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Guo, Xu and Lam, Kin and Wong, Wing-Keung and Zhu, Lixing

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Xu Guo *
Department of Mathematics
Hong Kong Baptist University, Hong Kong

Kin Lam
Department of Finance & Decision Sciences
Hong Kong Baptist University, Hong Kong

Wing-Keung Wong†
Department of Economics
Hong Kong Baptist University, Hong Kong

Lixing Zhu
Department of Mathematics
Hong Kong Baptist University, Hong Kong

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†*Correspondence:* Wing-Keung Wong, Department of Economics, Hong Kong Baptist University, WLB, Shaw Campus, Kowloon Tong, Hong Kong, Tel: (852) 3411-7542, Fax: (852) 3411-5580, e-mail: awong@hkbu.edu.hk

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Abstract

In this paper, we introduce a new pseudo-Bayesian model to incorporate the impact of a financial crisis and establish some properties of stock returns and investors' behavior during a financial crisis and subsequent recovery. Our approach provides a quantitative description for investors' representative and conservative heuristics by assuming that the earnings shock of an asset follows a modified random walk model to incorporate the impact of a financial crisis on the earnings of a firm. By using this model setup, we first establish some properties on the expected earnings shock and its volatility. Thereafter, we derive some properties of investors' behavior on the stock price and its volatility during a financial crisis and subsequent recovery. Last, we develop properties to explain some market anomalies, including short-term underreaction, long-term overreaction, and excess volatility during a financial crisis and subsequent recovery.

KEYWORDS: Bayesian model; Representative and conservative heuristics; Underreaction; Overreaction; Stock price; Stock return; financial crisis.

1 Introduction

Explaining market anomalies such as market excess volatility, overreaction, and underreaction is one of the most important issues in finance. Classical theorists of market rationality (e.g., Fama and French 1996) hypothesize that overreaction and underreaction can be explained by the efficient market paradigm. On the other hand, behaviorial economists (e.g., Barberis, Shleifer, and Vishny, 1998) combine psychological phenomena with finance theories to explain market anomalies such as the overreaction and underreaction phenomena.

There are five ingredients in the traditional asset-pricing model: (1) an economic structure for asset price dynamics, (2) rational agents' beliefs on asset prices, (3) the structure of market information, (4) rational agents' predictions by updating their views based on available information, and (5) rational agents' investment decisions. Most behavioral models use the bounded rationalism approach, which could violate some assumptions under rational expectations in traditional asset-pricing theory.

These assumptions include (1) the agents' knowledge of the economic structure of asset price dynamics is correct, (2) agents can process the homogeneously distributed information immediately, (3) investors update their beliefs using the Bayes rule, and (4) investors' choices are determined by Savage's notion of subjective expected utility.

Violating the assumptions imposed in the rationality paradigm due to behavioral biases could result in deviating from the traditional asset-pricing theory in a typical behavioral model. For example, Barberis, Shleifer, and Vishny (1998) assume that investors adopt conservative and representative heuristics and believe that the earnings announcements follow a random walk and fall into either a trending regime or a mean-reverting regime; this leads to both short-term underreaction and long-term overreaction in the market. Daniel, Hirshleifer, and Subrahmanyam (1998) document that the market exhibits short-term underreaction and long-term overreaction, since some investors who have private information are overconfident. On the other hand, Gervais and Odean (2001) argue that insider traders assign excessive weights to their past successful predictions of a security's dividend via a learning bias factor and use an updated probability that is larger than that derived from the Bayes rule. In addition, Grinblatt and Han (2005) believe that investors refuse to sell in a falling market because they are unwilling to admit their mistakes.

It has been observed that people are too conservative and slow in changing their prior beliefs when new information emerges. For example, Edwards (1968) argues that conservative investors might pay little attention, or even no attention, to the recent earnings announcements and still hold their prior beliefs based on past earnings in their valuation of shares. He formally establishes a Bayesian model that put less weigh on useful statistical evidence and put more weight on investors' priors. On the other hand, representative heuristics, the bias by which individuals expect main population parameters to be "represented" in recent data, have been used in many experimental studies. For example, Kahneman and Tversky (1973) find that a person following this heuristic evaluates the probability of an uncertain event by the degree to which the essential properties resemble its parent population and reflect the salient features of the process by which it is generated. To handle the issue, Tversky and Kahneman (1971) suggest that local representativeness is a belief in the "law of small numbers," and investors may find that even small samples are highly representative of the populations.

Griffin and Tversky (1992) combine conservatism with representativeness by assuming that people update their beliefs based on the "strength" (the salient and extreme aspects of the evidence) and "weight" (the statistical information, such as sample sizes). In this setup, when revising their forecasts, people overemphasize the strength of the evidence and de-emphasize its weight. Conservatism would follow when facing evidence with high weight but low strength, whereas overreaction occurs in a manner consistent with representativeness when the evidence has high strength but a low weight. Furthermore, Shefrin and Statman (1995) find that investors rely on representative heuristics in forming expectations because they tend to regard good stocks as the stocks of large companies with low book-to-market ratios. Barberis, Shleifer and Vishny (1998, henceforth BSV) extend their work by developing a Bayesian model to explain investors' behavioral biases by using both conservatism and representativeness heuristics in making decisions. Lam, Liu, and Wong (2010, 2012, henceforth LLW) further extend the model to assign weights by using a pseudo-Bayesian approach that reflects investors' behavioral biases.

In this paper, we extend the theory to study the impact of investors' behavior on stock price and its volatility before and during a financial crisis and subsequent recovery. We modify LLW's pseudo-Bayesian approach to provide a quantitative description of investors' representative and conservative heuristics by assuming that the earnings shock of an asset follows a modified random walk model to incorporate the impact of a financial crisis on the earnings of a firm. In addition, we assume that the likelihood function for earning shocks of the stock in a Bayesian paradigm is weighted by investors' behavioral biases. The degree of deviation of the weights could quantitatively reflect investors' level of behavioral biases. By using this model setting, we establish some properties on the expected earnings shock and its volatility. This information could then be used to derive some properties of investors' behavior on the stock price and its volatility during a financial crisis and subsequent recovery. Thereafter, we develop properties to explain some market anomalies, including short-term underreaction, long-term overreaction, and excess volatility during a financial crisis and subsequent recovery.

The rest of the paper is organized as follows. In the next section, we present the asset-pricing theory based on the modified random walk model for the earnings announcement of an asset. In Section 3, we first discuss the pseudo-Bayesian framework to update information about earning shocks in the asset-pricing theory. We then develop some properties for the stock returns based on the pseudo-Bayesian asset-pricing theory. Section 4 is devoted to discussing how cognitive biases are reflected in the weight assignment schemes. Thereafter, we will use our proposed model to explain excess volatility, short-term underreaction, long-term overreaction, and the magnitude effect during a financial crisis and subsequent recovery in Sections 5 to 7. The final section gives concluding remarks.

2 Asset Pricing Model With Financial Crisis

The pioneer work from Barberis, Shleifer, and Vishny (1998) considers a model of market sentiment in which a representative investor observes the earnings of an asset and updates her belief to value the asset and the earnings announcement of the asset at time t. N_t is assumed to follow a random walk:

$$N_t = N_{t-1} + y_t \ , \tag{1}$$

where y_t is an earnings shock at time t. Using a discounting model based on rational expectations (Wong and Chan, 2004), the asset is priced at time t as P_t given by:

$$\begin{aligned} P_t &= E_t\left[\frac{N_{t+1}}{1+r} + \frac{N_{t+2}}{(1+r)^2} + \cdots\right] \\ &= \frac{N_t}{r} + \frac{1+r}{r} \times \left[\frac{E_t[y_{t+1}]}{1+r} + \frac{E_t[y_{t+2}]}{(1+r)^2} + \cdots\right] \ , \end{aligned} \tag{2}$$

in which r is the discount rate, or the investor's anticipated return, which, for simplicity, is assumed to be a positive constant and $E_t[\cdot]$ represents the investor's conditional expectation given the information set Ω_t containing all information available to the investor at time t. We assume that y_t is Ω_t-measurable; that is, the value of y_t is known exactly given information Ω_t up to time t inclusively. Consequently, both N_t and P_t are Ω_t-measurable.

In this paper we extend the theory by considering the following modified random walk model with/without drifts to incorporate the impact of a financial crisis on the dynamics of the earnings announcement:

$$N_t = \begin{cases} N_{t-1} + y_t, & t < t_0 \quad , \quad t \geq t_2 \quad ; \\ \delta_0 + N_{t-1} + y_t, & t_0 \leq t < t_1 \quad ; \\ \delta_1 + N_{t-1} + y_t, & t_1 \leq t < t_2 \ ; \end{cases} \tag{3}$$

in which $\delta_0 < 0$ and $\delta_1 > 0$. In (3), we consider an economy with four states: normal economic conditions, economic conditions under a financial crisis, conditions under recovery, and eventually back to normal economic conditions. We suppose that when the economy is operating under normal conditions, the earnings announcement of the asset follows a random walk model, the same as in (1). If the economy is experiencing a financial crisis starting at time t_0, the earnings announcement of the asset after time t_0 is the random walk model with negative drift δ_0 in which the one-period conditional expected earning is discounted by the amount of $|\delta_0|$ with $\delta_0 < 0$. During recovery, the earnings announcement of the asset will follow another random walk with positive drift δ_1.

The rationale for using this model is to incorporate the impact of a downward trend and recovery from a financial crisis on the earnings announcement of the asset. Specifically, when the economy goes into a crisis at time t_0, earnings are expected to fall and the stock market starts to crash at that time. At time t_1, the economy is expected to recover, earnings are expected to rise, and thus, stock prices start to rise. From time t_2 onward, the economy then becomes stable and stock prices follow the random walk without drift again. We note that the random walk model after the recovery from a financial crisis could be different from the model before the financial crisis. However, without loss of generality, we assume these two models are the same because the conclusion drawn from different random walk models before and after the financial crisis is the same as that drawn from the same random walk model.

Barberis, Shleifer and Vishny (1998) assume that the earnings shock is independent and follows a distribution with equal chance on discrete values y_0 or $-y_0$. Lam, Liu, and Wong (2010, 2012) relax this assumption to let the earnings shock follow a normal distribution. In this paper, we further relax the assumption to assume that the earnings shock follows an exponential family distribution. We then apply the pseudo-Bayesian framework to study price behavior after a financial crisis and will present the results in Section 3. We first state our modified assumptions as follows:

Assumption 1: *The earnings announcement process $\{N_t\}$ may follow a random walk model in (1) or the random walk model with drifts δ_0 and δ_1 as stated in (3). Furthermore, the earnings shocks $\{y_t\}$ are a sequence of independent and identically distributed (i.i.d.) random variables following an exponential family distribution:*

$$y_t \sim f(y_t) = \exp\left\{(y_t\theta - b(\theta))/a(\phi) + c(y_t;\phi)\right\} , \tag{4}$$

where θ is the canonical parameter, $a(> 0), b$, and c are known functions, and ϕ is the dispersion parameter. The dispersion parameter is assumed to be a constant, either known or considered as a nuisance parameter.

Assumption 2: *The representative agent knows the nature of the random walk model, except that the mean μ is unknown. The agent estimates μ using observations about the earning shocks $\{y_t\}$. We assume that the agent knows the value of σ_y^2.*

Assumption 3: *The agent uses a "biased" statistical method to update her belief in a way that reflects the agent's behavioral bias.*

We note that in Assumption 1, we relax BSV's Bernoulli assumption and LLW's normality assumption to use the exponential family distribution. The advantage of using the exponential family distribution is that it is the most commonly used continuous distribution, including normal, gamma, and other distributions, and thus, it can fit into situations with symmetric as well as asymmetric distributions. We further note that this relaxation is very important because it is well known that market information could be asymmetric. A bear market will be more sensitive to bad news while a bull market is more sensitive to good news. We also note that, for an exponential family distribution, the mean $\mu = b'(\theta)$ and the variance $\sigma_y^2 = b''(\theta)a(\phi)$.

3 A Pseudo-Bayesian Approach and Properties of Stock Returns

By the standard and rational Bayesian approach to updating information on the mean level of the earnings shock, one could consider a vague, or improper, prior for the unknown mean μ; that is,

$$P_0(\mu) \propto 1 \ ,$$

see, for example, Matsumura, Tsui and Wong (1990) for related discussions. The likelihood function of μ given the observed earning shocks $\{y_t\}$ is:

$$L(y_1, y_2, \cdots, y_t|\mu) = \prod_{i=1}^{t} L\big(y_{t-i+1}|\mu\big) \ . \tag{5}$$

It is well known that by applying the Bayes formula, the posterior distribution of μ given $\{y_1, y_2, \cdots, y_t\}$ is:

$$P\big(\mu \mid y_1, y_2, \cdots, y_t\big) \propto \prod_{i=1}^{t} L\big(y_{t-i+1} \mid \mu\big) \ . \tag{6}$$

In this standard Bayesian approach, an equal weight is placed on each observation in $y_1, y_2, \cdots, y_t$. Consistent with the traditional efficient market hypothesis,[1] the rational expectations asset-pricing theory assumes that investors can have access both to the correct specification of the "true" economic model and to unbiased estimators of the model parameters (Friedman, 1979). If a rational investor is endowed with an objectively correct prior and the correct likelihood function, she will obtain the rational expectation equilibrium and, consequently, any structural irrationally inducing financial anomaly would disappear.

Violation of the rational expectation solution has been studied widely in the literature. For example, Blume and Easley (1982) show that if investors do not recognize the effect of learning on prices to obtain equilibrium, convergence of beliefs is not guaranteed within a general equilibrium learning model. Furthermore, Bray and Kreps (1987) observe that investors recognize and incorporate how their beliefs about

[1] Readers may refer to Chan, de Peretti, Qiao, and Wong (2012) and the references therein for more information.

the unknown essential features of an economy influence the structural model of the economy. However, the extreme knowledge required in these models is implausible.

As evidence has mounted against the traditional Bayesian model, theories to explain financial anomalies have been developed by relaxing some of those assumptions imposed in the standard theories. One approach is to assume that investors are plagued with cognitive biases (Slovic, 1972) and they may incorrectly assign different weights to different observations. To model such behavioral biases, in this paper we follow LLW to assume that investors place weight ω_1 on the most recent observation y_t, ω_2 on the second most recent observation y_{t-1} , and so on, with the possibility that ω_i's may not equal 1. We consider the following weighted likelihood function associated with the vector of weights $\omega := (\omega_1, \omega_2, \cdots, \omega_t)$:

$$L^{\omega}(y_1, y_2, \cdots, y_t|\mu) = \prod_{i=1}^{t} L\big(y_{t-i+1}|\mu\big)^{\omega_i} \ , \tag{7}$$

in which L^{ω} represents the weighted likelihood function depending on the subjective weighted ω. By the Bayes formula, the posterior distribution of μ given $\{y_t\}$ becomes:

$$P\big(\mu \mid y_1, \cdots, y_t\big) \propto \prod_{i=1}^{t} L\big(y_{t-i+1} \mid \mu\big)^{\omega_i} \ . \tag{8}$$

Consequently, obtaining the posterior mean and posterior variance of the unknown mean μ from the posterior distribution of μ, we establish the price and return dynamics of the stock under the behavioral model as shown in the following proposition:

Proposition 1 ***(Price and return dynamics in the pseudo-Bayesian approach)*** *Under a pseudo-Bayesian approach with a vague prior, the random walk N_t stated in (1) or (3), and an incorrect likelihood $L^{\omega}(\mu)$ stated in (7), for any $k \geq 1$, the predictive mean $E_t[y_{t+k}]$ of the future earning shock y_{t+k} given $\{y_1, y_2, \cdots, y_t\}$, and the posterior variance σ_t^2 of μ given $\{y_1, y_2, \cdots, y_t\}$ are, respectively, given by:*

$$E_t[y_{t+k}] = \frac{\omega_t y_1 + \cdots + \omega_1 y_t}{s_t} := d_t \quad \textit{and} \quad \sigma_t^2 = \frac{\sigma_y^2 s_t}{\sum_{i=1}^{t} \omega_i^2} \ , \tag{9}$$

where $s_t = \Sigma_{i=1}^{t} \omega_i$.

Lam, Liu, and Wong (2010, 2012) develop the results of the rational expectations pricing model in (2) when the random walk $\{N_t\}$ follows (1), whereas Fung, Lam, Siu, and Wong (FLSW, 2011) extend LLW's work by developing the results of the rational expectations pricing model when the random walk $\{N_t\}$ follows (3). In their work, they assume that the earnings shock is normally distributed. In this paper, by applying Proposition 1 and assuming the earnings shock y_t follows an exponential family distribution (4), we extend FLSW's work to relax the normality assumption and obtain the following proposition:

Proposition 2 *Under the assumptions stated in Proposition 1, we have*

a. *if the random walk $\{N_t\}$ follows (1), then the price at time t using the rational expectations pricing model in (2) becomes:*

$$E_t\left[\frac{P_{t+k}}{(1+r)^k}\right] = \frac{N_t}{r(1+r)^k} + \frac{[(1+k)r+1]d_t}{r^2(1+r)^k} \ , \quad \textit{and} \tag{10}$$

b. *if the random walk $\{N_t\}$ follows (3), then the price at time t using the rational expectations pricing model in (2) is given by:*

$$\begin{aligned} & E_t\left[\frac{P_{t+k}}{(1+r)^k}\right] \\ = \quad & \frac{N_t}{r(1+r)^k} + \frac{d_t[(k+1)r+1]}{r^2(1+r)^k} + \\ & \begin{cases} \frac{\delta_0}{r^2(1+r)^{b_t-2}}\left[(r+1)^{b_t-a_t}-1\right] + \frac{\delta_1}{(r+1)^{c_t-2}r^2}\left((r+1)^{c_t-b_t}-1\right) & \textit{if} \quad t<t+k<t_0 \\ \frac{\delta_0}{r^2(1+r)^{b_t-2}}\left[(r+1)^{b_t-k-2}([k+2-a_t]r+1)-1\right] + \frac{\delta_1}{(r+1)^{c_t-2}r^2}\left((r+1)^{c_t-b_t}-1\right) & \textit{if} \quad t<t_0\le t+k<t_1 \\ \frac{(b_t-a_t)\delta_0}{r(1+r)^k} + \frac{\delta_1}{r^2(1+r)^{c_t-2}}\left[(r+1)^{c_t-k-2}([k+2-b_t]r+1)-1\right] & \textit{if} \quad t<t_0, t_1\le t+k<t_2 \\ \frac{(b_t-a_t)\delta_0+(c_t-b_t)\delta_1}{r(1+r)^k} & \textit{if} \quad t<t_0, t_2\le t+k \\ \frac{\delta_0}{r^2(1+r)^{b_t-2}}\left[(r+1)^{b_t-k-2}([k+1]r+1)-1\right] + \frac{\delta_1}{(r+1)^{c_t-2}r^2}\left((r+1)^{c_t-b_t}-1\right) & \textit{if} \quad t_0\le t<t+k<t_1 \\ \frac{\delta_0(b_t-1)}{r(1+r)^k} + \frac{\delta_1}{(r+1)^{c_t-2}r^2}\left((r+1)^{c_t-k-2}([k+2-b_t]r+1)-1\right) & \textit{if} \quad t_0\le t<t_1\le t+k<t_2 \\ \frac{\delta_0(b_t-1)+\delta_1(c_t-b_t)}{r(1+r)^k} & \textit{if} \quad t_0\le t<t_1, t_2\le t+k \\ \frac{\delta_1}{(r+1)^{c_t-2}r^2}\left((r+1)^{c_t-2-k}([k+1]r+1)-1\right) & \textit{if} \quad t_1\le t<t+k<t_2 \\ \frac{\delta_1(c_t-1)}{r(1+r)^k} & \textit{if} \quad t_1\le t<t_2\le t+k \\ 0 & \textit{if} \quad t_2\le t<t+k \ , \end{cases} \end{aligned} \tag{11}$$

where $a_t = \max\{\lceil t_0-t\rceil, 0\}$, $b_t = \max\{\lceil t_1-t\rceil, 0\}$, $c_t = \max\{\lceil t_2-t\rceil, 0\}$, *and* $d_t = \frac{\omega_t y_1+\cdots+\omega_1 y_t}{s_t}$.

We note that FLSW consider the situation in which there is a crash but no recovery. We believe that this is not reasonable, and thus, we skip discussing this situation.

From Proposition 2, we observe that the conditional expected present value of the asset at time $t+k$ given information Ω_t depends on the current and the future earnings shocks. For example, in the simplest random walk model as stated in (1), the current earnings announcement depends only on the current earnings announcement and the current as well as the expected future earning shocks. However, it is generally believed that the price of the asset also depends on the economic situation. In the view of economic cycles, the economy will experience an expansion period after suffering a period of financial recession. Furthermore, the economy will eventually go back to normal. The present value of the asset is, therefore, proportional to the predictive mean of the future earning shocks, the current earnings announcement, the duration of the economic recovery, and the recovery rate of the economy. It is also inversely proportional to the risk-free interest rate, the duration of the economic downturn, and the deteriorating rate under economic crisis.

To describe this situation, we let the random walk $\{N_t\}$ follow (3). Under this circumstance, when the current economy is in the state just before the economic downturn or during the recession, the price of the asset depends not only on the current earnings announcement, the predictive mean of the future earning shocks, and the risk-free interest rate but also on how long and how serious is the impact of both the economic turmoil and the economic expansion on the price of the asset. If the duration of the economic turmoil is long or the effect of the economic turmoil is serious, the price of the asset falls. This is reflected in the coefficients of δ_0 and δ_1, respectively. Similarly, the coefficient of δ_1 is determined by the duration of the economic expansion, while the value of δ_1 depends on the level of the economic expansion. In particular, when we are in the economic expansion period, (i.e., $t_1 \leq t < t_2$), the effect of the term δ_0 vanishes, and only the term δ_1 reflects the effect of the economy's shift. Thereafter, if the bad days and the good days of the economy are all gone (i.e., $t_2 \leq t$), the estimation of the price of the asset is the same as that obtained from the random walk, $\{N_t\}$, from (1).

4 How are Cognitive Biases Reflected in the Weight Assignment Schemes?

In the model setup discussed in the previous sections, we incorporate general weights on observations into a simple asset-pricing model. This allows us to examine the price formation process under a rational expectations approach with biased weights in which investors, with or without cognitive biases, incorporate their prior beliefs into the historical data to estimate the valuation-relevant parameters that can lead to anomalous asset-price behavior.

Brav and Heaton (2002) consider weights given by $\omega_1 = \cdots = \omega_{\frac{t}{2}} = 1$ and $\omega_{\frac{t}{2}} = \cdots = \omega_t = 0$, where t is an even number. In this paper, we follow LLW to use a more general assumption that investors may use weights, $\omega_1, \omega_2, \cdots$, satisfying $0 \leq \omega_i \leq 1$ for all i. By allowing more flexibility in the choice of weights, we can represent investors' various behavioral biases quantitatively. Specifically, in (A), (B), and (C) below, we state the the following weight assignment schemes to characterize the conservative and/or representative heuristics in the following definition:

Definition 1

(A) *Investors using a conservative heuristic assign weights as:* $0 \leq \omega_1 \leq \omega_2 \leq \cdots \leq \omega_{n_0} = \omega_{n_0+1} = \cdots = 1$ *for an integer* $n_0 \geq 1$.

(B) *Investors using a representative heuristic assign weights as:* $1 = \omega_1 = \omega_2 = \cdots = \omega_{m_0} \geq \omega_{m_0+1} \geq \omega_{m_0+2} \geq \cdots \geq 0$ *where* m_0 *is a positive integer.*

(C) *Investors using both conservative and representative heuristics assign weights as:* $0 \leq \omega_1 \leq \omega_2 \leq \cdots \leq \omega_{n_0} = \omega_{n_0+1} = \cdots = \omega_{m_0} = 1 \geq \omega_{m_0+1} \geq \cdots \geq 0$ *for* $1 \leq n_0 \leq m_0$.

By using Scheme A, we could measure investors' conservative heuristics because those who are over-conservative will underweigh recent information and overweigh prior information. The parameter n_0 reflects the conservative heuristic in which the most recent n_0 observations are underweighed. If Edwards (1968) is right in noting that it takes two to five observations to do one observation's worth of work in inducing a subject to change her opinions, then some of ω_1, ω_2 , $\cdots$, ω_{n_0} could be substantially less than 1 for $n_0 \leq 5$. The smaller the weights, the more conservative the investors. Thus, the magnitudes of the weights ω_1, ω_2 , $\cdots$, ω_{n_0} can be used to measure the degree of conservatism. The evidence suggests that underreaction reflects the uncertainty regarding a possible structural change in the data and a lack of knowledge that a change occurred. This will result in a failure to fully incorporate the price implications of this change into the estimation of the valuation-relevant parameters.

The weight assignment in Scheme B is consistent with the psychological literature on the representative heuristic. The representative heuristic is often described as the tendency of experimental subjects to overweigh recent clusters of observations and underweigh older observations that would otherwise moderate beliefs. Heavy weights on recent data could be a reaction to a concern about structural changes. The representative heuristic is characterized by a parameter m_0 showing that the investor underweighs the observations beyond the most recent m_0 data points. Here, the parameter m_0 arises from the "law of small numbers" (Tversky and Kahneman, 1971) in the mind of the investor. Because of their representative heuristic, investors have the tendency to treat a small sample size, like m_0, as if it is large enough to represent the whole population. Consequently, they assign weights much smaller than 1 for observations beyond the most recent m_0 observations.

Our model formulation assumes that investors exhibit both conservative and representativeness heuristics. This is different from the regime-switching formulation in BSV in which investors are under the influence of one heuristic and then suddenly shift to another regime of being influenced by another heuristic. In other words, conservatism and representativeness are not mutually exclusive, and investors can be simultaneously influenced by both heuristics at any point in time. When the investor is under the influence of both heuristics, the model has two parameters n_0 and m_0 as described above. Here, conservatism is reflected by the existence of $n_0 > 0$ and the smallness of the sum $\omega_1 + \omega_2 + \cdots + \omega_{n_0-1}$, and representativeness is reflected by the existence of $m_0 < \infty$ and the smallness of the sum $\omega_{m_0+1} + \omega_{m_0+2} + \cdots$. Notice that investors of Type C described in Definition 1 degenerate into Type A investors when

$m_0 = \infty$ and degenerate into Type B investors when $n_0 = 0$. Also when $m_0 = \infty$ and $n_0 = 0$, all weights are equal to 1 and the investor has no behavioral bias. In this sense, the third type of investor embraces all of the other types. Therefore, in this paper we only consider investors of the third type.

5 Inference on Market Volatility

According to Proposition 2, whatever the random walk $\{N_t\}$ follows, we can have

$$E_t[P_{t+k}] = \frac{N_t}{r} + \frac{[r(1+k)+1]d_t}{r^2} + c_k$$

where c_k may vary in different situations as described in Proposition 2. For P_t, we can have $P_t = \frac{N_t}{r} + \frac{(r+1)d_t}{r^2} + c_0$ and $P_{t+1} = \frac{N_{t+1}}{r} + \frac{(r+1)d_{t+1}}{r^2} + c_1$, where c_0 and c_1 are constants and may have different values in different situations. Consequently, for the 1-period return $R_{t,t+1} = P_{t+1} - P_t$, we can have the following result:

$$\begin{aligned} R_{t,t+1} \quad = \quad & \frac{1+r}{r^2}\Big[(\frac{\omega_{t+1}}{s_{t+1}} - \frac{\omega_t}{s_t})y_1 + \cdots + (\frac{\omega_2}{s_{t+1}} - \frac{\omega_1}{s_t})y_t\Big] + (\frac{1}{r} + \frac{1+r}{r^2}\frac{\omega_1}{s_{t+1}})y_{t+1} \\ & + c_1 - c_0. \end{aligned}$$

From this result, we obtain the following proposition for the inference on market volatility:

Proposition 3 *Under the assumptions stated in Proposition 2, we have*

a. *if $s_t \to \infty$, then the market volatility $Var(R_{t,t+1}) \to \frac{\sigma_y^2}{r^2}$, and*

b. *if behavioral biases are severe, i.e., $s_t \to s_\infty < \infty$, then the market volatility $Var(R_{t,t+1})$ is given by*

$$\Big[\frac{1}{r^2} + 2\frac{1}{r}\frac{1+r}{r^2}\frac{s_1}{s_\infty}\Big]\sigma_y^2 + \frac{(1+r)^2}{r^4}\frac{1}{s_\infty^2}A_\infty\sigma_y^2 \ , \tag{12}$$

where $A_\infty = \omega_1^2 + \sum_{t=1}^{\infty}(\omega_{t+1} - \omega_t)^2$.

In addition, from Proposition 3, one could easily obtain some interesting observations about excess volatility as stated in the following property:

Property 4 *Under the assumptions stated in Proposition 2, we have*

Observation 1: *Excess volatility is a decreasing function of the discount rate or investors' anticipated return r.*

Observation 2: *Conservative heuristics will reduce excess volatility.*

Observation 3: *Representative heuristics will increase excess volatility.*

Observation 4: *Observations 1-3 hold regardless of the symmetric/asymmetric information on the signs of the earnings shock.*

Observation 5: *Observations 1-3 hold in the normal economic situation, as well as during the crash or during the recovery of the economy.*

One could obtain Observation 1 by conducting some simple computation. We provide some remarks for Observations 2 and 3 as follows: If investors adopt a conservative heuristic, then they will choose a positive integer n_0 and assign the following weights: $0 \leq \omega_1 < \omega_2 < \cdots < \omega_{n_0} = \omega_{n_0+1} = \cdots = 1$. This leads to $s_t \to \infty$, and thus, the excess volatility $Var(R_{t,t+1}) \to \frac{\sigma_y^2}{r^2}$. On the other hand, if investors select a representative heuristic, they will choose a positive integer m_0 and assign the weights $1 = \omega_1 = \omega_2 = \cdots = \omega_{m_0} > \omega_{m_0+1} > \omega_{m_0+2} > \cdots \geq 0$. In this situation, if the behavioral biases are severe, then ω_i are very close to 0 for any $i > m_0$ and $s_t \to s_\infty < \infty$. Thus, the excess volatility will appear in the form of (12) and can be larger than that in the conservative heuristic case. For the third case, investors adopt both conservative and representative heuristics, and thus, they will choose both n_0 and m_0 such that $1 \leq n_0 \leq m_0$ and assign the following weights: $0 \leq \omega_1 < \omega_2 < \cdots < \omega_{n_0} = \omega_{n_0+1} = \cdots = \omega_{m_0} = 1 > \omega_{m_0+1} > \cdots \geq 0$. Their market volatility will be larger than that in the conservative heuristic case but smaller than that in the representative heuristic case. If ω_i are very close to zero for any i with $m_0 < i < n_0$, then $s_t \to s_\infty < \infty$, and consequently, the excess volatility will also appear in the form of (12). In this situation, it will be larger than $\frac{\sigma_y^2}{r^2}$. However, when compared with the representative heuristic, because $0 \leq \omega_1 < 1$, the terms A_∞ and $\frac{s_1}{s_\infty}$ will be smaller than those of the representative heuristic, and thus, reduce its excess volatility.

One may think that different market situations and different shapes of the earnings shock would lead to different excess volatility. For example, one may think that during a market crash, excess volatility would increase sharply. However, Observations 4 and 5 tell us that this is not the case. We note that this does not mean that different market situations and different shapes of the earnings shock have no effect on excess volatility. These observations only tell us that the effects of different market situations and different shapes of the earnings shock are already reflected in the use of different heuristics. It does have an effect on excess volatility. For example, when the market is going to crash, investors realize that they could lose most of their investment if they do not sell their stocks. This means that investors will count the recent observations more, and thus, they select representative heuristics. In this situation, from Observation 3, excess volatility is increasing. Thus, the theory developed from our model could be used to explain the empirical situation as well.

6 Inference on Underreaction and Overreaction

To examine the underreaction and overreaction phenomenon, we first define the lag-one autocovariance, γ_1^k, of the k-period return as

$$\gamma_1^k = Cov(R_{t,t+k}, R_{t,t-k})$$

where $R_{t,t+k}$ is the k-period return and $R_{t,t-k}$ is the k-period return from time $t-k$ to time t. The lag-one autocorrelation, ρ_1^k, of the k-period return is then defined as

$$\rho_1^k = \frac{Cov(R_{t,t+k}, R_{t,t-k})}{\sqrt{Var(R_{t,t+k})Var(R_{t,t-k})}} \ . \tag{13}$$

Because underreaction is associated with positive autocorrelation and overreaction is associated with negative autocorrelation, we define short-term underreaction and long-term overreaction as follows:

Definition 2 *Prices of a single asset exhibit*

a. *a short-term underreaction if $\rho_1^k > 0$ for sufficiently small k, and*

b. *a long-term overreaction if $\rho_1^k < 0$ for sufficiently large k,* *where ρ_1^k is defined in (13).*

Based on the above definition, we establish the following proposition for the short-term underreaction and the long-term overreaction:

Proposition 5 *Under the assumptions stated in Proposition 2, if investors possess both conservative and representative heuristics, then prices exhibit short-term underreaction and long-term overreaction in terms of return autocorrelations. Specifically, there exist positive integers K_1 and K_2 such that for sufficiently large t , we have*

$$\begin{cases} \rho_1^k > 0 & k \le K_1, \\ \rho_1^k < 0 & k > K_2, \end{cases} \tag{14}$$

where ρ_1^k is defined in (13). Furthermore, the correlation coefficients above are non-trivial for sufficiently large t , i.e., the limiting correlation coefficients for $t \to \infty$ are non-zero.

In addition, the results in (14) hold regardless of the symmetric/asymmetric information on the signs of the earnings shock and they hold under normal economic conditions as well as during a crash or during a recovery.

Lam, Liu, and Wong (2010) have shown that the results in (14) hold under normal economic conditions. We note that it is natural that the results in (14) hold for the asymmetric information on the signs of the earnings shock and under normal economic conditions as well as during a crash or during a recovery. For example, during a crash it is generally common for stock prices to fall day after day. This is exactly what is shown in Proposition 5: that $\rho_1^k > 0$ for $k \le K_1$ for some small integers K_1; this condition holds during the recovery of the economy as well. On the other hand, during a crash, one would expect the market to recover after some time; this is exactly what is shown in Proposition 5: that $\rho_1^k < 0$ for $k > K_2$ for some large values K_2. Similarly, during a recovery, one would expect that a recession could arise in the future; this is also exactly what is shown in Proposition 5: that $\rho_1^k < 0$ for $k > K_2$ for some large values K_2. Similar arguments hold true for the asymmetric information on the signs of the earnings shock. For example, bad news arriving after bad news will result in a further decline in stock price. This is exactly what is shown in Proposition 5: that $\rho_1^k > 0$ for $k \le K_1$ for some small integers K_1.

We now adopt the event approach used by BSV to describe under- and overreaction. It is well known that sometimes the average return on a company's stock in a period following an announcement of good news is higher than that in a period following an announcement of bad news, whereas sometimes the average return following a series of good news announcements turns out to be lower than that following a series of bad news announcements. The market is said to be underreacting in the former situation and overreacting in the latter situation. We follow the approach used by BSV to quantify such under- and overreaction by considering the difference in average returns after a string of good or bad news denoted as follows:

$$\begin{aligned} U_t(s,j) &= E\{R_{t+1}|y_t > \mu + s\sigma_y, \cdots, y_{t-j+1} > \mu + s\sigma_y\} \\ &\quad -E\{R_{t+1}|y_t < \mu - s\sigma_y, \cdots, y_{t-j+1} < \mu - s\sigma_y\}, \end{aligned} \tag{15}$$

where j represents the time length of the string of good or bad news and s represents the intensity of the news content. The quantity $U_t(s,j)$ defined in (15) represents the expected profit of a momentum trading strategy that dictates buying when there is a string of good news and selling when there is a string of bad news. On the other hand, if one adopts a contrarian trading strategy of selling when there is a string of good news and buying when there is a string of bad news, the expected profit of such a contrarian trading strategy is represented by $-U_t(s,j)$. We can use the sign of $U_t(s,j)$ to measure underreaction and overreaction as stated in the following definition:

Definition 3 *Prices exhibit*

a. *a short-term underreaction if* $U_t(s,j) > 0$ *for sufficiently small* j*, and*

b. *a long-term overreaction if* $U_t(s,j) < 0$ *for sufficiently large* j*.*

Using the above definition, we establish the following proposition:

Proposition 6 *Under the assumptions stated in Proposition 2, if investors possess both conservative and representative heuristics, modeling by a weight assignment scheme (C), we have*

a. *prices exhibit short-term underreaction and long-term overreaction using an event approach. Specifically, there exist integers* J_1 *and* J_2 *such that for a given* $s > 0$ *and for large* t *, we have*

$$\begin{cases} U_t(s,j) > 0, & j \leq J_1, \\ U_t(s,j) < 0, & j \geq J_2, \end{cases} \tag{16}$$

where $U_t(s,j)$ *is defined in (15), and*

b. *the expected momentum trading profit* $U_t(s,j)$ *is non-trivial when* t *tends to infinity, i.e., the limiting trading profit is non-zero for* $t \to \infty$.

c. *In addition, the results in (a) and (b) hold regardless of the symmetric/asymmetric information on the signs of the earnings shock, and*

d. *the results in (a) and (b) hold under normal economic conditions as well as during a crash or during a recovery.*

I is observed that the representative heuristic contributes to the contrarian profit, whereas the conservative heuristic contributes to the momentum profit. Proposition 6 also links investors' irrational cognitive biases to financial anomalies of overreaction and underreaction by showing that overreaction occurs after long periods of good or bad performance, while underreaction happens after short periods of good or bad performance. In addition, Proposition 6 provides good insights into how the contrarian and momentum profits arise. The representative heuristic has to overpower the conservative heuristic for a contrarian profit to surface. The long-run assumption is necessary for a contrarian profit because under a long-run situation, the representativeness bias will become noticeable. Another interesting observation is that both momentum and contrarian profits are sensitive to the discount rate r. The smaller the discount rate, the larger the momentum and contrarian profits. This is because when r is small, future cash flows become important, and a mis-estimation of future cash flows will intensify the over- or underreaction phenomena. Last, Proposition 6 shows that these properties hold true for asymmetric as well as symmetric information on the signs of the earnings shock and they hold under normal economic conditions as well as during a crash or during a recovery.

7 Inference on Magnitude Effect

If the momentum (contrarian) profit $U_t(s,j)$ (-$U_t(s,j)$) increases as s or j increases, we say the profit possesses a magnitude effect in s or j. The following propositions state the magnitude effect for investors using both conservative and representative heuristics.

Proposition 7 *(a magnitude effect in s). Under the assumptions stated in Proposition 2, if investors possess both conservative and representative heuristics, both the long-term overreaction and the short-term underreaction established in Proposition 6 will exhibit a magnitude effect in s. Specifically, there exist integers J_1 and J_2 such that*

a. *the momentum profit $U_t(s,j)$ is positive and is monotonically increasing with s for any sufficiently small t and for any $j < J_1$, and*

b. *the contrarian profit $-U_t(s,j)$ is positive and is monotonically increasing with s for any sufficiently large t, and for any $j > J_2$.*

c. *In addition, the result of the momentum profit stated in (a) and the results of the contrarian profit stated in (b) hold regardless of the symmetric/asymmetric information on the signs of the earnings shock, and*

d. *the result of the momentum profit stated in (a) and the results of the contrarian profit stated in (b) hold under normal economic conditions as well as during a crash or during a recovery.*

Proposition 8 *(a magnitude effect in j). Under the assumptions stated in Proposition 7,*

a. *when j is sufficiently small, the momentum profit based on j consecutive good or bad news increases as j decreases; and*

b. *when j is sufficiently large, the contrarian profit based on j consecutive good or bad news increases as j increases.*

c. *In addition, the result of the momentum profit stated in (a) and the results of the contrarian profit stated in (b) hold regardless of the symmetric/asymmetric information on the signs of the earnings shock, and*

d. *the result of the momentum profit stated in (a) and the results of the contrarian profit stated in (b) hold under normal economic conditions as well as during a crash or during a recovery.*

8 Concluding Remarks

Barberis, Shleifer and Vishny (1998) and others have developed Bayesian models to explain investors' behavioral biases by using conservative heuristics and representative heuristics in making decisions. In this paper, we extend the theory to study the impact of investors' behavior on the stock price and its volatility before and during a financial crisis and subsequent recovery. We assume that (1) investors exhibit both conservative and representative heuristics that lead them to underweigh recent observations and past observations of the earnings shocks of corporations, (2) the earnings shock of an asset follows a modified random walk model to incorporate the impact of a financial crisis on the earnings of a firm, and (3) the likelihood function for earning shocks of the stock in a Bayesian paradigm is weighted by investors' behavioral biases. By using this model setting, we establish some properties on the expected earnings shock and its volatility. This information is then used to derive some properties of investors' behavior on the stock price and its volatility during a financial crisis and subsequent recovery. Thereafter, we use these properties to explain some market anomalies, including short-term underreaction, long-term overreaction, and excess volatility during a financial crisis and subsequent recovery.

Understanding investors' behavior will be useful in making decisions about investments. The information on companies, (Thompson and Wong, 1991, 1996), the economic and financial environment (Broll, Wahl and Wong, 2006; Fong, Lean, and Wong, 2008), technical analysis (Wong, Manzur, and Chew, 2003) could be used to make better investment decisions. Academic researchers and practitioners could incorporate the theory developed in this paper with the mean-variance rule (Wong, 2007; Wong and Ma, 2008; Bai, Hui, Wong, and Zitikis, 2012), CAPM statistics (Leung and Wong, 2008), VaR rule (Ma and Wong, 2010), portfolio optimization (Bai, Liu, and Wong, 2009, 2011; Egozcue and Wong, 2010; Broll, Egozcue, Wong, and Zitikis), or other advanced econometric techniques (Wong and Miller, 1990; Li and Lam, 1995; So, Lam and Li, 1998; Bai, Li, Liu, and Wong, 2011) to make better investment decisions. Another extension to improve investment decision-making is to study the behavior of different types of investors (Wong and Li, 1999; Li and Wong, 1999; Wong and Chan, 2008) or to incorporate stochastic dominance criteria (Gasbarro, Wong and Zumwalt, 2007; Post, 2003; Wong, Phoon, and Lean, 2008) to study investors' conservative and representative heuristics. For example, based on the empirical study on momentum profit from Fong, Wong, and Lean (2005), and Sriboonchitta, Wong, Dhompongsa, and Nguyen (2009) conclude that risk averters prefer to invest in winner portfolios, while risk seekers prefer to invest in loser portfolios. This finding could explain why the momentum profit could still exist after discovery. In addition, recently, Qiao, Clark, and Wong (2012) examine the Taiwan spot and futures markets and conclude that risk averters prefer to invest in the spot market, whereas risk seekers prefer to invest in the futures market.

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