theta(1-\beta(1-\delta))}\widehat{T}_t + \left(1 - \frac{\delta\beta}{\theta(1-\beta(1-\delta))}\right)\widehat{C}_t + \frac{\delta\beta}{\theta(1-\beta(1-\delta))}(\widehat{N}_{et}^H + \widehat{N}_{et}^F) \\
\widehat{Y}_t^F &= -\frac{(-3\gamma-\sigma)\beta(1-\delta)}{\theta(1-\beta(1-\delta))}\widehat{T}_t + \left(1 - \frac{\delta\beta}{\theta(1-\beta(1-\delta))}\right)\widehat{C}_t + \frac{\delta\beta}{\theta(1-\beta(1-\delta))}(\widehat{N}_{et}^H + \widehat{N}_{et}^F) \\
\widehat{T}_t &= \widehat{T}_{t-1} + \Delta\widehat{\varepsilon}_t + \pi_t^F - \pi_t^H \\
\widehat{N}_{et}^H &= \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\widehat{Y}_t^H + \left(1 - \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\right)\widehat{C}_t - \widehat{v}_t^H - \frac{(1-\delta)}{\delta}\widehat{nfa}_t \\
\widehat{N}_{et}^F &= \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\widehat{Y}_t^F + \left(1 - \frac{\theta(1-\beta(1-\delta))}{\beta\delta}\right)\widehat{C}_t - \widehat{v}_t^F + \frac{(1-\delta)}{\delta}\widehat{nfa}_t \\
\widehat{nfa}_t &= \widehat{Y}_t^H - \left(1 - \frac{\beta\delta(1-\delta)}{\theta(1-\beta(1-\delta))}\right)\widehat{C}_t - \frac{\beta\delta(1-\delta)}{\theta(1-\beta(1-\delta))}\widehat{N}_{et}^H - \gamma\widehat{T}_t \\
E_t\Delta\widehat{\varepsilon}_{t+1} &= \widehat{i}_t^H - \widehat{i}_t^F \\
\widehat{v}_t^J &= (\gamma - \sigma)\widehat{T}_t
\end{aligned}$$

The model is closed with the interest rate rules indicated in the text.

In the variant with labor entry costs, the log-linear model is as before except for the following equations:

$$\begin{aligned}
\widehat{v}_t^J &= \frac{1}{\varphi}\widehat{L}_t^J + \rho\widehat{C}_t^J - \widehat{Z}_t^J \\
\widehat{Y}_t^J &= \widehat{Z}_t^J + \widehat{L}_t^J + \widehat{P}_{t,t}^J \\
\widehat{Y}_t^H &= \frac{2(1-\gamma)(1-\beta(1-\delta))}{\delta\beta N}\widehat{T}_t + \left(1 - \frac{\delta\beta}{\theta(1-\beta(1-\delta))}\right)\widehat{C}_t \\
\widehat{Y}_t^F &= -\frac{2\gamma(1-\beta(1-\delta))}{\delta\beta N}\widehat{T}_t + \left(1 - \frac{\delta\beta}{\theta(1-\beta(1-\delta))}\right)\widehat{C}_t
\end{aligned}$$