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Abstract

In January 2011, a price regulation was established in the Austrian gasoline market which prohibits firms from raising their prices more than once per day. Similar restrictions have been discussed in New York State and Germany. Despite their intuitive appeal, this article argues that Austrian-type policies may actually harm consumers. In a two-period duopoly model with consumer search, I show that in face of the regulation, firms will distort their prices intertemporally in such a way that their aggregate expected profit remains unchanged. This implies that, as some consumers find it optimal to delay their purchase due to expected price savings, but find it inconvenient to do so, a friction is introduced that decreases net consumer surplus in the market.

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1 Introduction

Retail gasoline is a product that causes ongoing debates in the public, media and politics. High and volatile prices annoy consumers, and competition authorities are often suspicious about how prices are formed in this typically highly concentrated market. In fact, many politicians argue in favor of interventions such as price regulations to protect consumers from being exploited, and several countries across the world are either discussing regulation or have already adopted it in one way or the other.[1]

This paper is motivated by a regulatory price policy that has become law in Austria in 2011.[2] The law prohibits retail gasoline stations from *increasing* their prices more than once per day, and every such price increase must occur simultaneously at noon. In contrast, the stations are free to decrease their prices whenever they wish. The policy's main intent is to decrease consumer price uncertainty and make it easier for gasoline purchasers to assess and evaluate prices. This should foster competition and ultimately increase consumer welfare in the gasoline market.

The idea to the Austrian type of regulation is not new, but dates back to at least 2005, when a virtually identical law was passed by the New York State Senate[3], but later died in the New York State Assembly. In fact, the law is a recurring theme in New York State's legislation, as a new attempt for its establishment has been made in the New York State Senate in 2011[4], and is currently under review by the New York State Division of Consumer Protection. Moreover, in face of the Austrian policy, a public discussion has emerged in Germany whether German consumers could benefit from a similar regulation.[5] However, the German *Bundeskartellamt* (federal antitrust authority) so far has taken a skeptical view on the potential merits of the Austrian policy. For example, in Bundeskartellamt (2011, p.137), a comprehensive study of the German gasoline sector, it is stated that, as competitive structures and incentives are not

[1] For example, fuel price regulations are currently in place in Austria, Luxembourg, Western Australia, several Canadian states and Mexico. For further details, see Haucap and Müller (2012), Dewenter and Heimeshoff (2012) and Arteaga and Flores (2010).

[2] BGBl II Nr. 484 ("Verordnung des BM für Wirtschaft, Familie und Jugend betreffend Standesregeln für Tankstellenbetreiber über den Zeitpunkt der Preisauszeichnung für Treibstoffe bei Tankstellen"); established on January 1, 2011.

[3] Bill number S5969, introduced by Senator Marcellino, states that "[i]t is unlawful for any dealer [of gasoline or diesel fuel] to increase the price charged for the sale of motor fuel of any particular grade or quality more than once in any calendar day."

[4] Bill S603-2011, sponsored by Senator Peralta, is "[a]n act to amend the general business law, in relation to prohibiting more than one increase in the price of gasoline in any twenty-four hour period".

[5] For example, an initiative has been put forward by the German *Bundesrat* in Spring of 2012 that requests the German government to evaluate various possible pricing regulations in the German gasoline sector, including the Austrian policy. Several German newspapers have covered this story, e.g. *Handelsblatt* ("Ramsauer will Benzinpreise regulieren" / "Ramsauer wants to regulate fuel prices") on April 18, 2012 and *Die Welt* ("Benzinbranche wehrt sich gegen "Spritpreisbremse"" / "Fuel sector opposes fuel price regulation") on April 19, 2012.

affected, "an enduring positive effect [of price regulatory policies like in Austria] in the sense of improving competition [in the gasoline sector] is not implied."

The simple question one can ask is the following. Can consumer welfare in the gasoline market in fact be increased by restricting firms' pricing? Or is the New York State Assembly's 2005 decision of rejecting the New York State Senate's proposal and the German Bundeskartellamt's skeptical view on the price regulation warranted?

In this article, I argue that Austrian-type policies might in fact have a *negative* effect on consumer welfare. The principal reason for this is simple: if firms anticipate that they will not have the possibility to increase their prices whenever they wish, they might be reluctant to charge low prices in early stages of their (policy induced) twenty-four hour price setting cycle, in order to maintain more price flexibility later on. The distortions that are created by this can be harmful for consumers, as will become clear below.

Retail gasoline can be considered as an essentially homogeneous good that is traded in a market where search is important.[6] Some consumers find it worthwhile to shop around and compare prices whereas others purchase randomly whenever they are in need. From the firms' perspective, this creates a well-known tension between charging high prices and only selling to non-searching consumers at a high margin versus charging low prices and also attracting searching consumers, making larger sales. An elegant way to model this has been provided in Varian's (1980) seminal contribution on price dispersion. In order to capture the restrictive policy as established by the law, I extend a duopoly version of the Varian model to two periods. At the *beginning* of the first period (noon), the cycle is reset, implying that the firms are free to set any price they want. The chosen prices are assumed to be fixed in the short run and hence dictate each firms' demand over the course of the first period (a period covering several hours after noon). Then, at the start of the second period (e.g., at some point in the evening), the firms are given the chance to revise their prices for the rest of the pricing cycle (the remaining time until the following noon). However, because of the Austrian price regulation, the firms may only *decrease* their prices relative to their initial price choice. In addition, the consumers' intertemporal decision in which period to buy can be endogenized, as they may compare the expected gains of purchasing later in the cycle – at lower prices – with an idiosyncratic preference of purchase time.

Using this setup, I will derive two key properties of the Austrian regulation for the case

[6]For excellent introductions to the theoretical and empirical literature on retail gasoline markets, see Houde (2010) and Eckert (2011).

of unit demand. First, if the consumers' intertemporal distribution (the relative number of first to second period consumers) is an exogenous variable, the Austrian policy is ineffective in promoting consumer surplus. It merely leads to a redistribution of surplus from first to second period consumers, while it keeps firms' expected profits constant. Second, if the consumers may endogenously select their purchase time, the policy even leads to a reduction of consumer surplus, compared to the unregulated regime. This is because inefficient consumer switching across periods is induced, while firms can again preserve their pre-regulation equilibrium profits.

The first finding follows because, under the restrictive policy, firms' first period tradeoff between charging relatively low prices and attracting a large market share versus charging relatively high prices and "milking" non-searching consumers is shifted in favor of the latter. Intuitively, firms compete less aggressively in the first period because they have to face each other again and low initial prices imply harsh competition for all remaining consumers later. Hence, firms' pricing is more cautious in the first period, which can be interpreted as a desire for maintaining price flexibility and potentially large margins in the second period. Overall, this leads to higher first period prices, on average, compared to the situation where the policy is not in place. On the other hand, in the second period, firms' pricing tends to be more aggressive, as their pricing range is narrowed down due to the artificial price ceilings that are imposed. It turns out that for *any* exogenous intertemporal consumer distribution, firms' equilibrating strategies are such that these two effects exactly offset each other, leading to total profits that are unchanged when compared to the unrestricted regime. Hence, in the case of unit demand, expected aggregate consumer surplus is also the same as in the unrestricted regime, although there will be a redistribution of consumer welfare across periods.[7]

As the policy guarantees (weakly) lower prices in the second stage of the model, it is natural to let consumers endogenously decide upon their purchase time. After all, the lower second period prices should imply that more consumers wish to purchase then, and only those who have a sufficiently high (idiosyncratic) cost of doing so (or are ignorant about the regulatory policy) should continue purchasing in the first period. Intuition suggests that if some consumers

[7]None of the models' main results rely on the assumption of unit demand. For any well-behaved demand function, it is easy to argue that the firms can maintain their equilibrium profits if the price regulation gets implemented. Also, for a large class of demand functions, e.g. for $D(p) = 1 - p^r$, with $r > 0$, mathematical intuition can be provided why the aggregate two-period consumer surplus must unambiguously *decrease* facing the price regulation, even if the consumers' intertemporal distribution is fixed exogenously. This is because the additional deadweight loss created by firms distorting their prices upwards in the first period of the model may exceed the reduction of deadweight loss in the second period caused by artificial price ceilings. However, even for linear demand, comparing the expected consumer surplus across the restricted and unrestricted regime is so complicated that it requires numerical methods. This text thus focuses on the simple case of unit demand, although the mechanism generalizes to many downward sloping demand functions.

switch to the second period with lower average prices, the policy might be beneficial to consumers. The second main finding of this paper is that this intuition is wrong. Taking firms' equilibrating strategies into account, it is straightforward to see that the policy must be harmful for consumers. Namely, as aggregate consumer surplus remains constant for every intertemporal consumer distribution, an individual waiting consumer's realized *gross* gain of waiting resulting from lower second period prices is "financed" by an aggregate loss of equal size by all other consumers in the market, as they have to pay higher prices. This is true because waiting consumers exercise an indirect negative externality on all others: if fewer consumers purchase in the first period, firms compete less aggressively in both the first and second period. It is the combination of this negative externality and firms profit-preserving equilibrium strategies which leads to a loss of consumer welfare. Consumers' aggregate *gross* welfare remains unchanged, but waiting consumers do not realize the full gains of waiting, as they face a personal cost of doing so. Hence, the aggregate *net* consumer welfare in the market is reduced by the aggregate waiting cost that is incurred by waiters.

How is consumer welfare affected across consumer groups? As expected prices are higher (lower) in the first (second) period of the restricted regime, compared to the two identical periods of the unrestricted regime, one can see that all consumers who keep purchasing in the first period due to their high waiting cost must be strictly worse off after the policy gets implemented, whereas all consumers who have no disutility of purchasing in the second period must be strictly better off. The *critical* consumer, who is indifferent between the two policy regimes, is a switching consumer that faces some positive, but sufficiently low disutility of waiting. In other words, it is shown that the policy harms those (inflexible) consumers who have no other choice than to purchase in the first period. Moreover, it is apparent that consumers who are not aware of the price regulation – and hence purchase at random times – will tend to be harmed.

The model is mainly based on a two-period extension of Varian (1980) that allows for asymmetric strategy sets in the second period and an endogenous purchase time of consumers. The Varian model is characterized by homogeneous products, an exogenous search decision of consumers and non-sequential search. All of these features make it suitable for adaption to multiple periods and asymmetric strategy sets without introducing unnecessary complexity. Other important search models that employ richer consumer search behavior are, for example, given by Stahl (1989), who considers optimal sequential search for homogeneous goods; Wolinsky (1986) and Anderson and Renault (1999), who consider optimal sequential search for differentiated products; or Burdett and Judd (1983), who consider optimal fixed sample-size search for homogeneous products.

There is a small theoretical literature analyzing the effects of price regulations in markets governed by consumer search. Fershtman and Fishman (1994) and Armstrong et al. (2009) examine regulatory price ceilings in a market characterized by optimal (non-sequential) fixed sample-size search á la Burdett and Judd (1983). Both papers find that price ceilings have two effects on firms' equilibrium price setting. First, there is a direct effect of capping the upper range of firms' equilibrium price distributions, which has the intended result of reducing prices in the market. However, as in turn, consumers' expected gains from search decrease, there is also an indirect effect of reducing the amount of search in the market, leading to higher prices. Which effect dominates depends on the specific modeling assumption.[8] Rauh (2004) also analyzes regulatory price ceilings, but considers sequential search in a market where firms have heterogeneous marginal cost. In the model, introducing a price ceiling has three effects: it reduces the aggregate search cost because less people search in equilibrium; consequently, it transfers some production to the less efficient firm; and it increases output as more consumers find it optimal to enter the market. The net effect on welfare is shown to be ambiguous.

All of these articles are concerned with regulatory price ceilings, which are much different from the Austrian policy. In particular, while price ceilings can be regarded as direct price interventions with immediate effects on firms' equilibrium price distributions (e.g., by capping their upper range), it is a priori unclear how the Austrian regulation, which only indirectly affects firms' price setting by restricting the nature of price changes, alters the equilibrium outcome. Moreover, price ceilings can be modeled in a static way, as they do not make firms' pricing contingent on prior actions they have taken. In contrast, the Austrian policy can only be analyzed adequately in a dynamic model. Because of the technical complications that result, in contrast to the articles discussed above, I model consumers' contemporaneous (within-period) search behavior as exogenous. Instead, I allow consumers to optimally decide upon their purchase period, which can be interpreted as intertemporal search. Hence, another strand of related theoretical literature are dynamic models of consumer search. Examples include Fershtman and Fishman (1992), Yang and Ye (2008), and Tappata (2009).

Besides the theoretical contributions, there is also a recent empirical and experimental literature on the effects of price regulations in consumer search markets. Haucap and Müller (2012)

[8] In Fershtman and Fishman (1994), where consumers' search costs are homogeneous, the indirect effect always dominates, implying that the analyzed price regulation aversely affects consumer surplus in the market. Armstrong et al. (2009) employ a more general informational structure and allow for search costs to be heterogeneous across consumers. The latter results in an ambiguous effect of the policy on consumer surplus. For example, in the extreme case where some consumers have zero search cost while all others' search cost is prohibitively high, the regulation has no effect on consumers' search behavior, leading to an unambiguous decrease of prices.

analyze the consequences of the Austrian policy (among others) on price setting in the lab, and a study by Dewenter and Heimeshoff (2012) empirically assesses its effect on firms' pricing. The resulting evidence is mixed. While the former finds an increase of price levels caused by facilitated player coordination, the latter finds a significant negative effect on prices, compared to an inter-European control group. However, the empirical study employs weekly data, which is clearly not adequate to examine intertemporal price distortions that arise in the course of a day, as theory would predict for the Austrian regulation. In a broader context, Wang (2009) empirically studies a similar gasoline price regulation that was established in Western Australia in 2001. He finds that the policy significantly altered firms' equilibrium price setting, and that, while average prices tended to fall after the Australian law got established, they returned to their previous level after several months.

Finally, Noel (2012) empirically evaluates intertemporal consumer search strategies in the Toronto retail gasoline market. He finds that, while potential cost savings are sizable, few consumers exploit them. One reason might be that the non-monetary costs of doing so are substantial (e.g., because they involve regular price checks by consumers). This is not the case for the Austrian policy, where every consumer could easily infer that prices must be lowest short before noon.

The remainder of this article is structured as follows. In Section 2, the basic model with unit demand and an exogenous purchase period of consumers is introduced. In Section 3, consumers' purchase time is endogenized, allowing some of them to optimally switch periods. Section 4 concludes. Technical proofs are relegated to the appendix.

2 Model with Exogenous Purchase Times

Consider the following market. There are two ex-ante identical firms that supply a homogeneous good (retail gasoline) and compete through prices. Firms' constant unit cost is normalized to zero. There are two stages $t = 1, 2$ with an exogenous mass of $\kappa \in (0, 1)$ consumers in the first stage and $1 - \kappa$ consumers in the second stage. In this section, consumers may not select their purchase time. In each period, an exogenous fraction $\lambda \in (0, 1)$ of consumers shops around and observes both of their period's prices, buying at the cheaper firm (they purchase at either firm with equal probability if prices are the same). The remaining $1 - \lambda$ consumers buy at a random firm. Hence, Varian's (1980) well-known framework for non-sequential search in homogeneous goods markets is extended to two periods. Following standard terminology, I will refer to the λ

searching consumers as "shoppers" and to the $1-\lambda$ non-searching consumers as "non-shoppers".

At the beginning of $t = 1$, firms' pricing behavior is not restricted by the competition policy. Hence, they may choose any positive price to compete for the first-period consumers. However, given any price realization (p_1, p_2) in $t = 1$, the law forces firms not to *increase* their prices at the beginning of $t = 2$ (when they have the opportunity to revise their prices for the whole second period). In the remainder of this paper, w.l.o.g. define firm 1 as the firm which charged the higher price in period 1, that is, $p_1 \geq p_2$. Consequently, the firms' strategy sets in $t = 2$ are given by $S_{1,t_2} = [0, p_1]$ and $S_{2,t_2} = [0, p_2]$ with $S_{2,t_2} \subseteq S_{1,t_2}$. Importantly, it is assumed that firms observe each other's first period price realization at the beginning of the second period.[9]

I will solve this game by backward induction. First, given any possible realization (p_1, p_2) of stage 1, I will characterize the resulting equilibrium of stage 2 and show that it is unique. Using the expected equilibrium payoffs in this unique equilibrium of any $t = 2$ subgame, I will proceed to pin down the symmetric equilibrium actions of stage 1. Finally, I will determine the expected consumer surplus and firm profits of the whole game and prove that they are independent of the intertemporal consumer distribution κ.

I start with deriving the equilibrium strategies of any reachable second period subgame. Note that $p_1 \geq p_2$ are parameters of every subgame because they impose price ceilings on firms, hence restricting their strategy sets. As a first observation, it is obvious that firm 1 will never price in the range (p_2, p_1), as this is a strictly dominated strategy. By pricing higher than p_2, firm 1 can only attract the non-shoppers anyway (as firm 2 is prohibited to price above p_2), and hence could increase its profit by charging p_1 instead (which is the highest price it is allowed to set). Moreover, it is clear that firms' second period equilibrium strategies cannot depend on the number of consumers who actually buy in period 2, i.e., $1-\kappa$. This is because anything that has happened in period 1 is bygone and the only thing that matters for firms is to extract as much profit as possible from the remaining consumers in the market.

Next, one can see that the type of equilibrium that emerges in $t = 2$ must depend on the relative price difference resulting from stage 1 and the number of shoppers in the market. Note that, as in both periods, a fraction $1-\lambda$ of the consumers is uninformed and purchases at either firm with equal probability, each firm will attract a fraction $\frac{1-\lambda}{2}$ of second period consumers for certain. On top of that, the firm with the lower price in $t = 2$ will be visited by a fraction $\lambda + \frac{1-\lambda}{2} = \frac{1+\lambda}{2}$ of second period consumers because it additionally attracts the fraction λ of

[9] This assumption is justified because gasoline stations are clearly monitoring their rivals' prices – in fact, franchisees of the large corporations are sometimes even obliged to report rivals' prices on a regular (e.g. hourly) basis.

informed consumers in that period. Now clearly, if firm 1 sticks to its first period price p_1, and if the other firm's first period price p_2 is strictly smaller than p_1, firm 1 cannot win the informed consumers for any price firm 2 sets. But if p_1 is sufficiently large compared to p_2, firm 1 will never *want* to attract the shoppers because even when doing so for certain at the highest possible price $p_2 - \epsilon$, its profit would not exceed the profit when sticking to p_1. It is easy to see that this is the case whenever $\frac{p_1}{p_2} \geq \frac{1+\lambda}{1-\lambda}$.[10] However, as will be shown below, this pure strategy equilibrium of the second stage will never occur given the equilibrium actions taken in the first stage. Hence, in the following assume that

$$\frac{p_1}{p_2} < \frac{1+\lambda}{1-\lambda}. \tag{1}$$

The first question that arises is whether a pure strategy equilibrium of a subgame can exist if inequality (1) holds. It is easy to see that this is not the case: if firm 1 charges p_1, firm 2 maximizes its profit by pricing at p_2, which firm 1 wishes to undercut due to inequality (1). If firm 1 prices at any $p \leq p_2$ (recall that pricing in the range (p_2, p_1) is strictly dominated for firm 1), firm 2 either has an incentive to undercut slightly and attract all the informed consumers (which in turn leads firm 1 wanting to undercut the new price of firm 2), or, if p is low enough, to charge the highest possible price p_2. But in the latter case, firm 1 can again make a higher profit by slightly undercutting p_2, proving the statement.

Consequently, one has to look for an equilibrium in mixed strategies in which both firms might use a combination of mass points and continuous distributions. The following proposition characterizes the unique equilibrium of any subgame that satisfies inequality (1).

Proposition 1. *Suppose that in the first period, firms 1 and 2 chose prices p_1 and $p_2 \leq p_1$ that satisfy inequality (1). Then, no pure strategy equilibrium exists in the resulting second stage subgame. The unique mixed strategy equilibrium is characterized by*
(a) firms 1 and 2 choose p_1 respectively p_2 with identical probability mass

$$\alpha := \frac{1-\lambda}{2\lambda}\left(\frac{p_1}{p_2} - 1\right) \in (0,1), \tag{2}$$

(b) with the remaining probability, firms 1 and 2 draw prices randomly according to the same

[10] Denote by $\Pi_i(\tilde{p_1}, \tilde{p_2})$ the profit of firm i if firm 1 chooses a second period price of $\tilde{p_1} \leq p_1$ and firm 2 chooses a second period price of $\tilde{p_2} \leq p_2$. Then formally, if $\Pi_1(p_1, .) = (1-\kappa)\frac{1-\lambda}{2}p_1 \geq (1-\kappa)\frac{1+\lambda}{2}p_2 = \Pi_1(p_2 - \epsilon, p_2)$, which implies that $\frac{p_1}{p_2} \geq \frac{1+\lambda}{1-\lambda} > 1$, it is a dominant strategy for firm 1 to continue pricing at p_1, i.e., the price it charged in $t = 1$. Of course, given that firm 1 prices at $p_1 \notin S_{2,t_2}$, it only attracts the non-shoppers anyway, leading firm 2 to optimally price at the highest possible price it can charge, i.e., p_2. Hence, if $\frac{p_1}{p_2} \geq \frac{1+\lambda}{1-\lambda}$, the subgame will have a pure strategy equilibrium in which firm 1 sticks to charging p_1 and firm 2 sticks to charging p_2.

atomless distribution function

$$F(p) := \frac{1+\lambda-(1-\lambda)\frac{p_1}{p}}{1+\lambda-(1-\lambda)\frac{p_1}{p_2}} \tag{3}$$

with support $[\frac{1-\lambda}{1+\lambda}p_1, p_2)$.

Expected equilibrium profits are given by $(1-\kappa)\frac{1-\lambda}{2}p_1$ *for each firm.*

Proof. See Appendix. □

Corollary 1. *In the unrestricted regime where firms are free to choose any positive price in both periods, each stage is uniquely characterized by firms randomizing according to the identical distribution function*

$$F_0(p) := 1 - \frac{1-\lambda}{2\lambda}\left(\frac{v}{p}-1\right) \tag{4}$$

with support $[\frac{1-\lambda}{1+\lambda}v, v]$. *This distribution is independent of* κ. *Total expected equilibrium profit is given by* $\frac{1-\lambda}{2}v$ *for each firm.*

Proof. This can be seen directly by removing any price restrictions imposed on firms, i.e., by setting $p_1 = p_2 = v$ in Proposition 1 and noting that firms will never want to price above v in equilibrium, as this implies a profit of zero. Each firm's expected profit is then given by $\kappa\frac{1-\lambda}{2}v + (1-\kappa)\frac{1-\lambda}{2}v = \frac{1-\lambda}{2}v$. □

Note that Varian (1980), for unit demand, is a special case of the analyzed subgame where $p_1 = p_2 = v$. Hence the unrestricted regime's equilibrium price distribution $F_0(p)$ directly corresponds to Varian's equilibrium price distribution.

Proposition 1 shows in particular that if inequality (1) is satisfied (λ is sufficiently large compared to $\frac{p_1}{p_2}$), each firm's second stage equilibrium payoff is proportional to the *maximum price* that was chosen in the first period. This observation is sufficient to determine firms' equilibrium actions in the first stage, under the assumption that inequality (1) must hold for each possible outcome of stage 1. This will be assumed in the following and verified once the equilibrium actions of the first stage are determined. Moreover, since firms are ex-ante identical, I impose that they must choose ex-ante symmetric actions in stage 1.

The first question to ask is whether the firms might choose pure actions in the first stage. Since undercutting would pay due to the atom of shoppers in the market, this clearly cannot be the case.[11] Hence, firms must charge prices according to some (symmetric) distribution function

[11] Formally, if both firms were to price at some $p \leq v$, their expected payoffs for the whole game would be given by $\kappa\frac{1}{2}p + (1-\kappa)\frac{1-\lambda}{2}p$, since p would certainly be the maximum price of the first period. Now, by marginally

$G(p)$ in $t = 1$. By standard arguments, this distribution function can have no mass points (if there were any, there is a positive chance of ties, giving firms an incentive to undercut) and the upper support boundary must be equal to v (if the highest price in the support was some $\hat{p} < v$, a firm choosing $\hat{p}$ would only attract the non-shoppers anyway in $t = 1$ and hence, since the second period expected profit is proportional to the maximum price chosen in the first period, could unilaterally increase its profit by pricing at v instead).

A firm's total two-period profit when choosing $\overline{p} = v$ in $t = 1$ is easily calculated to be

$$\Pi_i^{tot}(v; G(p)) = \frac{1-\lambda}{2} v. \tag{5}$$

The reason is as follows. Because v is the highest price in the first stage pricing support and is chosen with zero probability mass, a firm charging v can only attract the uninformed consumers in the first stage, making a first-stage profit of $\kappa \frac{1-\lambda}{2} v$. In addition, given the equilibrium payoff that was calculated for the second period subgame (which is equal to $(1-\kappa)\frac{1-\lambda}{2}$ times the maximum price that was charged in period $t = 1$), the expected second period equilibrium payoff must be equal to $(1-\kappa)\frac{1-\lambda}{2} v$, which in total sums up to $\frac{1-\lambda}{2} v$.

In order for firms to be willing to randomize across prices in the support of $G(p)$, each price must yield the same expected profit *for the full game*. If one of the firms charges some price $p \le v$, its expected profit of the full game, given the expected equilibrium payoff of the second period, can be written as

$$\mathbb{E}\Pi_i^{tot}(p; G(p)) = G(p)\left[\kappa\frac{1-\lambda}{2}p + (1-\kappa)\frac{1-\lambda}{2}p\right] + (1-G(p))\left[\kappa\frac{1+\lambda}{2}p + (1-\kappa)\frac{1-\lambda}{2}\mathbb{E}_{\mathbb{G}}(\tilde{p}|\tilde{p} \ge p)\right], \tag{6}$$

where $\mathbb{E}_{\mathbb{G}}(\tilde{p}|\tilde{p} \ge p) = \frac{1}{1-G(p)}\int_p^v x dG(x)$ denotes the expected price the other firm charges in $t = 1$, given that it charges a higher price than p.

Equation (6) can be understood as follows. If firm i charges a price p in $t = 1$ while the other firm draws prices according to $G(.)$, there is a probability $G(p)$ that the other firm draws a lower price than p, leading to a first period profit of firm i of $\kappa\frac{1-\lambda}{2}p$. Moreover, since the second period expected profit is given by $(1-\kappa)\frac{1-\lambda}{2}$ times the *maximum* price charged in the first period (compare with Proposition 1) – which, in that case, is firm i's price p – firm i also makes an expected second period profit of $(1-\kappa)\frac{1-\lambda}{2}p$. With probability $1 - G(p)$, firm i draws the lower price in $t = 1$ and attracts the informed consumers, leading to a first period profit of

undercutting to $p' < p$, a firm could unilaterally increase its profit to $\kappa\frac{1+\lambda}{2}p' + (1-\kappa)\frac{1-\lambda}{2}p > \kappa\frac{1}{2}p + (1-\kappa)\frac{1-\lambda}{2}p$, as it can additionally win the $\kappa\frac{\lambda}{2}$ informed $t = 1$ consumers that would go to the other firm if prices were identical.

$\kappa\frac{1+\lambda}{2}p$. Again, as the second period expected profit is given by $(1-\kappa)\frac{1-\lambda}{2}$ times the maximum price charged in the first period (which, in that case, is the other firm's first period price), firm i's expected second period profit is given by $(1-\kappa)\frac{1-\lambda}{2}$ times the expected price the *other* firm charges, given that it charges a higher price than p.

Setting equation (6) equal to equation (5) and simplifying leads to the condition

$$G(p)p\left[1-\lambda-\kappa(1+\lambda)\right]+\kappa(1+\lambda)p+(1-\kappa)(1-\lambda)\int_p^v xG'(x)dx=(1-\lambda)v, \tag{7}$$

which has to hold for every price in the support of the unknown price distribution $G(p)$. Solving this functional equation, one finds

Proposition 2. *There exist no symmetric pure equilibrium actions of the first stage of the game. Both firms choose prices according to the identical atomless distribution function*

$$G(p)=\frac{\kappa(1+\lambda)-(1-\lambda)\left(\frac{v}{p}\right)^{\frac{\kappa(1+\lambda)-(1-\lambda)}{2\kappa\lambda}}}{\kappa(1+\lambda)-(1-\lambda)} \tag{8}$$

with $\overline{p_G}=v$ *and* $\underline{p_G}=v\left(\frac{1-\lambda}{\kappa(1+\lambda)}\right)^{\frac{2\lambda\kappa}{\kappa(1+\lambda)-(1-\lambda)}}$. *The equilibrium expected total profit of the full game is given by* $\frac{1-\lambda}{2}v$ *for each firm.*

Proof. The crucial point is that given $G(p)$, the pure strategy equilibrium of the second stage should never be reached, as in those cases, an incorrect equilibrium payoff for firm 2 would have been used in the calculation. Among other things, this is verified in the appendix. □

The finding on firm profits immediately gives rise to the central proposition of this section.

Proposition 3. *Under unit demand, implementing the restrictive policy is welfare neutral for firms and consumers, irrespective of the intertemporal consumer distribution. For every* $\kappa\in(0,1)$, *firms' expected total profit remains unchanged at* $(1-\lambda)v$ *compared to the unrestricted regime, and also consumers' expected total surplus is unaffected at* λv *across periods. Hence, the policy is ineffective in promoting consumer surplus.*

Proof. This is obvious when comparing firms' expected total payoff in the unrestricted regime (cf. Corollary 1) with their expected total payoff in the restricted regime (cf. Proposition 2) and noting that the full surplus of v must be distributed in total, as demand is inelastic and every consumer purchases in equilibrium. □

Note that Proposition 3 is *not* specific to inelastic consumer demand, but generalizes to many downward sloping demand functions. One principal reason is that for any strictly concave profit function $(p-c)D(p)$, one can argue that Austrian-type price regulations cannot decrease the equilibrium profits in the market.[12] Hence, the price distortions that are caused by the regulation can only be beneficial to consumers if the increased deadweight loss created by (probabilistically) higher equilibrium prices in the first period of the restricted regime is more than offset by the reduction of deadweight loss caused by lower prices in the second period. In a supplementary note, I show numerically that the opposite is true for linear demand. In that case, the policy-induced price distortion leads to both a redistribution of consumer welfare across periods *and* a welfare-decreasing allocative effect.

While the main result of policy ineffectiveness – Proposition 3 – can be stated without any further calculation, most of the economic intuition can only be gained when analyzing firms' first period actions in detail. On top of that, some results of Section 3, where consumers' purchase time is endogenized, follow directly from the interaction of firms' equilibrium pricing and consumers' intertemporal distribution. Thus, the remainder of this section focuses on firms' first period price setting.

As a start, it is important to analyze how changes in the intertemporal consumer distribution κ affect firms' pricing.

Lemma 1. *Let $G(p;\kappa)$ and $G(p;\kappa')$ be the equilibrium first period price distributions given $\kappa \in (0,1)$ and $\kappa' \in (0,1)$, with $\kappa' > \kappa$ (for any $\lambda \in (0,1)$). Then $G(p;\kappa)$ dominates $G(p;\kappa')$ in a first order stochastic sense.*

Proof. See Appendix. □

Lemma 1 states that, if fewer consumers purchase in $t = 1$ $(\kappa < \kappa')$, higher first period prices are drawn by firms, in a probabilistic sense. This is because firms anticipate that by pricing aggressively in the first period, their expected equilibrium profit in the second period diminishes, as the equilibrium payoff of any reachable subgame is proportional to the *maximum* price charged in the first period. But the fewer consumers purchase in the first period, the less important a high market share in the first period becomes. Hence, as the number of first period consumers

[12] This is because by pricing at the (unique) monopoly price in each period, each firm can guarantee itself a total profit that is not lower than $\frac{1-\lambda}{2}\Pi^m$ (where Π^m denotes the monopoly profit), which is equal to a firm's total profit in the equilibrium of the unregulated regime, as can easily be shown.

declines, firms counteract the effects of the regulation by softening price competition in the first stage.

Corollary 2. *The expected first and second period prices in the restricted regime are strictly decreasing in the fraction of first period consumers* κ*.*

Proof. The statement for the expected first period prices follows immediately from first order stochastic dominance of $G(p;\kappa)$ relative to $G(p;\kappa')$ for $\kappa' > \kappa$. For the second period expected prices, note that once the second period is reached, the intensity of firms' competition is independent of the absolute number of consumers in that period. Moreover, the expected second period profits are proportional to the maximum price charged in the first period. In expectation, this maximum price is clearly lower the more consumers purchase in the first period. □

The fact that the expected first period prices decrease in the fraction of first period consumers has the same interpretation as Lemma 1: the more consumers purchase in the first period, the more important it is for firms to attract a large market share. This intensifies competition and drives down prices in the first period. But due to the price regulation, low price realizations of the first stage imply a narrow scope for setting prices and harsh competition in the second stage. It follows that also firms' expected *second* period prices decrease in the fraction of first period consumers.

The combination of both results may seem surprising at first: how can aggregate consumer welfare remain constant irrespective of consumers' intertemporal distribution (cf. Proposition 3), but both periods' expected prices decrease in the number of first period consumers? To see why this is the case, recall that expected prices are strictly lower in the second period of the model than in the first. Hence, if more consumers purchase in the first period (and therefore, less in the second), more consumers pay relatively higher prices, although both period's prices fall. Firms' equilibrating strategies are such that the effect on aggregate consumer surplus is always zero, although the *individual* consumer surplus in each period rises with the number of first period purchasers. Namely, the *more* consumers purchase in the first period, the *more* aggressive firms' pricing becomes, leading to lower expected prices in the first and second period of the game.

Building on the above results, one can examine the limit behavior of firms' equilibrium pricing strategies as the fraction of first period consumers κ tends to zero and one, respectively. This is summarized in

Corollary 3. *As* $\kappa \to 0$*,* $G(p) = 0$ *for all* $p < v$ *and* $\underline{p_G} = v$*. As* $\kappa \to 1$*,* $G(p) = F_0(p) = 1 - \frac{1-\lambda}{2\lambda}\left(\frac{v}{p} - 1\right)$ *and* $\underline{p_G} = \underline{p_0} = \frac{1-\lambda}{1+\lambda}v$*.*

Proof. See Appendix. □

First, as the fraction of $t = 1$ consumers tends to zero, firms will charge the monopoly price in period 1. Intuitively, this must be the case because the expected second period equilibrium profit for both firms only depends on the maximum price charged in the first period. As the first period profits get less and less important as fewer and fewer consumers purchase in that period, it is best to price high in the first period and focus on second period profits. In the limit as nobody buys in the first period, it is optimal to have no competition in the first period and maximize the expected second period profit by pricing at v deterministically.

Second, as the fraction of $t = 1$ consumers tends to one, firms will draw prices according to the price distribution of the unrestricted regime. This is because, as fewer and fewer consumers purchase in $t = 2$, firms can effectively ignore the restrictive pricing policy in $t = 2$ and will compete more and more harshly in $t = 1$. In the limit, the situation becomes as if the second period did not exist, and firms price as competitively in the first period as they would do in the unrestricted regime.

Finally, it is easy to show

Corollary 4. *The equilibrium first period price distribution $G(p)$ of the restricted regime dominates the equilibrium price distribution $F_0(p)$ of the unrestricted regime in a first order stochastic sense. Expected prices in the first (second) period of the restricted regime are strictly higher (lower) than expected prices in both periods of the unrestricted regime.*

Proof. See Appendix. □

Corollary 4 exemplifies the two countervailing price distorting effects of the restrictive policy that were discussed in the introduction. Namely, the price regulation distorts firms' second period pricing downward, as intended by policymakers, but this comes at the cost of distorting firms' first period pricing upward, as they try to soften the increased competition in the second stage. This gives the key intuition why the Austrian policy is ineffective in promoting consumer surplus. Due to the intertemporal price distortions that arise, first period consumers are made worse off, whereas second period consumers benefit. Proposition 3 shows that the relative strength of these two effects is such that they exactly offset each other, leading aggregate consumer surplus to be unaffected across regimes.

Of course, while the expected total consumer surplus is unaffected in the restricted regime, there will be a redistribution of consumer surplus across time ($t = 1$ consumers will be worse off whereas $t = 2$ consumers will be better off), as is apparent from Corollary 4.

An example of the equilibrium price distribution in the unrestricted regime and several first period equilibrium price distributions of the restricted regime (for varying levels of κ) is provided in Figure 1.

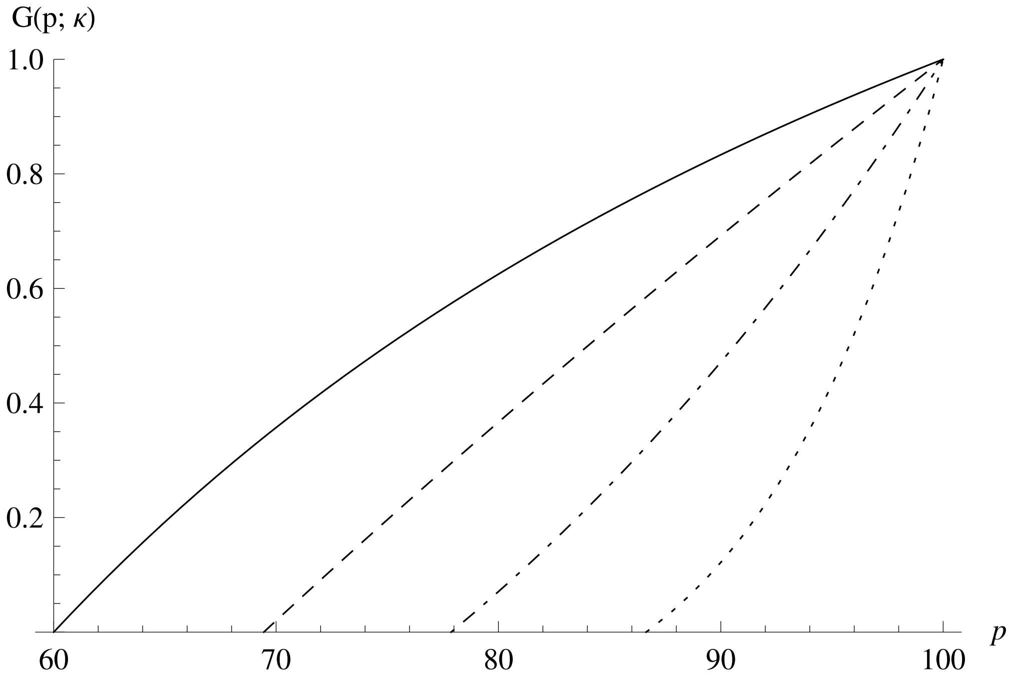

Figure 1: Depiction of the equilibrium distributions $F_0(p)$ (solid) and $G(p;\kappa)$ for $\kappa = 0.5$ (dashed), $\kappa = 0.25$ (dashed-dotted) and $\kappa = 0.1$ (dotted). The other parameters are $v = 100$ and $\lambda = 0.25$.

3 Model with Endogenous Purchase Times

Proposition 3 shows that if the intertemporal distribution of consumers is fixed exogenously, and their demand is inelastic, implementing the restrictive policy is welfare neutral in an aggregate sense. If all consumers were perfectly unaware of the establishment of the price regulation, such a fixed consumer distribution would be justified. However, it is more plausible to assume that at least some consumers are informed about the policy and its consequences on firms' equilibrium pricing. Thus, it seems desirable to allow consumers to endogenously decide upon their purchase period. Intuition suggests that this might make the price restriction more appealing, since, if many consumers delay their purchase until the second period with lower expected prices, consumer welfare might actually increase, compared to the unrestricted regime. It turns out that this intuition is wrong. Due to an indirect negative externality that switching consumers exercise on all others, the policy changes from being ineffective to actually being harmful to consumers.[13]

In order to illustrate these claims, I will explicitly model the consumers' decision in which period to buy, given exogenous and heterogeneous preferences regarding their purchase time. In particular, as in Section 2, I suppose that a fraction $\kappa \in (0,1)$ of consumers strictly prefers to buy in period one ("t_1 consumers"), whereas the remaining fraction of consumers (at least weakly) prefers to buy in period two ("t_2 consumers"), *given equal expected prices in both periods.* Moreover, shoppers and non-shoppers are assumed to have identical distributions regarding their favorite purchase times.[14]

The general mechanism that results in reduced consumer welfare now relies on two key properties of the baseline model. First, introducing the price regulation leads to a distortion of prices across periods, compared to the unrestricted regime. In expectation, the second period of the restricted regime is characterized by strictly lower prices than the first, while in the unrestricted regime, prices are drawn independently in each period. A direct implication of this is that all t_2 consumers will definitely keep buying in $t = 2$; that is, their preferred period. In contrast, t_1 consumers will face a tradeoff: by waiting for the second period, they can expect to find lower prices, however they find it inconvenient to do so.

Second, firms can maintain their pre-regulation equilibrium profits, no matter how consumers are distributed in the new equilibrium. Hence, in the case of unit demand, the aggregate expected consumer surplus cannot increase, compared to the unrestricted regime.

These properties together imply that consumer surplus and total social welfare must unambiguously *decrease* in face of the price regulation, as long as some t_1 consumers find it optimal to delay their purchase until the second period. This is because, by the nature of firms' equilibrating strategies, every t_1 consumer's *gross* gain of waiting for lower prices must be accompanied by an aggregate welfare loss of equal size by all other consumers in the market, as they are hurt by softened price competition. But the true individual gain of purchase-delaying t_1 consumers,

[13] As mentioned earlier, for many downward sloping demand functions (e.g. linear demand), the allocative effect of the price distortion under an exogenous consumer distribution is in fact such that the aggregate expected consumer surplus must unambiguously decrease, compared to the unrestricted regime. This implies that the adverse mechanism discussed in this section extends to a much wider class of demand functions. Clearly, if for *any* exogenous intertemporal consumer distribution, the aggregate consumer surplus in the market declines, and also, inefficient consumer switching is induced in the equilibrium with endogenous purchase times, the policy must be harmful to consumers.

[14] I also considered the case in which all shoppers have zero opportunity cost of time and therefore strictly prefer to purchase in $t = 2$ if the price regulation is in place. The result of reduced consumer welfare prevails. This finding is no coincidence, as will be argued below.

which is given by the gross gain of finding lower prices *minus* their idiosyncratic disutility of switching, cannot recover this aggregate loss of all other consumers in the market. As a result, the Austrian policy must lead to a net loss of consumer welfare that is equal to the aggregate reallocation disutility incurred by switching t_1 consumers. As the portrayed mechanism works under general conditions, it is summarized in the following proposition.

Proposition 4. *In the regulated regime, any equilibrium in which some consumers optimally delay their purchase until the second period, but face a disutility of doing so, must be characterized by strictly lower aggregate consumer welfare, compared to the unrestricted regime.*

Proof. Details can be found in the appendix. □

Proposition 4 states that the Austrian price regulation harms consumers in any equilibrium where some of them optimally delay their purchase despite facing a disutility of doing so. This leads to the questions (i) whether such equilibria exist, (ii) how large the loss in consumers' welfare is, and (iii) what the policy's distributional effects are. I will answer these questions for a specific interpretation of the general model.

In particular, assume from now on that all consumers are *ex-ante identical* regarding the nature of their search behavior. This is reasonable if consumers can only find out *after* entering the market which type (shopper or non-shopper) they are. For example, conditional on having entered the market, consumers might learn about events that keep them from comparing prices (e.g., delays due to traffic jams; work and family obligations; running out of gasoline early) or encourage it (price information from peers, advertisements, enjoyable weather). Hence, the parameter λ is interpreted as the *probability* that each consumer assigns of being a shopper (and observing both firms' prices) after entering the market.[15] The timing is as follows. First, consumers have to decide whether to enter in $t = 1$ or $t = 2$ (but not both).[16] Second, after having done so, each consumer finds out whether they are a shopper or non-shopper, accordingly observes one or both prices, and makes their purchase decision. Thus, before entering the market,

[15] A different modeling strategy would be to allow shoppers and non-shoppers to calculate their gains of waiting independently. However, the corresponding computation is very demanding because shoppers and non-shoppers have different incentives to wait. This is because the difference between the expected *minimum* price paid by a shopper across periods is typically not equal to the difference between the expected price paid by a non-shopper across periods. Moreover, even if it was possible to derive manageable expressions for these (heterogeneous) gains from waiting across consumer groups, one would have to extend the model to variable fractions of shoppers across periods, which leads to further complications. For example, if the model is extended in that way, the resulting first period equilibrium price distribution can only be defined iteratively for parts of the parameter space, which makes the analysis intractable. Since the main qualitative finding of this section (Proposition 4) does not depend on a fixed proportion of shoppers across periods, I do not follow that route.

[16] Sequential intertemporal search is ruled out: consumers may not enter in $t = 1$, observe that period's price(s), and then decide to wait for $t = 2$.

all consumers may simply compare the expected (average) prices in the first period of the model with the expected (average) prices in the second period. This yields

Lemma 2. *The ex ante gains from waiting, given that $x \in (0,1)$ consumers purchase in $t=1$, the remaining $1-x$ consumers purchase in $t=2$ and firms employ consistent equilibrium pricing strategies, are given by*

$$\begin{aligned} S(x,\lambda) \quad &:= \quad (1-\lambda)\frac{\int_{\underline{p_G}(x)}^{v} G(p;x)^2 dp}{x} \\ &= \quad \frac{v(1-\lambda)}{(1-x)[x-(1-\lambda)]}\left((1+\lambda)\left(\frac{x(1+\lambda)}{1-\lambda}\right)^{\frac{x(1-\lambda)-(1-\lambda)}{x(1+\lambda)-(1-\lambda)}} - \lambda - x\right). \end{aligned} \tag{9}$$

For all $\lambda \in (0,1)$ and $v>0$, $S(x,\lambda)$ is positive and strictly decreasing in x.

Proof. See Appendix. □

Using Lemma 2, one can subsequently endogenize the intertemporal distribution of consumers in the example. For the following arguments, it is easier to think in terms of the number of people who wait for $t=2$. Henceforth, let $z := 1-x$ denote the number of consumers who purchase in $t=2$ and let $\hat{S}(z) := S(1-z)$ denote the expected gains of waiting, given that z consumers purchase in $t=2$ and firms price consistently.

Denote by $W(z)$ the waiting cost of the marginal waiter, i.e., the maximum waiting cost out of all waiting consumers, given that z consumers wait. As a fraction $1-\kappa$ of consumers do not suffer any disutility from purchasing in $t=2$, it follows that $W(z)=0$ for all $z \in [0,1-\kappa)$. Clearly, $W(z)$ must be (weakly) increasing for all $z \geq 1-\kappa$. As a mild regularity condition, it is imposed that $W(1-\kappa)=0$. This means that the t_1 consumer with the lowest disutility of waiting actually has a negligible waiting cost, implying that there is no discontinuous jump of the waiting cost across consumers groups.[17] Now, for a given z, a t_1 consumer will only prefer to purchase in $t=2$ if her expected reduction in prices $\hat{S}(z)$ is larger than her disutility of waiting. Thus, as long as $\hat{S}(z)$ is larger than $W(z)$, even the consumer with the highest waiting cost benefits from waiting and more consumers will want to wait. It follows that an interior equilibrium is characterized by a value z^* such that $W(z^*) = \hat{S}(z^*)$. Moreover, such an equilibrium will only be stable if $W(z)$ crosses $\hat{S}(z)$ from below at z^*. Otherwise, slightly increasing (decreasing) z would imply that the gains from search start to strictly exceed (fall short of) the waiting cost

[17] In fact, all of the following results only rely on $W(1-\kappa) < \hat{S}(1-\kappa)$. That is, if there is a discontinuous jump of the waiting cost across consumer groups, it should not be too large.

of the marginal waiter and hence, a drift away from the equilibrium point would be caused. Given these observations, one can easily deduct the following proposition. An example and some graphic intuition is provided in Figure 2.

Proposition 5. *If the consumers are ex-ante identical regarding their search behavior, and* $W(1-\kappa) = 0$*, an equilibrium exists. In this equilibrium, either all consumers purchase in* $t = 2$ *(*$z^* = 1$*), or there exists at least one stable intertemporal consumer distribution* $z^* \in (1-\kappa, 1)$*. Equilibrium total social welfare and consumer surplus is reduced by* $\int_{1-\kappa}^{z^*} W(z)dz$*, compared to the unrestricted regime.*

Proof. See Appendix. □

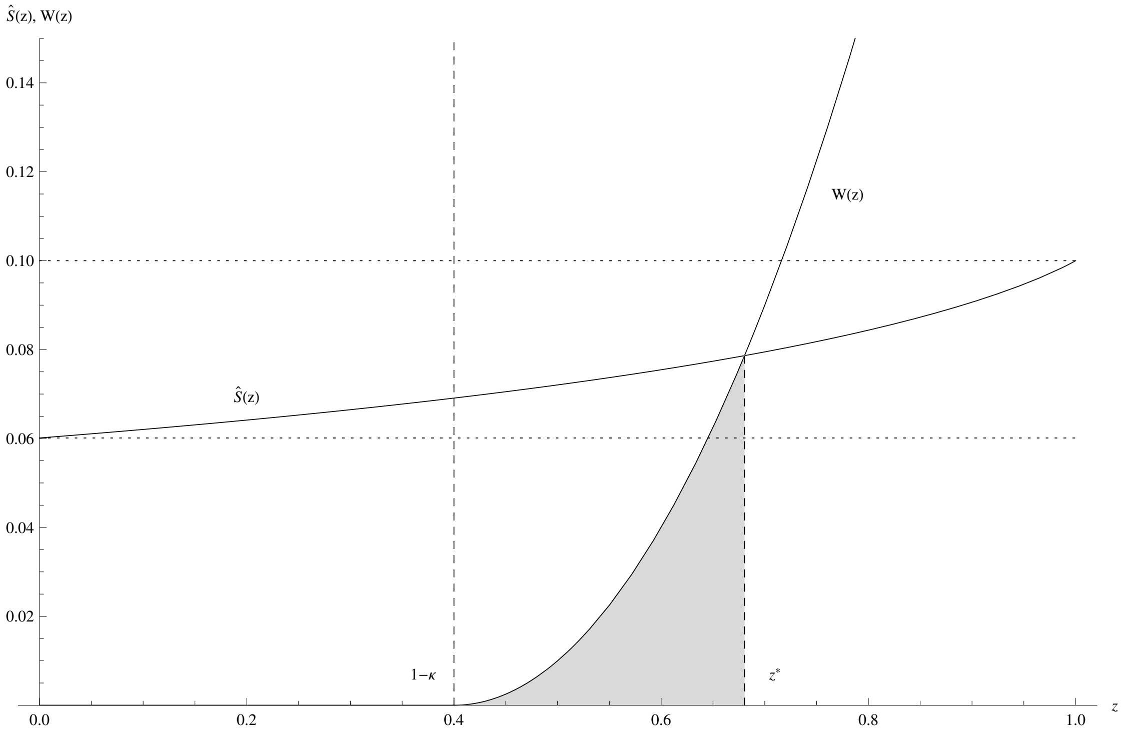

Figure 2: Determination of the equilibrium number of second period consumers z^* for $v = 1$, $\kappa = 0.6$, $\lambda = 0.1$, $W(z) = (z - 0.4)^2$ for $z \geq 0.4$. The resulting welfare loss, compared to the unrestricted regime, is given by the size of the shaded area.

Hence, in the example at least one stable equilibrium exists[18] where part of the t_1 consumers

[18]Without introducing additional assumptions, multiple stable equilibria may exist (although every stable equilibrium must be pure). This is because *both* the waiting cost of the marginal waiter and the gains from waiting are *increasing* in the number of waiters, which implies that the function $W(z)$ might intersect $\hat{S}(z)$ from below more than once. Thus, having identical fundamentals, a market might, in some circumstances, be

(who would, in the absence of any pricing distortions, strictly prefer to purchase in $t = 1$) optimally switch periods, but incur a disutility because of their waiting cost. By delaying their purchase, they exercise a negative externality on all other consumers, as both periods' expected prices strictly increase in the number of waiting consumers (see Corollary 2). The fact that consumers do not account for this negative externality, caused by firms' profit preserving equilibrium strategies, implies that the equilibrium net consumer welfare must unambiguously be lower than in the unrestricted regime.

Next, note that the aggregate welfare loss can be quite substantial. In the example where $\kappa = 0.6$ and $W(z) = (z - 0.4)^2$ for $z \geq 0.4$, one can numerically find that the unique equilibrium number of waiters is given by $z^* \approx 0.68$. Thus, the resulting welfare loss, as measured by the size of the shaded area in the Figure, can be calculated to be $\int_{0.4}^{0.68} W(z)dz = 0.28^3/3 \approx 0.0073$. This number is equal to roughly 7.3% of the total consumer surplus of $\lambda v = 0.1$ that would be generated in the unrestricted regime. This shows that the regulatory policy might even be harmful for consumers if part of firms' profits were redistributed to consumers, as long as sufficiently many of them delay their purchase.

An interesting feature of the model is that a decrease in consumers' disutility of waiting in the sense that the exogenous waiting cost curve $W(z)$ gets shifted downward might actually lead to a lower total consumer surplus. This is because there are two countervailing effects if consumers become more flexible in their choice of purchase period. First, those t_1 consumers who already found it worthwhile to wait for the second period before will cause a lower friction, as their costs of waiting are reduced. But second, in total more t_1 consumers will want to wait and incur the disutility, which increases the waiting friction. Which effect ultimately dominates depends on the shape of the new waiting cost function.

Finally, it is worthwhile to consider the distributional effects of the price regulation under an endogenous purchase timing. Recall that in the unrestricted regime, firms' pricing behavior is not distorted intertemporally: both firms choose prices according to $F_0(p)$ in either period, no matter how many consumers actually buy in it (compare with Corollary 1). The expected surplus per consumer is equal to λv in each period. Moreover, since there are no pricing distortions, each consumer clearly purchases in their preferred period.

In contrast, the first (second) period of the restricted regime has higher (lower) expected prices

characterized by both a large number of waiters and high gains from waiting or a low number of waiters and low gains from waiting. However, since both the first and second period's expected prices are strictly increasing in the number of waiters, different stable equilibria can be Pareto ranked in the sense that every consumer prefers equilibria with lower numbers of waiters. Ultimately, this suggests that consumers would not want to coordinate on equilibria in which many consumers wait.

than either period of the unrestricted regime. It follows that all consumers who, in equilibrium, keep purchasing in the first period are harmed by the restrictive policy, while all t_2 consumers (who do not suffer any disutility from purchasing in $t = 2$) are strictly better off. The *critical consumer*, who is indifferent between being in the restricted or unrestricted regime, is hence a consumer who optimally waits for the second period at a positive waiting cost. This critical consumer must have a waiting disutility W^{crit} that satisfies $v - (1-\lambda)\mathbb{E}_{x^*}(p_1) - W^{crit} = \lambda v$, where $v - (1-\lambda)\mathbb{E}_{x^*}(p_1)$ is the expected individual consumer surplus in $t = 2$, given that an equilibrium $x^* = 1 - z^*$ consumers purchase in $t = 1$.[19] This implies a critical waiting cost of

$$W^{crit} = (1-\lambda)\left[v - \mathbb{E}_{x^*}(p_1)\right]. \tag{10}$$

Because $\mathbb{E}_{x^*}(p_1)$ strictly decreases in x^* (see Corollary 2), the waiting cost of the critical consumer increases in x^*. In other words, if in equilibrium, many consumers find it optimal to keep purchasing in the first period (x^* is high), even relatively inflexible consumers benefit from the price regulation, although they have to be willing to switch periods.[20] Note moreover that, as $\mathbb{E}_{x^*}(p_1) = v$ holds only for $x^* = 0$, W^{crit} is a positive number unless everybody waits for the second period. Clearly, if *everybody* waits for the second period, the firms' equilibrium pricing in the second period is identical to the unrestricted regime's pricing (as they find it optimal to ignore the first period and charge v in $t = 1$ to maximize second period profits) and hence, the policy harms everybody except for the consumers with a zero disutility of purchasing in $t = 2$, who remain as well off as before.

The above findings are summarized in

Proposition 6. *If the consumers are ex-ante identical regarding their search behavior, all "flexible" consumers with a sufficiently low waiting cost benefit from the price regulation, while all others are harmed. If all consumers delay their purchase in equilibrium, nobody benefits from the policy, but consumers with a positive waiting cost are harmed.*

[19] The latter is true because the firms' total expected profit in $t = 2$, given that an equilibrium $1 - x^*$ consumers purchase in that period, is equal to $(1 - x^*)(1 - \lambda)$ times the maximum price that was chosen in $t = 1$ (compare with Proposition 1). It follows that the average profit per consumer is given by $(1 - \lambda)\mathbb{E}_{x^*}(p_1)$.

[20] The intuition to this is as follows. If equilibrium demand is high in the first period, many consumers are too inflexible to delay their purchase. Consequently, firms do not heavily distort their pricing upward in the first period of the game, as having a high market share in that period remains important. In turn, the second period's equilibrium pricing (which is more aggressive the more consumers purchase in the first period) will lead to significantly lower prices than in the unrestricted regime. This implies that the expected gains of consumers who purchase in $t = 2$ in the restricted regime, compared to the unrestricted regime, are high. Hence, everything else kept equal, consumers with a relatively higher waiting cost benefit from the price restriction, as long as they are flexible enough to delay their purchase.

Thus, one effect of the Austrian policy is that it harms consumers who are relatively inflexible in choosing their purchase period. Only those consumers who are significantly more flexible than others benefit. But paradoxically, in the case that everybody is flexible enough to purchase in the second period, nobody is made better off. Moreover, it is intuitive that the policy is likely to harm consumers who are unaware of its existence, since they will purchase at random points in time.

As a numerical example, take the market depicted in Figure 2, where $v = 1$, $\lambda = 0.1$, $x^* = 1 - z^* \approx 0.32$. The expected maximum price in the first period, $\int_{\underline{p_G}(x^*)}^{v} pdG(p;x^*)^2$, is then easily calculated to be 0.972. Using the above equation for W^{crit}, one finds that $W^{crit} \approx 0.025$. Hence, in this example, every consumer who has a disutility of waiting that is lower than 0.025 benefits from the restrictive policy, while all others are harmed.

4 Conclusion

This article studies the effectiveness of a real-world price regulation that was established in a homogeneous-goods consumer search market. More specifically, in January 2011, a law was passed in Austria that restricts retail gasoline stations from increasing their prices more than once per day at noon. Similar regulations have been discussed in Germany and New York State. Although the motivation for such policies is to reduce price uncertainty in the retail gasoline market, foster competition, and ultimately increase consumer surplus, a mechanism is provided that can plausibly lead to a *reduction* of consumer surplus in face of the pricing restriction.

In the basic two-period model, consumers have unit demand and their intertemporal distribution is fixed exogenously. Then, under the restrictive policy, it is shown that firms will distort their intertemporal pricing in such a way that aggregate firm profits and hence aggregate consumer surplus remains unchanged. This is because firms realize that when pricing aggressively in the first period of the price setting cycle, they effectively cut their profits in the second period by narrowing the range of admissible prices and intensifying competition. If firms' first period prices were chosen like in the unrestricted regime, profits would be reduced too much in the second stage of the model, which leads them to optimally soften price competition in the first period.

The baseline model is then extended to allow for an endogenous consumer purchase time. Surprisingly, even though prices are strictly lower in the second period of the model, this implies that the policy must be *detrimental* to consumer surplus. The reason for this is that because of the price distortions that are introduced to the market, consumers with a sufficiently low disutility of delaying their purchase will optimally do so, as they expect to find lower prices later in the cycle. But due to the firms' equilibrating strategies, this exerts an indirect negative externality of equal size on all other consumers, as firms' price competition is softened in both stages. In total, a loss for society is created because switching consumers' incurred disutility is not recovered. It is obvious that a loss for consumers would prevail even if firms' equilibrium profits where slightly reduced in favor of consumer surplus. Moreover, the adverse mechanism continues to hold for many downward sloping demand functions, as in that case, the intertemporal price distortions that are induced by the policy often exercise a negative allocative effect on consumer surplus. Another result is that the policy harms those consumers who are the least flexible or informed regarding the optimal purchase time, which seems undesirable for policymakers.

Future research might check the robustness of these results in several dimensions. For example, consumers' decision of whether to compare prices or not could be modeled explicitly. It has to be noted though that the qualitative findings of this article are very likely to be unaffected, as the Austrian regulation tends to *decrease* price dispersion and hence decrease the expected gains from search in the market. Consequently, less rather than more consumer search should be expected in equilibrium, implying that an even more pronounced loss of consumer welfare would occur. Other features worth exploring might be a sequential search protocol of consumers, or the incorporation of unexpected demand shocks. Also, an empirical analysis of high-frequency Austrian gasoline price data could help to reveal whether the predicted changes in firms' equilibrium behavior can be supported.

Ultimately, considering the generality of the portrayed mechanism, the paper casts serious doubts on whether Austrian-type price regulations can be an effective instrument for promoting consumer surplus in the retail gasoline market. Hence, the German antitrust authority's skeptical view towards adoption of similar restrictions and the rejection of a virtually identical policy by the New York State Assembly seem to be justified.

References

Simon P. Anderson and Regis Renault. Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model. *RAND Journal of Economics*, 30(4):719–735, Winter 1999.

Mark Armstrong, John Vickers, and Jidong Zhou. Consumer Protection and the Incentive to Become Informed. *Journal of the European Economic Association*, 7(2-3):399–410, 04-05 2009.

Julio Cesar Arteaga and Daniel Flores. Regulation, competition and fraud: evidence from retail gas stations in Mexico. MPRA Paper 34187, University Library of Munich, Germany, October 2010.

Bundeskartellamt. Sektoruntersuchung Kraftstoffe, Abschlussbericht Mai 2011. Technical report, Bundeskartellamt, 2011.

Kenneth Burdett and Kenneth L Judd. Equilibrium Price Dispersion. *Econometrica*, 51(4): 955–69, July 1983.

Ralf Dewenter and Ulrich Heimeshoff. Less pain at the pump? The effects of regulatory interventions in retail gasoline markets. Technical report, 2012.

Andrew Eckert. Empirical Studies of Gasoline Retailing: A Guide to the Literature. *Journal of Economic Surveys*, 2011. ISSN 1467-6419.

Chaim Fershtman and Arthur Fishman. Price Cycles and Booms: Dynamic Search Equilibrium. *American Economic Review*, 82(5):1221–33, December 1992.

Chaim Fershtman and Arthur Fishman. The 'perverse' effects of wage and price controls in search markets. *European Economic Review*, 38(5):1099–1112, May 1994.

Justus Haucap and Hans Christian Müller. The Effects of Gasoline Price Regulations: Experimental Evidence. Technical report, 2012.

Jean-Francois Houde. Gasoline Markets. In Steven N. Durlauf and Lawrence E. Blume, editors, *The New Palgrave Dictionary of Economics*. Palgrave Macmillan, Basingstoke, 2010.

Michael D. Noel. Edgeworth Price Cycles and intertemporal price discrimination. *Energy Economics*, 34(4):942–954, 2012.

Michael T. Rauh. Wage and price controls in the equilibrium sequential search model. *European Economic Review*, 48(6):1287–1300, December 2004.

Dale O. Stahl. Oligopolistic Pricing with Sequential Consumer Search. *American Economic Review*, 79(4):700–712, September 1989.

Mariano Tappata. Rockets and Feathers: Understanding Asymmetric Pricing. *RAND Journal of Economics*, 40(4):673–687, 2009.

Hal R. Varian. A Model of Sales. *American Economic Review*, 70(4):651–59, September 1980.

Zhongmin Wang. (Mixed) Strategy in Oligopoly Pricing: Evidence from Gasoline Price Cycles Before and Under a Timing Regulation. *Journal of Political Economy*, 117(6):987–1030, December 2009.

Asher Wolinsky. True Monopolistic Competition as a Result of Imperfect Information. *The Quarterly Journal of Economics*, 101(3):493–511, August 1986.

Huanxing Yang and Lixin Ye. Search with learning: understanding asymmetric price adjustments. *RAND Journal of Economics*, 39(2):547–564, 2008.

Appendix: Technical Proofs

Proof of Proposition 1. In the following, the proposition will be shown by a sequence of claims.

Claim 1. *Both firms make an equilibrium expected profit of at least* $\Pi^* := \frac{1-\lambda}{2}p_1$.

Proof. Firm 1's equilibrium profit cannot be lower than $\frac{1-\lambda}{2}p_1$, because the firm can always guarantee itself this profit by sticking to p_1. Next, consider firm 2. Clearly, firm 1 can never charge a price $p_L < \underline{p} := \frac{1-\lambda}{1+\lambda}p_1$ in equilibrium, because this is a strictly dominated strategy: even if the firm attracted the shoppers for sure at such a price, its expected profit of $\frac{1+\lambda}{2}p_L$ would fall short of $\frac{1-\lambda}{2}p_1$. But then, by elimination of strictly dominated strategies, it also follows that firm 2 cannot make an equilibrium profit that is less than $\frac{1-\lambda}{2}p_1$. This is because firm 2 can always attract the shoppers by pricing marginally below $\underline{p}$, hence firm 2 can get a guaranteed profit that is arbitrarily close to $\frac{1+\lambda}{2}\underline{p} = \frac{1-\lambda}{2}p_1$. □

Claim 2. *Firm 1 has a mass point with less than full probability at* p_1 *and makes an equilibrium expected profit of exactly* Π^*.

Proof. It has already been argued in the text that in equilibrium, firm 1 cannot charge p_1 for sure. Hence it remains to be shown why firm 1 cannot always price below p_1. Suppose to the contrary that it did. Then, as firm 1 finds pricing in (p_2, p_1) to be strictly dominated, this implies that it will charge some highest price $\hat{p} \leq p_2$ with positive density. Now one of the following must be true: either *one* of the firms has the highest price for sure if it prices at the upper boundary of its pricing support, or both firms have a mass point at the common upper support $\hat{p}$, allowing for ties. The former cannot be part of an equilibrium because such a firm makes an expected profit that is not larger than $\frac{1-\lambda}{2}p_2 < \Pi^*$, violating

Claim 1. The latter cannot be part of an equilibrium because marginally undercutting by either firm discretely increases its expected profit, as it avoids ties. Henceforth, in equilibrium it cannot be the case that firm 1 has no mass point at p_1. Thus, as firm 1 must choose p_1 with positive mass in equilibrium, every price in its pricing range must yield the same expected profit as pricing at p_1, i.e, Π^*. □

Claim 3. *Firm 2's upper pricing bound is* p_2.

Proof. Suppose the contrary, i.e., $\overline{p_2} < p_2$. But then, in those cases where firm 1 does not stick to p_1 (which it has to do with positive probability because of Claim 2), it would never want to put any probability mass at or above $\overline{p_2}$, since the expected profit of doing so would either be less than Π^* (if firm 2 has no mass point on $\overline{p_2}$) or could be increased discretely (by slightly undercutting a mass point of firm 2 at $\overline{p_2}$). In turn, this implies that firm 2 should not sample prices close to $\overline{p_2}$ at all, since it could earn a higher profit by charging $p_2 > \overline{p_2}$ instead. This is a contradiction. □

Claim 4. *Besides of its mass point at* p_1*, firm 1's upper pricing bound is* p_2.

Proof. Suppose the contrary. Then, as firm 1 finds pricing in (p_2, p_1) to be strictly dominated, firm 1 must have an upper bound $\overline{p_1} < p_2$ if it does not sample its mass point. In turn, firm 2 should never put any probability mass in the interval $[\overline{p_1}, p_2)$. This is because pricing in that range can only win the informed consumers if either firm 1 has a mass point at $\overline{p_1}$ (but then, undercutting that price would pay for firm 2), or if firm 1 sticks to its mass point at p_1. But in the latter case, firm 2 could attain a higher expected profit by transferring probability mass to p_2. The fact that firm 2 will avoid pricing in $[\overline{p_1}, p_2)$ implies that pricing close to $\overline{p_1}$ cannot be optimal in the first place for firm 1. This is because when doing so, firm 1 can only attract the shoppers if firm 2 charges p_2. However, the same would be true for a slightly higher price $\overline{p_1}' > \overline{p_1}$, resulting in higher expected profits. Ultimately it must thus be true that firm 1's upper bound, besides of its mass point at p_1, is p_2. □

Claim 5. *Firm 2 has a mass point at* p_2*, but firm 1 has not.*

Proof. If the former was not the case, firm 1 would make a profit close to $\frac{1-\lambda}{2}p_2$ when it charges a price that is close to p_2 (which is has to do because of Claim 4), which cannot be optimal since this is lower than Π^*. Hence firm 2 must have a mass point at p_2. It easily follows that firm 1 can have no mass point at p_2, since slightly undercutting would pay as it avoids ties. □

Claim 6. *Both firms' lowest price charged with positive density is* $\underline{p}$*, and it is charged with zero mass. Firm 2 makes an equilibrium expected profit of exactly* Π^*.

Proof. First, note that firm 2's lower support boundary must be $\underline{p}$. If it was some $p_L > \underline{p}$, firm 1 could profitably undercut and make an equilibrium expected profit that is strictly higher than Π^*, which is ruled out by Claim 2. Moreover, $p_L < \underline{p}$ cannot be part of an equilibrium as slightly increasing p_L would pay. Thus, it holds that $\underline{p_2} = \underline{p}$. Also, firm 2 cannot choose $\underline{p}$ with positive mass. If it did, firm 1 would never charge $\underline{p}$ with positive density, as the chance of not having the lowest price would give firm 1 an expected profit that is less than Π^*. Hence, firm 2 could profitably increase its lower support slightly. As the same holds vice versa (if firm 1 charged $\underline{p}$ with positive mass, firm 2's expected profit at $\underline{p}$ would be less than Π^*), firm 1 will charge $\underline{p}$ with zero mass (if at all). Because firm 2 charges $\underline{p}$ with positive density, and doing so guarantees attracting the shoppers as firm 1 prices at $\underline{p}$ with zero mass (if at all), it follows that also firm 2's expected profit is given by $\frac{1+\lambda}{2}\underline{p} = \Pi^*$ for every price it charges with positive density. This in turn implies that also firm 1's lower support boundary must satisfy $\underline{p_1} = \underline{p}$. If it did not, firm 2 could achieve a higher equilibrium profit than Π^* by marginally undercutting $\underline{p_1}$, which is in conflict with the above finding. ☐

Claim 7. *If a firm puts probability mass in some interval $[p_A, p_B] \subset [\underline{p}, p_2]$, the other firm must do so as well.*

Proof. Suppose to the contrary that one of the firms, say firm i, doesn't charge prices in some non-empty interval $[p_A, p_B] \subset [\underline{p}, p_2]$, but firm j does so with positive probability. But by pricing in that interval, firm j will only attract the shoppers in the event that firm i prices above p_B. It follows that firm j could increase its profit for all prices that are strictly less than p_B by pricing at p_B instead. Therefore, this cannot be the case. ☐

Claim 8. *Both firms cannot have a hole in their pricing range on some common interval $(p_A, p_B) \subset [\underline{p}, p_2]$ (where p_A and p_B are chosen with positive density by both firms).*

Proof. If there is a hole, it must be common to both firms because of Claim 7. Then, note first that it cannot be the case that one of the firms has a mass point at p_A (clearly, both firms cannot have a mass point at p_A because slightly undercutting would pay). If this was true, this firm would only attract the shoppers in the event that the other firm priced at or above p_B, hence it could increase its profit by charging a price closer to p_B instead. Thus both firms must charge p_A with positive density, but zero mass. But then, pricing at p_A is effectively the same as pricing marginally below p_B, as the shoppers are only attracted if the other firm prices at or above p_B. Hence, choosing p_A with positive density cannot be optimal, which is a contradiction. ☐

Claim 9. *Both firms cannot have a mass point below p_2.*

Proof. Clearly, both firms cannot have a mass point on some common price because slightly undercutting would pay. Suppose that just one firm, say firm i, has a mass point on $p_A < p_2$, and this mass point is unique on some non-empty interval $[p_A, p_B]$, where also $p_B < p_2$ (this must the case, as a firm can at most have countably many mass points). Then it cannot be optimal for firm j to charge p_A with positive density, as slightly undercutting would discretely increase its expected profit. Moreover, by continuity, the same argument applies for some (small) range $[p_A, p_C]$, with $p_C < p_B$. In turn, as firm j would not price in that range, it cannot be optimal for firm i to have a mass point on p_A at all, as pricing closer to p_C would give a strictly higher expected profit. Therefore, no firm can have a mass point on some price that is strictly less than p_2. $\square$

All of these observations together imply that (a) firm 1 must have a mass point with mass $\alpha \in (0,1)$ on p_1 (b) firm 2 must have a mass point with mass $\beta \in (0,1)$ on p_2 (c) if they don't price at their mass points, both firms must choose prices according to continuous and strictly increasing distributions $F_1(p)$ and $F_2(p)$ on the interval $[\underline{p}, p_2)$. From now, consider equilibria of this type.

Claim 10. *Firm 2 must choose its mass point at p_2 with probability $\beta := \frac{1-\lambda}{2\lambda}\left(\frac{p_1}{p_2} - 1\right) \in (0,1)$. With probability $1-\beta$, it chooses prices according to the continuous distribution function*

$$F_2(p) = \frac{1 + \lambda - (1-\lambda)\frac{p_1}{p}}{1 + \lambda - (1-\lambda)\frac{p_1}{p_2}}.$$

Proof. Clearly, firm 1 will only play the proposed mixed strategy if it gets exactly the same expected profit for every price it charges with positive density (or probability mass). In particular, if firm 1 prices at p_1, it makes a (deterministic) profit of $\frac{1-\lambda}{2}p_1$, while if it marginally undercuts p_2, it makes an expected profit of $\beta\frac{1+\lambda}{2}p_2 + (1-\beta)\frac{1-\lambda}{2}p_2$. Setting $\frac{1-\lambda}{2}p_1 = \beta\frac{1+\lambda}{2}p_2 + (1-\beta)\frac{1-\lambda}{2}p_2$, one can solve for β:

$$\beta = \frac{1-\lambda}{2\lambda}\left(\frac{p_1}{p_2} - 1\right) \in (0,1),$$

where $\beta < 1$ follows directly from inequality (1).

Suppose now that firm 1 charges some price p that is contained in the support of its pricing distribution $F_1(p)$. Its expected profit is then given by

$$\mathbb{E}\Pi_1(p;\beta,F_2(.)) = \beta\frac{1+\lambda}{2}p + (1-\beta)\left[(1-F_2(p))\frac{1+\lambda}{2}p + F_2(p)\frac{1-\lambda}{2}p\right].$$

The above equation holds because if firm 1 charges p, three scenarios can occur. With probability β, firm 2 chooses its mass point at p_2 and firm 1 attracts the shoppers for certain. With probability $1-\beta$, firm 2 charges p according to $F_2(p)$ and hence has a higher (lower) price than p with probability $1-F_2(p)$ $(F_2(p))$, which are the other two scenarios. Setting $\mathbb{E}\Pi_1(p;\beta,F_2(.))$ equal to $\frac{1-\lambda}{2}p_1$, one finds that

$$F_2(p) = \frac{1 + \lambda - (1-\lambda)\frac{p_1}{p}}{1 + \lambda - (1-\lambda)\frac{p_1}{p_2}}.$$

$\square$

Claim 11. *Firm 1 must choose its mass point at p_1 with probability $\alpha := \beta = \frac{1-\lambda}{2\lambda}\left(\frac{p_1}{p_2} - 1\right) \in (0,1)$.*

With probability $1-\alpha$, it chooses prices according to the continuous distribution function

$$F_1(p) := F_2(p) = \frac{1+\lambda-(1-\lambda)\frac{p_1}{p}}{1+\lambda-(1-\lambda)\frac{p_1}{p_2}}.$$

Proof. In order for the proposed strategy to be part of an equilibrium, firm 2 has to be indifferent between choosing any price in its support. If firm 2 charges the highest price p_2, its expected payoff is given by

$$\mathbb{E}\Pi_2(p_2;\alpha,F_1(.)) = \alpha\frac{1+\lambda}{2}p_2 + (1-\alpha)\frac{1-\lambda}{2}p_2.$$

If firm 2 charges the lowest price in its support $\underline{p}$, as shown by Claim 6, it will attract the shoppers with certainty (again, due to Claim 6) and make a (deterministic) payoff of $\frac{1+\lambda}{2}\underline{p_2} = \frac{1-\lambda}{2}p_1$. Setting $\frac{1-\lambda}{2}p_1 = \mathbb{E}\Pi_2(p_2;\alpha,F_1(.))$, one can readily observe that the same condition is imposed on α that was already imposed on β (see Claim 10). Hence, in the hypothesized equilibrium, it must hold that

$$\alpha = \beta = \frac{1-\lambda}{2\lambda}\left(\frac{p_1}{p_2}-1\right) \in (0,1).$$

If firm 2 charges any price in the support of $F_2(p)$, its expected profit is given by

$$\mathbb{E}\Pi_2(p;\alpha,F_1(p)) = \alpha\frac{1+\lambda}{2}p + (1-\alpha)\left[(1-F_1(p))\frac{1+\lambda}{2}p + F_1(p)\frac{1-\lambda}{2}p\right].$$

The intuition for this is the same as for firm 1: if firm 2 prices at some arbitrary p, it attracts the informed consumers either if firm 1 chooses its high price mass point or if firm 1 randomizes according to $F_1(p)$, but draws a higher price than p.

Finally, setting the above equation equal to $\Pi_2(\underline{p_2};\alpha,F_1(p)) = \mathbb{E}\Pi_2(p_2;\alpha,F_1(p)) = \frac{1-\lambda}{2}p_1$, one can see that this condition coincides with the condition that was imposed on $F_2(p)$ before. Hence,

$$F_1(p) = F_2(p) := F(p) = \frac{1+\lambda-(1-\lambda)\frac{p_1}{p}}{1+\lambda-(1-\lambda)\frac{p_1}{p_2}}.$$

□

This finalizes the proof of Proposition 1. □

Proof of Proposition 2. Differentiating the identity

$$G(p)p\left[1-\lambda-\kappa(1+\lambda)\right] + \kappa(1+\lambda)p + (1-\kappa)(1-\lambda)\int_p^v xG'(x)dx - (1-\lambda)v = 0$$

on both sides with respect to p leads to a differential equation that can be solved analytically. However, it is easy to see that the condition is fulfilled if one sets $G(p) = A + Bp^C$ and solves for the unknowns A, B and C. Doing so and also calculating $\underline{p_G}$ as the solution to $G(p) = 0$ yields to the expressions in the proposition.

Now, by construction, the strategy combination $(G(p), G(p))$ is a mutual best response, as every price in each firm's pricing support yields the same (maximum) expected profit for the whole game, given the strategy of the other firm. However, in order for Proposition 2 to be true, it must be the case that the pure strategy equilibrium of the second stage subgame can never be played following the firms' first stage equilibrium actions, as this would be characterized by different equilibrium payoffs. It is thus required that $\frac{\overline{p_G}}{\underline{p_G}} \leq \frac{1+\lambda}{1-\lambda}$ for all pairs (κ, λ), with $\kappa \in (0,1)$, $\lambda \in (0,1)$.[21] In order to prove this, one can first show that the LHS of the inequality $\frac{\overline{p_G}}{\underline{p_G}} \leq \frac{1+\lambda}{1-\lambda}$, which is given by $LHS := \left(\frac{\kappa(1+\lambda)}{1-\lambda}\right)^{\frac{2\lambda\kappa}{\kappa(1+\lambda)-(1-\lambda)}}$, is non-decreasing in κ. This is true because $\frac{\partial LHS}{\partial \kappa}$ has the same sign as

$$\kappa(1+\lambda) - (1-\lambda) - (1-\lambda)\ Log\left(\frac{\kappa(1+\lambda)}{1-\lambda}\right),$$

which can be dealt with in two separate cases. First, suppose that $\kappa(1+\lambda) - (1-\lambda) \geq 0$. Then, *keeping everything else fixed*, the above expression is clearly decreasing in the value of the logarithm, as the latter is nonnegative. Hence it follows that

$$\kappa(1+\lambda) - (1-\lambda) - (1-\lambda)\ Log\left(\frac{\kappa(1+\lambda)}{1-\lambda}\right) \geq \kappa(1+\lambda) - (1-\lambda) - (1-\lambda)\ \left[\frac{\kappa(1+\lambda)}{1-\lambda} - 1\right] = 0,$$

because it holds in general that $Log(z) \leq z - 1$ for every $z > 0$. Second, consider the case where $\kappa(1+\lambda) - (1-\lambda) < 0$. Then, *keeping everything else fixed*, the expression is again clearly decreasing in the value of the logarithm. This is true because for a negative logarithm, the value of the whole expression is lowest if the value of the logarithm is only slightly below zero, i.e., if it is as high as possible. Thus, the same argument as for $\kappa(1+\lambda) - (1-\lambda) \geq 0$ applies, which confirms that $\frac{\partial LHS}{\partial \kappa} \geq 0$.

To finalize the proof, it is sufficient to show that the inequality $LHS(\kappa) \leq \frac{1+\lambda}{1-\lambda}$ is satisfied for $\kappa \to 1$, which is trivially the case. □

Proof of Proposition 4. For a fixed proportion of shoppers across periods, as assumed throughout Section 2, the statement is obvious. It remains to be shown that Proposition 4 does not depend on a fixed proportion of shoppers across periods. To see this, note that under the regulation, for *any* (equilibrium) intertemporal distribution of shoppers and non-shoppers, a firm can always get a guaranteed profit that is not lower than in the unrestricted regime, as long as the total mass of shoppers stays the same as in the unrestricted regime. Namely, let $l_1 \leq \kappa$ and $l_2 \leq 1-\kappa$ be the equilibrium mass of shoppers that purchase in the first and second period, respectively. Moreover, let $l_1 + l_2 = \lambda$. Then the effective proportions of shoppers are given by $\lambda_1 := \frac{l_1}{\kappa}$ for the first period and $\lambda_2 := \frac{l_2}{1-\kappa}$ for the second. Hence, if a firm prices at v in both periods, even if it never attracts the shoppers, it makes an equilibrium profit

[21] If $\frac{\overline{p_G}}{\underline{p_G}} = \frac{1+\lambda}{1-\lambda}$, the pure strategy equilibrium will not be played with positive probability (as firms' equilibrium pricing involves no mass points), hence using the weak inequality sign is justified. Moreover, even in the case that $p_1 = v$ and $p_2 = \frac{1-\lambda}{1+\lambda}v$ and the pure strategy equilibrium is played in $t = 2$, the equilibrium payoffs used for calculating the equilibrium $t = 1$ actions remain valid.

that is not lower than

$$\begin{aligned}\underline{\Pi} := \underline{\Pi_i}(v,v) &= \kappa\frac{1-\lambda_1}{2}v + (1-\kappa)\frac{1-\lambda_2}{2}v \\ &= \kappa\frac{1-\frac{l_1}{\kappa}}{2}v + (1-\kappa)\frac{1-\frac{l_2}{1-\kappa}}{2}v = \frac{1-\lambda}{2}v,\end{aligned}$$

which is equal to each firm's expected total profit in the unrestricted regime. Hence, for any equilibrium in which some t_1 consumers switch periods (relative to the unregulated regime), firms profits are (at least) as high as before the regulation, but inefficient consumer switching takes place. $\square$

Proof of Lemma 1. In order to prove the above statement, one has to verify that $G(p;\kappa) \le G(p;\kappa')$ for all $p \in [\underline{p_G}, v]$. For this, it is sufficient to show that $\frac{\partial G(p)}{\partial \kappa}$ is nonnegative for all $p \in [\underline{p_G}, v]$. Note that

$$\frac{\partial G(p)}{\partial \kappa} = \frac{1}{[\kappa(1+\lambda)-(1-\lambda)]^2}\left\{(1+\lambda)\left[(1-\lambda)\left(\frac{v}{p}\right)^{\frac{\kappa(1+\lambda)-(1-\lambda)}{2\kappa\lambda}} - \kappa(1+\lambda)\right] + \right.$$
$$\left.[\kappa(1+\lambda)-(1-\lambda)]\left(1+\lambda-\frac{(1-\lambda)^2\left(\frac{v}{p}\right)^{\frac{\kappa(1+\lambda)-(1-\lambda)}{2\kappa\lambda}} Log(\frac{v}{p})}{2\kappa^2\lambda}\right)\right\},$$

which has the same sign as

$$h(p) := \left(\frac{v}{p}\right)^{\frac{\kappa(1+\lambda)-(1-\lambda)}{2\kappa\lambda}}\left\{1+\lambda-\frac{[\kappa(1+\lambda)-(1-\lambda)](1-\lambda)}{2\kappa^2\lambda}Log\left(\frac{v}{p}\right)\right\} - (1+\lambda).$$

One needs to show that $h(p)$ is nonnegative for all $p \in [\underline{p_G}, v]$. Taking its derivative with respect to p, it is easy to see that it must have the same sign as

$$(1-\lambda)\,Log\left(\frac{v}{p}\right) - 2\kappa\lambda,$$

which implies that the function has a *unique* extremal point at $\hat{p} := ve^{-\frac{2\kappa\lambda}{1-\lambda}}$. After some calculation, one can show that

$$\left.\frac{\partial^2 h(p)}{\partial p^2}\right|_{p=\hat{p}} = -\frac{e^{\frac{\kappa(1+5\lambda)-(1-\lambda)}{1-\lambda}}(1-\lambda)\left[\kappa(1+\lambda)-(1-\lambda)\right]^2}{4\kappa^3\lambda^2v^2} < 0,$$

hence, $\hat{p}$ is a (local) maximum of $h(p)$ and no local minimum exists. In particular, it follows that $h(p)$ attains its minimum over the relevant range $[\underline{p_G}, v]$ either at $\underline{p_G}$ or at v. In order to prove that $\frac{\partial G(p)}{\partial \kappa}$ is nonnegative, it thus suffices to show that both $h(\underline{p_G})$ and $h(v)$ are nonnegative. It can easily be seen that $h(v) = 0$, so the second is satisfied. Moreover,

$$h(\underline{p_G}) = \frac{\kappa(1+\lambda)}{1-\lambda}\left\{1+\lambda-\frac{1-\lambda}{\kappa}Log\left(\frac{\kappa(1+\lambda)}{1-\lambda}\right)\right\} - (1+\lambda).$$

Now, there are two cases. First, suppose that $\kappa(1+\lambda) \geq (1-\lambda)$, which implies that the value of the logarithm is nonnegative. Hence, keeping everything else fixed, the expression decreases in the value of the logarithm. Since $Log(z) \leq z-1$ holds for general $z>0$, at worst the expression attains a value that is not lower than

$$\frac{\kappa(1+\lambda)}{1-\lambda}\left\{1+\lambda-\frac{1-\lambda}{\kappa}\left(\frac{\kappa(1+\lambda)}{1-\lambda}-1\right)\right\}-(1+\lambda)=0.$$

Second, suppose that $\kappa(1+\lambda) < (1-\lambda)$. Then, the logarithm is strictly negative, which implies that the expression is *also* strictly decreasing in the value of the logarithm, since the positive term $-\frac{1-\lambda}{\kappa}Log\left(\frac{\kappa(1+\lambda)}{1-\lambda}\right)$ is smallest for a value of the logarithm that is only slightly below zero. Hence, the same argument as for $\kappa(1+\lambda) > 1-\lambda$ applies, which shows that also $h(\underline{p_G}) \geq 0$. Overall, this proves the statement. □

Proof of Corollary 3. The statement for $\kappa \to 1$ is immediate. For $\kappa \to 0$, note that

$$\lim_{\kappa\to 0}\frac{\kappa(1+\lambda)-(1-\lambda)\left(\frac{v}{p}\right)^{\frac{\kappa(1+\lambda)-(1-\lambda)}{2\kappa\lambda}}}{\kappa(1+\lambda)-(1-\lambda)}=\lim_{\kappa\to 0}\left(\frac{v}{p}\right)^{\frac{-(1-\lambda)}{2\kappa\lambda}}.$$

The exponent thus approaches $-\infty$, implying that for every $p<v$, $G(p)=0$, and $G(p)=1$ only for $p=v$. Finally, it holds that

$$\lim_{\kappa\to 0}\underline{p_G}=\lim_{\kappa\to 0}v\left(\frac{1-\lambda}{\kappa(1+\lambda)}\right)^{\frac{2\lambda\kappa}{\kappa(1+\lambda)-(1-\lambda)}}=\lim_{\kappa\to 0}v\left(\kappa^{\kappa}\right)^{\frac{2\lambda}{1-\lambda}}=\lim_{\kappa\to 0}v\left(e^{\frac{Log(\kappa)}{\frac{1}{\kappa}}}\right)^{\frac{2\lambda}{1-\lambda}},$$

which, after applying de L'Hôpital's rule once, collapses to v. □

Proof of Corollary 4. For the first part, one has to show that $G(p;\kappa) \leq F_0(p)$ for every $\kappa \in [0,1)$ and $p \in [\underline{p_G}, v]$. As it has been proven above that $G(p;\kappa)$ is weakly increasing in κ, it suffices to argue that $\lim_{\kappa\to 1} G(p;\kappa) \leq F_0(p)$ over the relevant range. As Corollary 3 shows that $\lim_{\kappa\to 1} G(p;\kappa) = F_0(p)$, this is clearly satisfied. The statement for the expected prices in the first period of the restricted regime then immediately follows from first order stochastic dominance of $G(p)$ relative to $F_0(p)$. The statement for the expected prices in the second period follows from the result that the total expected firm profits in any reachable subgame of the restricted regime are given by $(1-\kappa)(1-\lambda)p_1$ (see Proposition 2), which for any maximum price p_1 that is drawn in $t=1$, must always be lower than $(1-\kappa)(1-\lambda)v$, i.e., the total expected firm profit in each stage of the unrestricted regime (see Corollary 1). □

Proof of Lemma 2. From Proposition 1 it can be inferred that given a maximum price p_1 that is drawn in period one and a fraction x of $t=1$ consumers, *each* firm makes an expected second period profit of $(1-x)\frac{1-\lambda}{2}p_1$ in period two. Hence, the unconditional expected total industry profit in $t=2$ is given by

$$\mathbb{E}(\Pi_2^{tot}) := \int_{\underline{p_G}}^{v}(1-x)(1-\lambda)pdG(p)^2,$$

where $G(p)^2$ is the distribution function of the maximum of two independent draws out of $G(p)$. As a total surplus of $(1-x)v$ is generated in period two, the expected total consumer surplus in this period is given by $\mathbb{E}(CS_2) = (1-x)v - \mathbb{E}(\Pi_2^{tot})$. Moreover, it has been calculated above that the total expected consumer surplus for the full game is given by λv. Hence, the total expected consumer surplus in the first period must be given by $\mathbb{E}(CS_1) = \lambda v - \mathbb{E}(CS_2) = \lambda v - (1-x)v + \mathbb{E}(\Pi_2^{tot})$. Finally, the difference in expected consumer surplus for an individual consumer is given by the difference of the average consumer surplus in $t=1$ and $t=2$, that is, $\frac{\mathbb{E}(CS_2)}{1-x} - \frac{\mathbb{E}(CS_1)}{x}$. Now,

$$\begin{aligned}
\frac{\mathbb{E}(CS_2)}{1-x} - \frac{\mathbb{E}(CS_1)}{x} &= v - (1-\lambda)\int_{\underline{p_G}}^{v} p dG(p)^2 - \left[\frac{\lambda v}{x} - \frac{(1-x)v}{x} + \frac{(1-x)(1-\lambda)}{x}\int_{\underline{p_G}}^{v} p dG(p)^2\right] \\
&= \frac{v(1-\lambda)}{x} - \frac{(1-\lambda)\int_{\underline{p_G}}^{v} p dG(p)^2}{x} \\
&= (1-\lambda)\frac{\int_{\underline{p_G}(x)}^{v} G(p;x)^2 dp}{x},
\end{aligned}$$

where the last equality follows from integration by parts.

Next, it is straightforward to establish that

$$\int G(p;x)^2 dp =$$

$$\frac{1}{[x(1+\lambda)-(1-\lambda)]^2}\left((1+\lambda)^2 p x^2 - \frac{4\lambda(1+\lambda)v\left(\frac{v}{p}\right)^{\frac{x(1-\lambda)-(1-\lambda)}{2\lambda x}} x^2}{1-x} - \frac{(1-\lambda)^2\lambda v\left(\frac{v}{p}\right)^{\frac{x-(1-\lambda)}{\lambda x}} x}{x-(1-\lambda)}\right).$$

Inserting the integral boundaries and simplifying leads to the expression in the lemma.

Now clearly, any $S(x)$ must be positive because $S(x) = (1-\lambda)\frac{\int_{\underline{p_G}(x)}^{v} G(p;x)^2 dp}{x} > 0$. Proving monotonicity of $S(x)$ is difficult because of its complicated functional form that gives rise to an unhandy derivative. I have to resort to numerical methods to show that the derivative of $S(x)$ is strictly negative for all $x \in (0,1)$ and $\lambda \in (0,1)$. Figure 4 depicts the partial derivative of $-S(x,\lambda)$ with respect to x (light gray) over the relevant range and shows that it always stays above the black $f(x,\lambda) = 0$ surface.

□

Proof of Proposition 5. As $W(1-\kappa) = 0 < \hat{S}(1-\kappa)$, it cannot be an equilibrium that none of the t_1 consumers switches periods, as the gains of waiting for the consumer with the lowest disutility of waiting strictly outweigh her waiting cost. Moreover, it follows from Lemma 2 that $\hat{S}(z) = S(1-z)$ is strictly increasing in z. At the same time, $W(z)$ is a weakly increasing function of z. Hence there are two possibilities. First, if $W(z)$ stays strictly below $\hat{S}(z)$ for all $z \in [1-\kappa, 1]$, all t_1 consumers delay their purchase until $t=2$, as even the consumer with the highest disutility of waiting finds it worthwhile to do so. Second, if $W(z)$ surpasses $\hat{S}(z)$ in at least one point, the *first* of such points (say z^*) must

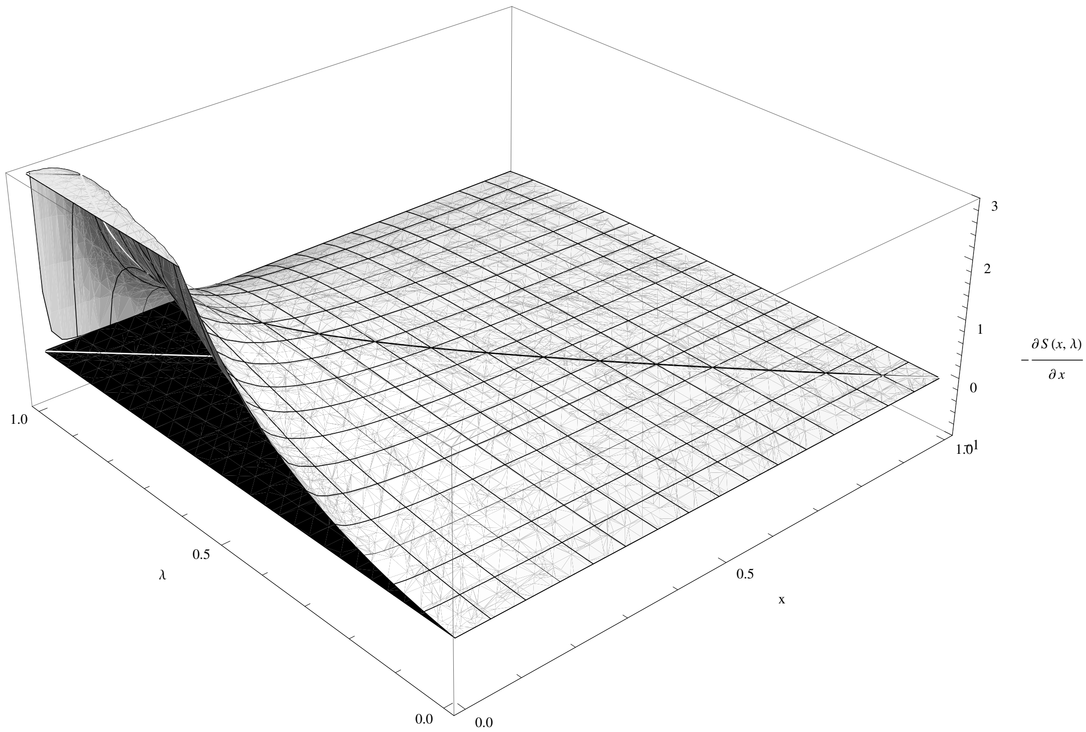

Figure 3: Numerical proof that $S(x, \lambda)$ is decreasing in x.

either be in a strictly increasing portion of $W(z)$ (as $W(z)$ starts from below $\hat{S}(z)$, and $\hat{S}(z)$ is strictly increasing), or there must be a discontinuous jump of $W(z)$ at z^* such that $W(z)$ suddenly surpasses $\hat{S}(z)$. In any case, $W(z)$ surpasses $\hat{S}(z)$ from below at z^*. This implies that z^* is stable in the sense that every consumer with a disutility of waiting that is higher (lower) than $W(z^*)$ finds it worthwhile to purchase in $t = 1$ (wait for $t = 2$), and that small perturbations of z around z^* lead to a convergence back to equilibrium if consumers optimally adapt their behavior. Moreover, since either $W'(z^*) > 0$ or there is a discontinuous jump at z^*, (at most) a negligible zero mass of consumers is indifferent in which period to buy.

Regarding total social welfare and consumer surplus, one can see the following. As in the game with an exogenous intertemporal distribution of consumers, expected firm profits are unaffected at $\mathbb{E}(\Pi^{tot}) = (1 - \lambda)v$ and also consumers' total expected *gross* surplus is given by λv (compare with Proposition 3). However, since $z^* > 1 - \kappa$, at least some t_1 consumers with a positive disutility of waiting will delay their purchase. The total friction that is introduced by this is given by $\int_{1-\kappa}^{z^*} W(z)dz > 0$. This leads to a decrease in expected *net* consumer surplus and hence, expected total social welfare. $\square$