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Abstract

Why don't people buy annuities? Several explanations have been provided by the previous literature: large fraction of preannuitized wealth in retirees' portfolios; adverse selection; bequest motives; and medical expense uncertainty. This paper uses a quantitative model to assess the importance of these impediments to annuitization and also studies three newer explanations: government safety net in terms of means-tested transfers; illiquidity of housing wealth; and restrictions on minimum amount of investment in annuities. This paper shows that quantitatively four explanations play a big role in reducing annuity demand: preannuitized wealth, minimum annuity purchase requirement, illiquidity of housing wealth, and bequest motives. The annuity purchase involves big upfront investment, especially if there is a minimum purchase restriction. This is binding for many, especially if housing is illiquid and part of wealth is preannuitized. While bequest motives significantly reduce the overall annuity demand, the model with a strong bequest motive cannot match the empirical fact that high-income people have the highest demand for annuities.

Keywords: annuity puzzle, longevity insurance, adverse selection

JEL Classification Codes: D91, G11, G22

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1 Introduction

A well-known prediction of the standard life-cycle model is that in the presence of lifespan uncertainty, people should invest in nothing but annuities (Yaari, 1965). In practice few people buy annuities. This empirical fact is called the "annuity puzzle". The literature seeking to explain this puzzle has mainly attributed the lack of interest in annuities to the following four factors: a substantial fraction of preannuitized wealth in retirees' portfolios, actuarially unfair prices, bequest motives, and uncertain health expenses. It is still an open question, however, what is the relative quantitative importance of different explanations for the annuity puzzle. The goal of this paper is therefore to provide a quantitative analysis of people's decisions to buy annuities in a model that nests all major impediments to annuitization.

I develop a quantitative model of saving after retirement in which individuals face lifespan uncertainties that create a demand for longevity insurance. At the same time the available annuities are illiquid, i.e., they entitle a person to a constant stream of income that cannot be converted back to liquid wealth. The other key features of the model are uncertain medical expenses, bequest motives, preannuitized wealth, and the government-provided minimum consumption level. Augmented in this way, the life-cycle model allows for states, when it is not optimal for an individual to lock his wealth in a constant stream of income.

Another important feature of the model is that annuity prices are determined in equilibrium. When modeling the annuity market I compare two information structures. In the first, the insurer and the annuity buyer have the same information about the mortality of the latter. In the second, there is asymmetric information, and the insurer can only observe the age of the annuity buyer. The latter scenario creates an environment for adverse selection which is intensified by the negative correlation between wealth and mortality. This happens because retirees with low mortality buy more annuities because they not only expect to live longer but are also wealthier.

The main quantitative exercise of this paper consists of comparing annuity market participation rates between the models that incorporate different impediments to annuitization. I study seven explanations for the annuity puzzle: four traditional ones and three factors that have been studied much less. The latter include government provided social assistance, difficulties with annuitizing housing wealth, and a minimum purchase requirement set by insurance companies.

The consumption minimum floor, among other things, provides financial support for people if they outlive their assets and thus offers some longevity insurance. This public longevity insurance may partially substitute for a private annuity, at least for low-income retirees.

Another possible impediment to annuitization arises because annuities pay off over a long period of time and, as such, involve a big upfront investment. When it comes to buying an annuity, liquidity constraints may therefore become an issue because, first, housing wealth may not be easily annuitized and, second, insurance firms place restrictions on the minimum amount that can be invested in an annuity.

Housing wealth has properties that distinguish it from other assets. It is less liquid and also it can provide utility benefits from homeownership. The fact that housing is decumulated at a substantially slower rate than other types of wealth suggests that retirees treat housing differently from other assets.[1] Since housing constitutes a significant portion of retirement wealth, this can also affect the retirees' willingness to annuitize.

Another consideration is that from the point of view of an economic model, an individual may find it rational to buy $1 worth of annuity. In reality insurance companies set some restrictions on the minimum amount of investment in an annuity. The minimum premium for a life annuity varies across insurance companies but can go up to $100,000.

I find that the following four factors play a major role in reducing annuity market participation rates: preannuitized wealth, minimum annuity purchase requirements, illiquidity of housing wealth, and bequest motives. Bequest motives also result in a non-monotone relationship between income and annuity demand which goes in contrast with the data. The consumption minimum floor can be very important if there is no preannuitized wealth. Adverse selection decreases the annuity demand only for people in low income quintiles, while for higher quintiles it has an opposite effect. Because of this its overall effect is small. Uncertain medical expenses have a small impact on annuity ownership rates but noticeably affect the life-cycle patterns of annuitization.

The paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model. Section 4 describes the data and calibration. Section 5 presents and discusses the results. Section 6 concludes.

2 Related literature

This paper is related to two strands of literature. First, it belongs to the literature that studies the annuity puzzle. The literature seeking to explain this puzzle has identified four factors that may play a major role in reducing the demand for annuities on the part of single retirees. First, individuals already have a substantial fraction of annuities in their portfolio provided by Social Security and Defined Benefits (DB) pension plans (Dushi and Webb, 2004). Second, the prices for annuities are actuarially unfair due to the presence of adverse selection (Mitchell, Poterba, and Warshawsky, 1997). Third, annuitized wealth cannot be bequeathed, thus individuals with bequest motives should

[1]Nakajima and Telyukova (2011) describe in detail the empirical patterns of housing and non-housing wealth decumulation after retirement.

have lower demand for annuities (Lockwood, 2012a). Fourth, the attractiveness of annuities can decrease in the presence of a health uncertainty. The possibility of incurring high medical expenses increases preferences for liquid wealth as opposed to an illiquid annuity (Turra and Mitchell, 2008). Also high medical expenses coincide with health deterioration, which increases mortality and decreases the value of an annuity (Sinclair and Smetters, 2004). The contribution of my study to the literature on the annuity puzzle is twofold. First, it extends the list of commonly studied factors contributing to the annuity puzzle. Second, it provides a relative quantitative assessment of all these impediments to annuitization.

The second strand of literature this paper is related to studies equilibrium in the annuity markets in the presence of adverse selection. Hosseini (2009) evaluates the benefits of the mandatory annuitization feature of Social Security. He considers an equilibrium where agents differ only by their mortality. Walliser (1999) studies the effects of Social Security on the private annuity market. He constructs an environment where agents are heterogeneous both by mortality and income and allows for the income-mortality correlation. I augment the heterogeneity of individuals by health and medical expenses, which allows me to get a more detailed picture of the effects of adverse selection on different categories of population.

3 Model

Consider a portfolio choice model of a single retiree who decides how much to save and how to split his net worth between bonds and annuities while facing uncertain lifespan and out-of-pocket medical expenses.[2]

Agents are heterogeneous by age, health status, initial endowment of wealth, and permanent income. Permanent income represents annuity-like income that an agent is entitled to receive during his retirement years. It consists of Social Security and DB pension wealth and it is an indicator of the agents' lifetime earnings. In addition, it affects survival probability, health evolution and medical expenses.

3.1 Households

3.1.1 Preferences

Denote the age of an individual by t, $t = 1, ...T$, where T is the last period of life. Households are assumed to have CRRA preferences:

[2]I consider only singles because annuitization decisions of married households can be different since couples have an option to buy joint annuities. This type of annuity continues payments as long as at least one of the spouses is alive and it allows for payout options that cannot be achieved by single-life annuity products (Brown and Poterba, 2000).

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

and enjoy leaving a bequest. Utility from the bequest takes the following functional form:

$$\upsilon(k_t) = \eta \frac{(\phi + k_t)^{1-\sigma}}{1-\sigma}$$

with $\eta > 0$. Here $\phi > 0$ is a shift parameter making bequests luxury goods, thus allowing for zero bequests among low-income individuals.

3.1.2 Health, survival, and medical expenses

In specifying medical expenses and survival uncertainty, I follow De Nardi, French, and Jones (2010) (hereafter DFJ). Their framework is well-suited for studying heterogeneity in annutization decisions because they explicitly model the relationships among several factors which affect the demand for annuities: income, life expectancy, and medical expenses.[3]

Each period an individual's health status m_t can be good ($m_t = 1$) or bad ($m_t = 0$). The transition between health states is governed by a Markov process with a transition matrix depending on age (t) and permanent income (I). The probability of being in bad health tomorrow given the current health status is denoted by $\Pr(m_{t+1} = 0 | m_t, t, I)$.

An individual survives to the next period conditional on being alive today with probability s_t, where $s_T = 0$. Survival is a function of age, permanent income and current health status: $s_t = s(m, I, t)$.

Each period, an agent has to pay medical costs, z_t, which are assumed to take the following form:

$$\ln z_t = \mu(m, t, I) + \sigma_z \psi_t, \tag{1}$$

The unconditional mean of medical expenses $(\exp(\mu(m, t, I) + 0.5\sigma_z^2))$ is a function of age, health, and permanent income. The stochastic part of medical expenses ψ_t consists of persistent and transitory components.

$$\psi_t = \zeta_t + \xi_t, \quad \xi_t \sim N(0, \sigma_\xi^2)$$

The persistent component is modeled as an AR(1) process:

$$\zeta_t = \rho_{hc} \zeta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{2}$$

[3] In the DFJ model there is an additional state variable that affects health uncertainty and mortality: gender. My model does not include gender, so, when using the DFJ estimates, the effect of gender on all the variables was averaged out. Adding gender will complicate the computation of the annuity market equilibrium. Since gender is an observed characteristic, adding it will double the number of prices I need to compute.

I denote the joint conditional distribution of ζ_t and ξ_t by $F(\zeta_t, \xi_t|\zeta_{t-1})$.

3.1.3 Government transfers

An agent who does not have enough resources to pay for his medical expenses receives a transfer from the government in the amount τ_t. This transfer maintains the agent's consumption at a minimum level guaranteed by the government $c_{\min}$.

3.1.4 Portfolio choice

Individuals have two investment options - a risk-free bond with return r and an annuity - and cannot borrow. Once the annuity is bought, it cannot be sold. The annuity is modeled in the following way: by paying the amount $q_t\Delta_{t+1}$ today, an individual buys a stream of payments Δ_{t+1} that he will receive each period, conditional on being alive. I denote the total annuity income an agent receives at age t by n_t.

3.1.5 Optimization problem

Each period an individual decides how to distribute his current wealth between consumption (c_t) and investments in bonds (k_{t+1}) and annuities (Δ_{t+1}), given that he has to pay medical expenses (z_t). I denote the set of state variables I, t, n_t, k_t as X_t, $X_t = (I, t, n_t, k_t)$. The recursive formulation of the optimization problem can be represented in the following form:

$$V(X_t, m_t, \zeta_t, \xi_t) = \max_{c_t, k_{t+1}, \Delta_{t+1}} \left\{ \begin{array}{c} u(c_t) + \beta s_t \Pr(m_{t+1} = 0|m_t, t, I)\times \\ \int\limits_{\zeta,\xi} V(X_{t+1}, 0, \zeta_{t+1}, \xi_{t+1}) dF(\zeta_{t+1}, \xi_{t+1}|\zeta_t)+ \\ \beta s_t \Pr(m_{t+1} = 1|m_t, t, I)\times \\ \int\limits_{\zeta,\xi} V(X_{t+1}, 1, \zeta_{t+1}, \xi_{t+1}) dF(\zeta_{t+1}, \xi_{t+1}|\zeta_t)+ \\ \beta(1 - s_t) v(k_{t+1}) \end{array} \right\} \tag{3}$$

s.t. the budget constraint:

$$c_t + z_t + k_{t+1} + q_t\Delta_{t+1} = k_t(1 + r) + n_t + \tau_t,$$

government transfers

$$\tau_t = \min\left\{0, c_{\min} - k_t(1 + r) - n_t + z_t\right\},$$

the annuities evolution equation

$$n_{t+1} = \Delta_{t+1} + n_t,$$

borrowing and annuity illiquidity constraints: $k_{t+1}, \Delta_{t+1} \geq 0$, and initial conditions k_0, m_0, and $n_0 = I$.

3.2 Insurance sector

I assume that annuity contracts are non-exclusive: individuals are free to buy an arbitrary number of contracts from different insurance companies. This makes it impossible to condition the contract design on the amount purchased. Contracts are linear, i.e. price of a unit of coverage does not depend on quantity.[4] Thus to purchase Δ units of annuity coverage, an individual pays $q\Delta$ in premiums.

I assume that insurance firms set a restriction on the minimum amount that can be invested in annuities equal to $\overline{\Delta}$. This restriction is motivated by the observation that annuity sellers do not allow for arbitrary small investments in annuities. For example, two big annuity distributors, Vanguard and Berkshire-Hathaway, put restrictions of $20,000 and $40,000, respectively, on the minimum premium for a life annuity.[5]

Another restriction that annuity buyers face is the maximum issue age $\bar{t}$. Individuals older than $\bar{t}$ cannot buy annuities. This restriction reflects the fact that in most states insurance companies are prohibited from selling annuities to individuals beyond a certain age (Levy et al., 2005).

To assess the importance of adverse selection I need to evaluate how much it changes the annuity prices. Adverse selection may arise when insurance firms have less information about the mortality of individuals buying annuities than the individuals do. In this environment, when setting the price insurers take into account the average mortality of all annuity buyers. This average mortality can be substantially lower than the average mortality of people who do not buy annuities. Moreover, the more annuities people with low mortality buy, the higher is the price and the more people with high mortality drop out of the market. To understand the quantitative importance of this problem, I need to compare an individual's decisions to buy annuities in two situations: i) when the annuity price reflects his own survival probability; ii) when the annuity price reflects the average survival probability of all annuity buyers.

To do this, I consider two scenarios. Under the first scenario insurance firms are allowed to observe all state variables of an individual that are relevant for forecasting his survival probability. As a result, annuities are individually-priced. I call this setup the "symmetric information scenario".

In the second scenario insurers know the aggregate distribution of individuals over states, but they cannot observe any characteristics of an annuity purchaser except age. I

[4]Pauly (1974) points out that exclusive contracts are hard to implement in a competitive insurance market making it impossible for insurers to price discriminate over units.

[5]An alternative modeling strategy would be to restrict minimum premium $q\Delta$. However, imposing the restriction on minimum premium rather than minimum purchase does not change the results.

call this setup the "asymmetric information scenario". In this environment all people of the same age buy annuities at a uniform price that reflects the average mortality of the pool of annuity buyers.[6]

I assume insurance firms act competitively: they take the price of an annuity q_t as given. Let $S_{t+i|t}$ denote the probability that an individual survives till period $t+i$ conditional on being alive at period t. Expected payout per unit of insurance sold to an individual of age t can be expressed as follows:

$$\pi_t(\Omega_t) = q_t(\Omega_t) - \gamma \sum_{i=1}^{T-t} \frac{\widehat{S}_{t+i|t}(\Omega_t)}{(1+r)^i}, \tag{4}$$

where $\gamma \geq 1$ is the administrative load, assumed to be proportional to the total expected payment for the contract[7], Ω_t is the information available to insurers about an individual of age t, and $\widehat{S}_{t+i|t}(\Omega_t)$ is the insurers' expectation of the future survival probability of an individual buying the annuity given all available information Ω_t. It can be expressed as follows:

$$\widehat{S}_{t+i|t}(\Omega_t) = E_t(S_{t+i|t}|\Omega_t).$$

In the symmetric information case, an insurer and an annuity buyer have the same information. Thus, Ω_t includes all variables relevant for determining the survival probability of a person of a given age:

$$\Omega_t = (m_t, I).$$

In the asymmetric information case, an insurer does not know anything about an individual except the age and the fact that he bought an annuity, so:

$$\Omega_t = (\Delta_{t+1}(k, n, m, I, t, \zeta, \xi) \geq \overline{\Delta}\ ).$$

In this case $\widehat{S}_{t+i|t}$ represents a firm's belief about the probability that an individual who buys an annuity will survive until period $t+i$. In equilibrium, $\widehat{S}_{t+i|t}$ has to be consistent with the optimal behavior of individuals.

Firms chooses the amount of annuity to sell (N_t) by solving the following maximization problem:

$$\max_{N_t} N_t \pi_t. \tag{5}$$

[6]This outcome resembles the current situation in the market for longevity insurance in the U.S. - annuity prices are usually conditioned only on age and gender.

[7]An alternative modeling strategy would be to set the load as an additive term. The results are not sensitive to this modeling assumption as the robustness check in Appendix C illustrates.

3.3 Competitive equilibrium

Let V denote the state variables of individuals, $V = (k, n, m, I, \zeta, \xi)$, $V \in \mathcal{V} = \mathcal{K} \times \mathcal{N} \times \mathcal{M}$ $\times \mathcal{I} \times \mathcal{Z} \times \Xi$ where $\mathcal{K} = R^{+} \cup \{0\}$, $\mathcal{N} = R^{+} \cup \{0\}$, $\mathcal{M} = \{0, 1\}$, $\mathcal{I} = \{I_1, I_2, I_3, I_4, I_5\}$, $\mathcal{Z} = R$, $\Xi = R$. Denote the distribution of individuals of age t over states by $\Gamma_t(V)$.

The competitive equilibrium for the asymmetric information case can be defined as follows.

Definition 1 *A competitive equilibrium is:*
(i) a set of belief functions $\left\{ \widehat{S}_{t+i|t},\ i = 0, .., T - t \right\}_{t=1}^{T}$
(ii) a set of annuity prices $\{q_t\}_{t=1}^{T}$
(iii) a set of decision rules for households $\{c_t^*(V), k_{t+1}^*(V), \Delta_{t+1}^*(V),\ V \in \mathcal{V}\}_{t=1}^{T}$ *and for insurance firms* $\{N_t^*\}_{t=1}^{T}$

such that:

1. Each annuity seller earns zero profit:

$$N_t^* \pi_t = 0$$

2. Firms' belief functions are consistent with households' decision rules:

$$\widehat{S}_{t+i|t} = \frac{\int\limits_{\mathcal{V}} \Delta_{t+1}^*(V) \Gamma_{t+i}^t(V) dV}{\int\limits_{\mathcal{V}} \Delta_{t+1}^*(V) \Gamma_t(V) dV} \tag{6}$$

where $\Gamma_{t+i}^t(V)$ is the measure of people of age $t+i$ who bought an annuity in the amount $\Delta_{t+1}^*(V)$ at age t. It can be defined recursively in the following way:

$$\Gamma_{t+1}^t(V) = s(m, I, t) \Gamma_t(V)$$

$$\Gamma_{t+i}^t(V) = \int\limits_{\widetilde{m} \in \mathcal{M}} s(\widetilde{m}, I, t + i - 1) \Gamma_{t+i-1}^t(V, \widetilde{m}) d\widetilde{m}$$

Here $\Gamma_{t+i-1}^t(V, \widetilde{m})$ is the distribution of people aged $t + i - 1$ who bought an annuity in the amount $\Delta_{t+1}^*(V)$ at age t across their current health status $\widetilde{m}$. It can be recursively expressed as follows:

$$\Gamma_{t+1}^t(V, \widetilde{m}) = \Pr(\widetilde{m}|m, t, I) s(m, I, t) \Gamma_t(V)$$

$$\Gamma_{t+i}^t(V, \widetilde{m}) = \int\limits_{\overline{m} \in \mathcal{M}} \Pr(\widetilde{m}|\overline{m}, t + i - 1, I) s(\overline{m}, I, t + i - 1) \Gamma_{t+i-1}^t(V, \overline{m}) d\overline{m}$$

3. Given annuity prices $\{q_t\}_{t=1}^{T}$, households' decision rules solve optimization problem (3) and N_t^* solves equation (5).

4. The market clears

$$N_t^* = \int\limits_{\mathcal{V}} \Delta_{t+1}^*(V)\Gamma_t(V)dV.$$

The definition of the competitive equilibrium for the symmetric information scenario is similar, with the following modifications: the annuity prices now depend on m_t and I and the second condition for the equilibrium takes the form:

$$\widehat{S}_{t+i|t}(\Omega_t) = E_t(S_{t+i|t}|m_t, I_t).$$

4 Data and calibration

4.1 Parameters calibration

The model period is two years. Retirees in the model start their life at age 70 and live at a maximum to age of 100.

The annual interest rate r is set to 2%. The administrative load γ is assumed to be equal to 10%. This number is based on the study of Mitchell et al. (1999) which showed that on average, U.S. insurance companies add 10% to the annuity price because of administrative costs. The maximum issue age is set to be equal to 88 years. In general, the maximum issue age varies by state and ranges from 80 years old to the mid-90s (Levy et al, 2005).

The minimum purchase requirement is set to \$2,500. This means that in order to buy an annuity, an individual should be willing to initiate a contract that will bring him at least \$2,500 per year or \$208 per month. Given prices produced by the model, this is equivalent to a minimum initial premium ($q\Delta$) of approximately \$28,000 for a 70 year old and \$12,000 for an 88 year old. This is in line with the restrictions on minimum premiums set by big annuity distributors such as Vanguard and Berkshire-Hathaway.

The parameters governing the evolution of health, survival, and medical expenses come from papers by DFJ and French and Jones (2004). The persistence parameter (ρ_{hc}) is set to 0.849. The innovation variance of the persistent component σ_ε^2 is equal to 0.133 and the innovation variance of the transitory component σ_ξ^2 is 0.524. The sum of persistent and transitory components is normalized to one.[8] The variance of the log medical expenses σ_z^2 is equal to 1.78.

[8] DFJ's estimates of the stochastic component of medical expenses are based on the results of French and Jones (2004). The difference between these two studies is that DFJ uses one year medical expenses while French and Jones (2010) use two-year averages. Since in my model one period is two years, I use French and Jones's (2004) estimates and adjust the mean. The corresponding parameters for the one year process are: $\rho_{hc} = 0.922$, $\sigma_\varepsilon^2 = 0.050$, and $\sigma_\xi^2 = 0.665$.

For the discount factor β, preference parameters σ, η, and ϕ, and the minimum consumption floor $c_{\min}$ I use structural estimates from the DFJ study. In particular, β is set to 0.97, σ to 3.84, and $c_{\min}$ to \$2,665. The strength of the bequest motive η is set to 2,360 and the shift parameter ϕ to \$273,000.[9] Table 1 summarizes all the parameter values.[10]

Parameter	**Value**
Risk aversion σ	3.84
Discount factor β	0.97
Strength of bequest motive η	2,360
Shift parameter ϕ	\$273,000
Interest rate r	2%
Administrative load γ	10%
Consumption floor $c_{\min}$	\$2,665
Maximum issue age $\bar{t}$	88 years
Minimum purchase $\overline{\Delta}$	\$2,500
Persistence ρ_{hc}	0.849
Variance of medical costs σ_z^2	1.78
Variance of transitory shock σ_ξ^2	0.524
Variance of persistent shock σ_ε^2	0.133

Table 1: Parameters of the model

4.2 Initial distribution

To calibrate the initial distribution of retirees over the state variables k_0, n_0, m_0 I use the Health and Retirement Study (HRS) dataset. I use the RAND version of all the variables except for the annuity ownership rates. Individuals in the model start their retired life at age 70, so I used the cohort aged 65-75 in 1998 (wave 4) to calibrate the initial distribution.[11] The sample used for simulations includes only single retirees.

[9] The results for several alternative values of the coefficients of risk aversion, discount factor and bequest parameters are reported in Appendices A and B.

[10] All parameters correspond to annual values except for parameters describing the evolution of stochastic medical expenses.

[11] The model of this study builds on the DFJ framework, however I cannot use the same cohort as DFJ for the calibration of the initial distribution. For simulations I need a large sample of people around age 70 because few people buy annuities. The AHEAD dataset that is used by DFJ includes people older than age 70. The only wave of AHEAD that has a large sample of people younger than 75 is wave 1 (1993), however this wave has a serious wealth underreporting problem (Rohwedder et al, 2006). I chose to use the fourth wave of HRS because wave three has some inaccuracy in wealth data (Lockwood, 2012b). All other sample selection criteria are the same as in DFJ.

	All	Income quintiles				
		1st	2nd	3rd	4th	5th
Mean						
Age	70.4	68.8	70.7	70.6	71.0	70.7
Total wealth	161,311	70,942	78,213	173,047	197,731	287,082
Non-housing wealth	107,047	47,147	41,916	116,132	134,201	196,168
Income	12,964	4,403	7,766	10,667	14,820	23,206
Median						
Age	71	68	71	71	72	71
Total wealth	64,000	3,000	22,500	71,000	112,000	176,950
Non-housing wealth	13,000	43	2,300	15,650	44,000	87,200
Income	10,663	4,773	7,802	10,671	14,720	23,725
Percent						
Healthy	59.2	39.7	54.2	61.5	66.3	74.3
Own life annuity	5.0	0.4	1.0	5.3	6.4	12.2

Table 2: Sample characteristics

Singles are defined as people who are divorced, never married or widowed and who reported having no partner. A person is defined as retired if his annual earnings were below \$3,000. The resulting sample size is 1,483. I convert all dollar variables to constant 1998 dollars using the Consumer Price Index (CPI).

Initial wealth (k_0) includes the value of housing and real estate, vehicles, value of business, IRAs, Keoghs, stocks, bonds, checking, saving and money market accounts, minus mortgages and other debts. Preexisting annuity holdings (n_0) correspond to annuity-like income that an individual is entitled to receive during his retirement years and it also proxies permanent income (I). I follow DFJ in defining permanent income (or annuity-like income) as the sum of Social Security benefits, DB pensions, veteran benefits, food stamps, and annuities that individuals receive each year and then take the average over all years that individuals are observed in the data. Table 2 displays means and medians for the total and non-housing wealth and income in the sample used for simulations. The displayed statistics illustrate a substantial heterogeneity across income quintiles. People in the top quintile receive income that is on average almost five times higher than people in the bottom quintile. The disparity in wealth holdings is even more pronounced. Median total wealth of retirees in the bottom quintile is only \$3,000, while for the top quintile it is close to \$180,000.

Initial health (m_0) is defined based on self-reported health. Individuals who report being in excellent, very good and good health are classified as healthy, while individuals who report their health being fair and poor are classified as unhealthy. The resulting fraction of healthy people is reported in the ninth row of Table 2. Overall, most people report being healthy. However, there is substantial heterogeneity by income quintiles. The fraction of healthy people in the top income quintile (74%) is almost twice the fraction of healthy people in the bottom quintile (38%).

When constructing annuity ownership rates for my sample, I define a person as having life annuity if the following two conditions hold. First, he answers 'yes' to the question of whether he receives income from an annuity other than pensions or Social Security. Second, he reports that at least one of his two largest private annuities continues for life. The resulting annuity ownership rates are shown in the last row of Table 2. Overall, only 5% of people own life annuities.[12] There is a substantial heterogeneity in annuity market participation rates by income quintiles. While almost no retirees in the bottom quintile own annuities, in the top quintile the annuity ownership rate exceeds 12%.

5 Results

This section describes the model predictions about the annuity market participation rates and compares them to the data. To evaluate the combined effect of all factors behind non-annuitization I consider two versions of the model: the model that has no impediments to annutization (hereafter called the simple model), and the model with all seven impediments to annuitization (hereafter called the full model). The full model is as described in Section 3 (with asymmetric information equilibrium) while the simple model represents its stripped-down version where all features that can negatively affect the annuity demand are assumed away.

To evaluate the relative quantitative importance of different factors behind non-annuitization, I consider two sets of experiments. I start with the full model and remove impediments for annuitization one at a time, keeping all other model features constant. I then compare the annuity market participation rates between the full model and the model where one impediment to annuitization is missing. In the next set of experiments, I start with the simple model and I add different impediments to annuitization one at a time. Then I compare the annuity market participation rates between the simple model and the model where only one impediment to annuitization is present.

The experiments that switch on/off different factors affecting the annuity demand are designed in the following way.

Adverse selection. The model without adverse selection assumes symmetric information equilibrium in the annuity market , i.e. retirees face individually priced annuities. The model with adverse selection assumes asymmetric information equilibrium, i.e. there is only one pooling price for each age.[13]

[12] The percentage of people who own both life and period-certain annuities constitutes 7.3%.

[13] It is important to note that in the model without medical expenses the equilibrium in the annuity market is not unique. This happens because, as will be shown later, in the absence of medical expenses most people buy annuities only once in the first period. However, there may exist a group of people that buy annuities several times and the timing of their second purchase depends on the equilibrium prices for ages above 70. To be consistent in comparisons, when considering versions of the model without

Consumption floor. The consumption floor $c_{\min}$ estimated by DFJ ($2,665) is on the low side of what is commonly used in the literature (see Kitao and Jeske, 2009 and Kopecky and Koreshkova, 2011). In the model with high consumption floor this number is raised to $6,000.

Illiquid housing. In the model with illiquid housing initial wealth k_0 is redefined as total wealth minus housing wealth.[14]

Preannuitized wealth. In the model with preannuitized wealth retirees receive annuity-like income n_0 from Social Security and DB plans as observed in the data. In the model without preannuitized wealth it is assumed that all annuity-like income is converted to liquid wealth. This conversion is done by assuming that individuals sell all annuity-like income they are entitled to receive at market prices for the case of symmetric information equilibrium.[15]

Minimum purchase requirements, medical expenses, and bequest motives. In the model without minimum purchase requirement $\overline{\Delta}$ is set to zero, i.e. people can buy any amount of annuities. In the model without medical expenses $z_t = 0$ for all t. In the model without bequest motives the strength of bequest η is set to zero.

5.1 Combined effect of all the impediments to annuitization

Table 3 compares annuity market participation rates for people at age 70 for the data, the simple model and the full model.[16] The simple model predicts that almost 97% of people buy annuities. Those few people who do not participate in the annuity market come from the bottom income quintile: in this quintile 82.8% of people buy annuities, while in all other quintiles the participation rate is 100%. The non-participating individuals are those who start retirement with almost zero wealth and who rely mostly on government means-tested transfers.

The full model goes a long way towards decreasing the annuity demand: once all potential explanations for the annuity puzzle are included, the participation rate drops from 97% to around 20%.[17] However, it still remains higher than in the data (5%).

medical expenses I only focus on equilibriums where people buy annuities only once in the first period. This does not affect the main results because the group that chooses to buy additional annuities later is very small.

[14] Ideally, one would want to allow people to adjust their housing wealth inside the model. However, this involves adding another state variable which makes the model computationally intractable. To get some idea of the importance of housing wealth, I compare two extreme assumptions: i)housing has the same liquidity as other types of wealth, ii) housing is absolutely illiquid.

[15] If I assume individuals sell preexisting annuities at prices observed under equilibrium with asymmetric information the resulting amount of liquid wealth will differ from one experiment to another due to the changing prices.

[16] I show the participation rates only at age 70 because, as shown later, people start buying annuities in the first period of retirement and in most cases they do it once.

[17] The equilibrium annuity prices produced by the full model are reported in Appendix D.

Income quintile	Data	No impediments to annuitization	All impediments to annuitization
All	5.0	96.6	20.3
1	0.4	82.8	7.0
2	1.0	100.0	14.6
3	5.3	100.0	26.2
4	6.4	100.0	30.7
5	12.2	100.0	23.2

Table 3: Participation in the annuity market: data, model with no impediments to annuitization, and model with all impediments to annuitization

Section 5.4 discusses what changes in the benchmark calibration can move the model closer to the data.

5.2 Relative importance of different impediments to annuitization

5.2.1 Analyzing the full model

The top panel of Table 4 shows how much annuity market participation rates change when only one impediment to annutization is removed from the full model. The most quantitatively important factor is preannuitized wealth. Elimination of this factor increases the annuity ownership rate more than twofold. If there is no preannuitized wealth around 42% of retirees will buy annuities even if all other impediments to annuitization are present. The next three most important factors are illiquid housing, minimum purchase requirement and bequest motives: without each of these factors the annuity ownership rates will be more than 35%. It is important to note that illiquidity of housing, preannuitized wealth and minimum purchase requirement act through a similar mechanism: they decrease the possibilities for annuitization. Annuities pay out for a long period of time and thus involve big upfront investments. Minimum purchase requirement increases these upfront costs, and illiquidity of housing and preannuitized wealth decrease the amount of wealth available for annuitization. This suggests that disposable wealth is an important factor affecting the demand for annuities, especially if there is a restriction on the minimum amount that can be invested.[18]

There is some heterogeneity in terms of how different impediments to annuitization affect people in different income quintiles. For the two bottom quintiles, illiquidity of

[18] Inkman et al (2011) also find that wealth is a quantitatively important factor affecting households' annuity demand.

Income quintile	Full model	No adv. selection	No med. expense	Low c_{min}	No bequest	Liquid housing	No min. purchase	No preann wealth
All	20.3	19.9	23.6	27.2	36.3	39.0	35.8	42.0
1	7.0	10.5	10.1	8.8	8.1	18.7	12.1	3.9
2	14.6	20.6	20.9	15.8	17.4	32.7	29.3	6.9
3	26.2	33.7	32.9	29.9	34.8	50.3	48.5	29.0
4	30.7	30.1	33.3	43.6	53.5	55.0	51.5	75.7
5	23.2	5.0	20.9	38.1	67.7	38.5	37.8	94.7
Income quintile	Simple model	+ adv. selection	+ med. expense	High c_{min}	+ bequest	Illiquid housing	+ min. purchase	+ preann wealth
All	96.9	92.2	92.8	84.1	96.6	95.7	95.1	83.5
1	82.8	60.9	65.5	32.9	82.8	78.5	75.8	59.6
2	100.0	100.0	98.7	88.2	100.0	100.0	100.0	77.4
3	100.0	100.0	100.0	99.7	100.0	100.0	100.0	89.8
4	100.0	100.0	100.0	100.0	100.0	100.0	100.0	95.9
5	100.0	100.0	100.0	100.0	100.0	100.0	100.0	94.6

Table 4: Annuity market participation rates for the modifications of the full model (top panel) and the simple model (bottom panel)

housing wealth has the largest quantitative impact on the fraction of annuity market participants. The removal of preannuitized wealth, however, decreases their demand for annuities. When Social Security and DB pension income is converted to liquid wealth, the annuity ownership rate for the bottom income quintile goes down from 7.2% to 3.9%. This happens because of the interaction of preannuitized wealth and a high consumption floor which will be discussed later when analyzing the simple model. For the top income quintile, the two most important factors affecting the annuity demand are preannuitized wealth and bequest motives. This happens because people in this group hold a large amount of preannuitized wealth and its transformation into liquid wealth has a large impact on resources available for annuitization. As for bequests, these are luxury goods and this strongly affects people in the top income quintile because they have higher wealth (see Table 2). Note that no other income quintile is affected by bequest motives to such an extent as the fifth quintile.

Another important observation from the top panel of Table 4 is that the removal of adverse selection from the full model has almost no impact on the overall annuity ownership rate. However, this hides a substantial heterogeneity in how this factor affects people in different income quintiles. In the absence of adverse selection there is a drop in the annuity market participation rate from the top two income quintiles and an increase in the participation from the bottom three quintiles.

The heterogeneous effect of adverse selection arises because the survival probability is positively correlated with income. In the asymmetric information equilibrium when annuities are priced only based on age, people in high income quintiles (and low survival probabilities) enjoy annuity prices that are lower than they would face if their survival probability was observed. In other words, higher income quintiles get an implicit subsidy from low income quintiles. Table 5 provides a quantitative assessment of this subsidy. For people in the lowest income quintile and in bad health, the pooling equilibrium price is around 57% higher than the price they face if insurance firms observe their mortality. At the other extreme, people in the highest income quintile and in good health pay almost 12% less for annuities than in the symmetric information equilibrium.

Note that the removal of medical expenses has a small effect on the overall annuity ownership rates. Without medical expenses the demand for annuities from the bottom and middle income quintiles goes up, while it decreases for the top quintile. The effect of medical expenses will be discussed in more detail later when I analyze the simple model.

5.2.2 Analyzing the simple model

The bottom panel of Table 4 shows annuity market participation rates when the simple model features only one impediment to annuitization. It is evident that only one

Income quintile	Bad health	Good health
1	56.5	21.1
2	40.7	11.2
3	26.5	2.5
4	13.9	-5.1
5	3.2	-11.7

Table 5: Percentage change in price in the pooling equilibrium comparing to the symmetric information equilibrium

impediment to annuitization has little power in decreasing annuity ownership rates: when the life-cycle model features only one factor that can potentially decrease the annuity demand the participation rate is always above 80%. This suggests that to explain low demand for annuities it is important to take into account the interaction between different factors behind non-annuitization.

Two factors have the largest quantitative impact in the simple model: preannuitized wealth and the consumption minimum floor. Both factors decrease the demand for annuities from 97% to around 84%. However, the mechanism behind the effect of these two factors is different. Preannuitized wealth substantially decreases annuity market participation rates because many people have almost no financial resources except for Social Security and DB income. Table 6 illustrates this further by showing the share of annuity income in total amount of available resources $\frac{n_0}{k_0 + n_0}$ separately for people who bought annuities and those who did not in the simple model with preannuitized wealth. Those retirees who decide not to invest in annuities have few resources except for annuity income: the percentage of annuity income in total available funds is more than 90%. For people who chose to invest in annuities this percentage is much lower: it does not exceed 40%.

Quintile	**Retirees who bought annuity**	**Retirees who didn't buy annuity**
1	32.5	93.5
2	37.7	99.7
3	27.0	98.6
4	25.3	99.6
5	26.2	98.5

Table 6: Average shares of annuity-like income in available resources

As for the consumption minimum floor, it provides public annuity-like income for people who outlive their assets. In the presence of preannuitized wealth most people have pension income that exceeds the consumption floor. Once preannuitized wealth is converted to liquid wealth, people may use this opportunity to consume out of this liquid wealth and then rely on the consumption floor as opposed to buying private annuities. This strategy becomes more attractive as the consumption floor increases. In other words, when the consumption floor is high and there is no preannuitized wealth, low-income people have less incentives to buy annuities. This explains why the removal of preannuitized wealth from the full model decreases the demand for annuities from the lowest income quintiles as shown in the top panel of Table 4. This result also emphasizes the importance of taking into account the interaction between government means-tested transfers and preannuitized wealth when considering the consequences of the transition from Defined Benefits to Defined Contribution pension plans or possible privatization of Social Security.

Another important observation from the second panel of Table 4 is that except for preannuitized wealth, no other factor can affect all income quintiles. In the top two income quintiles everyone buys annuities regardless of what impediments to annuitization are present.

5.2.3 Sequential analysis

In the next set of experiments I cumulatively add to the simple model the four impediments to annuitization that turned out to be the most quantitatively important in the full model. The first panel of Table 7 considers the quantitative implications of adding one impediment to annuitization to the simple model that already features preannuitized wealth.

In this setup the impact of different factors becomes more pronounced. The two most important factors are now the minimum purchase requirement and bequest motives: each of these factors decreases the participation rate from around 84% to less than 66%. Another important observation is that bequest motive introduces strong non-monotonicity in the relationship between the annuity market participation rate and income quintile. For example, in the presence of bequest motives only around 26% of people in the top income quintile buy annuities while among the third quintile the participation rate is around 82%. This goes in sharp contrast with the data, where the relationship between annuity ownership rates and income is strongly monotone (see the first column of Table 3). Notice also that in the model where retirees have preannuitized wealth the consumption floor has much less impact on the annuity market participation rate. This happens because most retirees have pension income which is above the consumption floor and thus this safety net does not impact their decisions to annuitize.

The second panel of Table 7 illustrates the relative quantitative importance of differ-

Income quintile	Simple model	+ adv. selection	+ med. expense	High $c_{\min}$	+ bequest	Illiquid housing	+ min. purchase
Simple model+preann wealth							
All	83.5	78.5	80.8	81.0	65.2	72.6	65.8
1	59.6	49.5	58.2	47.5	58.6	39.7	37.4
2	77.4	66.3	72.0	77.4	72.4	66.3	48.1
3	89.8	86.8	86.2	89.8	87.5	82.4	72.2
4	95.9	94.9	93.5	95.9	81.5	85.2	85.2
5	94.6	95.2	94.2	94.6	26.3	89.5	86.4
Simple model+preann wealth+min purchase							
All	65.8	64.1	61.3	65.0	40.3	40.1	-
1	37.4	33.3	35.0	33.3	37.4	13.5	-
2	48.1	43.1	43.3	48.1	47.8	23.9	-
3	72.2	71.9	65.8	72.2	72.2	38.6	-
4	85.2	84.5	77.7	85.2	40.7	59.3	-
5	86.4	87.8	84.8	86.4	3.4	65.2	-
Simple model+preann wealth+min purchase+illiquid housing							
All	40.1	39.5	36.7	39.8	19.8	-	-
1	13.5	12.1	12.4	12.1	13.5	-	-
2	23.9	21.5	21.1	23.9	23.2	-	-
3	38.6	37.9	35.8	38.6	38.6	-	-
4	59.3	58.2	49.8	59.3	21.5	-	-
5	65.2	67.9	64.6	65.2	2.4	-	-
Simple model+preann wealth+min purchase+illiquid houisng+bequest							
All	19.8	24.4	23.6	19.6	-	-	-
1	13.5	12.1	12.4	12.1	-	-	-
2	23.2	20.9	20.3	23.6	-	-	-
3	38.6	32.9	33.6	38.6	-	-	-
4	21.5	33.3	39.1	21.5	-	-	-
5	2.4	22.9	12.9	2.4	-	-	-

Table 7: Annuity market participation rates: sequential analysis of the simple model. Each panel considers adding one impediment to annuitization at a time to different versions of the simple model

ent impediments to annuitization in the simple model that already features preannuitized wealth and minimum purchase requirement. In this setup the strongest impact is produced by illiquidity of housing wealth and bequest motives. In total, the combination of minimum purchase requirement, preannuitized wealth and one of these two factors can decrease the annuity ownership rates to 40%. This version of the model also highlights the interaction between the illiquidity of housing wealth and minimum annuity purchase requirement. When people face no restrictions on how much they can invest in annuities, the illiquidity of housing wealth has a moderate impact on the annuity demand (see the first panel of Table 7). However, once minimum purchase restriction is introduced, the illiquidity of housing wealth reinforces this constraint and substantially decreases the demand for annuities.

The third panel of Table 7 repeats the analysis for the model that already features illiquid housing, preannuitized wealth and minimum annuity purchase requirement. Consistent with the previous results, in this environment bequest motives stand out as the most important impediment to annuitization. In total the combination of bequest motives, illiquid housing, preannuitized wealth and minimum annuity purchase requirement can reduce the demand for annuities to around 20%, which is the same as in the full model. However, as in the previous set of experiments, bequest motives result in non-monotone relationship between annuity ownership and income.

The last panel of Table 7 shows how adding adverse selection, medical expenses and increasing the consumption floor changes the demand for annuities in the model that already features bequest motives, minimum purchase requirement, illiquidity of housing and preannuitized wealth. In general, adding additional factors to the model that already features the four most powerful impediments to annuitization cannot decrease the annuity demand any further.

The sequential analysis illustrates that the effect of medical expenses changes depending on what other features are present in the model. In the versions of the simple model without bequest motives (panel 1-3 of Table 7) adding medical expenses decreases annuity ownership rates in each income quintile. However, when medical expenses are added to the simple model with bequest motives the annuity ownership rate increases and this is driven by the top two income quintiles. In general, in the presence of uncertain medical expenses people increasingly use liquid wealth to insure against medical shocks. However, in the presence of a bequest motive this liquid wealth has an additional value of being bequeathable. People with a strong bequest motive want to avoid the situation of surviving until old age and having their wealth depleted by a sequence of bad medical shocks, so they use annuities to insure against such an event. Thus, annuities are used not only to insure old age consumption but also liquid wealth at old ages.

5.3 Life-cycle pattern of annuity purchase

Figure (1) illustrates how age profiles of annuity purchases are affected by different impediments to annuitization. To highlight the importance of different factors the experiments are performed for the simple life-cycle model with preannuitized wealth.[19] The patterns of annuity purchase are simulated for individuals who initially are in good health and who were given the initial wealth and annuity income that correspond to the median values of the initial distribution for each permanent income quintile.[20]

In most experiments people buy annuities only once in the first period. It can be shown (see Appendix G) that, under certain conditions, the one-time purchase of annuities in the first period can be a general result. The conditions under which this result holds include the following:

1) There is no uncertainty except the time of death

2) Medical expenditures are zero

3) $\beta(1+r) < 1$

4) $n_0 > c_{\min}$.

The last condition ensures that an individual is already guaranteed income that exceeds the minimum consumption floor.[21]

The intuition behind this theoretical result is as follows. There are two ways to finance an annuity purchase: using financial wealth or existing annuity income. The second way would imply an increasing consumption profile, which is not optimal given $\beta(1+r) < 1$. Thus, if an individual buys an annuity, he will use his financial wealth; if an individual waits to buy annuities, he has to save in bonds. But this strategy is dominated by buying annuities from the start, because over the long-run an annuity brings a higher return.

The most noticeable changes in the life-cycle pattern of annuity purchase are introduced by medical expenses. People still buy annuities at the beginning of retirement but they also increase annuity holdings towards the end of life. This happens since retirees now have to finance not only their consumption but also medical expenditures that are increasing steeply over time. In this case, retirees use their annuity income to buy more annuities.

The bottom right graph in Figure (1) shows the life-cycle annuity purchase profile

[19] The graphs look very similar for the case when there is no preannuitized wealth but the amount of annuity purchased is larger and the effect of each factor is less pronounced. On average, in the absence of preannuitized wealth people buy twice as much annuity income.

[20] The graphs for individuals who start retirement in bad health are omitted because the patterns look the same except for the difference in the magnitude of purchase.

[21] This theoretical result illustrates the intuition behind the life-cycle profiles of annuity purchase. It is important to point out that the assumption 1) does not hold in the simple model because health is uncertain and thus future survival probabilities are uncertain as well. The deterioration in health in the equilibrium with symmetric information decreases the price of annuities and this can lead to additional annuity purchases. However, few people engage in additional purchase of annuities that is why it does not have an effect on annuity purchase profiles for most retirees except those in the bottom quintile.

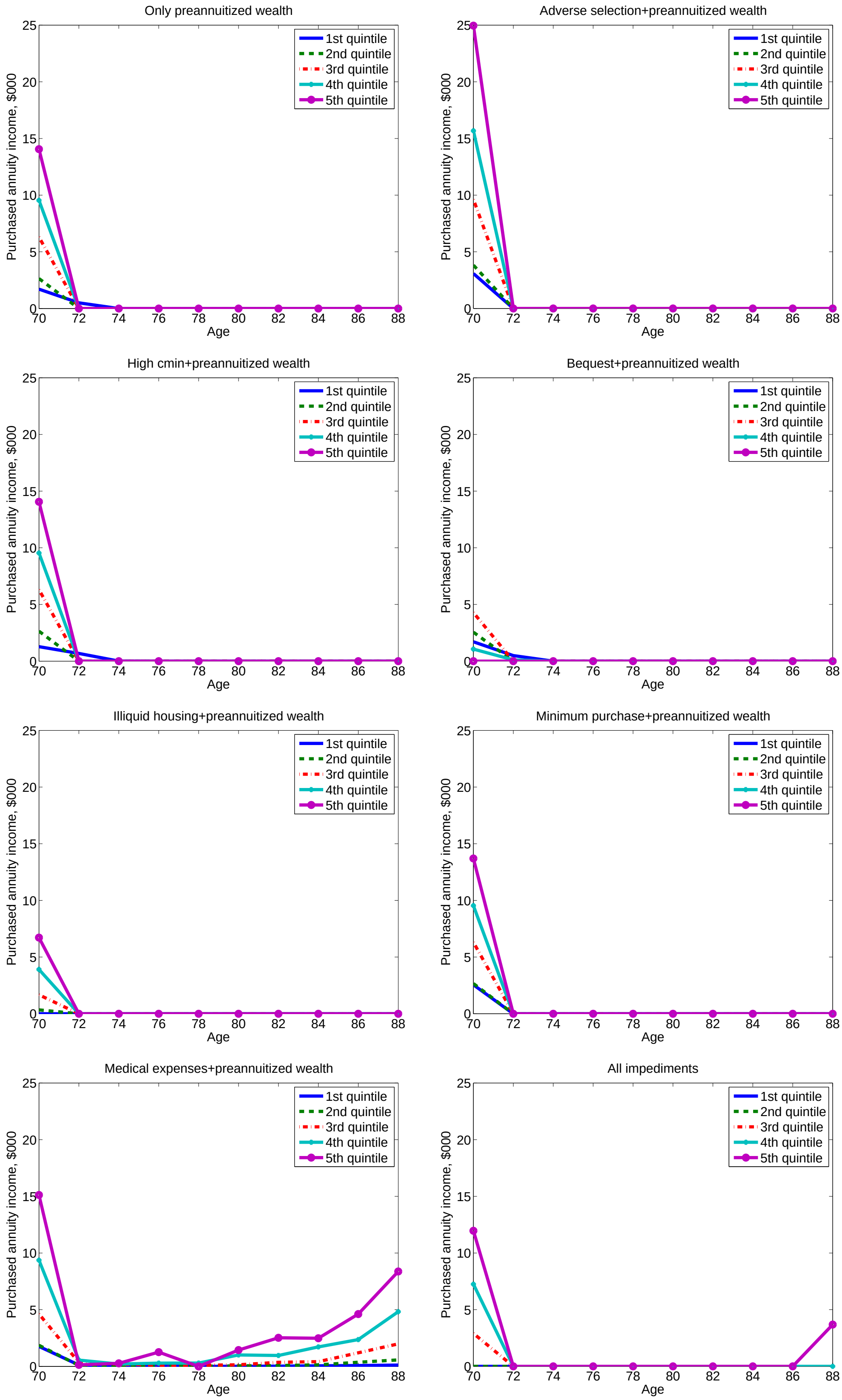

Figure 1: The effect of different factors on the annuity purchases for a retiree with median wealth and in good initial health

for the full model, i.e. the model with all the impediments to annuitization. Adding all other impediments to annuitization eliminates the pattern of increasing annuity purchases produced by medical expenses. In other words, medical expenses make it optimal for people to build up their income by annuitizing out of existing annuities, but other impediments to annuitization almost eliminate the demand for annuities for ages above 70.

5.4 Can the annuity market participation rate be decreased further?

As Table 3 illustrates, even the full model with all the impediments to annuitization overpredicts annuity ownership rates comparing to the data. This section discusses in what directions the model can be modified in order to produce lower annuity demand.

The quantitative analysis above indicates that disposable wealth is one of the most important factors in determining the demand for annuities, especially when combined with minimum annuity purchase requirement. To understand whether these two factors are powerful enough to reduce the demand for annuities close to the level that we observe in the data, I introduce two modifications to the full model. First, I redefine the initial wealth k_0 by excluding from the total non-housing wealth the value of cars and businesses. The motivation for this exercise is that given the special properties of these two types of assets it may be possible that retirees do not consider them as a potential target for annuitization. Second, I increase the minimum purchase requirement twice - from \$2,500 to \$5,000 annual annuity income. The results of this modified full model are reported in Table 8 alongside the data and the results from the original full model.

Income quintile	Data	Full model	Modified full model
All	5.0	20.3	7.5
1	0.4	7.0	2.3
2	1.0	14.6	4.8
3	5.3	26.2	9.5
4	6.4	30.7	12.2
5	12.2	23.2	8.8

Table 8: Participation in the annuity market: data, full model and modified full model

Comparing to the full model annuity ownership rates go down by more than half: from 20% to less than 8%. This suggests that an important direction in the future exploration of the annuity puzzle is to better understand the properties of the different types of assets that retirees own. If people consider all their assets equally suitable for annuitization, then it is hard to justify the observed low annuity demand. However, if people prefer not to annuitize their less liquid wealth then the standard life-cycle model augmented with high minimum purchase requirement and other impediments to annuitization can come close to accounting for low demand for annuities.

6 Conclusion

This study considers different explanations for the annuity puzzle in a quantitative heterogeneous agent model with equilibrium in the annuity market. It shows that in the absence of any impediments to annuitization all but the poorest retirees buy annuities. The life-cycle model that incorporates all impediments to annuitization decreases the demand for annuities almost five times. The most quantitatively important impediments to annuitization are preannuitized wealth, illiquid housing, minimum purchase requirement and bequest motives. Adverse selection noticeably decreases the demand for annuities amongst people in the bottom income quintiles but increases the demand from the top quintiles, thus producing a small overall effect. Medical expenses have a small effect on annuity ownership rates but they can substantially change the life cycle pattern of annuity purchases.

In general, retirees are willing to buy annuities but the amount of annuities they are actually buying is small. Annuities bring returns for many years and the upfront investment, even when buying a small stream of annuity income, is large. When insurance firms restrict the minimum amount an individual can invest in annuities this substantially decreases the number of retirees in the market, especially if housing wealth is illiquid and part of wealth is preannuitized. Increasing minimum purchase requirement to $5,000 and restricting people from annuitizing their businesses and cars can bring the annuity ownership rates close to the data. This suggests the importance of better understanding of how easily different components of retirees' wealth can be annuitized in future studies of the annuity puzzle.

A Alternative specifications of bequest motives

To understand how sensitive the demand for annuities to difference in bequest motives is, I use several alternative bequest specifications from the following studies: De Nardi (2004), Lockwood (2012b), Ameriks et al (2012) and Nakajima and Telyukova (2011). Bequest specifications for each of these studies are shown in Table 9 alongside with DFJ's bequest specification that is used in the benchmark calibration (in Table 9 I keep the notation as in the original studies).

	Bequest form	Parameter values
De Nardi et al. (2010)	$\theta\frac{(k+b)^{1-\sigma}}{1-\sigma}$	$\theta = 2,360, k = 273,000, \sigma = 3.84$
De Nardi (2004)	$\phi_1(1+\frac{b}{\phi_2})^{1-\sigma}$	$\phi_1 = -9.5, \phi_2 = 11.6, \sigma = 1.5$
Lockwood (2012b)	$\left(\frac{\phi}{1-\phi}\right)^{\sigma}\frac{(\frac{\phi}{1-\phi}c_b+b)^{1-\sigma}}{1-\sigma}$	$\phi = 0.919, c_b = 20,416, \sigma = 3$
Ameriks et al. (2011)	$\frac{\varpi}{1-\sigma}(\phi+\frac{b}{\varpi})^{1-\sigma}$	$\varpi = 47.6, \phi = 7,280, \sigma = 3$
Nakajima et al. (2011)	$\gamma\frac{(\zeta+b)^{1-\sigma}}{1-\sigma}$	$\gamma = 3.77, \zeta = 9,675, \sigma = 2.68$

Table 9: Bequest motives in various studies. Notation is the same as used by the authors, b denotes the amount of wealth bequeathed

The parameterizations of bequest motives specified in Table 9 are not directly comparable because each of these studies uses a different coefficient of risk-aversion and normalizes nominal variables to a different base year. To compare these bequest motives in the unified framework I use the following approach. First, I consider a simple one-period consumption-saving model augmented with a bequest motive. Assume an agent has wealth K. Tomorrow an agent dies with probability 1 and he can leave a bequest in the amount b. His consumption today will be $K - b$. The optimization problem looks as follows:

$$\max_b\{\frac{(K-b)^{1-\sigma}}{1-\sigma} + v(b)\} \tag{7}$$

The solution to this problem produces two outcomes: i) the threshold value of wealth above which a bequest motive becomes operational ($\overline{K}$), i.e. people with wealth below the threshold will not leave a bequest[22]; ii) marginal propensity to bequeath (MPB), i.e. the fraction of each additional dollar of wealth that will be bequeathed once bequest is operational. This can be defined as $\frac{\partial b^*}{\partial K}$ where b^* is a solution to problem (7).

[22] $\overline{K}$ can be defined as follows: $\overline{K} = (v'(0))^{-\frac{1}{\sigma}}$

Next, I compute the threshold and MPB implied by each bequest specification in Table 9. Both $\overline{K}$ and $\frac{\partial b^*}{\partial K}$ are influenced not only by the parameters of bequest functions but also by the risk aversion. In addition, the threshold values are nominal variables and thus depend on the base year used in each study. I convert them to one base year which is the same as used in my study. The resulting values for MPBs and for threshold values can be compared across studies and they are reported in the first two columns of Table 10. Note that the bequest motives under consideration cover a wide spectrum of possible bequest preferences. The least strong bequest motives (Nakajima and Telyukova, 2011) have MPB equal to 0.62, while the strongest bequest motives (Ameriks et al, 2011) have MPB equal to 0.98, which is equivalent to a linear bequest motive since almost the entire marginal wealth above the threshold is bequeathed. In terms of being luxury goods, the highest threshold among all specifications is \$64,078 (De Nardi, 2004) while the lowest are \$6,063 and \$6,093 (Ameriks et al, 2011 and Nakajima and Telyukova, 2011). In other words, the form of De Nardi (2004) implies that bequests are luxury goods to a much larger extent than the forms of Ameriks et al (2011) and Nakajima and Telyukova (2011).

	$\overline{K}$ (in 1998\$)	$\frac{\partial b^*}{\partial K}$	η	ϕ
De Nardi et al. (2010)	$36,124$	0.88	2,360	273,000
De Nardi (2004)	$64,708$	0.86	1,244	413,911
Lockwood (2012b)	$15,265$	0.92	11,832	175,552
Ameriks et al. (2011)	$6,063$	0.98	2,767,009	288,597
Nakajima et al. (2011)	$6,093$	0.62	6.7	9,998

Table 10: Bequest motives in various studies: unified framework

As a next step I need to find bequest parameters η and ϕ that can produce the same thresholds and MPBs as listed in Table 10 when plugged into the bequest function $v(k_t) = \eta\frac{(\phi + k_t)^{1-\sigma}}{1-\sigma}$ together with the risk aversion used in my benchmark calibration. The resulting values of η and ϕ are shown in the last two columns of Table 10. Next I solve four versions of the full model with these parameters of the bequest function. The results are shown in Table 11.

One can see that bequest specifications with MPB greater than 0.9 can completely eliminate annuity demand when all other impediments to annuitization are present: bequest motives specified in Ameriks et al (2012) and Lockwood (2012b) result in zero annuity ownership rates. The bequest parametrization of De Nardi (2004) has almost no effect on the demand for annuities. The full model with bequest motives as in De Nardi (2004) produces almost the same annuity ownership rate as the full model without

Income quintile	DFJ (2010)	De Nardi (2004)	Lockwood (2012b)	Ameriks (2009)	Nakajima (2011)
All	20.3	34.8	0.0	0.0	25.8
1	7.0	7.9	0.0	0.0	8.1
2	14.6	17.4	0.0	0.0	11.3
3	26.2	34.2	0.0	0.0	27.4
4	30.7	53.4	0.0	0.0	35.6
5	23.2	61.3	0.0	0.0	46.6

Table 11: Annuity market participation rates for the full model with different parameterizations of bequest motives.

bequest motives (see the top panel of Table 4). Even though the MPB in the bequest specification of De Nardi (2004) is almost the same as the MPB used in my benchmark calibration, the former bequest motives have a higher threshold and thus affect fewer people. Another important observation is that the bequest parametrization of Nakajima and Telyukova (2011) can reproduce the monotone relationship between annuity ownership and income.

As a final exercise I consider if it is possible to reduce overall annuity demand while keeping the monotone relationship between income quintile and annuity ownership rate. To do this I increase the strength of the bequest motive specified by Nakajima and Telyukova (2011) since it is the only bequest motive that can produce this monotone relationship due to its low threshold. Table 12 displays the results of the full model that has bequest motives with the same threshold as in Nakajima and Telyukova (2011) but with higher MPB. In particular, I increase MPB to 0.7, 0.8 and 0.88 (the last MPB is the same as in the benchmark calibration). The results show that increasing the strength of a bequest motive has a noticeable effect on annuity ownership rates, but once MPB exceeds 0.8 the monotone relationship between annuity ownership and income disappears. This suggests that only bequest specification where both MPB and threshold are small (bequest is much less of a luxury good) is consistent with the empirical fact that annuity ownership rates are positively correlated with income. Bequest motives with high MPB can account for low overall annuity demand but they cannot explain why high-income people have the highest demand for annuities.

Income quintile	MPB=0.62 (original)	MPB=0.7	MPB=0.8	MPB=0.88 (as in DFJ)
All	25.8	22.3	13.9	0.8
1	8.1	7.1	5.6	0.3
2	11.3	9.5	6.0	0.0
3	27.4	23.5	17.2	0.8
4	35.6	30.8	20.7	2.0
5	46.6	40.6	20.4	0.8

Table 12: Annuity market participation rates for the bequest parametrization of Nakajima and Telyukova (2011) with higher MPB

B Alternative specification of other preference parameters

This section checks the sensitivity of results to changes in two parameters: discount factor (β) and risk aversion (σ). The first two columns of Table 13 display participation rates in the annuity market for the full model when β is first raised to 0.98 and then decreased to 0.90. Note that in the first experiment β was increased to the point when $\beta(1+r) = 1$.

Income quintile	Full model	$\beta = 0.98$	$\beta = 0.90$	$\sigma = 3$	$\sigma = 6$
All	20.3	20.9	7.1	13.9	27.6
1	7.0	6.7	4.5	4.2	7.9
2	14.6	13.9	6.0	9.7	17.2
3	26.2	26.7	14.5	20.4	32.9
4	30.7	33.1	7.5	22.5	43.7
5	23.2	24.0	3.0	12.6	36.1

Table 13: The sensitivity of annuity ownership rates to changes in the discount factor and the risk aversion

A decrease in the discount factor decreases annuity market participation rates while an increase in the discount factor has almost no effect. Impatient individuals save less and so they are less interested in annuities: when the discount factor goes down to 0.90, the participation rate drops more than twofold.

In the next experiment the risk aversion in the full model was first decreased to 3 and then raised to 5. The last two columns of Table 13 display the results of these experiments. When the risk aversion decreases, agents want to save less and thus they buy less annuities.[23] An increase in the risk aversion has the opposite effect, thus higher risk aversion makes the annuity puzzle harder to explain.

C Alternative specifications for the administrative loads

When modeling the annuity market I made the assumption that the administrative load is proportional to the amount of insurance sold. This section shows how this assumption can affect the results. To do this I consider an alternative specification for the administrative costs by assuming they do not depend on the total value of insurance contract but represent fixed additive costs. In this case equation (4) can be rewritten as follows:

$$\pi_t(\Omega_t) = q_t(\Omega_t) - \sum_{i=1}^{T-t} \frac{\widehat{S}_{t+i|t}(\Omega_t)}{(1+r)^i} - \widetilde{\gamma} \tag{8}$$

where $\widetilde{\gamma}$ is lump sum administrative costs.

To be consistent with the results of Mitchell et al (1999) I set $\widetilde{\gamma}$ so that the fraction of administrative costs in the total value of the contract is equal to 10% for people of age 70. The results of the full model with this alternative specification of administrative costs is reported in Table 14. Comparing to the full model with proportional administrative costs, annuity ownership rates are very similar.

Income quintile	Full model	Additive load
All	20.3	19.7
1	7.0	6.9
2	14.6	13.9
3	26.2	26.2
4	30.7	29.8
5	23.2	21.8

Table 14: The sensitivity of annuity ownership rates to the different specification of administrative loads

[23] Inkman et al (2011) also find that lower risk aversion can substantially decrease the demand for annuities.

D Equilibrium annuity prices

This section compares prices produced by the full model with the prices observed in the data. Figure 2 shows that prices in the model line up well with what we see in the data[24], though for most ages the model underpredicts the real prices.

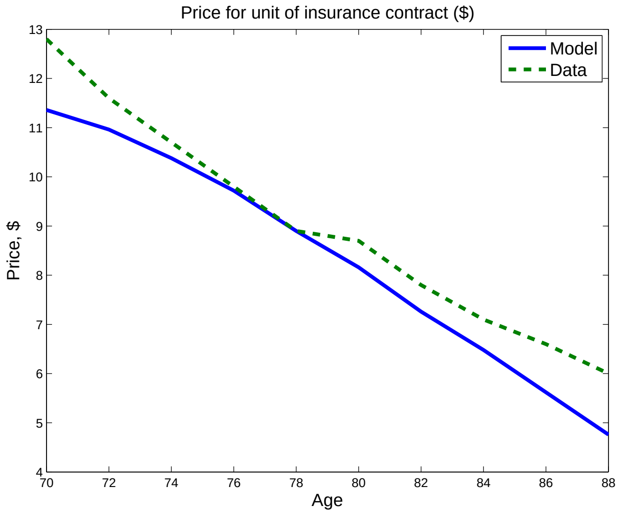

Figure 2: Prices in the full model and in the data

There are at least three explanations for this downward bias of the prices in the model. First, the model considers only single individuals, while real prices are based on the aggregate statistics that include couples. Second, the HRS dataset underrepresents wealthy individuals. Given that wealth is negatively correlated with mortality and that rich individuals tend to buy more annuities, the lack of wealthy individuals in the model biases the prices downwards. Third, this paper makes an assumption that insurance firms are perfectly competitive and sets administrative load to 10%. It may be that the markup insurance firms set is more than 10% due to higher administrative expenses or violation of the perfect competition assumption.

E Annuitization levels

This section discusses how different factors affect the optimal level of annuitization. Table 15 shows the median fractions of liquid wealth annuitized by retirees in the simple model that features preannuitized wealth. When there is no other impediment to annuitization except Social Security and DB plans, median annuitization level is 60% for retirees in the bottom quintile and around 82% for the top income quintile. Most

[24]The observed price corresponds to the price of a lifetime annuity with fixed income payment with inflation adjustment. The quotes were obtained from Vanguard at www.aigretirementgold.com in 2009.

impediments strongly affect people in the bottom quintile. Medical expenses decrease their median annuitization level to less than 10%, while in the presence of adverse selection, high consumption floor, illiquidity of housing and minimum purchase requirement median annuitization level of this group becomes zero. For the top income quintile the strongest impact is produced by bequest motives, which result in zero annuitization level. Illiquidity of housing and minimum purchase requirement have very small negative impact on the optimal annuitization level for this group. Medical expenses and adverse selection actually increase the fraction of wealth annuitized in the top income quintile.

Income quintile	Simple model	+ adv. selection	+ med. expense	High $c_{\min}$	+ bequest	Illiquid housing	+ min. purchase
1	59.3	0.0	9.7	0.0	39.2	0.0	0.0
2	76.6	66.1	51.5	76.1	50.7	46.4	0.0
3	80.0	79.5	75.4	80.0	36.8	74.1	79.8
4	81.5	82.5	81.8	81.0	14.8	78.9	81.2
5	81.9	83.9	87.3	81.9	0.0	79.1	78.5

Table 15: Median fraction of liquid wealth annuitized in the simple model with preannuitized wealth

F Welfare

Another dimension to consider is how each impediment to annuitization changes people's valuation of annuities. Table 16 illustrates the welfare gains of people from having access to the annuity market. Welfare gains are measured as the consumption equivalent variation (CEV). This represents the percentage of the annual consumption that a retiree who does not have access to the annuity market is willing to give up in order to get an option to buy annuities. The results are presented for the simple model with preannuitized wealth. Overall, if there are no impediments to annuitization except for Social Security and DB plans, gaining access to the annuity market is equivalent to increasing consumption by 2% each period. People in high income quintiles gain more than people in low quintiles since they have more wealth to invest in annuities and hence can change their consumption patterns in a more significant way when the annuity market is available.

Adding impediments to annuitization in most cases decreases the value of the annuity market for retirees. The most noticeable decline in CEV (to 0.7%) happens when housing wealth is illiquid, emphasizing the importance of disposable wealth for annuity valuation. Another factor that has a noticeable overall effect is adverse selection, which

Income quintile	Only prean wealth	+ adv. selection	+ med. expense	High $c_{\min}$	+ bequest	Illiquid housing	+ min. purchase
All	2.0	1.1	1.6	2.5	1.6	0.7	1.7
1	1.7	0.8	1.1	1.8	1.6	0.5	1.4
2	2.8	1.7	1.7	2.8	1.8	1.2	2.4
3	4.8	3.9	3.8	4.8	1.6	2.1	4.6
4	5.8	6.0	6.7	5.7	0.4	2.5	5.6
5	4.5	6.1	8.6	4.6	0.0	2.3	4.4

Table 16: Consumption equivalent variation for the simple model with different impediments to annuitization

decreases the CEV from 2.0% to 1.1%. The effect of adverse selection on the annuity valuation is positive for people in high income quintiles and negative for people in low income quintiles, and the latter effect dominates. Similar effects are produced by medical expenses: the CEV for high-income groups goes up while for low-income groups it goes down. People in high quintiles value annuities more since they can use them to insure against uncertain medical expenses. For low income quintiles medical expenses substantially increase the risk of falling under the consumption minimum floor, and in this state annuities do not have any value.

Another factor that has a heterogeneous effect on different income quintiles is bequest motives, which have almost no effect on the bottom quintiles but decrease the CEV of the highest income quintile to almost zero.

Surprisingly, higher consumption minimum floor increases the value of the annuity market from 2.0% to 2.5% and this happens because of the change in the CEV of low-income group. This is mostly driven by a change in the welfare of people whose preexisting annuity income is below the high consumption floor, and without an annuity market they quickly decumulate their assets and rely on government transfers. After gaining access to the annuity market they can increase their annuity income above the level of c_{min} and this gives them higher lifetime consumption.

G Theory

This section proves that certain assumptions retirees buy annuities only once in the first period.

G.1 Auxiliary propositions

Auxiliary proposition 1 Assume there is no uncertainty except the time of death. Then the following is true:

$$\frac{1+q_{t+1}^{AF}}{q_t^{AF}} = \frac{1+r}{s_t}$$

where q_t^{AF} is an actuarially fair annuity price defined as follows:

$$q_t^{AF} = \sum_{i=1}^{T-t} \frac{S_{t+i|t}}{(1+r)^i}$$

Proof Using the definition of actuarially fair annuity prices and the fact that $S_{t+i|t} = s_t S_{t+i|t+1}$ we can write:

$$q_t^{AF} = \frac{s_t}{(1+r)} \left(1 + \sum_{i=2}^{T-t} \frac{S_{t+i|t+1}}{(1+r)^{i-1}}\right) = \frac{s_t}{(1+r)} \left(1 + q_{t+1}^{AF}\right)$$

This proves Auxiliary proposition 1.

Auxiliary proposition 2 Assume there is no uncertainty except the time of death. Then the following is true:

$$\frac{1+q_{t+1}}{q_t} < \frac{1+r}{s_t}$$

where q_t is the annuity price that includes administrative load, i.e. $q_t = \gamma q_t^{AF}$.

Proof We can write the following relationship:

$$\frac{1+q_{t+1}}{q_t} = \frac{1+\gamma q_{t+1}^{AF}}{\gamma q_t^{AF}} = \frac{1/\gamma + q_{t+1}^{AF}}{q_t^{AF}} < \frac{1+q_{t+1}^{AF}}{q_t^{AF}}$$

The last inequality follows from the fact that $\gamma \geq 1$. By Auxiliary proposition 1 $\frac{1+q_{t+1}^{AF}}{q_t^{AF}} = \frac{1+r}{s_t}$. Thus $\frac{1+q_{t+1}}{q_t} < \frac{1+r}{s_t}$, which proves Auxiliary proposition 2.

G.2 Assumptions

The proofs below rely on the following assumptions:

1. There is no uncertainty except the time of death.
2. Medical expenses are zero (z_t=0).
3. $\beta(1+r) < 1$.

4. Annuity prices satisfy the following condition: $\dfrac{1+q_{t+1}}{q_t} > 1+r$.[25]
5. The initial annuity income is above the consumption floor ($n_0 > c_{min}$.)
6. There are no bequest motives.

G.3 Proof that people buy annuities only once in the first period

Proposition 1 Assume conditions 1-5 hold. Then if an agent's optimal investments in annuities at time $t+1$ is positive ($\Delta_{t+2} > 0$) then he does not invest in bonds at time t ($k_{t+1} = 0$).

Proof Suppose not. Consider Euler equations for an agent's investments in annuities and bonds at time t. Given assumptions 5 and 6 we can write these equations as follows:

$$u'(c_t)q_t - \mu_t^n = \beta s_t u'(c_{t+1})(1+q_{t+1}) - \mu_{t+1}^n \tag{9}$$

$$u'(c_t) - \mu_t^k = \beta s_t (1+r) u'(c_{t+1}) \tag{10}$$

Here μ_t^n and μ_t^k are Lagrange multipliers on constraints $\Delta_{t+1} \geq 0$ and $k_{t+1} \geq 0$ and they satisfy the following conditions:

$$\mu_t^n \geq 0, \qquad \mu_t^n \Delta_{t+1} = 0$$

$$\mu_t^k \geq 0, \qquad \mu_t^k k_{t+1} = 0$$

Since $\Delta_{t+2} > 0$, we have $\mu_{t+1}^n = 0$. By supposition, $\mu_t^k = 0$. Thus equations (9) and (10) can be rewritten in the following way:

$$\frac{\mu_t^n}{q_t} = u'(c_t) - \beta s_t u'(c_{t+1}) \frac{1+q_{t+1}}{q_t}$$

$$u'(c_t) = \beta s_t (1+r) u'(c_{t+1})$$

Combining these equations we get:

$$\frac{\mu_t^n}{q_t} = \beta s_t u'(c_{t+1}) \left((1+r) - \frac{1+q_{t+1}}{q_t} \right)$$

Given assumption 4 this implies $\mu_t^n < 0$ which contradicts the definition of Lagrange multiplier. This contradiction proves Proposition 1.

[25]Note, this inequality is always true if prices are actuarially fair (see Auxiliary proposition 1).

Proposition 2 Assume conditions 1-5 hold. Then if at some time t an agent has only annuity income and no liquid wealth he will not invest either in bonds or in annuities.

Proof If an agent does not invest neither in bonds nor in annuities he has: $c_t = n_t$ and $c_{t+1} = n_t$. Consider if it is optimal for an agent to deviate from the strategy of not investing at all and relying only on his annuity income. Consider first the decision to invest in bonds. If it is not optimal to have zero investments in bonds, than based on the Euler equation (10) we can write the following:

$$u'(n_t) < \beta s_t(1+r)u'(n_t)$$

Rearranging this expression we get:

$$0 < (\beta s_t(1+r) - 1)u'(n_t)$$

This cannot be true because by assumption 3, $\beta s_t(1+r) < 1$. Thus it is not optimal to deviate from no investments in bonds strategy.

Second, consider the decision to invest in annuities. We can rewrite equation (9) in the following way:

$$\begin{gathered}
\frac{\mu_t^n}{q_t} = u'(c_t) - \beta s_t u'(c_{t+1})\frac{1+q_{t+1}}{q_t} + \frac{\mu_{t+1}^n}{q_t} = \\
u'(n_t)\left(1 - \beta s_t\frac{1+q_{t+1}}{q_t}\right) + \frac{\mu_{t+1}^n}{q_t} > \\
u'(n_t)\left(1 - \beta(1+r)\right) + \frac{\mu_{t+1}^n}{q_t} > 0
\end{gathered}$$

The last inequality follows from assumption 3 and the inequality before last - from Auxiliary proposition 2. Thus $\mu_t^n > 0$, i.e. it is not optimal to invest in annuities. This finishes the proof of Proposition 2.

Proposition 3 Assume conditions 1-5 hold. If an agent buys annuities he will start doing it in the first period ($\Delta_2 > 0$).

G.3.1 Proof

Suppose not. Without loss of generality, assume that an agent starts buying annuity at time $t = 2$. Then by Proposition 1 he does not invest in bonds in period $t = 1$. Then at time $t = 2$ he does not have liquid wealth. Then by Proposition 2 he does not invest in annuities. This logic can be applied to every period. It proves Proposition 3.

Proposition 4 Assume conditions 1-5 hold. If an agent buys annuities he will do it only once in the first period ($\Delta_2 > 0, \Delta_3 = .. = \Delta_T = 0$).

Proof Suppose not. We know that an agent will start buying annuities in the first period. Suppose he also chooses to buy annuities at some period $\widetilde{t} > 1$. Then at time $\widetilde{t} - 1$ his investments in bonds are zero by Proposition 1. Then by Proposition 2 his investments in annuities at time $\widetilde{t}$ are zero. This contradiction proves Proposition 4.

H Discussion of several features missing from the model

This section discusses how some assumptions of the model can affect its main results.

H.1 Exogenous medical expenses

I assume that medical expenses are exogenous, i.e. people have no control over their medical spending. In reality, medical spending has both discretionary and non-discretionary parts. The non-discretionary part represents a risk which creates precautionary demand for liquid assets and thus can decrease interest in illiquid annuities. The results of this paper show that when comparing the model with and without exogenous medical expenses, the annuity ownership rates change little, suggesting that this mechanism does not represent a quantitatively important impediment to annuitization.

The discretionary part of medical expenses is usually modeled as investments in health that increase utility (see, for example, Scholz and Seshadri, 2010). As such, it becomes part of a portfolio choice problem. Yogo (2009) shows that when modeled in this way, medical expenses do not represent an impediment to annuitization. On the contrary, when gaining access to a frictionless annuity market, people can decrease their investments in health in order to invest in annuities. If people are allowed to move their investments from health to annuities, this means they have a way to increase their disposable wealth by reducing medical expenses. The results of my study show that disposable wealth is an important determinant of annuity demand. Thus this mechanism would make the annuity puzzle harder to explain because we need to understand not only why people do not convert their wealth to annuities, but also why they choose to invest in health rather than annuities.

H.2 Long-term care insurance market

People who survive to very old ages are likely to face high medical expenses and a substantial part of these expenses is represented by the costs of long-term care (Kopecky and Koreshkova, 2011). The risk of nursing home costs can be privately insured in the market for long-term care insurance. I abstract from modeling this market. The presence of explicit insurance against long-term care costs can change my results in two directions.

On the one hand, as Figure (1) shows, some retirees use annuities to insure their old age medical care. Long-term care insurance can crowd out demand for annuities that are used for this purpose. This can decrease the overall demand for annuities, especially for people in high income quintiles.

On the other hand, there are some features of long-term care insurance contracts that can increase demand for annuities. In particular, this contract requires annual payments of premiums which will continue until a person goes to a nursing home. Given that such long-term annual payments are cheaper to finance by buying annuities than by paying for them out of savings, retirees can combine a purchase of annuities with purchase of long-term care. In this case, long-term care insurance does not necessarily crowd out the demand for annuities.

H.3 Absence of working-age stage of life

This study analyzes the decisions of retirees without considering the earlier stages of their life-cycle. As shown by Scholz et al (2006), people in the US save optimally for retirement. In other words, the initial distribution of wealth at retirement reflects the optimal decision making of people over the course of their working lives. Some counterfactual experiments in this study involve a significant change in the environment that people are living in and this can lead to different behavior during earlier stages of life. In particular, as shown by Kopecky and Koreshkova (2011), medical expenses in retirement are an important factor increasing life-cycle savings. Eliminating this risk will substantially reduce the initial endowment of wealth of retirees. This study shows that the initial endowment of resources is an important determinant of the annuity demand. Therefore allowing for a full life-cycle will likely lead to a more noticeable negative impact of medical expenses on retirees' participation in the annuity market.

I Computation

I solve the model using backward induction. In the last period ($t = T$), the value function (and policy functions) can be solved analytically. For every age t prior to T and for each point in the state space, I optimize with respect to bonds and annuities. I solve the two-dimensional optimization problem by applying, first, a coarse grid search and then the Brent method along both dimensions. The numerical integration of tomorrow's value function was performed using Gauss-Legendre quadrature. This numerical integration was complicated by the fact that the value function is not concave. The non-concavity arises because each period the agent can run out of assets due to medical expense shocks. The probability of getting into this state is endogenous. To address this problem, in numerical integration a "kink point"for each grid of tomorrow's assets is identified. The "kink point"is a value of tomorrow's medical shock that lets an agent finance his medical expenses and be left with an amount exactly equal to the consumption minimum floor. Then the value function was integrated separately to the left and to the right of the "kink point". I evaluate the value function for points outside the state space grid using a linear interpolation. I discretize persistent and transitory shocks using the Tauchen-Hussey algorithm. To create the simulation sample I draw with replacement 10,000,000 individuals from the sample described in Section 4. Each of these individuals is endowed with the state vector (k_0,n_0,m_0) drawn from the data. In addition, each individual is assigned random shocks ζ_0 and ξ_0 where ζ_0 is drawn from an invariant distribution.

To solve for the equilibrium in the annuity market I start by guessing a vector of prices q^0. To ensure that I trap the equilibrium price from below, I start with a very low guess that corresponds to the price that people in the lowest quintile and bad health face if their mortality is observed. Given this price I compute decision rules of households and find the new equilibrium price q^1 implied by these decision rules using the equilibrium condition (6). To ensure that the price vector changes very slowly from iteration n to iteration $n+1$ I use the following update rule: $q^{n+1} = 0.9q^{n-1} + 0.1q^n$

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