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Working Paper No. 1

Analysis on Runs of Daily Returns in Istanbul Stock Exchange

Analysis on Runs of Daily Returns in Istanbul Stock Exchange

Ahmet ŞENSOY*

Abstract:

The aim of this paper is to obtain some statistical properties about runs of daily returns of ISE30, ISE50 and ISE100 indices and compare these results with the empirical stylized facts of developed stock markets. In this manner, all time historical daily closing values of these indices are studied and the following observations are obtained;

Exponential law fits pretty well for the distribution of both run length and magnitude of run returns.

Market is equally likely to go up or go down everyday.

Market depth has improved over recent years.

Large magnitudes of run returns are more likely to be seen in positive runs.

As in the developed stock markets, daily returns in Istanbul Stock Exchange don't have significant autocorrelations but absolute values (i.e. magnitudes) of daily returns exhibit strong and slowly decaying autocorrelations up to several weeks suggesting volatility clustering. Similar to the absolute daily returns, absolute value of run returns display strong and slowly decaying autocorrelations which again supporting the existence of volatility clustering. Unlike magnitudes of run returns, lenghts of runs don't have significant autocorrelations.

1. Introduction

Empirical analysis is the first and best way to understand the world around us. By collecting data and studying statistical properties there of we can learn about the underlying distributions governing many phenomena. Then, once sufficient empirical data have been collected, idealized models may be constructed to try and account for the data (Block, 2000)). Inspired by this approach, lots of data sets have been analyzed many times in the history of financial markets, in particular the ones about asset prices.

In financial markets, fluctutations in asset prices produce what we call financial time series and these series have been deeply investigated for making inferences and forecastings. Especially in recent decades, empirical studies on financial time series indicate that if we examine these series from a statistical point of view, the seemingly random variations of asset prices do share some quite nontrivial statistical properties. Such properties, common across a wide range of instruments, markets and time periods are called *stylized empirical facts* (Cont, 2001). Researchers have now come to agree on several stylized facts about financial markets: heavy tails in asset return distributions, absence of autocorrelations in asset returns, volatility clustering and asymmetry between rises and falls... (Cont (2001), Engle and Patton

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Keywords: Stylized facts, return runs, autocorrelation, volatility clustering, stock market efficieny

(2001), Abergel and others (2009), Mantegna and Stanley (2000), Bouchaud and Potters (2003))

Most of the time, these studies mainly focus on analysing daily or weekly individual returns of the assets, but sometimes just the sign of these returns can be a useful tool for understanding the market structure (Marumo and others (2002)). Moreover, instead of individual returns, considering the cumulative returns of specific sequences may give us nontrivial information about the market or even help us to reveal some stylized facts about stock price movements.

In financial markets, a *run* is a consecutive series of price movements without a sign reversal, hence a positive (negative) run is an uninterrupted sequence of positive (negative) returns and this run continues until a negative (positive) return comes out. For example, consider daily closing values of ISE30 index for 12 days from 27.01.1997 to 13.02.1997 . These values are 1818.21, 1648.34, 1690, 1755.14, 1699.25, 1673.01, 1544.48, 1577.23, 1650.83, 1647.3, 1683.58, 1703.56 which give us the following daily returns -0.09343, 0.025274, 0.038544, -0.03184, -0.01544, -0.07683, 0.021205, 0.046664, -0.00214, 0.022024, 0.011868 . Signs of these returns generates the sequence "$- + + - - - + + - + +$" which contains three positive and three negative runs. The lengths of the three negative runs are $1, 3, 1$ and similarly lengths of the three positive runs are $2, 2, 2$. The cumulative returns (which we will call *run returns*) obtained in the positive runs are 0.064792, 0.068858, 0.034153 and the cumulative returns obtained in the negative runs are -0.09343, -0.12003, -0.00214 .[1]

Runs are simple constructs, but little research has been done on them in finance. Most of these researchs aim to examine the informal efficiency of stocks (however this paper does not have such a purpose) because of the distinctive[2] run length of a random walk. Fama (1965) investigated the runs of several stocks, and found little evidence for violations of efficiency based on serial dependence in returns. Similar research have been done by Moore (1978) and Grafton (1981) to test the efficient market hypothesis. Easley and others (1997) used runs to examine dependence in intra-day data.

In this paper, we will conduct a detailed runs analysis similar to work of Gao and Li (2006) on Dow Jones Industrial; first we will analyze the distributions of run lengths and run returns of ISE30, ISE50 and ISE100 indices then we will talk about some of the stylized facts observed in Istanbul Stock Exchange, and finally we will investigate the time correlation of the run lengths and magnitudes of run returns.

2. Analysis

We consider daily closing values of ISE30 , ISE50 and ISE100 indices from the day they have been introduced to the date 24.04.2012 . The daily return of an index is found by

$$r_t = \frac{s_t - s_{t-1}}{s_{t-1}} \qquad (E1)$$

where s_t is the index' closing value of day t.

[1] The possibility is very small but if there happens to be a day with zero return, it is ommited
[2] For pure random walks, average run length is two

2.1 Distribution of the Run Length

Using $(E1)$ and the definition of a run, we obtain several information from the empirical data. Tables 1.a and 1.b shows us the longest positive and negative runs, their corresponding date periods and their returns and table 2 shows the frequencies of all runs with different lengths;

Table 1.a: All time longest positive runs of ISE30, ISE50 and ISE100

	Longest Positive Runs	**Returns**
ISE30	02.09.1997 – 17.09.1997 12 days	0,22338
ISE50	14.01.1997 – 27.01.1997 10 days	0,601792
	02.11.1999 – 15.11.1999 10 days	0.292414
	18.08.2005 – 01.09.2005 10 days	0,158245
ISE100	13.02.1989 – 02.03.1989 12 days	0,62534
	15.09.1989 – 04.10.1989 12 days	0,29406
	13.08.1993 – 31.08.1993 12 days	0,39589

Table 1.b: All time longest negative runs of ISE30, ISE50 and ISE100

	Longest Negative Runs	**Returns**
ISE30	09.01.2008 – 23.01.2008 11 days	-0,19799
	14.11.2011 – 24.11.2011 9 days	-0,12934
ISE50	09.01.2008 – 23.01.2008 11 days	-0,19919
ISE100	16.08.1988 – 01.09.1988 12 days	-0,08515
	23.06.1988 – 07.07.1988 11 days	-0,17495
	18.04.1994 – 02.05.1994 11 days	-0,37174
	09.01.2008 – 23.01.2008 11 days	-0,20093

Table 2: Frequencies of runs with different lengths

		LENGTH												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**TOTAL**
ISE30	Positive Run	465	223	136	65	33	19	5	-	4	3	-	1	954
	Negative Run	483	233	135	59	25	8	7	2	1	-	1	-	954
ISE50	Positive Run	375	188	102	55	26	18	3	2	2	2	-	-	773
	Negative Run	390	194	111	45	17	8	6	2	-	-	1	-	774
ISE100	Positive Run	633	354	202	108	56	35	12	4	6	9	-	3	1422
	Negative Run	697	344	210	97	34	18	14	4	1	-	3	1	1423

Considering run length distributions (obtained from table 2) in fig. 1.a, 1.b and 1.c; we suggest that the number of observations $N(L)$ of a run with length L can be expressed as the following exponential form;

$$N(L) = \alpha e^{-\beta L} \qquad (E2)$$

Fig. 1.a: Run length distribution of ISE30

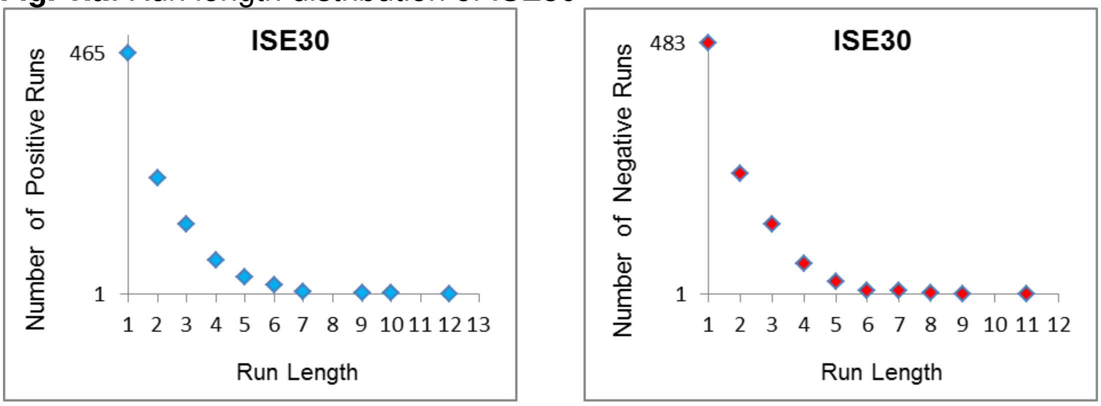


Fig. 1.b: Run length distribution of ISE50

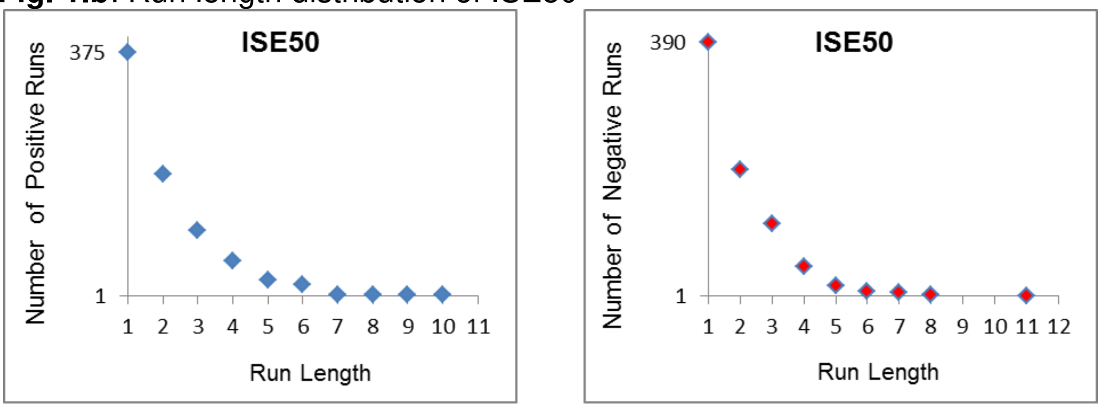
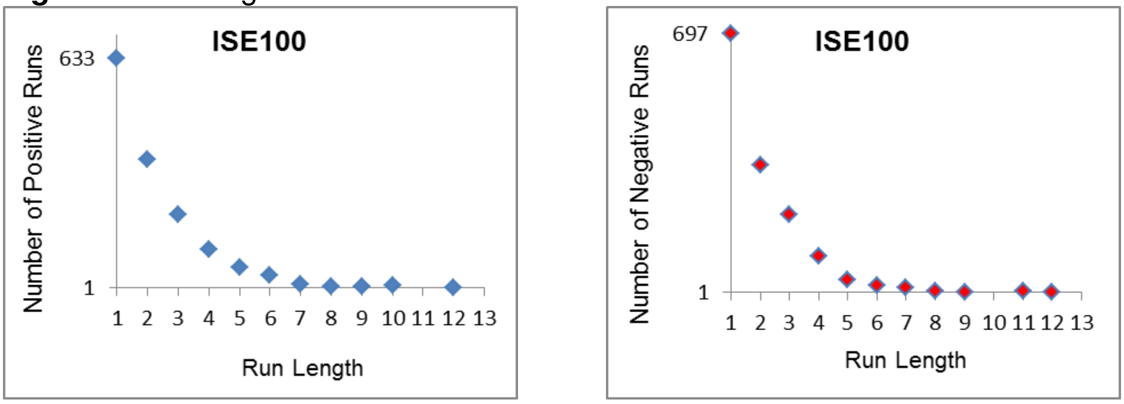


Fig. 1.c: Run length distribution of ISE100



For positive and negative runs of each index, fitting an exponential form of $(E2)$ to the datas in table 2 gives us the following results;

Table 3.a

POSITIVE RUNS				
	β	R^2	adjusted R^2	%95 confidence interval for β
ISE 30	0,66098	0,99778	0,99751	(0,61654 , 0,70542)
ISE50	0,65982	0,99936	0,99928	(0,63561 , 0,68403)
ISE100	0,58594	0,99953	0,99948	(0,56924 , 0,60266)

Table 3.b

NEGATIVE RUNS				
	β	R^2	adjusted R^2	%95 confidence interval for β
ISE 30	0,69224	0,99836	0,99816	(0,65154 , 0,73294)
ISE50	0,68613	0,99808	0,99781	(0,63846 , 0,73379)
ISE100	0,66049	0,99758	0,99731	(0,61576 , 0,70522)

As we see from table 3.a and 3.b; R^2 and $adj.R^2$ values show that for both positive and negative runs of each index, exponential law fits pretty well for the frequency of run lengths.

Consider a simple random process with two equally likely outcomes; in such a process the probability density function of run length L should follow an exponential distribution of the form $(\frac{1}{2})^L = e^{-(ln2)L} \cong e^{-(0.69315)L}$.

We see that for negative runs of ISE30, ISE50 and ISE100 and positive runs of ISE30 indices, 0.69315 is in the %95 confidence interval for estimated β but for positive runs of ISE50 and ISE100, it is not in the %95 confidence interval (for ISE50 it is pretty close though; see table 3.a) hence we can roughly conclude that, ignoring the magnitudes and considering just the signs of the daily returns, market is equally likely to go up or go down everyday.

2.2 Distribution of the Run Returns

Tables 4.a and 4.b give some historical information about largest negative and positive run returns of ISE30, ISE50 and ISE100 indices;

Table 4.a: Largest negative run returns of ISE30, ISE50 and ISE100

LARGEST NEGATIVE RUN RETURNS		
	Return	**Duration**
ISE30	-0,36078	23.11.2000 – 04.12.2000 8 days
	-0,27329	21.03.2001 – 29.03.2001 7 days
	-0,26727	28.05.2002 – 03.06.2002 5 days
ISE50	-0,36568	23.11.2000 – 04.12.2000 8 days
	-0,22093	06.07.2001 – 11.07.2001 4 days
	-0,20937	09.09.2008 – 18.09.2008 7 days
ISE100	-0,37174	18.04.1994 – 02.05.1994 11 days
	-0,36565	23.11.2000 – 04.12.2000 8 days
	-0,31315	21.01.1994 – 18.01.1994 6 days

Table 4.b: Largest positive run returns of ISE30, ISE50 and ISE100

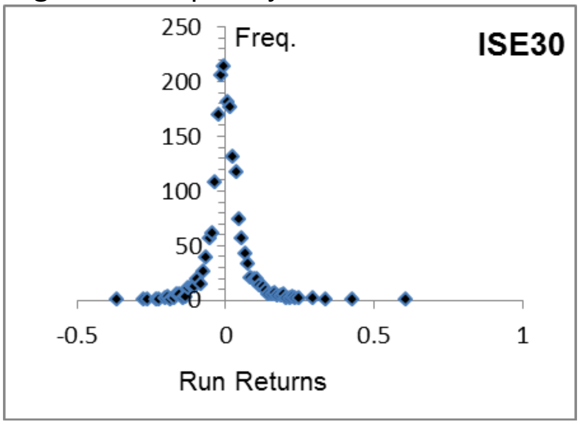
LARGEST POSITIVE RUN RETURNS		
	Return	**Duration**
ISE30	0,60179	14.01.1997 – 27.01.1997 10 days
	0,42027	05.12.2000 – 06.12.2000 2 days
	0,33490	09.12.1999 – 13.12.1999 3 days
ISE50	0,41564	05.12.2000 – 06.12.2000 2 days
	0,24112	22.02.2001 – 26.02.2001 3 days
	0,23099	26.04.2001 – 30.04.2001 3 days
ISE100	0,62534	15.09.1989 – 04.10.1989 12 days
	0,49633	14.01.1997 – 27.01.1997 12 days
	0,41718	05.12.2000 – 06.12.2000 2 days

Before starting the analysis, an interesting observation is (also as suggested by tables 1.a, 1.b, 4.a and 4.b) as getting close to present day we still observe considerable amount of long runs, but in these long runs, magnitudes of run returns seem significantly smaller compared to those of the long runs in earlier dates suggesting that market depth has improved over recent years.

To analyse the distribution of run returns, we consider the frequency distributions as the following: First we create intervals with 0,01 increments as .... , [-0,02 , -0,01) , [-0,01 , 0) , [0 , 0,01) , [0,01 , 0,02) , ... and for each index, we count the number of observed run returns belonging to each interval (see LHS of fig. 2.a, 2.b and 2.c). Then we take absolute values of the observed run returns and count the number of these absolute values belonging to each interval mentioned above (see RHS of fig.2.a, 2.b and 2.c).

First thing to notice here is one of the stylized facts of financial markets: just like the distribution of the daily returns, distribution of the run returns display heavy tails and sharp peaks.

Fig. 2.a: Frequency distributions of run returns and absolute run returns of ISE30



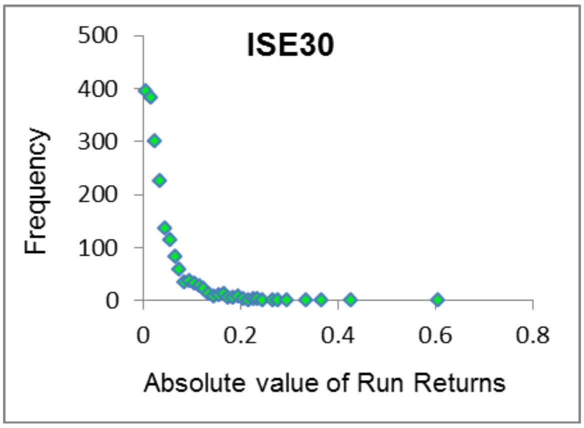
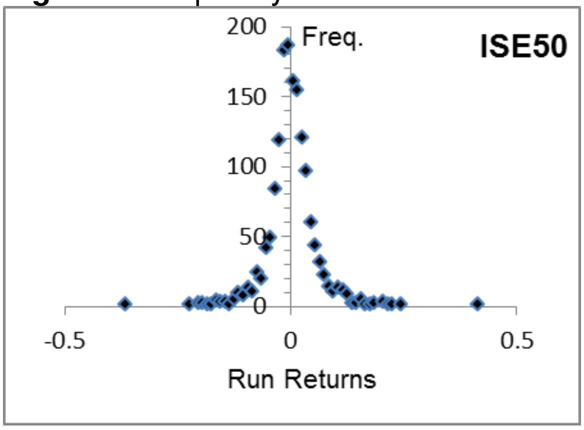


Fig. 2.b: Frequency distributions of run returns and absolute run returns of ISE50



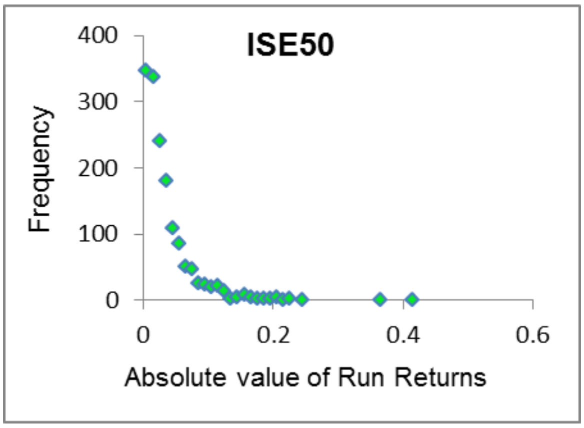
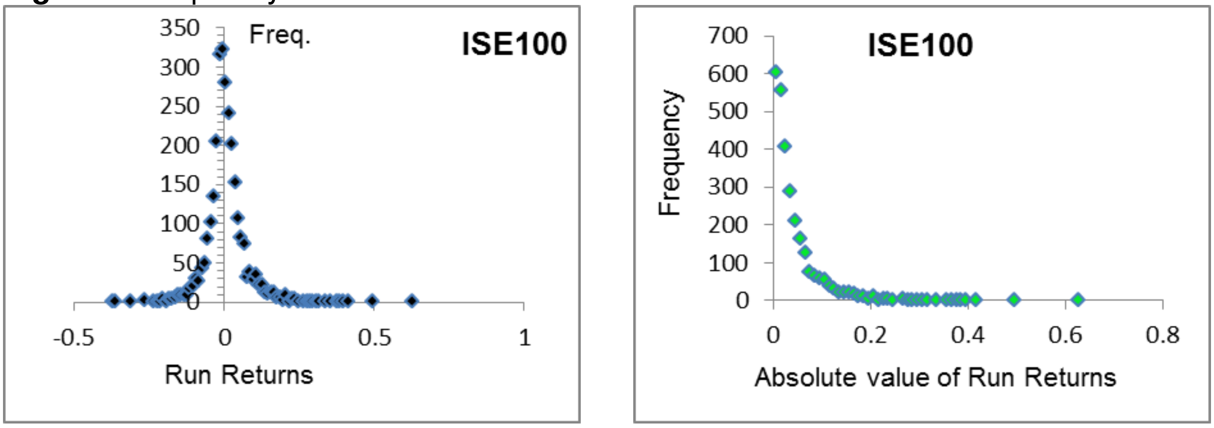


Fig. 2.c: Frequency distributions of run returns and absolute run returns of ISE100



Theory suggests the idea of fitting normal distributions and exponential distributions to the datas in the LHS and RHS of fig. 2.a, 2.b and 2.c. respectively.

To understand which one would be a better fit, we compare the actual frequency and fitted frequency values with the same returns (see fig. 3.a, 3.b and 3.c; dashed lines have unit slope and pass through the origin).

Fig. 3.a: actual freq. vs normal fit freq. actual freq. vs exponential fit freq.

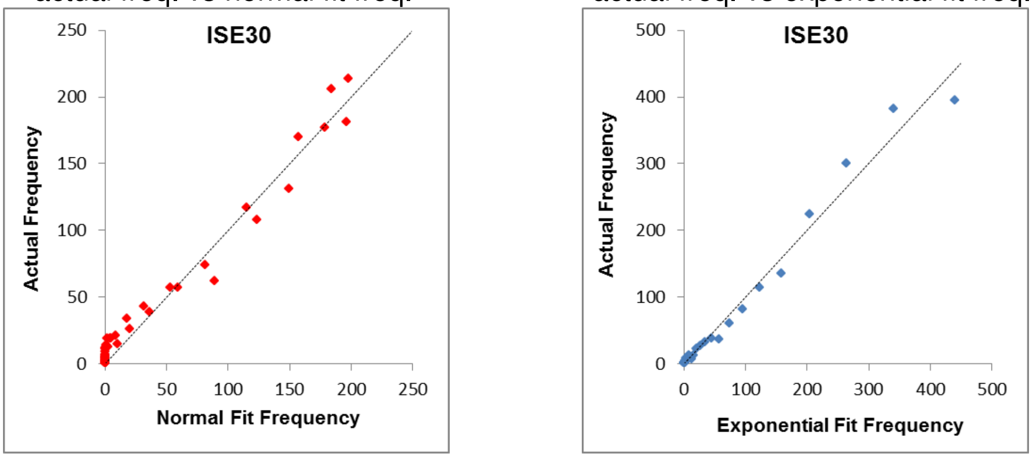


Fig. 3.b: actual freq. vs normal fit freq. actual freq. vs exponential fit freq.

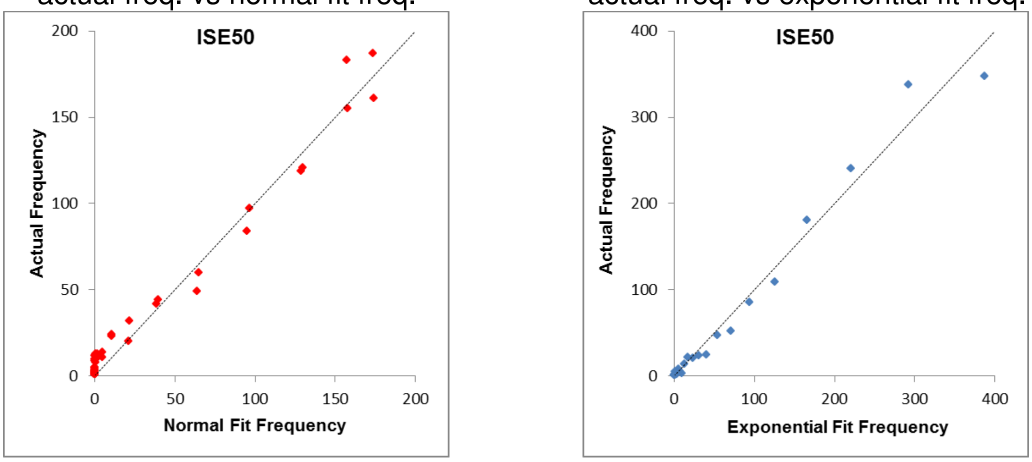
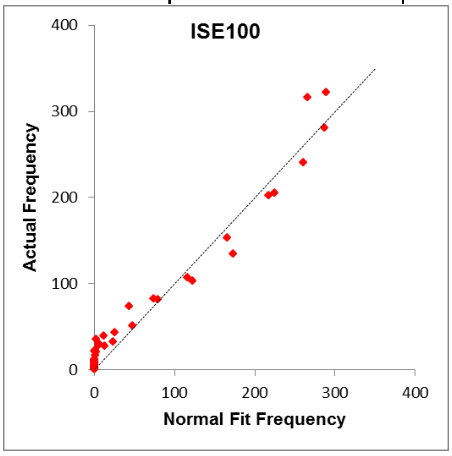
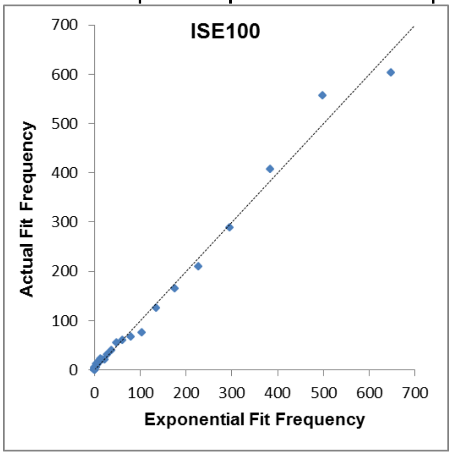


Fig. 3.c: actual freq. vs normal fit freq. actual freq. vs exponential fit freq.





We observe that for each index, exponential distribution fits better than normal distribution in explaining the relationship between magnitudes of run returns and their frequency. R^2 and $adj.R^2$ values of these fits also suggest us a similar idea (see table 5);

Table 5: R^2 and $adj.R^2$ values of exponential and normal fit

		R^2	adjusted R^2
ISE30	Exponential Fit:	0,98096	0,98033
	Normal Fit:	0,97265	0,97160
ISE50	Exponential Fit:	0,97912	0,97825
	Normal Fit:	0,97635	0,97527
ISE100	Exponential Fit:	0,99047	0,99022
	Normal Fit:	0,96420	0,96308

2.3 Asymmetry of Gains and Losses in a Run

Using the frequency data obtained in the last subsection, for each index we compare the number of positive and negative runs according to their return magnitudes.

Fig. 4.a

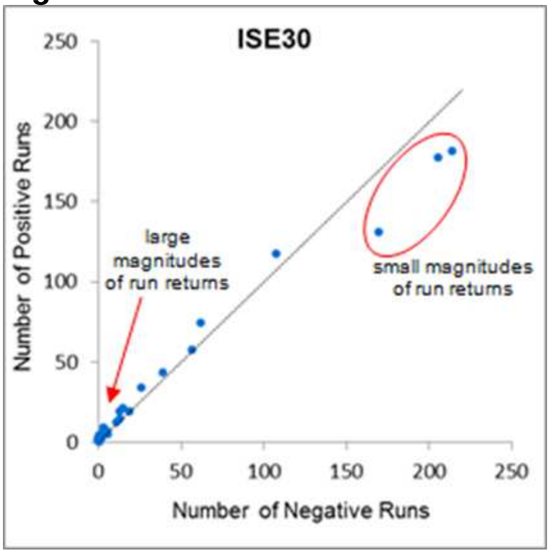
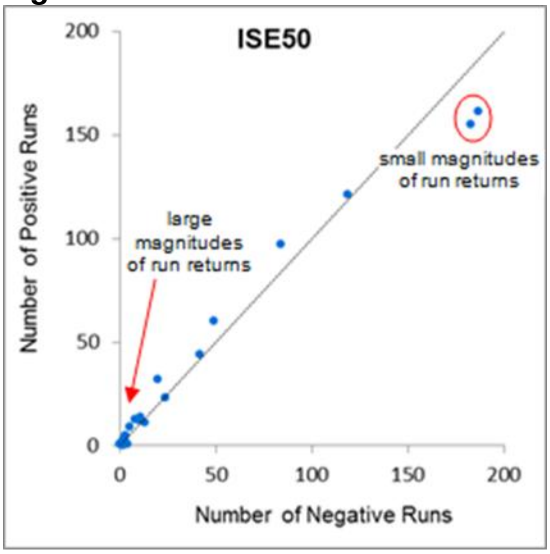


Fig. 4.b



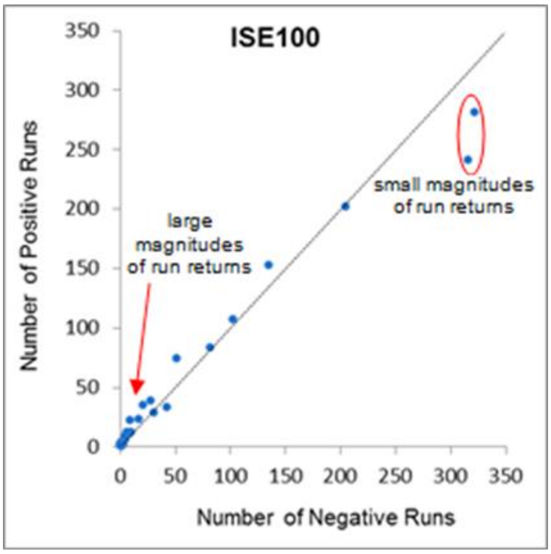


Fig. 4.c

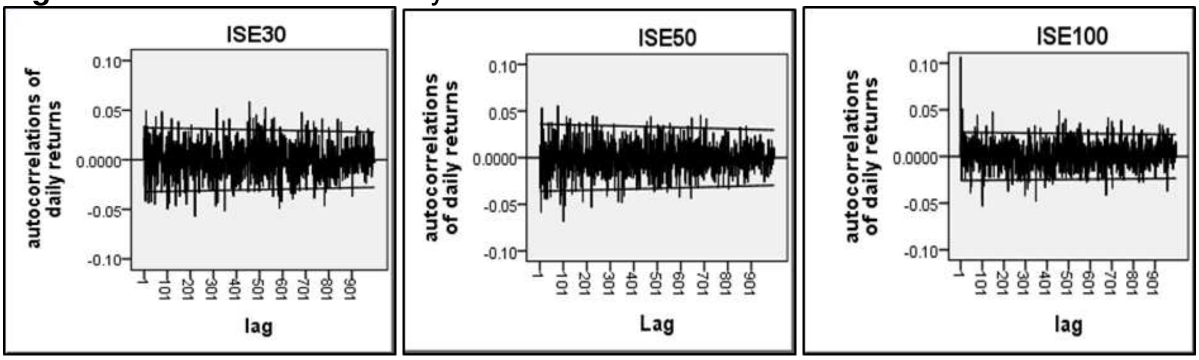
Just like in the previous case, dashed lines pass through the origin with unit slope and as it is easily understood magnitude of run returns gets larger as points get closer to the origin. We see that for each index there is a significant asymmetry, suggesting that large magnitudes of run returns is more likely to be seen in positive runs, in other words It is more likely to see big gain rather than big loss in a run.

2.4 Autocorrelation Analysis

2.4.1 Daily returns

One of the stylized facts of liquid markets is that daily returns do not exhibit any significant autocorrelation and ISE is no exception as we see from fig. 5;

Fig. 5: Autocorrelations of daily returns



The absence of significant autocorrelations in asset returns has been studied in detail (Fama (1971), Pagan (1996)) and it is usually used to support random walk models in which the returns are considered to be independent random variables (Fama (1991)). But as Cont (2001) states "independence implies that any nonlinear function of returns will also have no significant autocorrelation: For example, absolute values or squares of daily returns should also have no significant autocorrelation" but various empirical studies (Bollerslev and others (1992), Comte and Renault (1996), Bouchaud and others (1997), Cont (1998), Ding and others (1983), Ding and Granger (1994), Engle (1995)) show that in this case, autocorrelations remains significantly positive for several weeks and decays slowly[3] . This situation is usually interpreted as there is a correlation in volatility of returns but not the returns themselves. This is a quantitative manifestation of one of the sylized facts in financial markets called *volatility clustering*.[4] To see if we observe this stylized fact in Istanbul Stock Exchange, we consider the autocorrelations of absolute daily returns of our indices and obtain very similar results (see fig. 6)

2.4.2 Run Returns

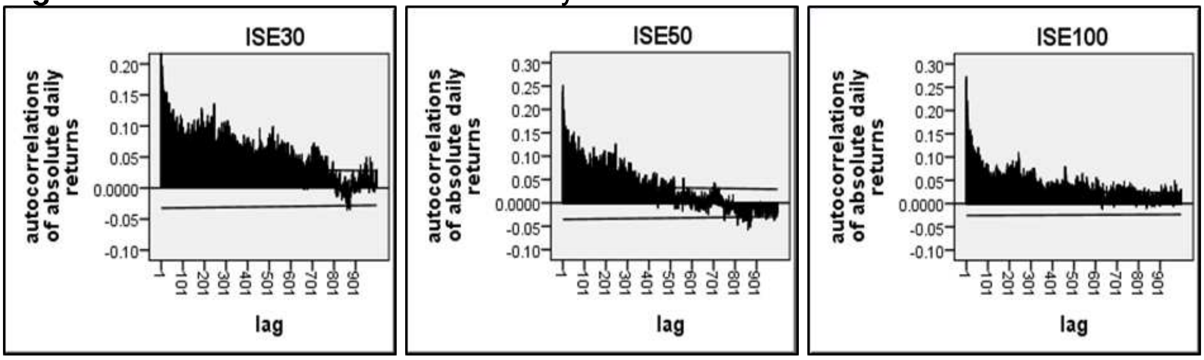
In the next step, we investigate if there exists significant autocorrelations in magnitudes of run returns. In this case, the autocorrelation function is defined as;

$$AC_1(n) = corr(\,|R_i|\,,|R_{i+n}|\,) \qquad (E3)$$

[3] Sometimes this slow decay is considered as an indicator of longe-range dependence in volatility, but arguments still continue on whether it should imply longe time memory of financial time series (Cont (2007), Taqqu and others (1999))

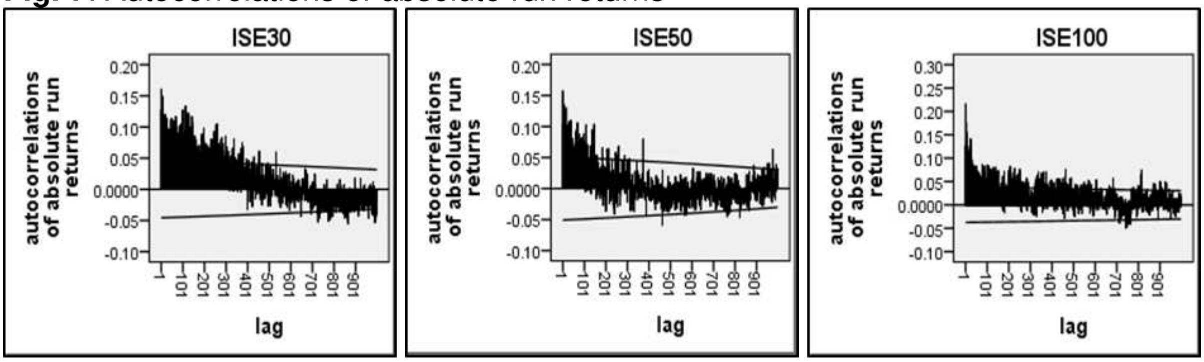
[4] In finance, volatility clustering refers to the observation that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes." (Mandelbrot, 1963)

Fig. 6: Autocorrelations of absolute daily returns



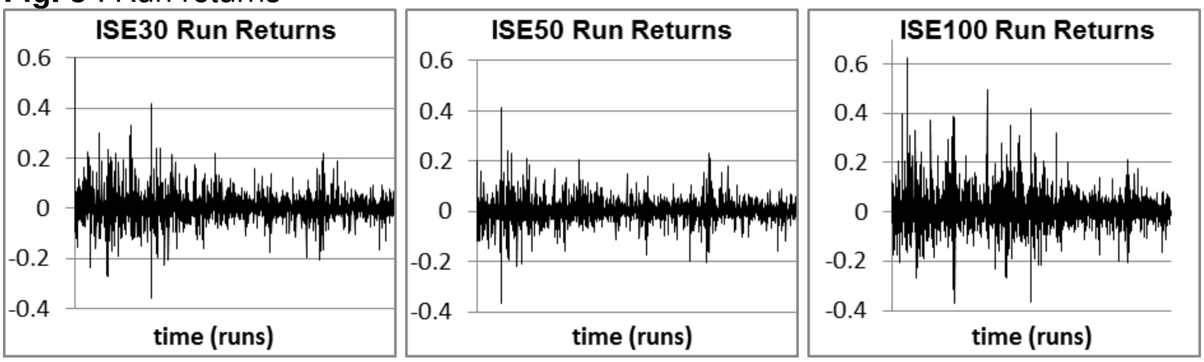
where R_i is the return of i^{th} run and n denotes the lag. Analysing this case gives us a similar result as of absolute daily returns. (see fig. 7)

Fig. 7: Autocorrelations of absolute run returns



As it is seen from fig. 7, there exists strong correlation decaying slowly (persisting up to several months) which again suggesting volatility clustering that also can be directly observed from progress of run returns in fig. 8;

Fig. 8 : Run returns



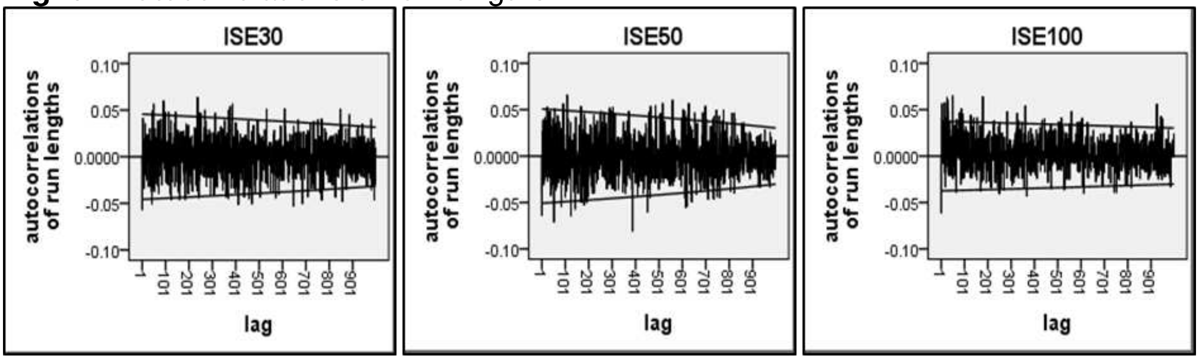
2.4.3 Run Lengths

We ask ourselves if we can we find a similar result for run lenghts. Here the autocorrelation function is defined as;

$$AC_2(n) = corr(L_i, L_{i+n}) \qquad (E4)$$

where L_i is the length of i^{th} run. We see that the autocorrelation function fluctuates around zero in the %95 confidence interval meaning there do not exist significant time correlation in run lengths. (see fig. 9)

Fig. 9 : Autocorrelations of run lengths



3. Conclusion

In this work, we conducted a detailed analysis on runs of daily retuns of three popular Istanbul Stock Exchange indices (ISE30, ISE50 and ISE100) hoping to find some meaningful properties. As a result we have the following observations;

Exponential law fits pretty well for the distribution of both length and return magnitude of the runs.

Market is equally likely to go up or go down everyday.

Market depth has improved over recent years.

It is more likely to see big gain rather than big loss in a run.

Just like in most of the developed stock markets, in Istanbul Stock Exchange there is an absence of significant autocorrelations in daily returns but the autocorrelations of absolute daily retuns are strong and slowly decaying (persisting up to several months) suggesting volatility clustering. Similarly, significant correlation exists in the absolute run returns which also supports the same deduction. Hoping to find a similar relation, we investigated the autocorrelations of runs length but in this case there seems no significance.

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