A generalized directional distance function in data envelopment analysis and its application to a cross-country measurement of health efficiency

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Abstract

Economic activity produces not only desirable outputs but also undesirable outputs that are usually called negative externalities in economic theory. Negative externalities are usually omitted from efficiency assessments (i.e., applications of Data Envelopment Analysis) which fail to express the true production process. In the present paper we develop a generalized directional distance function method for handling asymmetrically both desirable and undesirable outputs in the assessment process. Unlike the existing directional distance function-based approaches, the proposed method is units-invariant even in case assumptions for the direction vectors are relaxed. The new method is applied to data from national health systems of 160 countries. Desirable and undesirable outputs are incorporated to obtain a clear view of the efficiency status of the national health systems.

Keywords: Data envelopment analysis; Directional distance function; Undesirable outputs; Units-invariant; Health systems

1. Introduction

Data Envelopment Analysis (DEA) is a nonparametric methodology for evaluating the production process of operational units, or, as they are usually called in DEA literature, decision making units (DMUs). Drawing on the seminal paper of Charnes et al. [1], the scope of DEA is the comparative efficiency assessment of DMUs defining the minimum inputs engaged or the maximum outputs produced. At that study, outputs were regarded as a non-homogeneous entity with a unitary (positive) impact for every DMU.

Nowadays, becoming more sensitive to the negative impact of human activity (e.g., pollution, health system inequalities, medical complications, negative effects of policy making), a distinction between good and bad outputs should not be neglected, if such is present. Characteristically, in recent years, because of the growing interest in incorporating both desirable and undesirable outputs in performance measurements, an increased number of scholars in the DEA literature are engaged with the development of methods for handling asymmetrically the two types of outputs [2-6]. In the presence of undesirable outputs, their omission from the evaluation process is regarded as a misspecification error which yields misleading results [3, 7-9].

The methods that deal with good and bad outputs in the DEA literature can be classified into three groups according to their methodological framework. Each group introduces the following: (a) transformations of conventional DEA models (i.e., hyperbolic efficiency measure [3], separating measures for good and bad outputs [4], linear monotone decreasing transformation of the bad outputs [5], and handling of the bad outputs as inputs [7]); (b) modifications on the slacks-based measure (SBM) [6, 9]; and (c) modifications on the directional distance function [2]. In practice, the directional distance function is applied mostly for handling desirable and undesirable outputs [9, 10].

The proposed method draws on the directional distance function introducing a modified definition for the efficiency score. Utilizing a new definition of the efficiency score, unlike the existing directional distance function-based approaches, the new method complies with the units-invariance property, regardless of the value of the direction vectors. In addition, it always defines an inefficiency score between null and unity keeping the same restrictions with the existing directional distance function-based measures.

In our study, we apply the new method in conjunction with super-efficiency DEA in order to rank the evaluated DMUs in our numerical example.

This paper is organized as follows. Section 2 presents the foundations of the new method and discusses a limitation of the directional distance function. Section 3 discusses the generalized form of efficiency score in the directional distance function introduced in this paper, and Section 4 demonstrates the mathematical formulation of the proposed method. In Section 5, the new method is applied to real-world data as referred to the national health system of 160 countries. The scope of this section is the measurement of efficiency in the presence of both desirable and undesirable outputs, the ranking of the evaluated DMUs and the determination of optimal input and output levels. Section 6 concludes.

2. Foundations of the new model

The directional distance function can be seen as a generalized form of the radial model

$$\begin{aligned} & \max \quad \beta \\ & s.t. \quad X\lambda + \beta g_x \le x_0 \\ & \qquad Y\lambda - \beta g_y \ge y_0 \\ & \qquad \lambda \ge 0 \qquad (1) \end{aligned}$$

which is formulated appropriately when undesirable outputs exist

$$\begin{aligned} & \max \quad \beta \\ & \text{s.t.} \quad X\lambda + \beta g_x \le x_0 \\ & \qquad Y\lambda - \beta g_y \ge y_0 \end{aligned}$$

$$B\lambda + \beta g_b \leq b_0$$

$$\lambda \geq 0 \qquad (2)$$

In model (1), g_x and g_y denote the direction vectors associated with inputs (x) and outputs (y), respectively, and β is the measure of inefficiency. Model (2) differs from model (1) in that it introduces a distinction between good outputs, denoted by y, and bad outputs, expressed by b. Accordingly, an additional direction vector (g_b) is incorporated that refers to bad outputs (b).

Note that the direction vectors, g_x, g_y and g_b in (2), are all non-negative. If g_b is allocated a negative value to indicate that the outputs are undesirable, the constraint for the undesirable outputs is expressed as

$$B\lambda - \beta g_b \leq b_0 \qquad (3).$$

Both the constraints for the undesirable outputs in model (2) and model (3) indicate that the undesirable outputs are reduced to reach the frontier. In order to make the formulas easy to read, in this paper we use the direction vector in model (2) for undesirable outputs, i.e., all of the direction vectors have non-negative values.

One drawback of the directional distance function is that although β stands for a measure of inefficiency, it is not restricted within the interval null and unity. This restriction applies only if g_x=x_0, g_y=y_0 and g_b=b_0. In practice, when the direction vectors are considered equal to the observed inputs and outputs, the directional distance function is identical to a radial model. On the other hand, if such a simplification is not applied, β is not an appropriate measurement of inefficiency as it can be greater than unity. Therefore, the limitation of the existing directional distance function-based measures is twofold. In addition, another drawback of the directional distance function-based measures is that they violate a fundamental property of data envelopment analysis, i.e., units-invariance.

3. A generalized directional distance function

In this section, we develop a generalized definition of the efficiency score for the directional distance function even if no simplification in the direction vectors is applied.

By assuming that no undesirable outputs are present, the generalized directional distance function can be defined as

$$\min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i / x_{io}}{1+\frac{1}{s}\sum_{r=1}^{s}\beta g_r / y_{ro}}$$

$$s.t. \quad X\lambda + \beta g_x \leq x_0$$

$$Y\lambda - \beta g_y \geq y_0$$

$$\lambda \geq 0 \qquad (4)$$

and when undesirable outputs are produced by the production process model (4) is rewritten as

$$\min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i / x_{io}}{1+\frac{1}{s+p}(\sum_{r=1}^{s}\beta g_r / y_{ro} + \sum_{t=1}^{p}\beta g_t / b_{to})}$$

$$s.t. \quad X\lambda + \beta g_x \leq x_0$$

$$Y\lambda - \beta g_y \geq y_0$$

$$B\lambda + \beta g_b \leq b_0$$

$$\lambda \geq 0 \qquad (5)$$

where s denotes the number of good outputs and p the number of bad outputs. The ratio $\beta g_i/x_{i0}$ indicates the proportion of inputs decrease. Accordingly, the ratios $\beta g_r/y_{r0}$ and $\beta g_t/b_{t0}$ express the proportion of good output increase and the proportion of bad output decrease, respectively.

The proposed mathematical formulation (5) can be combined with the super-efficiency DEA model in order to increase its discriminatory power on efficient DMUs and to obtain ranking properties. For an efficient DMU_k, the super-efficiency model becomes

$$\min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i / x_{ik}}{1+\frac{1}{s+p}(\sum_{r=1}^{s}\beta g_r / y_{rk}+\sum_{t=1}^{p}\beta g_t / b_{tk})}$$

$$\text{s.t.} \sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j x_{ij}+\beta g_i \le x_{ik}, \quad i=1,2,\dots,m$$

$$\sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j y_{rj}-\beta g_r \ge y_{rk}, \quad r=1,2,\dots,s$$

$$\sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j b_{tj}+\beta g_t \le b_{tk}, \quad \mathrm{t}=1,2,\dots,p$$

$$\lambda_j \ge 0, \quad j=1,2,\dots,n \quad (j\neq k) \qquad (6)$$

The generalization of the proposed method lies in the fact that both radial and slack-based measures can be seen as special cases of the introduced generalized directional distance function.

In particular, let's assume that undesirable outputs are not present in order to deal with a simplified expression of the new model. In the case that

(i) $g_x=x_0$, and $g_y=y_0$, model (5) is reduced to the non-oriented radial measure proposed by Chen et al. [11], as follows

$$\min \frac{1-\beta}{1+\beta}$$

$$\text{s.t.} \quad X\lambda \le (1-\beta)x_0$$

$$Y\lambda \ge (1+\beta)y_0$$

$$\lambda \ge 0 \qquad (7)$$

(ii) $g_x= x_0$, and $g_y=0$, model (5) is reduced to the classic input-oriented radial measure [1]

$$\min \ 1-\beta$$

$$\text{s.t.} \quad X\lambda \le (1-\beta)x_0$$

$$Y\lambda \ge y_0$$

$$\lambda \geq 0 \qquad (8)$$

(iii) $g_x=0$, and $g_y=y_0$, model (5) is reduced to the classic output-oriented radial measure [1]

$$\max \ 1+\beta$$
$$\text{s.t.} \ X\lambda \leq x_0$$
$$Y\lambda \geq (1+\beta)\, y_0$$
$$\lambda \geq 0 \qquad (9)$$

(iv) the slack-based measure (SBM) put forth by Tone [12] is expressed as

$$\min \ \frac{1-\frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}}{1+\frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro}}$$
$$s.t. \ X\lambda + s^- \leq x_0$$
$$Y\lambda - s^+ \geq y_0$$
$$\lambda, s^-, s^+ \geq 0 \qquad (10)$$

where s^{-*} and s^{+*} are the optimal solutions of the SBM model (10).

If $g_x=s^{-*}$, and $g_y=s^{+*}$ in model (5), the optimal value of β will be 1, and consequently the objective value of model (5) is the same as that of the SBM model.

The proposed generalized directional distance function has the following features:

1. The efficiency scores computed with this method are independent of the length of the direction vector, although the values of β are dependent on the length of the direction vector. For example, the computed efficiency scores are the same irrespective of whether the direction vector (1, 1, 1, 1) or the direction vector (2, 2, 2, 2) is used.
2. It produces results that are consistent with the measures used in the radial model. For example, if the direction vector is defined as the input (or output) values of the evaluated DMU, the directional distance function model will be equivalent to the radial model, and the efficiency scores of the two models will be the same.
3. The inputs and outputs can be attached with user-defined weights (w) to indicate

their relative importance.

$$min \frac{1-\frac{1}{m}\sum_{i=1}^{m} w_i \beta g_i \ / \ x_{io}}{1+\frac{1}{s}\sum_{r=1}^{s} w_r \beta g_r \ / \ y_{r0} + \frac{1}{p}\sum_{t=1}^{p} w_t \beta g_t \ / b_{t0}}$$

$$\text{s.t. } Y\lambda + \beta g_x \le x_0$$

$$Y\lambda - \beta g_y \ge y_0$$

$$B\lambda + \beta g_b \le b_0$$

$$\lambda,\ g \ge 0$$

$$\sum_{i=1}^{m} w_i = 1, \sum_{r=1}^{s} w_r + \sum_{t=1}^{p} w_t = 1 \qquad (11)$$

This extension of the proposed generalized directional distance function model is particularly useful and has significant practical implications when the relationship between the variables is known. Essentially, it further enhances the accuracy and the applicability of the new model.

Although the generalized directional distance function determines an appropriate measurement of inefficiency, that is limited within the interval null and unity, irrespective of the value of the direction vectors, the units-invariance property is still violated. We overcome this deficiency by normalizing the data (inputs and outputs) using the method proposed by Cheng and Qian [13]. The normalization procedure is as follows:

$$\hat{x}_{ij} = x_{ij} \ / \ x_{i0}, \quad i = 1, 2, \ldots, m$$

$$\hat{y}_{rj} = y_{rj} \ / \ y_{r0}, \quad r = 1, 2, \ldots, s$$

$$\hat{b}_{tj} = b_{tj} \ / \ b_{t0}, \quad t = 1, 2, \ldots, p \qquad (12)$$

where $\hat{x}_{ij}$, $\hat{y}_{rj}$ and $\hat{b}_{tj}$ denote the normalized *i*th input, *r*th good output, and *t*th bad output, respectively, of the *j*th DMU. Accordingly, x_{ij}, y_{rj} and b_{tj} represent the original inputs, and the original desirable and undesirable outputs. The symbols indicated by the subscript '$_0$' differ from the aforementioned symbols in that they refer to the evaluated DMU. What essentially happens with the above data normalization is that the values of the evaluated DMU serve as the measurement unit. In this context, the efficiency scores from originally units-invariant DEA models, such as the radial model and SBM model, remain unchanged after the data normalization is applied, i.e., it is a normalization compatible with existing DEA models.

Subsequent to the data normalization procedure, model (4) is formulated as follows:

$$\min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i / \hat{x}_{io}}{1+\frac{1}{s}\sum_{r=1}^{s}\beta g_r / \hat{y}_{ro}}$$

$$s.t.\quad \hat{X}\lambda + \beta g_x \le \hat{x}_0$$

$$\hat{Y}\lambda - \beta g_y \ge \hat{y}_0$$

$$\lambda \ge 0 \qquad (13)$$

Since $\hat{x}_{i0}$ and $\hat{y}_{r0}$ are normalized to one, model (13) can be rewritten as

$$\min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i}{1+\frac{1}{s}\sum_{r=1}^{s}\beta g_r}$$

$$s.t.\quad \hat{X}\lambda + \beta g_x \le 1$$

$$\hat{Y}\lambda - \beta g_y \ge 1$$

$$\lambda \ge 0 \qquad (14)$$

Accordingly, model (5) becomes

$$\min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i}{1+\frac{1}{s+p}\left(\sum_{r=1}^{s}\beta g_r + \sum_{t=1}^{p}\beta g_t\right)}$$

$$s.t.\quad \hat{X}\lambda + \beta g_x \le 1$$

$$\hat{Y}\lambda - \beta g_y \ge 1$$

$$\hat{B}\lambda + \beta g_b \le 1$$

$$\lambda \geq 0 \qquad (15)$$

and model (6) is rewritten

$$\min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i}{1+\frac{1}{s+p}(\sum_{r=1}^{s}\beta g_r+\sum_{t=1}^{p}\beta g_t)}$$

$$\text{s.t.} \sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j \hat{x}_{ij}+\beta g_i \leq 1, \quad i=1,2,\ldots,m$$

$$\sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j \hat{y}_{rj}-\beta g_r \geq 1, \quad r=1,2,\ldots,s$$

$$\sum_{\substack{j=1\\ j\neq k}}^{n}\lambda_j \hat{b}_{tj}+\beta g_t \leq 1, \quad \text{t}=1,2,\ldots,p$$

$$\lambda_j \geq 0, \quad j=1,2,\ldots,n \quad (j\neq k) \qquad (16)$$

4. Solving the model

From the constraints of model (15) we know that the relationship $0 \leq \beta g \leq 1$ exists in the objective function, so the numerator of the objective function monotonously decreases as β increases, and the denominator of the objective function monotonously increases as β increases. Consequently, the objective function monotonously decreases as β increases. Therefore, model (15) is equivalent to model (17)

$$Score = \min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\beta g_i}{1+\frac{1}{s+p}(\sum_{r=1}^{s}\beta g_r+\sum_{t=1}^{p}\beta g_t)}$$

$$\max \quad \beta$$

$$\text{s.t.} \quad \hat{X}\lambda+\beta g_x \leq 1$$

$$\hat{Y}\lambda-\beta g_y \geq 1$$

$$\hat{B}\lambda+\beta g_b \leq 1$$

$$\lambda, g \geq 0 \quad (17)$$

In the mathematical formulations of the proposed generalized directional distance function measure we assume strong disposability of inputs, and of good and bad outputs. However, the linear programming can easily be modified so that it serves the weak disposability assumption of the undesirable outputs. The purpose for assuming strong disposability of all the variables lies in the nature of our data, which differs from that of the data utilized in most of the studies that measure efficiency in the presence of desirable and undesirable outputs. To be more precise, unlike the air pollution that is usually referred to as bad output in the literature, the bad outputs in this study (i.e., under-five mortality rate and maternal mortality ratio) are not direct or exclusive products of the production process, particularly, of the operation of the health system. In addition, the underlying relationship of the variables we utilize is much different from that applied in respective studies. That is to say, in our case, undesirable outputs are not symmetrically jointly produced with desirable outputs. To be more precise, in the utilized example, a decrease of bad outputs results in an increase of the good output.

5. An application to health data with undesirable outputs

In this section, we illustrate an application of the generalized directional distance function model to a cross-country dataset on health and compare the results with other applicable models. The data are from the World Bank and the United Nations, and they consist of one input and three outputs. The input is measured by health expenditure per capita, in US dollars, and the outputs are life expectancy, under-five mortality rate, i.e., the mortality rate of children less than five years of age, and maternal mortality rate. Life expectancy has been used as an output in many empirical analyses of the efficiency of health systems using DEA [14-17], and it is also a general indicator for the measurement of population health but also a vital component in the evaluation of socioeconomic conditions [18-20]. The mortality rate of children less than five years of age and the maternal mortality rate, which are used extensively in the extant literature in evaluating a health system and human development, are regarded as undesirable outputs [21-28].

The primary focus of this analysis is on the technical efficiency of the health systems of 160 countries. This analysis is useful because it will increase our understanding of how each national health system performs relative to others. The Millennium Development Goals emphasize that, between 1990 and 2015, the under-five years of age mortality rate should be reduced by two thirds (67%) and that the maternal mortality ratio should be reduced by three quarters (75%). In some areas, including Sub-Saharan Africa, South-East and Southern Asia, and Oceania, insufficient progress has been made towards the attainment of these goals [29]. In particular, the highest level of child and maternal mortality rates always occurs in Sub-Saharan Africa. It has been reported that one in eight children in this region die before the age of five. This is more than twice the average rate of the rest of the world, and it is far greater (19 times) than the average rate in high income regions. Sub-Saharan Africa has the worst performance in reducing both child and maternal mortality even though it does not have the lowest health expenditure per capita. As a comparison, South Asia has the lowest health expenditure per capita, but neither its child mortality rate nor its maternal mortality ratio is the highest in the world. The data indicate that, in addition to insufficient investment in healthcare, technical inefficiency may be another major cause of poor performance in Sub-Saharan Africa.

Table 1 summarizes the latest available data (from 2008) provided by the official websites of the World Bank and the United Nations. These data, which were obtained from 160 different countries, were used in this analysis.

Table 1 Summary of input and output indicators by geographical and economic regions

	Input	**Output**		
Region	**Health expenditure per capita (USD)**	**Life expectancy**	**Under-five mortality rate (1/1000)**	**Maternal mortality ratio (1/100,000)**
North America	6895.65	78.24	7.56	22.78
Latin America & Caribbean	543.99	73.64	23.27	85.13
Europe & Central Asia	2286.65	75.08	15.47	21.39
East Asia & Pacific	386.48	72.77	25.42	84.10
South Asia	40.23	64.63	70.79	290.00

Middle East & North Africa	290.72	71.98	33.62	80.10
Sub-Saharan Africa	75.27	53.24	127.91	640.00
OECD members	3983.14	78.92	9.00	24.67
European Union	3519.98	79.12	5.49	8.53
Low income	23.83	58.02	112.54	590.00
Middle income	185.28	68.54	54.33	210.00
High income	4452.57	79.36	6.74	15.18
World	863.77	69.14	60.85	260.00

A peculiarity of the health data we utilize in this paper, compared to the datasets used in other papers dealing with the measurement of efficiency in the presence of undesirable outputs, is that the bad outputs (under-five mortality rate, and maternal mortality ratio) are inversely related to the good output (life expectancy) at a significance level of 0.01 (Table A2 – Appendix). For the calculation of the correlation coefficients we used Spearman's measure as the data are not normally distributed (Table A3 – Appendix).

For the application of the generalized directional distance function model we used direction vector (1,1,1,1), i.e., $g_x=(1)$, $g_y=(1)$, $g_b=(1,1)$. In addition, we apply the super-efficiency DEA model under variable returns to scale (VRS) in conjunction with the proposed generalized directional distance function method in order to identify a realistic measurement of efficiency for the evaluated health systems. The combination of the super-efficiency DEA under VRS and the generalized directional distance function models enables the ranking of the DMUs under evaluation.

Table 2 Application of the generalized directional distance function to health data

No.	DMU	Score	Beta	Projections: Health expenditure per capita	Life expectancy	Mortality rate under 5	Maternal mortality ratio
1	Afghanistan	0.372624	0.457063	25.78	69.267	47.645	289.893
2	Albania	1.026887	-0.013265	284.243	75.604	13.483	31.411
3	Algeria	0.889877	0.05827	255.868	76.607	14.867	45.324
4	Angola	0.321866	0.513013	71.965	75.42	17.186	45.996
5	Argentina	0.905022	0.049857	660.789	79.046	9.893	42.1
6	Armenia	1.05751	-0.027951	146.803	71.442	22.307	29.811
7	Australia	0.971123	0.01465	3189.784	82.588	3.4	6

8	Austria	0.960315	0.020244	3167.816	81.858	3.73	4.899
9	Azerbaijan	0.840403	0.086718	218.788	76.144	19.134	34.705
10	Bahamas, The	0.851317	0.080312	1362.093	80.693	3.197	10.852
11	Bahrain	0.886557	0.060132	981.248	79.254	9.099	17.857
12	Bangladesh	1.224242	-0.100817	18.185	62.25	59.334	247.212
13	Barbados	0.914298	0.044769	930.141	79.679	7.321	30.098
14	Belarus	1.447378	-0.182799	414.942	73.166	8.516	17.742
15	Belgium	0.942689	0.029501	3166.893	81.828	3.744	4.852
16	Belize	0.977316	0.011472	199.354	76.242	15.58	45.53
17	Benin	0.588108	0.25936	23.802	68.997	48.981	300.597
18	Bhutan	0.752118	0.141476	85.96	75.51	17.01	45.945
19	Bolivia	0.760613	0.135968	64.79	74.586	21.297	78.823
20	Bosnia and Herzegovina	1.030471	-0.015007	513.335	73.979	8.243	9.135
21	Botswana	0.371315	0.458454	286.905	76.808	14.476	45.21
22	Brazil	0.832848	0.091198	654.795	79.032	9.95	42.367
23	Brunei Darussalam	0.982632	0.00876	826.432	78.343	7.633	20.816
24	Bulgaria	0.937996	0.031994	466.127	75.298	10.493	12.584
25	Burkina Faso	0.556226	0.28516	26.583	69.376	47.103	285.55

Table 2 displays in column 3 the super-efficiency scores of 25 out of 160 evaluated DMUs (the full Table – Table A4 – is at the Appendices), beta (β) values in column 4, and the target values for the input, and the good and bad outputs in the following columns.

Based on the selected input and output indicators, the most efficient health system among the 25 illustrated in Table 2 is that of Belarus (DMU 14) with a score of 1.4474, followed by the Bangladeshi (DMU 12) and the Armenian (DMU 6) health systems which obtain 1.2242 and 1.0575 respectively. At the last places of this ranking are Angola (DMU 4) and Botswana (DMU 21) with scores of 0.3219 and 0.3713, respectively.

The least efficient health system of the sample illustrated in Table 2, the health system of Angola, could be regarded as (weak) efficient even if health expenditure is decreased by 51.3% compared to the original expenditure level. At the same time, in the pursuit of efficiency attainment, life expectancy should be increased by 51.3%, and also mortality under 5 and maternal mortality should be scaled down by the same proportion.

The data analysis displayed in Table 2, as well as in Table A4 (Appendices), proves that the proposed method yields reasonable inefficiency measures between null and unity.

6. Conclusion

In this paper, we developed a generalized directional distance function measure for assessing efficiency in the presence of desirable and undesirable outputs. A new efficiency definition is introduced to overcome the inappropriateness that is associated with the measurement of inefficiency (beta) when the direction vectors are not identical to the inputs and outputs of the evaluated DMU. Unlike the existing directional distance function-based measures, the proposed generalized approach is units-invariant. The proposed method is consistent with the radial and the slack-based measures as the new method is a generalized expression of the last two measures. In addition, the generalized approach we put forth is flexible as it can be combined with super-efficiency DEA models and can be extended with the incorporation of weights in its objective function. This flexibility enhances the assessment power of the model and its applicability to real case studies. Therefore, the proposed approach has significant managerial implications. The properties of the new approach are presented in a numerical example utilizing health data from 160 countries.

Appendices

Table A1 Descriptive statistics of input and output indicators

	Health expenditure per capita	Life expectancy	Mortality rate under five	Maternal mortality ratio
N	160	160	160	160
Median	257.23	72.36	22.75	64
St. deviation	1736.46	10.3	49.23	277.92
Minimum	9.6	45.93	2.6	2
Maximum	8018.86	82.59	185.4	1400

Table A2 Nonparametric correlation of input and output indicators

		Life expectancy	Mortality rate under five	Maternal mortality ratio
Life expectancy	Corr. Coefficient	1.000	-0.942**	-0.890**
	p-value	.	0.000	0.000
Under five mortality rate	Corr. Coefficient	-0.942**	1.000	0.944**
	p-value	0.000	.	0.000
Maternal mortality ratio	Corr. Coefficient	-0.890**	0.944**	1.000
	p-value	0.000	0.000	.

** significant at the 0.01 level

Table A3 Testing the normality of the distributions of input and output indicators

	Health expenditure per capita	Life expectancy	Mortality rate under five	Maternal mortality ratio
Kolmogorov-Smirnov Test	2.042	2.596	3.495	3.558
p-value	0.000	0.000	0.000	0.000

Table A4 Application of the generalized directional distance function to health data (Part A)

No.	DMU	Score	Beta	Projections			
				Health expenditure per capita	Life expectancy	Mortality rate under five	Maternal mortality ratio
1	Afghanistan	0.372624	0.457063	25.78	69.267	47.645	289.893
2	Albania	1.026887	-0.013265	284.243	75.604	13.483	31.411
3	Algeria	0.889877	0.05827	255.868	76.607	14.867	45.324
4	Angola	0.321866	0.513013	71.965	75.42	17.186	45.996
5	Argentina	0.905022	0.049857	660.789	79.046	9.893	42.1
6	Armenia	1.05751	-0.027951	146.803	71.442	22.307	29.811
7	Australia	0.971123	0.01465	3189.784	82.588	3.4	6
8	Austria	0.960315	0.020244	3167.816	81.858	3.73	4.899
9	Azerbaijan	0.840403	0.086718	218.788	76.144	19.134	34.705
10	Bahamas, The	0.851317	0.080312	1362.093	80.693	3.197	10.852
11	Bahrain	0.886557	0.060132	981.248	79.254	9.099	17.857
12	Bangladesh	1.224242	-0.100817	18.185	62.25	59.334	247.212
13	Barbados	0.914298	0.044769	930.141	79.679	7.321	30.098
14	Belarus	1.447378	-0.182799	414.942	73.166	8.516	17.742
15	Belgium	0.942689	0.029501	3166.893	81.828	3.744	4.852
16	Belize	0.977316	0.011472	199.354	76.242	15.58	45.53
17	Benin	0.588108	0.25936	23.802	68.997	48.981	300.597
18	Bhutan	0.752118	0.141476	85.96	75.51	17.01	45.945
19	Bolivia	0.760613	0.135968	64.79	74.586	21.297	78.823

20	Bosnia and Herzegovina	1.030471	-0.015007	513.335	73.979	8.243	9.135
21	Botswana	0.371315	0.458454	286.905	76.808	14.476	45.21
22	Brazil	0.832848	0.091198	654.795	79.032	9.95	42.367
23	Brunei Darussalam	0.982632	0.00876	826.432	78.343	7.633	20.816
24	Bulgaria	0.937996	0.031994	466.127	75.298	10.493	12.584
25	Burkina Faso	0.556226	0.28516	26.583	69.376	47.103	285.55
26	Burundi	0.498206	0.33493	12.441	65.379	64.033	277.834
27	Cambodia	0.741542	0.148408	36.439	70.72	40.446	232.223
28	Cameroon	0.414126	0.414301	38.072	70.943	39.343	223.386
29	Canada	0.960704	0.020042	3189.784	82.588	3.4	6
30	Cape Verde	0.93234	0.035014	146.966	75.904	16.24	45.722
31	Central African Republic	0.421613	0.406853	11.767	64.945	65.71	267.548
32	Chad	0.397473	0.431154	27.949	69.562	46.181	278.16
33	Chile	1.006833	-0.003405	764.549	78.379	9.031	26.089
34	China	0.939244	0.03133	141.536	75.117	18.277	36.809
35	Colombia	0.8983	0.053574	300.36	76.894	14.307	45.161
36	Comoros	0.7431	0.147381	23.615	68.684	50.135	289.891
37	Congo, Dem. Rep.	0.568158	0.275382	9.655	60.654	66.797	274.032
38	Congo, Rep.	0.529908	0.307268	56.207	73.415	27.094	125.261
39	Costa Rica	1.017856	-0.008849	623.62	78.248	9.043	44.389
40	Cote d'Ivoire	0.498041	0.335077	40.424	71.263	37.754	210.658
41	Croatia	0.901974	0.051539	1166.374	79.825	5.691	13.278
42	Cuba	1.052709	-0.025678	688.745	76.53	6.462	27.391
43	Cyprus	0.945278	0.028131	1854.975	81.244	2.952	8.242
44	Czech Republic	0.938192	0.031889	1421.812	79.43	3.872	7.745
45	Denmark	0.918366	0.042554	3165.591	81.785	3.764	4.787
46	Djibouti	0.545312	0.294237	56.495	73.455	26.9	123.704
47	Dominican Republic	0.900077	0.052589	247.586	76.554	14.972	45.354
48	Ecuador	0.969168	0.015657	212.517	76.327	15.414	45.482
49	Egypt, Arab Rep.	0.918446	0.042511	93.129	75.556	16.919	45.919
50	El Salvador	0.87024	0.069382	201.875	76.259	15.548	45.521
51	Equatorial Guinea	0.311484	0.524989	258.861	76.626	14.83	45.313
52	Eritrea	1.114811	-0.054289	10.123	63.887	69.794	242.491
53	Estonia	0.878384	0.064745	1004.373	78.547	6.173	11.223
54	Ethiopia	0.765378	0.132902	11.986	65.086	65.165	270.892
55	Fiji	1.159988	-0.074069	165.123	73.174	19.548	27.926
56	Finland	0.935085	0.033546	3630.087	82.238	3.189	5.737
57	France	0.958365	0.02126	3189.784	82.588	3.4	6
58	Georgia	0.907104	0.048711	245.43	76.54	14.999	45.362
59	Germany	0.930959	0.035755	3189.784	82.588	3.4	6

60	Ghana	0.74186	0.148198	47.23	72.191	33.158	173.837
61	Greece	3.981956	-0.598551	4971.478	78.905	4.545	3.197
62	Guatemala	0.850236	0.080943	169.092	76.047	15.961	45.641
63	Guinea	0.565927	0.277198	15.413	67.292	56.649	323.138
64	Guinea-Bissau	0.457528	0.372186	10.977	64.437	67.671	255.515
65	Guyana	0.818498	0.099808	110.096	75.666	16.705	45.857
66	Haiti	0.736249	0.151909	33.744	70.353	42.266	246.801
67	Honduras	0.908696	0.047836	114.851	75.697	16.645	45.839
68	Hungary	0.868217	0.07054	1040.177	78.901	6.528	12.083
69	Iceland	1.48	-0.193548	4022.1	81.025	3.103	5.968
70	India	0.810287	0.104797	40.528	71.174	38.052	205.897
71	Indonesia	0.884197	0.06146	47.727	72.259	32.822	171.144
72	Iran, Islamic Rep.	0.917682	0.042926	243.387	75.32	19.807	28.712
73	Iraq	0.796049	0.113555	96.51	75.578	16.877	45.906
74	Ireland	0.964651	0.017993	3128.861	80.565	4.316	2.946
75	Israel	1.000946	-0.000473	2093.662	80.964	3.28	7.003
76	Italy	0.987328	0.006376	3169.2	81.904	3.71	4.968
77	Jamaica	0.890426	0.057963	241.606	76.515	15.047	45.376
78	Japan	1.033366	-0.016409	3242.127	81.232	3.456	5.853
79	Jordan	0.897968	0.053759	307.646	76.941	14.215	45.135
80	Kazakhstan	0.747103	0.144752	284.554	76.724	17.033	38.486
81	Kenya	0.595027	0.253898	24.705	69.12	48.371	295.709
82	Korea, Rep.	0.985961	0.007069	1236.152	80.397	4.399	16.463
83	Kuwait	0.917526	0.043011	947.257	77.577	5.878	8.613
84	Kyrgyz Republic	0.944074	0.028768	52.589	71.242	39.141	78.67
85	Lao PDR	0.887995	0.059325	31.971	70.111	43.464	256.397
86	Latvia	0.838988	0.087555	893.35	78.76	9.672	18.249
87	Lebanon	0.860372	0.075054	559.026	77.455	14.99	24.049
88	Lesotho	0.32212	0.512722	29.465	69.482	46.182	258.257
89	Liberia	0.598864	0.250888	19.85	68.458	51.65	321.979
90	Lithuania	0.849351	0.08146	855.124	77.662	6.889	11.941
91	Luxembourg	0.943339	0.029156	3396.938	82.423	3.301	5.876
92	Madagascar	0.919653	0.041855	20.825	68.591	50.992	316.703
93	Malawi	0.574995	0.269846	13.07	65.784	62.47	287.421
94	Malaysia	1.073747	-0.035562	365.764	70.952	7.249	17.405
95	Maldives	0.951102	0.025062	450.484	77.711	15.402	36.073
96	Mali	0.451937	0.37747	24.111	69.039	48.773	298.926
97	Malta	0.998139	0.000931	1372.227	79.506	3.78	7.993
98	Mauritania	0.677789	0.192045	21.94	68.743	50.239	310.67
99	Mauritius	0.890055	0.05817	378.24	76.792	14.504	33.906
100	Mexico	0.939051	0.031432	569.573	78.633	10.912	44.178
101	Moldova	0.854352	0.078544	166.463	73.717	18.613	29.487
102	Mongolia	0.814621	0.102159	65.922	74.381	22.29	58.36
103	Montenegro	0.942798	0.029443	592.221	76.213	8.541	14.558

104	Morocco	0.880016	0.063821	139.94	75.859	16.329	45.748
105	Mozambique	0.482401	0.349162	13.362	65.972	61.745	291.87
106	Myanmar	1.265659	-0.117254	11.127	60.465	66.26	268.141
107	Namibia	0.598061	0.251517	212.816	76.329	15.41	45.481
108	Nepal	0.957322	0.021804	23.827	69	48.964	300.462
109	Netherlands	0.943421	0.029113	3189.784	82.588	3.4	6
110	New Zealand	0.953967	0.023559	2848.565	82.244	3.285	6.573
111	Nicaragua	0.933882	0.034189	101.49	75.61	16.814	45.888
112	Niger	0.580692	0.265269	15.709	67.482	55.913	327.647
113	Nigeria	0.410455	0.417982	42.742	71.579	36.189	198.117
114	Norway	0.951691	0.024753	3189.784	82.588	3.4	6
115	Oman	0.931642	0.035388	438.014	75.582	10.514	19.292
116	Pakistan	0.932477	0.034941	20.855	66.983	56.72	250.915
117	Panama	0.938271	0.031847	477.465	78.038	12.074	44.514
118	Papua New Guinea	0.770666	0.129519	33.955	69.693	44.58	217.62
119	Paraguay	0.892626	0.056733	152.323	75.939	16.173	45.702
120	Peru	0.924112	0.039441	192.331	76.197	15.669	45.556
121	Philippines	0.837425	0.08848	62.007	74.005	23.896	85.683
122	Poland	1.249808	-0.111035	1079.158	75.038	7.089	6.666
123	Portugal	0.930742	0.035871	2346.55	81.341	3.953	6.749
124	Qatar	0.929876	0.036336	1710.46	80.601	3.152	7.709
125	Romania	0.882694	0.062308	484.582	77.087	15.003	25.318
126	Russian Federation	0.756497	0.13863	489.347	77.255	11.542	33.593
127	Rwanda	0.545962	0.293693	32.084	70.126	43.388	255.786
128	Saudi Arabia	0.888261	0.059175	635.996	77.746	13.836	22.58
129	Senegal	0.615533	0.237982	47.191	72.186	33.184	174.047
130	Serbia	1.205675	-0.093248	545.165	75.143	8.431	8.746
131	Sierra Leone	0.345076	0.486905	24.044	69.03	48.818	299.287
132	Singapore	2.299574	-0.393861	1956.497	78.229	3.903	12.545
133	Slovak Republic	0.941865	0.029938	1353.425	76.941	5.766	5.82
134	Slovenia	0.931713	0.035351	2158.96	81.55	3.054	7.731
135	Solomon Islands	0.812305	0.103567	60.556	73.579	25.19	89.643
136	South Africa	0.340331	0.492169	232.934	76.459	15.157	45.408
137	Spain	0.971971	0.014214	3087.836	82.329	3.587	5.915
138	Sri Lanka	1.125073	-0.058856	87.459	73.807	18.636	41.295
139	Suriname	0.803269	0.109097	376.811	77.388	13.343	44.882
140	Swaziland	0.277404	0.565675	61.391	74.122	23.593	97.213
141	Sweden	0.9881	0.005985	3981.455	81.585	3.181	4.97
142	Switzerland	0.985594	0.007255	3189.784	82.588	3.4	6
143	Syrian Arab Republic	1.281653	-0.123443	79.603	74.106	19.323	51.678
144	Tajikistan	1.341162	-0.145724	42.839	66.786	57.025	73.326
145	Tanzania	0.637781	0.221164	17.25	68.104	53.407	336.047
146	Thailand	0.994101	0.002958	163.342	74.808	13.859	41.087

147	Timor-Leste	0.662131	0.203275	56.852	73.503	26.659	121.773
148	Trinidad and Tobago	0.747529	0.144473	776.443	79.318	8.789	36.947
149	Tunisia	0.942311	0.029701	240.711	76.509	15.059	45.379
150	Turkey	0.89163	0.057289	587.569	77.319	13.747	21.682
151	Turkmenistan	0.722759	0.160929	68.445	75.084	18.828	59.045
152	Uganda	0.50374	0.330017	29.296	69.746	45.27	270.87
153	Ukraine	0.874697	0.066839	250.063	72.813	13.064	24.262
154	United Arab Emirates	0.893919	0.056011	1347.031	80.491	3.363	9.44
155	United Kingdom	0.927651	0.037532	3189.784	82.588	3.4	6
156	United States	0.887428	0.059643	3189.784	82.588	3.4	6
157	Uruguay	0.939041	0.031438	701.785	78.369	11.623	26.151
158	Uzbekistan	2.6333	-0.449536	74.486	73.038	26.518	43.486
159	Yemen, Rep.	0.745385	0.145879	56.8	73.496	26.694	122.052
160	Zambia	0.337952	0.494822	34.534	70.335	42.179	237.434

References

[1] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. Eur J Oper Res. 1978;2(6):429-44.

[2] Chung YH, Färe R, Grosskopf S. Productivity and undesirable outputs: A directional distance function approach. J Environ Manage. 1997;51(3):229-40.

[3] Fare R, Grosskopf S, Lovell CAK, Pasurka C. Multilateral Productivity Comparisons When Some Outputs Are Undesirable - a Nonparametric Approach. Rev Econ Stat. 1989;71(1):90-8.

[4] Scheel H. Undesirable outputs in efficiency valuations. Eur J Oper Res. 2001;132(2):400-10.

[5] Seiford LM, Zhu J. Modeling undesirable factors in efficiency evaluation. Eur J Oper Res. 2002;142(1):16-20.

[6] Tone K. Dealing with Undesirable Outputs in DEA:A Slacks-based Measure (SBM) Approach. The Operations research Society of Japan2004. p. 44-5.

[7] Yang H, Pollitt M. Incorporating both undesirable outputs and uncontrollable variables into DEA: The performance of Chinese coal-fired power plants. Eur J Oper Res. 2009;197:1095–105.

[8] Yu M-M, Hsu S-H, Chang C-C, Lee D-H. Productivity growth of Taiwan's major domesthic airports in the presence of aircraft noise. Transportation Research Part E. 2008;44:543–54.

[9] Lozano S, Gutierrez E. Slacks-based measure of efficiency of airports with airplanes delays as undesirable outputs. Computers & Operations Research. 2011;38(1):131-9.

[10] Podinovski VV, Kuosmanen T. Modelling weak disposability in data envelopment analysis under relaxed convexity assumptions. 2011;211:577–85.

[11] Chen J-X, Deng M, Gingras S. A modified super-efficiency measure based on

simultaneous input–output projection in data envelopment analysis. Computers and Operations Research. 2011;38(2):496-504.
[12] Tone K. A slacks-based measure of efficiency in data envelopment analysis. Eur J Oper Res. 2001;130(3):498-509.
[13] Cheng G, Qian Z. Data Normalization for Data Envelopment Analysis and Its Application in Directional Distance Function. Systems Engineering. 2011;29(7):70-5.
[14] Zaim O, Fare R, Grosskopf S. An economic approach to achievement and improvement indexes. Soc Indic Res. 2001;56(1):91-118.
[15] Retzlaff-Roberts D, Chang CF, Rubin RM. Technical efficiency in the use of health care resources: a comparison of OECD countries. Health Policy. 2004;69(1):55-72.
[16] Afonso A, St Aubyn M. Non-parametric approaches to education and health efficiency in OECD countries. J Appl Econ. 2005;8(2):227-46.
[17] Grosskopf S, Self S, Zaim O. Estimating the efficiency of the system of healthcare financing in achieving better health. Applied Economics. 2006;38(13):1477-88.
[18] Lantz PM, Ubel PA. The Use of Life Expectancy in Cancer Screening Guidelines. Journal of General Internal Medicine. 2005;20(6):552-3.
[19] Wilson J, Graham J, MacDonagh R. Patient life-expectancy: a vital element in planning treatment? BJU International. 2004;93(4):461-3.
[20] Weinstein M, Appadoo S. CREATIVE APPROACHES TO MODELING LIFE EXPECTANCY GAINS FOR ECONOMIC EVALUATION USING PUBLISHED DATA. Value Health. 2001;4(2):189-90.
[21] Rajaratnam JK, Tran LN, Lopez AD, Murray CJL. Measuring Under-Five Mortality: Validation of New Low-Cost Methods. PLoS Med. 2010;7(4):e1000253.
[22] Hegyvary ST, Devon MB, Murua A. Clustering Countries to Evaluate Health Outcomes Globally. Journal of Public Health Policy. 2008;29(3):319-39.
[23] Rutherford ME, Mulholland K, Hill PC. How access to health care relates to under-five mortality in sub-Saharan Africa: systematic review. Tropical Medicine & International Health. 2010;15(5):508-19.
[24] Loudon I. The Transformation Of Maternal Mortality. BMJ: British Medical Journal. 1992;305(6868):1557-60.
[25] Shiffman J. Can Poor Countries Surmount High Maternal Mortality? Studies in Family Planning. 2000;31(4):274-89.
[26] Kao S, Chen L-M, Shi L, Weinrich MC, Miller CA. Maternal Mortality in Taiwan: Rates and Trends. International Family Planning Perspectives. 1997;23(1):34-8.
[27] Mauldin WP. Maternal Mortality in Developing Countries: A Comparison of Rates from Two International Compendia. Population and Development Review. 1994;20(2):413-21.
[28] Bhatia JC. Levels and Causes of Maternal Mortality in Southern India. Studies in Family Planning. 1993;24(5):310-8.
[29] United Nations. Millennium development goals report 2011. New York2011.