Satisficing decision procedure and optimal consumption-leisure choice

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Abstract

The paper argues that when a consumer searches for a lower price, a satisficing decision procedure equalizes marginal costs of search with its marginal benefit. The consumer can maximize the utility of his consumption-leisure choice with regard to the equality of marginal values of search. Therefore, the satisficing decision procedure results in the optimizing consumer behavior.

JEL Classification: D11, D83.

Introduction

The discussion between the search-satisficing concept and the neoclassical paradigm has a long story. In 1957 H.Simon revived the Scottich word *satisficing* to denote decision making "***that sets an aspiration level, searches until an alternative is found that is satisfactory by the aspiration level criterion, and selects that alternative***". The confrontation between two approaches had reached its peak in 1977 when H.Simon presented his Richard T. Ely Lecture. Then, the discussion went into decline, but from time to time researchers in different fields animated it (see for example Slote (1989), Schwartz et al. (2002), Fellner et al. (2006)). As a result, the theory of consumer behavior has accepted the strict distinction between "maximizers" and " satisficers" (Lewer et al. (2009)) Unfortunately, opponents forget the fact that H.Simon himself paid attention to the possibility of matching the satisficing and optimizing procedures. In 1972 he wrote:

"A satisficing decision procedure can be often turned into a procedure for optimizing by introducing a rule for optimal amount of search, or, what amounts to the same thing, a rule for fixing the aspiration level optimally." (Simon (1972), p.170)

This note tries to restore the methodological equilibrium. The rule for optimal amount of search is derived from the reserve maximization model, which emphasizes the role of the need to save for daily expenses and purchases (Malakhov (2011b) and which is briefly described here in Appendix. This paper shows how a satisficing decision procedure results in an optimal search-stopping rule and in an optimal consumption-leisure choice.

Satisficing Price Decision and Optimal Search-Stopping Rule

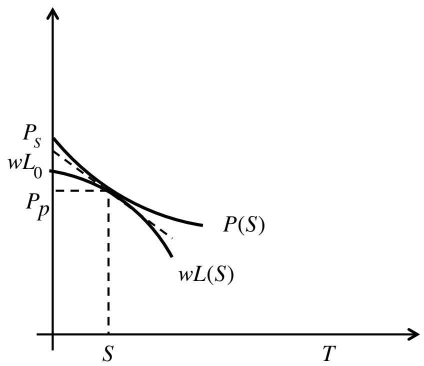
Let us start with the famous distinction between an optimizing model and a satisficing model. In 1978 H.Simon wrote:

"In an optimizing model, the correct point of termination is found by equating the marginal cost of search with the (expected) marginal improvement in the set of alternatives. In a satisficing model, search terminates when the best offer exceeds an aspiration level that itself adjusts gradually to the value of the offers received so far" **(Simon (1978, p.10)).**

Suppose a consumer who reserves the labor income wL_0 for the purchase of an item $Q=1$. He begins to search for a cheaper price from the starting price of the search $P_S > wL_0$ and he concludes the search at the satisficing purchase price $P_P < wL_0$.

Let us analyze the intersection of two curves, the expenditures $P(S)$ curve and the labor income $wL(S)$ curve, where T is the time horizon of the consumption-leisure choice, the value $w\times\partial L/\partial S<0$, because the best offer P_P exceeds the aspiration level wL_0 and the value $\partial P/\partial S$, which is exposed at the moment of purchase by the tangent dotted line, is also negative, now due to the diminishing marginal efficiency of the search S (Fig.1):

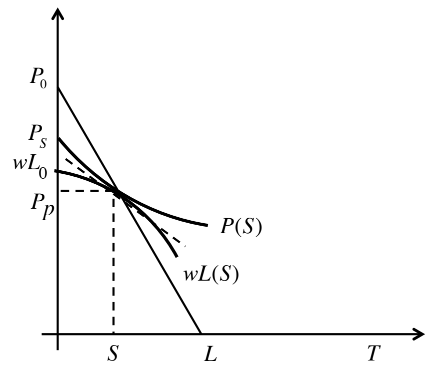
Fig.1



If the value P_P is equal to the *disposable* labor income $wL(S)$ at the moment of purchase, the angle (-w) gives us the value of the labor time L on horizontal axis. However, it also gives us the value P_0 on the vertical axis, which is equal to the *potential* labor income (Fig.2), or

$$P_0=w(L+S) \quad (1)$$

Fig.2



The Fig.2 shows that the absolute value of the decrease in the potential labor income at the moment of purchase is greater than the absolute value of the decrease in the disposable labor income, or $w > |w \times \partial L/\partial S|$. This consideration attracts attention to the core function $L(S)$. Indeed, when we take the values $\partial P/\partial S<0$, $\partial^2 P/\partial S^2>0$, we simply follow the assumption of the diminishing marginal efficiency of the search. However, the behavior of the $L(S)$ function is not so clear.

When the search S "squeezes out" the labor L and the leisure H from the time horizon T, the $\partial L/\partial S$ rate directly depends on the value $\partial H/\partial S$. However, the value $\partial H/\partial S$ can be determined b y the very simple rule. If we take the differential $dH(S)$, we can see that the absolute rate of the decrease in leisure time is equal to its share in the time horizon, or $|\partial H/\partial S|=H/T$ and $H/T= -\partial H/\partial S$. From here we get the value of the ***propensity to search $\partial L/\partial S$.*** It is negative because the labor and the search represent ***alternative sources of income***. We can also get its derivatives, which are very important for our analysis:

$$L(S) = T - H(S) - S;$$
$$\partial L / \partial S = -\partial H / \partial S - 1;$$
$$dH(S) = dS \frac{\partial H}{\partial S} = -dS \frac{H}{T}$$
$$\Downarrow$$
$$\frac{\partial L}{\partial S} = -\frac{\partial H}{\partial S} - 1 = \frac{H}{T} - 1 = \frac{H-T}{T} = -\frac{L+S}{T} \quad (2)$$
$$\frac{\partial L}{\partial S} = \frac{H-T}{T} \Rightarrow \partial^2 L / \partial S \partial H = 1/T;$$
$$\frac{\partial L}{\partial S} = -\frac{L+S}{T} \Rightarrow \partial^2 L / \partial S^2 = -\frac{\partial L / \partial S + 1}{T} < 0$$ [1]

[1] Here, the value of the propensity to search is limited by $-1<\partial L/\partial S<0$. When the propensity to search becomes $\partial L/\partial S<-1$, the value $\partial H/\partial S$ becomes positive. The 'price of time' $\mu=Q\times\partial P/\partial S$ (see Aguiar and Hurst 2007, p.1536), here the value $w\times\partial L/\partial S$, becomes greater than the wage rate. The marginal value of leisure MU_H becomes negative and it produces the Veblen effect (Malakhov 2012b). So, it seems that the satisficing decision procedure cannot correspond to the choice of the high price, which compensates the excess leisure time.

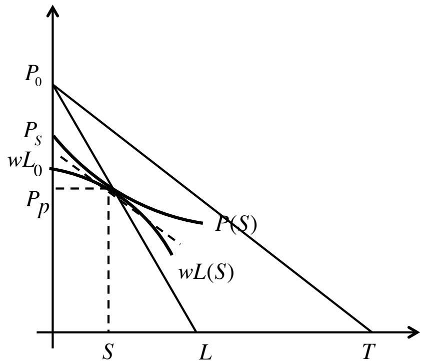
If at the moment of purchase the marginal loss in labor income is equal to the marginal benefit of the search, we have:

$$\frac{\partial P}{\partial S}=w\frac{\partial L}{\partial S}=w\frac{H-T}{T}=-w\frac{L+S}{T} \quad (3)$$

If we re-arrange the Equations (1) and (3), we get that the value of potential labor income is equal to the value of the time horizon times the absolute value of the price reduction at the moment of purchase (Fig.3), or

$$P_0 = -T\times\partial P/\partial S = w\times(L+S) \qquad (4)$$

Fig.3



Now we can proceed to the indirect proof of the correspondence of the optimal search-stopping rule to the satisficing choice.

Let us presuppose that at the moment of purchase the price P_P doesn't equalize marginal values of search and the absolute value of the marginal decrease in the labor income is still less than the marginal benefit of the search, or

$$\left|w\frac{\partial L}{\partial S}\right|<\left|\frac{\partial P}{\partial S}\right| \quad (5)$$

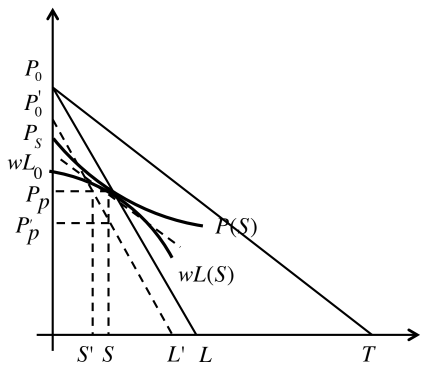
The Equation (4) tells us that this case should result in the hypothetical value P_0', where

$$P_0' = w(L'+S') < P_0 = w(L+S) \quad (6)$$

However, due to the rule $\partial^2L/\partial S^2<0$, the inequality $(L'+S')<(L+S)$ produces the following inequalities: $L' > L$ and $S' < S$. And we can see that ***our assumption is false***, because either the hypothetical amount of search S' results in a greater purchase price and it should be less than the actual amount of search S, or the actual amount of search S should result in the value $P'_P < P_P$.

We can graphically confirm these considerations, if we take the [$P'_0;L'$] line, which is parallel to the [$P_0;L$] line due to the same wage rate (Fig.4):

Fig.4



The same indirect proof can be used when it is supposed that at the purchase price level the marginal costs of search are decreasing already faster than its marginal benefit. The only difference is that this case can be eliminated from the analysis by definition, because it requires recognition that the chosen price is not satisficing. Indeed, we can reproduce the set of inequalities, which describe the dissatisfying choice, when a high price corresponds to unexpectedly low savings on purchase:

$$\left| w\frac{\partial L}{\partial S} \right| > \left| \frac{\partial P}{\partial S} \right| \quad (7)$$

$$dS\left| w\frac{\partial L}{\partial S} \right| > dS\left| \frac{\partial P}{\partial S} \right| \quad (8)$$
$$dwL(S) > dP(S)$$

Now we can say that ***when the consumer chooses the satisficing price, his decision automatically equalizes the marginal loss in the labor income with the marginal benefit of the search.***

However, the Fig.4 provides us with another interesting consideration. Let us pay attention to the situation when the same amount of search S results in a price $P'_P<P_P$, i.e., when the best offer, an unexpected price discount, for example, significantly exceeds the aspiration level – the case that really challenges the optimizing approach. Here we realize that the absolute value of the actual price reduction $|\partial P/\partial S|$ is greater than its planned value $|\Delta P/\Delta S|$. It seems that if the consumer accepts this price, he doesn't equalize marginal costs of the search $|w\times\partial L/\partial S|$ with its marginal benefit $|\partial P/\partial S|$.

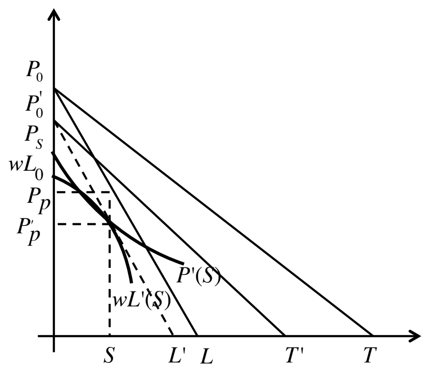
However, this decision changes not only the value of the marginal benefit of the search but it also changes both the propensity to search and the marginal loss in the labor income. It happens because the choice of the lower price decreases, as Fig.4 shows and the Equation (1) proves it, the value T of the time horizon of the consumption-leisure choice.

The time horizon of the consumption-leisure choice depends on the products' lifecycles. The lower price can exhibit the coming expiration date for pork sausages, for example.

If we go back to the Friedman's metaphor, we should say that billiards is played by two people. The seller doesn't bother about consumer's marginal values of search, but he either cut the price for yesterday's "fresh" sausages, or he offers packed pork sausages with extended shelf life. In addition, if the consumer buys yesterday's "fresh" sausages, he should quickly eat them, i.e., to cut leisure time H, reserved for the consumption.

If the consumer doesn't accept this lower price because he estimates it as too high price for the shorten shelf life, we meet again inequalities (7) and (8) of the dissatisfying choice. So, the producer should support the shorten shelf life by the corresponding price discount $|P'_P - P_P|$ (Fig.5):

Fig.5



Now, the new search path $P'(S)$ meets the decrease in the labor income $wL'(S)$ at the point $(S;P'_P)$. The $P'(S)$ curve becomes steeper due to the price discount and the $wL'(S)$ curve becomes steeper due to the shorten time horizon T'. So, for the same amount of the search S we have

$$\frac{\partial P'}{\partial S} = w\frac{\partial L'}{\partial S} = -w\frac{L'+S}{T'} \quad (9)$$

The time horizon T' is cut here not only by the decrease in the labor time L but also by the decrease in time of consumption, i.e., leisure time H. So, the absolute value of the propensity to search $|\partial L/\partial S|$ becomes greater. In addition, the $[P'_0;T']$ line also becomes steeper than the initial $[P_0;T]$ line because the absolute value of the equilibrium price reduction $|\partial P/\partial S|$ becomes greater.

General relationship between savings on purchase, the time horizon of the consumption-leisure choice, and the potential labor income

Our analysis discovers the general relationship between savings on purchases $\Delta P/\Delta S$, the time horizon of the consumption-leisure choice, and the potential labor income:

$$-\frac{\Delta P}{\Delta S}=\frac{P_0}{T} \quad (10)$$

We can change the vector of our analysis and to start from aspiration levels or $\Delta P/\Delta S$-expectations. There, we can find that high aspiration levels as well as unrealistic $\Delta P/\Delta S$-expectations usually result in dissatisfying decisions and/or in corner solutions. It means, that the market, when it "sells" products' lifecycles or shelf lives, tries to adjust $\Delta P/\Delta S$-expectations in order to restore the equation (10). In 1979 Kapteyn et al. presented the brilliant example of this kind of adjustment. The authors demonstrated that purchase decisions concerned durables were satisficing rather than maximizing (Kapteyn et al. 1979, p.559.). Now we can presuppose that the consumers' reports for that study had simply documented the adjustment of aspiration levels. The equation (10) demonstrates how, other things being equal, the increase in the time horizon of the consumption-leisure choice reduces the absolute value of planned savings on purchase $|\Delta P/\Delta S|$ and, therefore, moderates the aspiration level.[2]

However, like it happens with 'lemons', and like Fig.5 illustrates this phenomenon, sellers of high-quality products with long-term lifecycles and guarantees can leave the market because sellers of low-quality products without guarantees or with short shelf lives reduce prices.

Finally, we should pay attention to the price equivalent of the potential labor income. Obviously, it represents some willingness to pay, but it might also correspond to the monopoly price.

Let us suppose that a consumer has no liquidity constraint, for example, due to his strong precautionary motive (Carroll 2001). Then, we can solve the static optimization problem of his consumption-leisure choice with regard to the equality of the marginal values of search, or

$$\Lambda=U(Q,H)+\lambda(w-Q\frac{\partial P/\partial S}{\partial L/\partial S}) \quad (11)$$

If we take the value $\partial P/\partial S$ as the value given by a particular local market, either by a convenient store or a supermarket, we can solve the optimization problem $U(Q,H)$ of the consumption-leisure choice for this particular market. And the solution of this optimization problem gives us another illustration to the value of the potential labor income:

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=MRS(HforQ)=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H;$$
$$\frac{\partial U/\partial H}{\partial U/\partial Q}=MRS(HforQ)=-\frac{Q}{T\times\partial L/\partial S}=\frac{Q}{L+S}=-\frac{w}{T\times\partial P/\partial S}=\frac{w}{P_0} \quad (12)$$

[2] The analysis of the paradox of little pre-purchase search for durables is presented in (Malakhov 2012a).

The *MRS (H for Q)* is determined not by the purchase price but by the price equivalent of the potential labor income.

Conclusion

The last consideration initiates further analysis with regard to both consumers' and sellers' heterogeneity. In this paper we simply touch a little this problem when we examined the decrease in the time horizon of the consumption-leisure choice. If we make one more step in this direction we can see that the Equation (10) provide sellers with a trade-off between advertising costs, which minimize consumer's search costs, and price discounts. However, all these questions, which had been successfully observed by J.Stiglitz (Stiglitz 1987), go beyond the scope of the present analysis. Here we have tried to confirm the assumption that consumer's satisficing price decision automatically equalizes marginal costs of the search with its marginal benefit.

Appendix.

Suppose that the general relationship between the benefits and costs of a search is given by

$R(S) = wL(S) - QP(S)$, where

$wL(S)$ – labor income wL, diminishing during the search S $(\partial L/\partial S < 0)$,

$QP(S)$ – expenditures on fixed or pre-allocated quantity ($\partial P/\partial S<0$),

$R(S)$ – reserve (saving) for daily expenses and for purchases.

When the consumer concludes the search, he maximizes the reserve for purchases:

$$\frac{\partial R}{\partial S} = 0 \Leftrightarrow Q\frac{\partial P}{\partial S} = w\frac{\partial L}{\partial S}$$

The consumer can optimize the consumption-leisure choice *(Q,H)* with respect to the equality of the marginal values of the search and with respect to the given value of price reduction $\partial P/\partial S$, which corresponds to a particular local market. We can re-write the equation (1) to obtain the following constraint:

$$w = Q\frac{\partial P/\partial S}{\partial L/\partial S}$$

Setting the Lagrangian expression $\Lambda = U(Q,H) + \lambda(w - Q\frac{\partial P/\partial S}{\partial L/\partial S}.)$, the first-order conditions for a maximum are

$$\frac{\partial \Lambda}{\partial Q} = \frac{\partial U}{\partial Q} - \lambda\frac{\partial P/\partial S}{\partial L/\partial S} = 0; \quad \frac{\partial \Lambda}{\partial H} = \frac{\partial U}{\partial H} - \lambda Q \frac{\partial \frac{\partial P/\partial S}{\partial L/\partial S}}{\partial H} = 0$$

.

Trying to determine the marginal rate of substitution of leisure for consumption, we get[3]

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=\frac{\partial Q\frac{\partial P/\partial S}{\partial L/\partial S}\Big/\partial H}{\frac{\partial P/\partial S}{\partial L/\partial S}}=-Q\frac{\partial P/\partial S\times\partial L^2/\partial S\partial H}{\frac{\partial P/\partial S}{\partial L/\partial S}(\partial L/\partial S)^2}=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H;$$

$$Q\frac{\partial P}{\partial S}=w\frac{\partial L}{\partial S}\Rightarrow\frac{Q}{\partial L/\partial S}=\frac{w}{\partial P/\partial S}\Rightarrow$$

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=MRS(HforQ)=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H.$$

We can present the *MRS (H for Q)* in two interrelated forms, a physical form and a monetary form:

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=MRS(HforQ)=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H.$$

Both of these forms include the value $\partial^2 L/\partial S\partial H$. If we denote $\partial L/\partial S=\partial L/\partial S(H)$, we get $\partial^2 L/\partial S\partial H=1/T$, where the value T represents the time horizon of the consumption-leisure choice:

$$L(S)=T-H(S)-S\Rightarrow\partial L/\partial S=-\partial H/\partial S-1;$$

$$dH(S)=dS\frac{\partial H}{\partial S}=-dS\frac{H}{T}\ \Rightarrow\ \frac{\partial L}{\partial S}(H)=-\frac{\partial H}{\partial S}-1=\frac{H-T}{T}\Rightarrow\partial^2 L/\partial S\partial H=1/T.$$

The physical form of the *MRS (H for Q)* results in the following equation:

$$MRS(HforQ)=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=-\frac{Q\times T}{T(H-T)}=\frac{Q}{L+S}.$$

Now we can present the *MRS (H for Q*) with regard to the elasticity of substitution between leisure and consumption:

$$MRS(HforQ)=-\frac{dQ}{dH}=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H;$$

$$\frac{dQ}{dH}=\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H=\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=\frac{QT}{(H-T)}\frac{1}{T}=\frac{Q}{H}\frac{H}{(H-T)}=\frac{Q}{H}\frac{(H-T+T)}{(H-T)};$$

$$\frac{dQ}{dH}=\frac{Q}{H}(1+\frac{T}{H-T})=\frac{Q}{H}(1+\frac{1}{\partial L/\partial S})=\frac{Q}{H}(\frac{\partial L/\partial S+1}{\partial L/\partial S});$$

$$if\ \partial L/\partial S=-\alpha\Rightarrow\frac{dQ}{dH}=-\frac{Q}{H}(\frac{1-\alpha}{\alpha})$$

$$MRS(HforQ)=(\frac{1-\alpha}{\alpha})\frac{Q}{H}=(\frac{-\partial H/\partial S}{-\partial L/\partial S})\frac{Q}{H}$$

We can get the same result for the following Cobb-Douglas utility function:

$$U(Q,H)=Q^{-\partial L/\partial S}H^{-\partial H/\partial S}$$

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[3] If we presuppose that an individual can always adjust price reduction to a pre-allocated quantity *($\partial P/\partial S=\partial P/\partial S(Q)$)* and to target leisure time *($\partial P/\partial S=\partial P/\partial S(H)$)*, consumption and leisure become perfect complements. The model implies that consumers can choose a market with certain price dispersion, but they are still price-takers there—now, *price-reduction takers*.

If we follow the $\partial L/\partial S+\partial H/\partial S+1=0$ rule, the elasticity of substitution between leisure and consumption is $\sigma = 1$.

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