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Non-Linear Fiscal Regimes and Interest Rate Policy

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Abstract

Much empirical evidence finds that governments react to fiscal imbalances in a non-linear way, through an increasing marginal response of primary surpluses to changes in debt. This paper shows that non-linear fiscal regimes alter equilibria under active and passive monetary-fiscal policies. The Fisher equation combined with non-linear fiscal policies leads to multiple steady states. Under passive interest rate rules, even if the steady state at which fiscal policy is active is locally saddle-path stable, there exist infinite equilibrium paths originating in the neighborhood of that steady state which converge into a high-debt trap. Under active interest rate rules, even if the steady state at which fiscal policy is active is locally unstable, there exists a saddle connection with the high debt equilibrium along which inflation is uniquely determined.

JEL Classification: E63; E52; E31.

Keywords: Non-Linear Fiscal Rules; Interest Rate Policy; Multiple Equilibria; Global Dynamics.

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1 Introduction

A large body of empirical studies shows that fiscal authorities react to public debt accumulation in a non-linear way.¹ Governments tend to enhance the adoption of corrective actions as fiscal imbalances worsen, through an *increasing* marginal response of primary surpluses to changes in debt. What are the implications for monetary-fiscal policy interactions?

This paper shows that non-linear fiscal regimes give rise to steady-state multiplicity and alter dynamic equilibria under “active” and “passive” monetary-fiscal policies.² The long-run Fisher equation combined with a fiscal policy stance displaying convex non-linearity in the surplus-debt relationship yields two types of steady states: a low-debt steady state at which fiscal policy is active and a high-debt steady state at which fiscal policy is passive. It is demonstrated that the two steady states are dynamically connected under either a passive or an active monetary policy stance. Under a passive interest rate policy, even if the steady state at which fiscal policy is active is locally a unique stable equilibrium (Leeper, 1991; Woodford, 2003), there exists an infinite number of equilibrium paths originating in the neighborhood of that steady state which converge to a high-debt trap. Under an active interest rate policy, even if the steady state at which fiscal policy is active is locally unstable (Leeper, 1991; Woodford, 2003), there exists a saddle connection with the high-debt equilibrium along which inflation is uniquely determined.

Consequently, a non-linear fiscal policy stance *per se* renders global dynamic properties of interest rate rules fundamentally different from local dynamic properties. This is, relatedly, critical for questions of determinacy. In particular, if governments respond to debt accumulation non-linearly, inflation turns out to be pinned down only by an active monetary policy stance, even near a steady state at which fiscal policy is active. The passive monetary, active fiscal local regime does not ensure, as instead in the context of standard theoretical formulations of monetary-fiscal interactions focused on local analysis around a single steady state, equilibrium determinacy.

¹See, for example, Bohn (1998), Sarno (2001), Arestis, Cipollini and Fattouh (2004), Bajo-Rubio, Diaz-Roldan and Esteve (2004, 2006), Arghyrou and Luintel (2007), Chortareas, Kapetanios and Uctum (2008), Considine and Gallagher (2008), Cipollini, Fattouh and Mouratidis (2009), Arghyrou and Fan (2011), and Legrenzi and Milas (2012a, 2012b).

²Fiscal policy is “passive” (“active”) in Leeper’s (1991) terminology or “locally Ricardian” (“locally non-Ricardian”) in Woodford (2003)’s terminology if it implies (does not imply) local stability government liabilities for all paths of the other endogenous variables in the neighborhood of a steady state. Monetary policy is “active” (“passive”) in Leeper’s (1991) terminology or satisfies (does not satisfy) the “Taylor (1993) principle” in Woodford (2003)’s terminology if the nominal interest rate set by the central bank increases by more (less) than one-for-one with respect to an increase in the inflation rate. See Canzoneri, Cumby, and Diba (2011) for a recent review of literature on the interactions between monetary and fiscal policy.

The paper is organized as follows. Section 2 presents the monetary model with non-linear fiscal policy rules. Section 3 analyzes the issue of equilibrium dynamics and states the main results. Section 4 provides concluding comments.

2 The Model

In this section we shall use a continuous-time macroeconomic environment *à la* Benhabib, Schmitt-Grohé and Uribe (2001) in order to show how a non-linear fiscal policy stance can easily lead to multiplicity of steady-state equilibria.

2.1 Households

We consider an economy populated by a large number of identical infinitely lived households deriving utility from consumption and real money holdings. The lifetime utility function of the representative household is given by

$$\int_0^{\infty} e^{-rt} u(c(t), m(t)) dt, \quad (1)$$

where $r > 0$ denotes the rate of time preference, $c(t)$ consumption, and $m(t)$ real money balances at instant of time t . The utility function $u(\cdot, \cdot)$ is strictly increasing and strictly concave in both arguments. Consumption and real balances are Edgeworth complements, so that $u_{cm} > 0$. The representative household's instant budget constraint is given by

$$\dot{a}(t) = (R(t) - \pi(t)) a(t) - R(t) m(t) + y(t) - c(t) - \tau(t), \quad (2)$$

where $a(t)$ denotes real financial wealth, consisting of interest-bearing government bonds and money balances, $y(t)$ an endowment of perishable goods, $\tau(t)$ lump-sum taxes net of public transfers, $R(t)$ the nominal interest rate on government bonds, and $\pi(t)$ the inflation rate. Households are subject to the borrowing limit condition precluding Ponzi's games, given by

$$\lim_{t \rightarrow \infty} e^{-\int_0^t [R(j) - \pi(j)] dj} a(t) \geq 0. \quad (3)$$

Optimality implies

$$u_c(c(t), m(t)) = \lambda(t), \quad (4)$$

$$u_m(c(t), m(t)) = \lambda(t) R(t), \quad (5)$$

$$\dot{\lambda}(t) = \lambda(t) (r + \pi(t) - R(t)), \quad (6)$$

$$\lim_{t \rightarrow \infty} e^{-\int_0^t [R(j) - \pi(j)] dj} a(t) = 0, \quad (7)$$

where $\lambda(t)$ is the costate variable associated with the flow budget constraint.

2.2 Monetary and Fiscal Policy

The monetary authority adopts an interest rate policy described by a feedback rule of the form

$$R(t) = r + \pi^* + \psi(\pi(t) - \pi^*), \quad (8)$$

where π^* denotes the target inflation rate and ψ a positive parameter. Monetary policy is “active” if $\psi > 1$ and is “passive” if $\psi < 1$.

The government’s instant budget constraint is given by

$$\dot{a}(t) = (R(t) - \pi(t)) a(t) - s(t), \quad (9)$$

where $s(t) = \tau(t) + R(t) m(t)$ denotes the primary surplus inclusive of interest savings from the issuance of money.³ We depart from the related theoretical literature in the description of the fiscal policy stance followed by the government. We assume that the fiscal authority adjusts the primary surplus according to a non-linear feedback policy of the form

$$s(t) = \sigma(a(t)), \quad (10)$$

where function $\sigma(\cdot)$ is continuous, positive, and satisfies $\sigma' > 0, \sigma'' < 0$.

³Public consumption is set equal to zero, for simplicity.

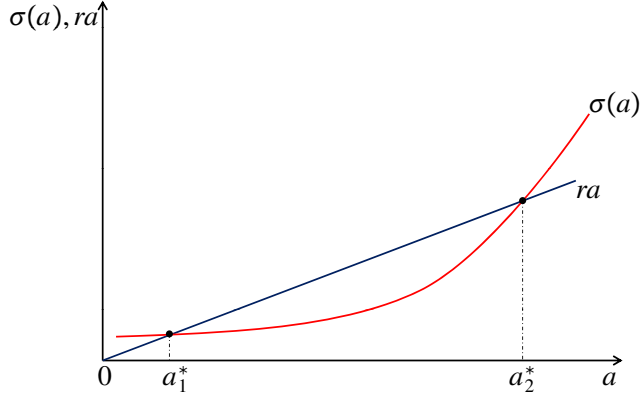


Figure 1: Multiple steady states under a non-linear fiscal regime.

2.3 Equilibrium

Equilibrium in the goods market requires that consumption is equal to the endowment, $c(t) = y(t)$. Assuming that the endowment is constant over time, $y(t) = y$ for each $t \in [0, \infty)$, equations (4) and (5) imply

$$\lambda(t) = L(R(t)), \quad (11)$$

with $L' = \lambda / (u_{mm}/u_{cm} - R) < 0$. Combining (6), (8) and (11), equilibrium dynamics of inflation are given by

$$\dot{\pi}(t) = -\frac{L(r + \pi^* + \psi(\pi(t) - \pi^*))}{\psi L'(r + \pi^* + \psi(\pi(t) - \pi^*))} [(\psi - 1)(\pi(t) - \pi^*)]. \quad (12)$$

Substituting (10) and (8) into (9), the law of motion of government liabilities can be expressed as

$$\dot{a}(t) = [r + (\psi - 1)(\pi(t) - \pi^*)]a(t) - \sigma(a(t)). \quad (13)$$

2.4 Steady States and Active-Passive Fiscal Policies

Setting $\dot{\pi}(t), \dot{a}(t) = 0$ in (12) and (13) yields $\pi = \pi^*$ and $\sigma(a) = ra$. Because the fiscal policy reaction function $\sigma(\cdot)$ is positive and obeys $\sigma', \sigma'' > 0$, the steady-state relation $\sigma(a) = ra$ has, in general, two solutions, $a_1^*, a_2^* > 0$. Setting $a_1^* < a_2^*$, it follows $\sigma'(a_1^*) < r$ and $\sigma'(a_2^*) > r$ (see Figure 1). Hence, using Leeper (1991)'s terminology, fiscal policy is “active” in the neighborhood of the steady state (π^*, a_1^*) , because from (13) $\partial \dot{a}(t) / \partial a(t)|_{(\pi^*, a_1^*)} = r - \sigma'(a_1^*) > 0$, and is “passive” in the neighborhood of the steady state (π^*, a_2^*) , because from (13) $\partial \dot{a}(t) / \partial a(t)|_{(\pi^*, a_2^*)} = r - \sigma'(a_2^*) < 0$.

3 Equilibrium Dynamics

We now examine local and global equilibrium dynamics under both passive and active monetary policies. Consider first the dynamic effects of a passive interest rate rule ($\psi < 1$). The following proposition holds.

Proposition 1 *Suppose that fiscal policy is non-linear ($\sigma', \sigma'' > 0$) and monetary policy is passive ($\psi < 1$). Then: (a) locally, the steady state (π^*, a_1^*) is a saddle point and the steady state (π^*, a_2^*) is a sink; (b) globally, there exist infinite equilibrium paths originating in the neighborhood of (π^*, a_1^*) and converging asymptotically to (π^*, a_2^*) ; the saddle manifold associated to (π^*, a_1^*) is the boundary of the basin of attraction of (π^*, a_2^*) .*

Proof. (a) Let $J_{(\pi^*, a_1^*)}$ be the Jacobian matrix of the equilibrium system (12)-(13) evaluated at the steady state (π^*, a_1^*) . We have $\det J_{(\pi^*, a_1^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_1^*)]}{\psi L'(r+\pi^*)} < 0$, because $L'(r+\pi^*) < 0$, $\psi < 1$, and $\sigma'(a_1^*) < r$. Therefore, the steady state (π^*, a_1^*) is a saddle point, with the stable arm given by $\pi(t) = \pi^* + \frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*) - [r-\sigma'(a_1^*)]}{(\psi-1)a_1^*} (a(t) - a_1^*)$. Let $J_{(\pi^*, a_2^*)}$ be the Jacobian matrix of the system (12)-(13) evaluated at the steady state (π^*, a_2^*) . We have $\text{tr} J_{(\pi^*, a_2^*)} = \frac{L(r+\pi^*)(\psi-1)}{\psi L'(r+\pi^*)} + [r - \sigma'(a_2^*)] < 0$ and $\det J_{(\pi^*, a_2^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_2^*)]}{\psi L'(r+\pi^*)} > 0$, because $L'(r+\pi^*) < 0$, $\psi < 1$, and $\sigma'(a_2^*) > r$. Therefore, the steady state (π^*, a_2^*) is a sink. (b) From (12) and (13), the two isoclines $\dot{\pi}(t) = 0$ and $\dot{a}(t) = 0$ are $\pi(t) = \pi^*$ and $\pi(t) = \pi^* + \frac{[\sigma(a(t))/a(t)] - r}{(\psi-1)}$, respectively. We have $\left. \frac{d\pi(t)}{da(t)} \right|_{\dot{a}(t)=0} = \frac{\sigma'(a(t)) - [\sigma(a(t))/a(t)]}{(\psi-1)a(t)}$, which is positive at the steady state (π^*, a_1^*) , where $\sigma'(a_1^*) < [\sigma(a_1^*)/a_1^*] = r$, equal to zero for $\sigma'(a(t)) = \sigma(a(t))/a(t)$, and negative at the steady state (π^*, a_2^*) , where $\sigma'(a_2^*) > [\sigma(a_2^*)/a_2^*] = r$. Thus, in the phase plane $(\pi(t), a(t))$, the isocline $\dot{\pi}(t) = 0$ is horizontal, the isocline $\dot{a}(t) = 0$ is inverted U-shaped, and the two isoclines intersect at (π^*, a_1^*) and (π^*, a_2^*) . From (12) and (13), $\dot{\pi}(t) > (<) 0$ if $\pi(t) < (>) \pi^*$ and $\dot{a}(t) > (<) 0$ if $\pi(t) < (>) \pi^* + \frac{[\sigma(a(t))/a(t)] - r}{(\psi-1)}$. Figure 2 shows the global dynamics of the system. The stable arm of the saddle point passing through (π^*, a_1^*) , SS , has a positive slope, given by $\frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*) - [r-\sigma'(a_1^*)]}{(\psi-1)a_1^*}$, which is greater than the slope of the isocline $\dot{a}(t) = 0$ evaluated at (π^*, a_1^*) , given by $\frac{\sigma'(a_1^*) - r}{(\psi-1)a_1^*}$. Hence, in the neighborhood of the steady state (π^*, a_1^*) , for a given initial condition $a(0)$, there exists an infinite number of equilibrium initial values $\pi(0) < \pi(0)_S = \pi^* + \frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*) - [r-\sigma'(a_1^*)]}{(\psi-1)a_1^*} (a(0) - a_1^*)$, for example $\pi(0)_1$ and $\pi(0)_2$ in Figure 2, such

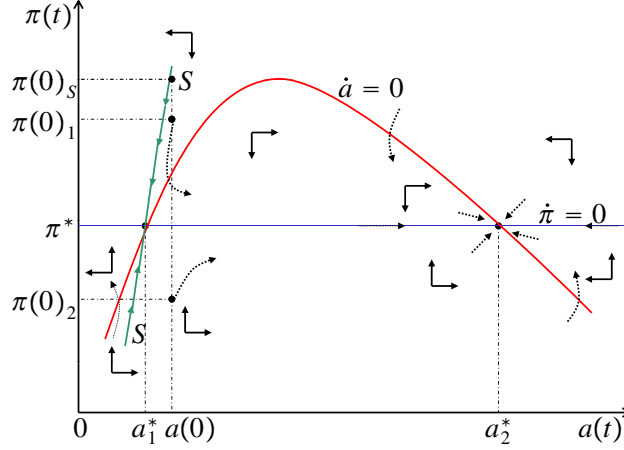


Figure 2: Dynamic behavior of $(\pi(t), a(t))$ with a non-linear fiscal regime and a passive interest rate rule.

that $(\pi(t), a(t))$ will converge asymptotically to the steady state (π^*, a_2^*) . The saddle manifold associated to (π^*, a_1^*) is thus the boundary of the basin of attraction of (π^*, a_2^*) . ■

Leeper (1991) and Woodford (2003) demonstrate an active fiscal, passive monetary regime yields local determinacy. In Figure 2, this result corresponds to the existence of a saddle path associated to the steady state (π^*, a_1^*) . If one restricts attention to local dynamics around (π^*, a_1^*) , for a given initial condition $a(0) \neq a_1^*$, there in fact exists a unique value of inflation, $\pi(0)_S = \pi^* + \frac{-L(r+\pi^*)(\psi-1)/\psi L'(r+\pi^*) - [r - \sigma'(a_1^*)]}{(\psi-1)a_1^*} (a(0) - a_1^*)$, such that $(\pi(t), a(t))$ will converge to the steady state (π^*, a_1^*) . Nevertheless, (π^*, a_1^*) is not the only steady-state equilibrium if the conduct of fiscal policy is non-linear. Proposition 1 shows that, when the fiscal regime is non-linear, even if the steady state (π^*, a_1^*) at which fiscal policy is active is locally a unique stable equilibrium, there exists an infinite number of equilibrium paths originating in the neighborhood of (π^*, a_1^*) which converge to the high-debt steady state (π^*, a_2^*) . Inflation no longer needs to “jump” on to saddle path ensure global stability. All initial values $\pi(0)$ below the saddle path are equilibrium values which make $(\pi(t), a(t))$ converge to the steady state (π^*, a_2^*) .⁴ This implies that the dynamic system is globally indeterminate.

Consider now the dynamic effects of an active interest rate rule ($\psi > 1$). The following proposition holds.

⁴Notice that the inflation rate follows a non-monotonous trajectory if $\pi(0)$ lies in the region between the saddle path and the isocline $\dot{a}(t) = 0$.

Proposition 2 *Suppose that fiscal policy is non-linear ($\sigma', \sigma'' > 0$) and monetary policy is active ($\psi > 1$). Then: (a) locally, the steady state (π^*, a_1^*) is a source and the steady state (π^*, a_2^*) is a saddle point; (b) globally, there exists a saddle connection joining the two steady states (π^*, a_1^*) and (π^*, a_2^*) , along which the inflation rate is uniquely determined ($\pi = \pi^*$).*

Proof. (a) Let $J_{(\pi^*, a_1^*)}$ be the Jacobian matrix of the equilibrium system (12)-(13) evaluated at the steady state (π^*, a_1^*) . We have $\text{tr}J_{(\pi^*, a_1^*)} = \frac{L(r+\pi^*)(\psi-1)}{\psi L'(r+\pi^*)} + [r - \sigma'(a_1^*)] > 0$ and $\text{det}J_{(\pi^*, a_1^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_1^*)]}{\psi L'(r+\pi^*)} > 0$, because $L'(r + \pi^*) < 0$, $\psi > 1$, and $\sigma'(a_1^*) < r$. Therefore, the steady state (π^*, a_1^*) is a source. Let $J_{(\pi^*, a_2^*)}$ be the Jacobian matrix of the system (12)-(13) evaluated at the steady state (π^*, a_2^*) . We have $\text{det}J_{(\pi^*, a_2^*)} = \frac{-L(r+\pi^*)(\psi-1)[r-\sigma'(a_2^*)]}{\psi L'(r+\pi^*)} < 0$, because $L'(r + \pi^*) < 0$, $\psi > 1$, and $\sigma'(a_2^*) > r$. Therefore, the steady state (π^*, a_2^*) is a saddle point, with the stable arm given by $\pi(t) = \pi^*$. (b) From (12) and (13), the two isoclines $\dot{\pi}(t) = 0$ and $\dot{a}(t) = 0$ are $\pi(t) = \pi^*$ and $\pi(t) = \pi^* + \frac{[\sigma(a(t))/a(t)] - r}{(\psi-1)}$, respectively. We have $\left. \frac{d\pi(t)}{da(t)} \right|_{\dot{a}(t)=0} = \frac{\sigma'(a(t)) - [\sigma(a(t))/a(t)]}{(\psi-1)a(t)}$, which is negative at the steady state (π^*, a_1^*) , where $\sigma'(a_1^*) < [\sigma(a_1^*)/a_1^*] = r$, equal to zero for $\sigma'(a(t)) = [\sigma(a(t))/a(t)]$, and positive at the steady state (π^*, a_2^*) , where $\sigma'(a_2^*) > [\sigma(a_2^*)/a_2^*] = r$. Thus, in the phase plane $(\pi(t), a(t))$, the isocline $\dot{\pi}(t) = 0$ is horizontal, the isocline $\dot{a}(t) = 0$ is U-shaped, and the two isoclines intersect at (π^*, a_1^*) and (π^*, a_2^*) . From (12) and (13), $\dot{\pi}(t) > (<) 0$ if $\pi(t) > (<) \pi^*$ and $\dot{a}(t) > (<) 0$ if $\pi(t) > (<) \pi^* + \frac{\sigma(a(t)) - ra(t)}{(\psi-1)a(t)}$. Figure 3 shows the global dynamics of the system. There exists a unique trajectory originating in the neighborhood of the steady state (π^*, a_1^*) and converging to the other steady state (π^*, a_2^*) . The saddle connection joining the two steady states is given by the isocline $\dot{\pi}(t) = 0$ ($\pi = \pi^*$), which is also the stable arm of the saddle-path stable steady state (π^*, a_2^*) , *SS*. ■

If monetary policy is active, the steady state (π^*, a_1^*) at which fiscal policy is active now delivers local instability, consistently with the results obtained in Leeper (1991) and Woodford (2003). Therefore, restricting attention to local dynamics around (π^*, a_1^*) , no stable equilibria exist. However, Proposition 2 shows that, in the case in which the fiscal policy stance is non-linear, even if the steady state (π^*, a_1^*) is locally unstable, there exists a saddle connection with the high-debt steady state (π^*, a_2^*) . As it emerges from Figure 3, the saddle connection corresponds to the isocline $\dot{\pi}(t) = 0$, $\pi = \pi^*$. Thus, under a non linear fiscal regime, the dynamic system is globally determinate and inflation is pinned down to the target rate even around the steady state at which fiscal policy is

active displays local instability, but there exists a saddle connection with the high-debt equilibrium along which inflation is uniquely pinned down.

Thus, the analytical results derived in this paper suggest that considering potential non-linearities in governments' budgetary policies may be essential for a global characterization of monetary-fiscal policy interactions. Extensions of the present framework aimed to incorporate, for example, the zero-lower bound problem on nominal interest rates, nominal rigidities, distortionary taxation, sovereign risk, and/or agents' learning, may constitute the object of further research.

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