# Vertical Integration and Fuel Diversity in Electricity Generation – a theoretical study in imperfect competition

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# Vertical Integration and Fuel Diversity in Electricity Generation
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**Abstract**

This paper contains an analysis of stylised natural gas and electricity supply sectors. Power plants operate either on natural gas or on a competing fuel – e.g. oil. The competing fuel is assumed to be traded at world market price whereas natural gas is sold by a national monopoly.

The paper conducts a number of analyses using different market structures, ranging from perfect competition, over oligopolistic competition among power plants using natural gas only, to situations with both gas and oil and vertical integration in the natural gas part of the sector.

The paper provides a number of results, some of which are quite surprising: (1) The natural gas monopoly price is only vaguely correlated with the oil price although the fuels are competing; (2) A merger between the gas supplier and a gas-fired power plant is likely to increase social welfare, and consumer surplus; and (3) Competition between fuels has more positive effect on social welfare than increased competition between gas-fired plants.

# 1 Introduction

The starting point of this analysis is a strong well-established electricity sector incorporating power plants that use either natural gas or a competing fuel, such as coal or oil. The model is short-term; and permanent technology is adopted so that the individual power plants cannot make changes to the fuel thereby dividing the electricity generating sector into two. It is further assumed that the oil can be purchased at a world market price whereas the natural gas is a local raw material subject to imperfect competition due to national protection. Hence, the natural gas supplier will be a monopoly supplier on several gas power plants that operate within the electricity market, competing with the oil-based electricity supplied. All power plants are assumed to have equal access to all customers in the electricity market. It is further assumed that the competition the gas market and the electricity market

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are quantity competition, i.e. Cournot competition according to Cournot (1938) and Tirole (1988).

The analysis is concerned with two main points, namely:

- What happens if vertical integration becomes part of the gas market, and
- What effect will it have on the market structure and profitability of the gas section if there are competing electricity production technologies?

The analyses show many surprising results, for example, the price of natural gas is only partly linked to the price of oil, and that seemingly enhanced competition, such as more gas-fired plants, is not necessarily in the best interest of the electricity consumers.

This paper is organised as follows: Firstly, the gas part of the market is analysed – at first with two competitive power plants and later with three power plants. Secondly, the oil-based share of the electricity sector is introduced. Vertical integration in respect of the gas part is thereafter introduced. All the earlier results are reviewed on the basis of this vertical integration. Lastly, the six analyses of imperfect competition are all compared to the benchmark that corresponds to perfect competition.

# 2 Two Gas-Fired Power Plants

As an introduction to the equilibrium analysis a simple case is considered, namely an electricity sector, where only gas-fired power plants exist. Their only option is to purchase natural gas from the monopoly supplier of natural gas. The supplier is assumed to fixed marginal cost, $c$. The supplier sells the natural gas to the two gas-fired power plants at a common wholesale price, $w$. The quantities of gas sold are $q_1$ and $q_2$ for plant 1 and 2 respectively. The gas supplier's profit is thus:

$$\Pi_g(q_1, q_2, w) = (w - c)(q_1 + q_2).$$

To facilitate calculations the output unit of the power are normalised, such that output equals input. The power plants face a common electricity demand described by a correspondence between quantities offered, $q_1$ and $q_2$, and the market prices of electricity, $p$:

$$q_1 + q_2 = 1 - p.$$

In order to assure an equilibrium, it has to be assumed that the cost parameter $c$ is less than unity otherwise the electricity demand will equal zero. Figure 1 illustrates market relations.

The Cournot equilibrium of the electricity market will according to Tirole (1988) be a symmetric Nash equilibrium with the following properties:

$$\begin{aligned}
q_1 = q_2 &= \frac{1-w}{3}, \\
q = q_1 + q_2 &= \frac{2}{3}(1-w), \\
p &= w + \frac{1-w}{3}, \\
\Pi_1(w) = \Pi_2(w) &= \frac{(1-w)^2}{9},
\end{aligned}$$

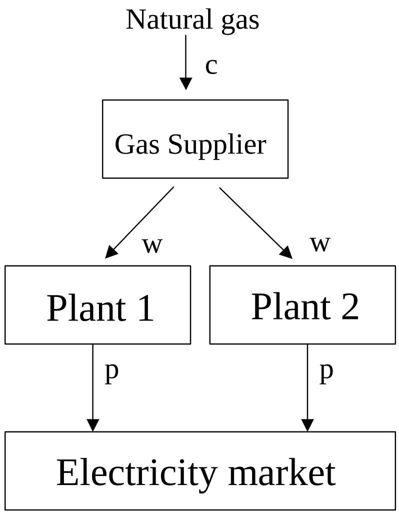



Figure 1. Market Relation I: A single gas supplier, serving a power plant duopoly

where $\Pi_1(w)$ and $\Pi_2(w)$ are the profit margins of power plant 1 and 2, respectively, as a function of the wholesale price of natural gas $w$. It is derived that the mark up at the power plant is $(1-w)/3$.

The supplier of natural gas is now subject to demand, $D_g(w)$, defined by the following equation:

$$D_g(w) \;=\; q_1 + q_2 \;=\; \frac{2(1-w)}{3}.$$

We can easily derive his optimal wholesale price in that his profit can likewise be expressed as a function of $w$ alone:

$$\begin{aligned} \Pi_g(w) &= (w-c)D_g(w) \;=\; \frac{2(1-w)(w-c)}{3} \;\Rightarrow \\ \frac{\partial \Pi_g(w)}{\partial w} &= \frac{2(1-2w+c)}{3} = 0 \;\Rightarrow \\ w &= \frac{1+c}{2}. \end{aligned}$$

Inserting his optimal wholesale price into the demand equations leads to:

$$\begin{aligned} q_1 = q_2 &= \frac{1-c}{6} \;\Rightarrow \\ q_1 + q_2 &= \frac{1-c}{3} \\ \Rightarrow p &= \frac{2+c}{3}. \end{aligned}$$

On the basis of the deduced quantities, the profits and the cunsumer suprplus, CS, in the sector will be as follows:

$$\begin{aligned} \Pi_1 = \Pi_2 &= \frac{(1-c)^2}{36}, \\ \Pi_g &= \frac{(1-c)^2}{6}, \end{aligned}$$

$$\begin{aligned}\Pi + \Pi_1 + \Pi_2 &= \frac{8(1-c)^2}{36}, \\ \text{CS} &= \frac{q(1-p)}{2} = q^2/2 \\ &= \frac{(1-c)^2}{18}.\end{aligned}$$

The social welfare, $W$, is thus:

$$\text{W} = \text{CS} + \Pi_g + \Pi_1 + \Pi_2 = 10(1-c)^2/36.$$

Because there are market imperfections at two levels, it is not feasible to calculate traditional indicators for market power, such as the Herfindahl or Lerner indexes. An indicator of the size of the market imperfections could instead be to calculate the dead weight loss, DWL, and thereafter calculate the relative loss, namely the relationship, DWLR, between the dead weight loss and the social welfare in social optimum.

The maximum achievable social welfare, maxW, corresponds to a situation where resources are optimally utilised, i.e., when marginal cost, marginal value and price coincide.

$$\begin{aligned}\text{maxW} = \text{CS}_{|p=c} &= \frac{q(1-p)}{2} \\ &= \frac{(1-p)^2}{2} = \frac{(1-c)^2}{2}. \\ \text{DWL} &= \text{maxW} - \text{W} \\ &= \frac{(1-c)^2}{2} - \frac{10(1-c)^2}{36} \\ &= \frac{2(1-c)^2}{9}. \\ \text{DWLR} &= \frac{\text{DWL}}{\text{maxW}} \\ &= \frac{2(1-c)^2/9}{(1-c)^2/2} = 4/9 = 44\%.\end{aligned}$$

The dead weight loss constitutes, therefore, nearly half the potential social welfare. The magnitude of the loss is due to mark-ups at both stages of the supply chain. The gas supplier is responsible for 75% of the profit and is, therefore, also believed to be the main contributor to the dead weight loss. The figures from the analysis are shown in the column headed GG in Table 1 at the end of this article.

We will in the following sections look at what occurs when the power plants are subject to a higher degree of competition.

# 3 Three Gas-Fired Power Plants

Even though the gas power stations jointly only accounted for 25% of the profit that was due to the imperfect competition, we will look at what it would mean to have greater competition between the gas power stations.

Greater competition will be analysed through an example where three gas power stations are present on the market. Their profit will be as follows:

$$\Pi_i(q_i;q_j,q_k,w) = q_i(p-w) = q_i(1-q_1-q_2-q_3-w),\ i=1,2,3,\ i\neq j\neq k.$$

Applied notation as before, the following description of the balance is obtained:

$$\begin{aligned}
q_i &= \frac{1-w}{4},\ i=1,2,3,\\
q_1+q_2+q_3 &= \frac{3(1-w)}{4},\\
\Pi_g &= (w-c)\frac{3(1-w)}{4},\\
w &= \frac{1+c}{2},\\
q_i &= \frac{1-c}{8},\ i=1,2,3,\\
q = q_1+q_2+q_3 &= \frac{3(1-c)}{8},\\
p &= \frac{5+3c}{8},\\
\Pi_i &= \frac{(1-c)^2}{64},\ i=1,2,3,\\
\Pi_g &= \frac{3(1-c)^2}{16},\\
\Pi_1+\Pi_2+\Pi_3+\Pi_g &= \frac{15(1-c)^2}{64},\\
\mathrm{CS} &= q^2/2 = \frac{9}{128}(1-c)^2.
\end{aligned}$$

If we compare the results from the situation with only two gas-fired power plants, we will not surprisingly see that the dead weight loss has now become smaller. The electricity production has increased with 12.5% from $(1-c)/3$ to $3(1-c)/8$. This level is, however, still below half of the quantity associated with maximum welfare equilibrium in which the quantity would have been $1-c$. The consumer profit has increased by a factor of 5. The gas supplier's profit has slightly increased, so it is only the power stations, both individually and jointly, that have lost profit. Despite the gas supplier's income increase, the model finds that the consumers and the power plants as a whole have gained. Consumers have gained. Power plants have lost.

Considering the movement of incomes, it is surprising that the optimal wholesale price from the gas supplier is still the same as the previous scenario. So, even though the natural gas supplier profits from the increased competition among his costumers, which is unambiguously expected to imply larger sales, it is not optimal for him to increase the price.

The welfare analysis give:

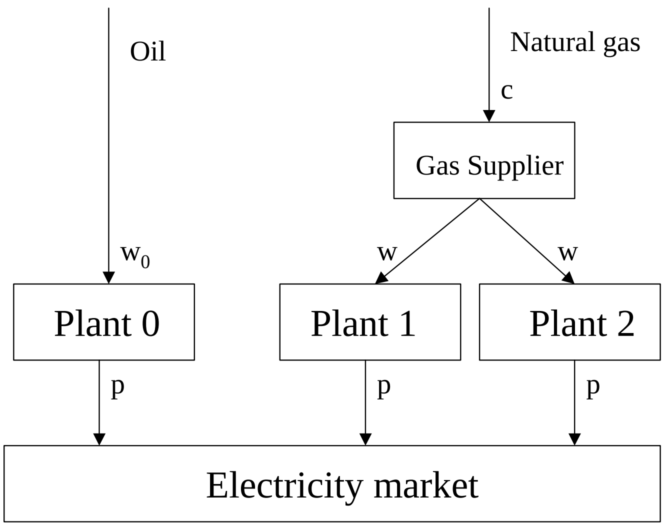
$$\begin{aligned}
\mathrm{W} &= \Pi_g+\Pi_1+\Pi_2+\Pi_3+\mathrm{CS}\\
&= \frac{39(1-c)^2}{128},\\
\mathrm{DWL} &= \max\mathrm{W}-\mathrm{W}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(1-c)^2}{2} - \frac{39(1-c)^2}{128} \\
&= \frac{25(1-c)^2}{128}. \\
\text{DWLR} &= \frac{\text{DWL}}{\text{maxW}} \\
&= \frac{25(1-c)^2/128}{(1-c)^2/2} = 25/64 = 39\%.
\end{aligned}
$$

The existence of three competitive power stations gives an increase in the total production of 13% in comparison to the situation with only two power stations. The dead weight loss has been reduced by 11%, but is still quite high. The natural gas supplier more than triples his profit. The figures from this analysis are shown in the column headed GGG in Table 1 at the end of this article.

# 4 Two Gas-Fired and One Oil-Fired Power Plant

Consider now the natural gas scenario as previously with an addition of competing power plants using a fuel other than gas. This fuel could, for example, be oil. It is assumed that the fuel technology i fixed, i.e. ta hat gas cannot be substituted by oil. To facilitate calculations the oil-based part of the sector is aggregated to be just one single plant that acquires oil at a world market price, $w_0$. Oil prices are assumed to be normalised to be the relative fuel cost of generating one unit of electricity. The marginal costs from transforming oil to electricity are assumed to be zero and the oil-fired power plant provides the electricity volume $q_0$. The market structure for this scenario is illustrated in Figure 2.




Figur 2. Market Relation II: Two gas-fired plants and a single oil-fired plant.

The natural gas part of the electricity market is as previously: The supplier is assumed to have fixed marginal costs, $c$, and to market the natural gas to the two gas-fired power plant at a common wholesale price, $w$, in the volumes $q_1$ and $q_2$, respectively. The natural gas supplier's profit is:

$$\Pi_g(q_1, q_2, w) = (w - c)(q_1 + q_2).$$

It is further assumed that the power plants, both gas and oil based, are facing a commond demand curve with the following correlation between marketed quantities of electricyt and the common electricity market price, $p$:

$$q_0 + q_1 + q_2 = 1 - p.$$

The two gas-fired power plants have the following profits:

$$\Pi_i(q_i; q_0, q_j, w) = (p - w)q_i = (1 - q_0 - q_1 - q_2 - w)q_i, \ i = 1, 2.$$

The oil-fired power plant will have the profit:

$$\Pi_0(q_0; q_1, q_2, w_0) = (p - w_0)q_0 = (1 - q_0 - q_1 - q_2 - w_0)q_0.$$

The individual power plants maximise their profit in accordance with the wholesale price for the fuels and the other plants production levels. The first order conditions for the profit functions lead to the power plants' reaction functions:

$$\begin{aligned} q_0 &= \frac{1 - q_1 - q_2 - w_0}{2}, \\ q_1 &= \frac{1 - q_0 - q_2 - w}{2}, \\ q_2 &= \frac{1 - q_0 - q_1 - w}{2}. \end{aligned}$$

The Nash-equilibrium within this supply game, where plant 1 and 2 are identical, will be:

$$\begin{aligned} q_0 &= \frac{q_1 + q_2}{2} + (w - w_0), \\ q_1 = q_2 &= q_0 - (w - w_0) \\ \Rightarrow q_0 &= \frac{1 - 3w_0 + 2w}{4}, \\ q_1 = q_2 &= \frac{1 + w_0 - 2w}{4}. \end{aligned}$$

The joint demand for gas, $D_g(w; w_0) = q_1 + q_2$, will be:

$$D_g(w; w_0) = \frac{1 + w_0 - 2w}{2}.$$

The natural gas supplier's profit, $\Pi_g$ is:

$$\Pi_g = (w - c)D_g(w; w_0),$$

leading to a the following natural gas monopoly wholesale price:

$$w = \frac{1 + 2c + w_0}{4}.$$

Using the optimal value of $w$ in the expressions for $q_0$, $q_1$ and $q_2$, the following is obtained:

$$\begin{aligned} q_0 &= \frac{3+2c-5w_0}{8}, \\ q_1 \;=\; q_2 &= \frac{1-2c+w_0}{8} \\ \Rightarrow q \;\equiv\; q_0+q_1+q_2 &= \frac{5-2c-3w_0}{8}, \\ p &= \frac{3+2c+3w_0}{8}. \end{aligned}$$

To study the cross impact of $c$ and $w_0$, the marginal reaction can be calculated:

$$\begin{aligned} \frac{\partial q_0}{\partial c} &= .25, \\ \frac{\partial q_1}{\partial w_0} \;=\; \frac{\partial q_2}{\partial w_0} &= .125. \end{aligned}$$

Hence, the two parts of the electricity sector are not very sensitive to the cost conditions regarding the competing fuel.

Not surprisingly, both basic costs parameters, $w_0$ and $c$, are included in the equation describing the electricity price. The following three relations explain how the costs constants $c$ and $w_0$ affect the diverted prices $w$ and $p$:

$$\begin{aligned} \frac{\partial w}{\partial w_0} &= .25, \\ \frac{\partial p}{\partial c} &= .25 \;=\; -\frac{\partial q}{\partial c}, \\ \frac{\partial p}{\partial w_0} &= .375 \;=\; -\frac{\partial q}{\partial w_0}. \end{aligned}$$

Compared to the situation of the two gas power plants alone, we see that $c$ has less impact on the electricity price. The coefficient, $\partial p/\partial c$ was previously approximately 0.67 , but it is now reduced to approximately 0.25, implying that in the presence of oil, the gas cost becomes much less relevant in determining the electricity price. In return, the oil price will have an impact with a coefficient around 0.63, i.e. almost three times the impact of the gas costs. Hence, gas and oil prices have very different impact on the electricity prices.

The last two derivatives show that cost increase, such as fuel taxes, will only be partly passed on to the electricity market in that a cost increase of one monetary unit on both fuels will result in an increase in the electricity price of just 0.625 monetary units. Had there been total competition, the tax rise would have been passed entirely to the electricity price and the decrease in demand would have been greater. Given the market relations the fuels prove good tax objects in the sense, that both consumers and producers will share the tax burden.

The volumes derived at lead to the following profits and consumer surplus:

$$\begin{aligned} \Pi_0 &= (p-w_0)q_0 \\ &= \frac{(3+2c-5w_0)^2}{64}, \end{aligned}$$

$$
\begin{aligned}
\Pi_g &= \frac{(1+w_0-2c)^2}{16}, \\
\Pi_1 = \Pi_2 &= \frac{(1+w_0-2c)^2}{64}, \\
\text{CS} &= q^2/2 = \frac{(5-2c-3w_0)^2}{128}.
\end{aligned}
$$

$$
\begin{aligned}
\text{Profit to the oil-fired plant} &= \Pi_0 \\
&= \frac{(3+2c-5w_0)^2}{64}, \\
\text{Profit to the gas segment} &= \Pi_g + \Pi_1 + \Pi_2 \\
&= \frac{3(1+w_0-2c)^2}{32}.
\end{aligned}
$$

It is difficult to compare the welfare results here with the results from pure gas analysis, as both $c$ and $w_0$ are included in the expressions. However, an indication on the effects can be produced by letting $w_0 = c$, i.e. by assuming that none of the fuels have a cost advantage.

$$
\begin{aligned}
\text{Case:} &\quad w_0 = c. \\
\text{Profit to oil-fired plants} &= \frac{9}{64}(1-c)^2, \\
\text{Profit to the gas segment} &= \frac{3}{32}(1-c)^2, \\
\Pi_0 + \Pi_g + \Pi_1 + \Pi_2 &= \frac{15}{64}(1-c)^2, \\
\text{CS} &= \frac{25(1-c)^2}{128}, \\
\text{W} &= \text{CS} + \Pi_0 + \Pi_g + \Pi_1 + \Pi_2 \\
&= \frac{55(1-c)^2}{128}, \\
\text{DWL} &= \frac{(1-c)^2}{2} - \frac{55(1-c)^2}{128} \\
&= \frac{9(1-c)^2}{128}, \\
\text{DWLR} &= \frac{9(1-c)^2/128}{(1-c)^2/2} = 9/64 = 14\%.
\end{aligned}
$$

The competition from the oil-based plant reduces the market power of the natural gas supplier, and and oveerall electricity production will increase by almost 88% compare to the case with to gas-fired plants alone. This is quite high in comparison to the rise of just 13% related to introducing a third gas-fired plant. This implies that diversity is preferential to multiplicity.

The natural gas supplier's profit decreases by 2/3rd in comparison to the scenario with three gas power stations. The electricity sector's total profit is the same as with the situation relating to the three gas power stations. The two original power stations earn individually the same as the situation involving three works, but the consumer surplus has nearly tripled. The shifts have meant that the dead weight loss is reduced by 2/3rd in comparison to the three gas power station scenario. The result will to a certain degree hold even though oil is more expensive than gas, because the natural gas supplier will react moderately on the oil price changes.

The figures from the analysis where $w_0$ equals $c$ are shown in the column headed GGO in Table 1 at the end of this article.

# 5 Vertical Integration — Two Gas-Fired Plants

The next three sections will be devoted to cases where the natural gas supplier merges with one of the gas-plant power plants (plant number 1), but continues to offer natural gas to other gas-fired competitive plants, possibly to an alternative (higher) price than prior to the merger. This market structure in its simplest form illustrated in Figure 3. We will see that many could benefit from such a configuration.

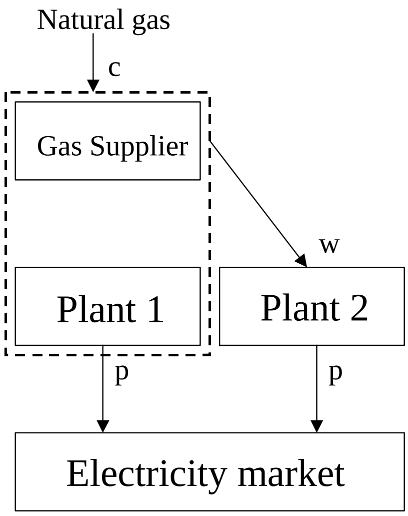



Figure 3. Market Relation III: Vertical Integration — Two gas-fired plants.

The natural gas supplier's internal transfer price with plant number 1 will disappear in the optimisation and the profit, $\Pi_g$, of the integrated enterprise will in a scenario with just one competing gas-fired plant be:

$$\begin{aligned}\Pi_g &= (p-c)q_1 + (w-c)q_2 \\ &= (p-w)q_1 + (w-c)(q_1+q_2),\end{aligned}$$

where the latter expression clearly shows changes in the object function.

The profit of the independent power plant is described by:

$$\Pi_2 = (p-w)q_2.$$

Including the electricity demand function yields:

$$\begin{aligned}\Pi_g(q_1, w) &= (1-q_1-q_2-w)q_1 + (w-c)(q_1+q_2), \\ \Pi_2(q_2) &= (1-q_1-q_2-w)q_2.\end{aligned}$$

Solving this equation system for a fixed $w$, leads to the following first-order conditions:

$$\begin{aligned} \frac{\partial \Pi_g(q_1, w)}{\partial q_1} &= 1 - 2q_1 - q_2 - c = 0, \\ \frac{\partial \Pi_2(q_2)}{\partial q_2} &= 1 - q_1 - 2q_2 - w = 0, \end{aligned}$$

indicating that they react on their individual cost levels. The two equations are reduced to

$$q_1 + c = q_2 + w \text{ and } q_1 = \frac{1 - c - q_2}{2},$$

where the first expression clearly shows that different costs lead to different volumes even during quantity competition. Since $w \geq c$, we can infer that it will be the integrated organisation that will produce the most. If the equation system is further elaborated, the following is obtained:

$$\begin{aligned} q_1 &= \frac{1 - 2c + w}{3}, \\ q_2 &= \frac{1 + c - 2w}{3}, \\ q_1 + q_2 &= \frac{2 - c - w}{3}, \\ p &= \frac{1 + c + w}{3}, \\ \Pi_g(w) &= \frac{(1 - 2c + w)^2}{9} + \frac{(w - c)(1 + c - 2w)}{3}, \\ \Pi_2(w) &= \frac{(1 + c - 2w)^2}{9}. \end{aligned}$$

The optimal value of $w$ is obtained by differentiating $\Pi_g$ with respect to $w$:

$$\begin{aligned} \frac{\partial \Pi_g(w)}{\partial w} &= \frac{5(1 + c - 2w)}{9} = 0 \\ \Rightarrow \quad w &= \frac{1 + c}{2}, \end{aligned}$$

leading to the same natural gas wholesale price as in the model with no integration. The difference between the two models is only clearly observed when we look more closely at the optimal volumes. To facilitate the comparison, some of the values from the analysis of two gas-based works under integration will be brought forward. These values are shown by a tilde (e.g., $\tilde{q}_1$).

$$\begin{aligned} q_1 &= \frac{1 - c}{2} > \tilde{q}_1 = \frac{1 - c}{6}, \\ q_2 &= 0. \end{aligned}$$

The integrated enterprise has in this way been successful in totally driving out the competitor - even without increasing the wholesale price. It is thus difficult for the competition authority to prove that the new integrated enterprise abuses its new dominating position. The reason for why the competitor no longer finds it feasible to participate in the market is because the integrated enterprise supplies a electricity quantity large enough to reduce the electricity price to

$$p = \frac{1+c}{2} = \tilde{w},$$

implying that it is no longer profitable for the competitor to demand gas at the wholesale price prevailing prior to the merger.

The relationship between the volumes offered before and after the integration is:

$$\tilde{q}_1 + \tilde{q}_2 = \frac{1-c}{3} < q_1 = \frac{1-c}{2},$$

which correspond to a 50% increase in production. This increase will result in a reduction of the dead weight loss of the remaining market imperfections at the electricity market by nearly 50%. Two real world condition could question the magnitude of the impacts found. Firstly, the result assumes that power plant 1 can triple its production without investments. Secondly, a reduction of active plants can be jeopardised the physical supply structure.

The welfare analysis shows that:

$$\begin{aligned}
\text{CS} &= \frac{(1-c)^2}{8}, \\
\Pi_g &= \frac{(1-c)^2}{4}, \\
\text{W} &= \Pi_g + \text{CS} \\
&= \frac{5(1-c)^2}{8}, \\
\text{DWL} &= \frac{(1-c)^2}{8}, \\
\text{DWLR} &= 25\%.
\end{aligned}$$

We can see that the dead weight loss reduced by almost 50% compared with cases with solely gas-fired plants involving no vertical integration.

The source for this progress is elimination of the so-called "double marginalisation" (see Spengler (1950)), namely two consecutive market imperfections in the supply chain — the raw material supplier's monopoly followed by the distributors' duopoly. The market imperfections will after integration be reduced to one single monopoly. It is important here to underline that according to Lipsey & Lancaster (1956), it is not a generic characteristic that it is incremental to social welfare to reduce the number of market imperfections unless they can all be removed. Here we were just lucky that one monopolist served the society better than three players divided into two layers.

Besides the welfare gain, the new monopoly does also quite well:

$$\begin{aligned}
\Pi_g &= \frac{(1-c)^2}{4} \\
&> \tilde{\Pi}_g + \tilde{\Pi}_1 \\
&= \frac{7(1-c)^2}{36}
\end{aligned}$$

The profit for the integrated enterprise is 3% higher than prior to the integration.

One last question is if it is possible for the integrated enterprise to compensate the obsolete plant for his loss. The sector's total profit without the integration would be:

$$\tilde{\Pi}_g + \tilde{\Pi}_1 + \tilde{\Pi}_2 = \frac{(1-c)^2}{9},$$

which is less than half of the integrated sector's profit. The difference between the two scenarios is

$$\Pi_g - (\tilde{\Pi}_g + \tilde{\Pi}_1 + \tilde{\Pi}_2) = \frac{(1-c)^2}{4} - \frac{(1-c)^2}{9} = \frac{5(1-c)^2}{36} > \tilde{\Pi}_2 = \frac{(1-c)^2}{36},$$

leaving excess profit that could be used to reach an agreement of suitable compensation to the company behind plant number 2.

# 6 Vertical Integration – Three Gas-Fired Plants

If the natural gas supplier is integrated with plant number 1 out of three, we will obtain the following equation system:

$$\begin{aligned}
\Pi_g &= (p-w)q_1 + (w-c)(q_1+q_2+q_3) \\
&= (1-q_1-q_2-q_3-w)q_1 + (w-c)(q_1+q_2+q_3), \\
\Pi_i &= (1-q_1-q_2-q_3-w)q_i, \ \ i = 2,3.
\end{aligned}$$

Firstly, the demand for gas of the independent power stations is determined:

$$\begin{aligned}
\frac{\partial \Pi_2}{\partial q_2} &= 1 - q_1 - 2q_2 - q_3 - w = 0 \\
\Rightarrow q_2 &= \frac{1-q_1-w}{3}.
\end{aligned}$$

For symmetry reasons $q_2$ equals $q_3$.

Inserting the reaction functions for the independent power plants into the integrated enterprise's profit function, the following is obtained:

$$\begin{aligned}
\Pi_g &= \left(1-q_1-w-\frac{2(1-q_1-w)}{3}\right)q_1 + (w-c)\left(q_1+\frac{2(1-q_1-w)}{3}\right) \\
&= \frac{1-q_1-w}{3}q_1 + (w-c)\left(\frac{q_1}{3}+\frac{2(1-w)}{3}\right) \\
\Rightarrow \frac{\partial \Pi_g}{\partial q_1} &= \frac{1-q_1-w}{3} - \frac{q_1}{3} + \frac{w-c}{3} = \frac{1-2q_1-c}{3} = 0 \\
\Rightarrow q_1 &= \frac{1-c}{2},
\end{aligned}$$

which is the same quantity as in the case with only one single independent power station.

The optimal wholesale price offered by the integrated enterprise is found by considering the integrated enterprise's profit, $\Pi_g$, as a function of the natural gas wholesale price:

$$
\begin{aligned}
\frac{\partial \Pi_g}{\partial w} &= 1 - 2w + c = 0 \\
\Rightarrow w &= \frac{1+c}{2}. \\
q_2 \; = \; 2_3 &= \frac{1 - q_1 - w}{3} \\
&= \frac{1 - \frac{1-c}{2} - \frac{1+c}{2}}{3} \; = \; 0.
\end{aligned}
$$

The merged enterprise totally forces out the two independent power plants. The independent power plants will still formally be offered natural gas at the same price as prior to the merger. The unchanged listed price will again make it difficult for the competition authority to intervene.

The consumer have benefited from the merger because the production of electricty has increased by 33%. The merger will cause the sector's profit to change:

$$
\Pi_g \;=\; \left(1 - \frac{1-c}{2} - c\right)\frac{1-c}{2} \;=\; \frac{(1-c)^2}{4}.
$$

The new profit level is only 1.5% above the sector's profit prior to the integration. There are, as in the case with two gas-fired plants, resources available to compensate the two dethroned power plants.

The welfare analysis, i.e., CS, W, DWL and DWLR, are in line with the results from the case with only two gas-fire plants. The is a welfare gain by vertical integration as it removes a double marginal problem. For policy consideration some remarks should be raised:

- The modelled situation is unrealistic because it is implicitly assumed that plant number 1 at the situation's starting point has enough idle capacity to be able to quadruple its production.
- The competition regulating authority can also expect a complaint from the dethroned power stations, which are on the other side placed legally weak because they will following the merger be offered the product at the same price as before.
- Even though the integration improves welfare, there is still a dead weight loss of 25% and it can be discussed if this is not enough reason for the competition authority to interfere.

# 7 Vertical Integration with Competing Fuels

As the analysis of the gas section shows, it was not only a great incitement for the natural gas supplier to merge with one of the power plants, but this merger was also beneficial to the consumers. The natural gas supplier's feasible strategies will, in a mixed sector, be limited by the indirect relation to the world market price of oil. Figure 4 shows vertical integration in a mixed sector.

Allow, as previously, $\Pi_0$, $\Pi_g$ and $\Pi_2$ to denote the profits at the oil-based power plants, and the merger of the natural gas supplier and power plant 1, and power

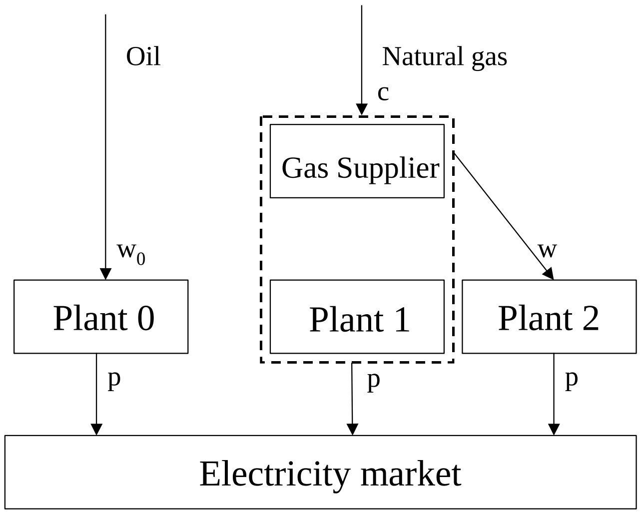



Figure 4. Market Relations IV: Vertical integration in the gas-fired part of a mixed electricity sector.

plant 2, respectively. These profits will become:

$$\begin{aligned}
\Pi_0 &= (p - w_0)q_0 = (1 - q_0 - q_1 - q_2 - w_0)q_0, \\
\Pi_g &= (p - c)q_1 + (w - c)q_2 = (1 - q_0 - q_1 - q_2 - c)q_1 + (w - c)q_2, \\
\Pi_2 &= (p - w)q_2 = (1 - q_0 - q_1 - q_2 - w)q_2.
\end{aligned}$$

As the three market participants attempt to maximise their own profit given the other participants' behaviour, the equilibrium will be characterised by the following relations:

$$\begin{aligned}
\frac{\partial \Pi_0}{\partial q_0} &= 1 - 2q_0 - q_1 - q_2 - w_0 = 0 \\
\Rightarrow \; q_0 &= \frac{1 - q_1 - q_2 - w_0}{2}, \\
\frac{\partial \Pi_g}{\partial q_1} &= 1 - q_0 - 2q_1 - q_2 - c = 0 \\
\Rightarrow \; q_1 &= \frac{1 - q_0 - q_2 - c}{2}, \\
\frac{\partial \Pi_2}{\partial q_2} &= 1 - q_0 - q_1 - 2q_2 - w = 0 \\
\Rightarrow \; q_0 &= \frac{1 - q_0 - q_1 - w}{2}.
\end{aligned}$$

If these three first-order conditions are compared, the following will be obtained:

$$\begin{aligned}
q_0 &= \frac{1 - 3w_0 + c + w}{4}, \\
q_1 &= \frac{1 + w_0 - 3c + w}{4}, \\
q_2 &= \frac{1 + w_0 + c - 3w}{4}.
\end{aligned}$$

There exists one further first-order condition related to $w$:

$$\frac{\partial \Pi_g}{\partial w} = q_2.$$

This condition will lead to either $q_2$ being equal to zero, or $w$ being infinite, implying $q_2$ equal to zero in all circumstances. Setting $q_2 = 0$ in the expression for $q_2$ yields:

$$\begin{aligned}
w &= \frac{1 + w_0 + c}{3}, \\
q_0 &= \frac{1 - 2w_0 + c}{3}, \\
q_1 &= \frac{1 + w_0 - 2c}{3}, \\
q_2 &= 0, \\
q_0 + q_1 + q_2 &= \frac{1 - w_0}{3} + \frac{1 - c}{3}.
\end{aligned}$$

Simplifying by setting $w_0 = c$ yields:

$$\begin{aligned}
w &= \frac{1 + 2c}{3}, \\
q_0 &= q_1 = \frac{1 - c}{3}, \\
\Pi_0 &= \Pi_g = \frac{(1 - c)^2}{9}, \\
\text{CS} &= q^2/2 = \frac{2(1 - c)^2}{9}, \\
\text{W} &= \Pi_0 + \Pi_g + \text{CS} = \frac{4(1 - c)^2}{9}, \\
\text{DWL} &= \frac{(1 - c)^2}{18}, \\
\text{DWLR} &= 1/9 = 11\%.
\end{aligned}$$

There is still an economic incentive for the natural gas supplier to carry out the integration in that his integrated profit will be 19% greater than the profit in the gas sector prior to the integration. The society's gain by allowing vertical integration here is not so great. The integration only contributes by reducing the dead weight loss by 27%, compared to 43% and 36% in the unmixed case with two and three gas-fired plants respectively. There is, therefore, still competition from the oil sector, which provides the largest contribution to reducing the affect of the imperfections.

# 8 Welfare Analyses and Policy Recommendations

The main results from the analyses are organised in columns in Table 1. The first column, SO, corresponds to the social optimum where social welfare is optimised by letting marginal value equal marginal cost. In the social optimum in the present analysis all social welfare is assigned to the consumers. This is due to the cost linearity assumed. Non-constant cost would alter this result without tampering with the overall conclusions.

| | | No integration | | | Vertical integration | | |
|---|---|---|---|---|---|---|---|
| | SO | GG | GGG | GGO | GG | GGG | GGO |
| $p$ | $c$ | $\frac{2+c}{3}$ | $\frac{5+3c}{8}$ | $\frac{3+5c}{8}$ | $\frac{1+c}{2}$ | $\frac{1+c}{2}$ | $\frac{1+2c}{3}$ |
| $w$ | $c$ | $\frac{1+c}{2}$ | $\frac{1+c}{2}$ | $\frac{1+3c}{4}$ | $\frac{1+c}{2}$ | $\frac{1+c}{2}$ | $\frac{1+2c}{3}$ |
| $p-c$ | 0 | $\frac{2(1-c)}{3}$ | $\frac{5(1-c)}{8}$ | $\frac{3(1-c)}{8}$ | $\frac{1-c}{2}$ | $\frac{1-c}{2}$ | $\frac{1-c}{3}$ |
| $q_0$ | n.a. | n.a. | n.a. | $\frac{3(1-c)}{8}$ | n.a. | n.a. | $\frac{1-c}{3}$ |
| $q_1$ | $\frac{1-c}{3}$ | $\frac{1-c}{6}$ | $\frac{1-c}{8}$ | $\frac{1-c}{8}$ | $\frac{1-c}{2}$ | $\frac{1-c}{2}$ | $\frac{1-c}{3}$ |
| $q_2$ | $\frac{1-c}{3}$ | $\frac{1-c}{6}$ | $\frac{1-c}{8}$ | $\frac{1-c}{8}$ | 0 | 0 | 0 |
| $q_3$ | $\frac{1-c}{3}$ | n.a. | $\frac{1-c}{8}$ | n.a. | n.a. | 0 | n.a. |
| $\sum q_i$ | $1-c$ | $\frac{1-c}{3}$ | $\frac{3(1-c)}{8}$ | $\frac{5(1-c)}{8}$ | $\frac{1-c}{2}$ | $\frac{1-c}{2}$ | $\frac{2(1-c)}{3}$ |
| $\Pi_0$ | n.a. | n.a. | n.a. | $\frac{9(1-c)^2}{64}$ | n.a. | n.a. | $\frac{(1-c)^2}{9}$ |
| $\Pi_g$ | 0 | $\frac{(1-c)^2}{6}$ | $\frac{3(1-c)^2}{16}$ | $\frac{(1-c)^2}{16}$ | $\frac{(1-c)^2}{4}$ | $\frac{(1-c)^2}{4}$ | $\frac{(1-c)^2}{9}$ |
| $\Pi_1$ | 0 | $\frac{(1-c)^2}{36}$ | $\frac{(1-c)^2}{64}$ | $\frac{(1-c)^2}{64}$ | n.a. | n.a. | n.a. |
| $\Pi_2$ | 0 | $\frac{(1-c)^2}{36}$ | $\frac{(1-c)^2}{64}$ | $\frac{(1-c)^2}{64}$ | 0 | 0 | 0 |
| $\Pi_3$ | 0 | n.a. | $\frac{(1-c)^2}{64}$ | n.a. | n.a. | 0 | n.a. |
| $\sum \Pi_i$ | 0 | $\frac{2(1-c)^2}{9}$ | $\frac{15(1-c)^2}{64}$ | $\frac{15(1-c)^2}{64}$ | $\frac{(1-c)^2}{4}$ | $\frac{(1-c)^2}{4}$ | $\frac{2(1-c)^2}{9}$ |
| CS | $\frac{(1-c)^2}{2}$ | $\frac{(1-c)^2}{18}$ | $\frac{9(1-c)^2}{128}$ | $\frac{25(1-c)^2}{128}$ | $\frac{(1-c)^2}{8}$ | $\frac{(1-c)^2}{8}$ | $\frac{2(1-c)^2}{9}$ |
| W | $\frac{(1-c)^2}{2}$ | $\frac{5(1-c)^2}{18}$ | $\frac{39(1-c)^2}{128}$ | $\frac{55(1-c)^2}{128}$ | $\frac{3(1-c)^2}{8}$ | $\frac{3(1-c)^2}{8}$ | $\frac{4(1-c)^2}{9}$ |
| DWL | 0 | $\frac{2(1-c)^2}{9}$ | $\frac{25(1-c)^2}{128}$ | $\frac{9(1-c)^2}{128}$ | $\frac{(1-c)^2}{8}$ | $\frac{(1-c)^2}{8}$ | $\frac{(1-c)^2}{18}$ |
| DWLR | 0 | 44% | 39% | 14% | 25% | 25% | 11% |

Table 1. Summary of the results from the analyses.

The six scenarios could be compared on two criteria: (1) their effectiveness, i.e. their ability to avoid dead weight losses; and (2) their distributional effect.

Comparing all scenarios there is hardly any difference in the electricity and gas sectors' total profit. The total profits range from $0.22(1-c)^2$ to $0.25(1-c)^2$. The consumer surplus varies somewhat more, i.e., by nearly a factor four from the lowest value, $0.06(1-c)^2$ to the highest, $0.22(1-c)^2$. It should be noted, that consumer surplus in the oligopoly scenarios never exceed the profits to the electricity and gas sectors.

With regard to the distribution criteria, there is hardly any difference in the order of priority of the seven scenarios as to whether one uses consumer surplus (CS) or the relative dead weight loss (DWLR) as criteria. The distributional result thus makes it easier to implement forced changes in the structure.

The most effective alternative to perfect competition is scenario GGO under vertical integration. The double marginalisation problem is eliminated here – the gas-supplier is under pressure from the world market price of oil. Only 11% of the potential social welfare is lost here, which can seem surprising because production is only 2/3rd of the optimal level. That this loss in production does not amount to more is due to the decreasing marginal value of the electricity production. The social welfare is distributed equally between the producers sector and the consumers. The two power plants share equally and will, therefore, receive 25% of the social welfare.

The profit of the oil-fired power plants is reduced by 12% and the gas-fired power plant number 2 has to shut down. However, a secondary result of the integration is that the oil-fired power plant has to turn reduce the production by 11%, which may prove positive for the nation's greenhouse gas accounts. Unfortunately, if, for environmental reasons, one wishes to shut the oil-fired production completely down, the dead weight loss will increase by 100%.

# References


Cournot, A. (1838): Recherces sur les Principes Mathématiques de la Théorie des Richesses.

Lipsey, R.G., & K. Lancaster (1956): The General Theory of the Second Best, *Review of Economic Studies,* 24 pp. 11-32.

Spengler, J. (1950): Vertical Integration and Anti-trust Policy, *Journal of Political Economy* 58