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Intertwined Real and Monetary Stochastic Business Cycles

Egmont Kakarot-Handtke*

Abstract

There is no such thing as a real economy. The task, therefore, is to consistently reconstruct the fluctuations of employment and output from the interactions of real and nominal variables. The present paper does exactly this. No nonempirical concepts like utility, equilibrium, rationality, decreasing returns or perfect competition are applied. The analysis runs rigorously in objective structural axiomatic terms. Therefrom follows that it is the factor cost ratio, i.e. the relation of the nominal variables wage rate and price and the real variable productivity that, for any given level of effective demand, drives the fluctuations of employment and output.

JEL E10, E24, E32

Keywords new framework of concepts; structure-centric; axiom set; profit; distributed profit; Say's regime; supersymmetric price; Slutsky-cycle; transaction money; general multiplier

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It is certainly an irritating state of affairs, yet economists definitively have no proper understanding of the pivotal economic phenomenon profit. Between orthodoxy and heterodoxy there is no significant difference on this point. The defining characteristic of profit is that it cannot appear in a real economy but only in a monetary economy. Hence real models cannot, as a matter of principle, explain how the economy works. The first task of theoretical economics is to resolve the profit conundrum and to constitute the notion of income. Without a rigorous formal expression of profit and income, as Keynes recognized, ‘we shall be lost in the wood’ (1973, p. 297).

Theories have a logical architecture consisting of premises and conclusions or, in a purely formal context, of axioms and theorems. To change a theory means to change the premises. Therefore, the accustomed formal points of departure are in the present paper replaced by *structural* axioms. From this secure formal foundation the multitude of phenomena that makes the business cycle is consistently derived.

Section 1 states the formal starting point. In Section 2 the market clearing and budget balancing price is objectively derived for the initial period. Subsequently, its path in a random environment is determined. The symmetric random changes produce inflationary and deflationary Slutsky-cycles that cancel out in the very long run due to the law of large numbers. From this analysis follow the conditions of price stability in a stationary and in a growing consumption economy. In Section 3 the formal properties of Say’s regime and the conditions of full employment are established. Say’s regime is a formal benchmark with the product and labor market continuously clearing. In Section 4 the price becomes the independent variable and this change of dependency gives rise to inventory cycles. The relationship between money, credit and the stock of transaction money on the one hand and the real and nominal key variables on the other is established in Section 5. This makes it possible to give, in Section 6, a comprehensive account of employment and output fluctuations in the consumption and investment economy. Section 7 concludes.

1 Simply the simplest

1.1 Axioms

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. All quantitative and temporal extensions have to be deferred until the implications of the most elementary economic configuration are perfectly understood. Axiomatization is about ascertaining the *minimum* number of premises. Four suffice for our present purposes.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L , and distributed profit, i.e. the product of dividend D and the number of shares N .

$$Y = WL + DN \quad |t \quad (1)$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \quad (2)$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X .

$$C = PX \quad |t \quad (3)$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other activity of the government sector. All axiomatic variables are measurable in principle. No nonempirical concepts like utility, equilibrium, rationality, decreasing returns or perfect competition are put into the premises.

The economic meaning is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. Profit and distributed profit are quite different things.

The period values of the axiomatic variables are connected formally by the familiar growth equation, which is added to the structural set as the 4th axiom:

$$Z_t = Z_{t-1} \left(1 + \ddot{Z}_t \right). \quad (4)$$

The path of the representative variable Z_t , which stands for the axiomatic variables, is then determined by the initial value Z_0 and the rates of change \ddot{Z}_t for each period:

$$Z_t = Z_0 (1 + \ddot{Z}_1) (1 + \ddot{Z}_2) \dots (1 + \ddot{Z}_t) = Z_0 \prod_{t=1}^t (1 + \ddot{Z}_t) \quad (5)$$

abridged
 $Z_t = Z_0 \Pi \ddot{Z}_t$

Given convenient initial values, eq. (5) describes the paths of the variables with the *rates of change* \ddot{Z}_t as unknowns. These unknowns are in need of determination and explanation. The explanation of the rates of change is, in principle, to be found between the limiting cases of perfect determinism and perfect randomness (for details see 2011a, Sec. 2). For methodological reasons we have to choose the random hypothesis first because:

The simplest hypothesis is that variation is random until the contrary is shown, the onus of the proof resting on the advocate of the more complicated hypothesis ... (Kreuzenkamp and McAleer, 1995, p. 12)

The random hypothesis does not preclude that a more specific behavioral hypothesis is introduced at a later stage. It is of importance, however, to bear in mind that economics is – basically and essentially – not a science of behavior (Hudík, 2011).

1.2 Definitions

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (6) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \quad Y_D \equiv DN \quad |t. \quad (6)$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \quad (7)$$

A sales ratio $\rho_X = 1$ indicates that the quantity sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \quad (8)$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditure C are equal to total income Y , in other words, that the household sector's budget is balanced.

2 Price fluctuations

2.1 The supersymmetric product price in period t

If distributed profit Y_D is set to zero in the 1st axiom then $Y = Y_W$ and the market clearing price P is determined as shown in Figure 1. The four quadrant positive rational diagram, 4QPR-diagram for short, makes the simplified consumption economy immediately comprehensible. The four axes represent the positive rational values of the variables employment L , income Y , consumption expenditures C , quantity bought X and output O , respectively. The bisecting line in the northwestern quadrant mirrors income from the horizontal to the vertical axis. The quadrants are numbered according to the axioms they enclose.

The market price follows from the axioms (1) to (3) and the conditions of market clearing, budget balancing and zero distributed profit:

$$P = \frac{W}{R} \quad (9)$$

$$\text{if } \rho_X = 1, \rho_E = 1, Y_D = 0 \quad |t.$$

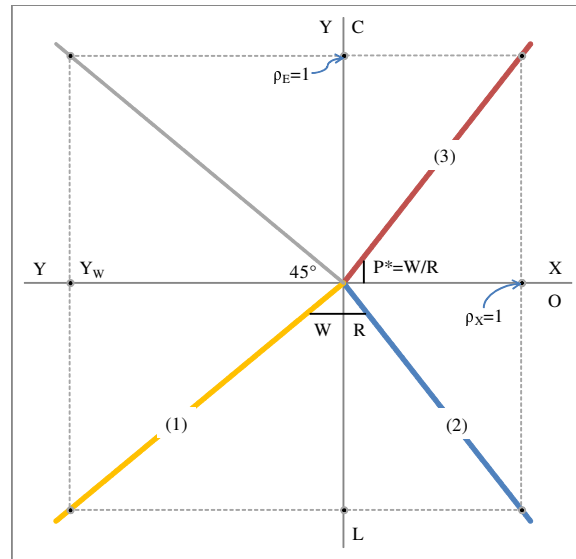


Figure 1: Market price of the consumption good under the conditions $X = O$, $C = Y$ and $Y_D = 0$

The market price is, under the given conditions, equal to unit wage costs, that is, profit per unit is zero at any level of employment. All changes of the wage rate and the productivity affect the market price. The elementary consumption economy with full price flexibility on the product market is reproducible for an indefinite time span at any level of wage rate, productivity and employment.

From (9) follows the real wage immediately as:

$$\frac{W}{P} = R \quad (10)$$

$$\text{if } \rho_X = 1, \rho_E = 1, Y_D = 0 \quad |t.$$

The real wage in period t is equal to the productivity. Note that the real wage is not determined in the labor market but *uno actu* with the supersymmetric product price. It is the wage rate W that is established in the labor market as nominal numéraire. This follows directly from the structural axioms.

Real and nominal flows determine the events in the product market. Therefore, the outcome ratio – defined as the ratio of the ratios of real and nominal flows – offers itself as a straightforward measure:

$$\omega \equiv \frac{\rho_E}{\rho_X} \quad |t. \quad (11)$$

The outcome ratio in each period is given by the configuration of the expenditure ratio ρ_E and the sales ratio ρ_X . This is the most compact numerical characterization of the situation in the product market. The configurations that produce exactly an outcome ratio of unity are called symmetric. One single configuration among these is supersymmetric, i.e. $\rho_E = 1$, $\rho_X = 1$. This, clearly, is an analytical limiting

case. It is rather improbable that the economy finds itself in this unique state that is characterized by the complete disappearance of real and nominal residuals. Figure 2 summarizes the interrelations.

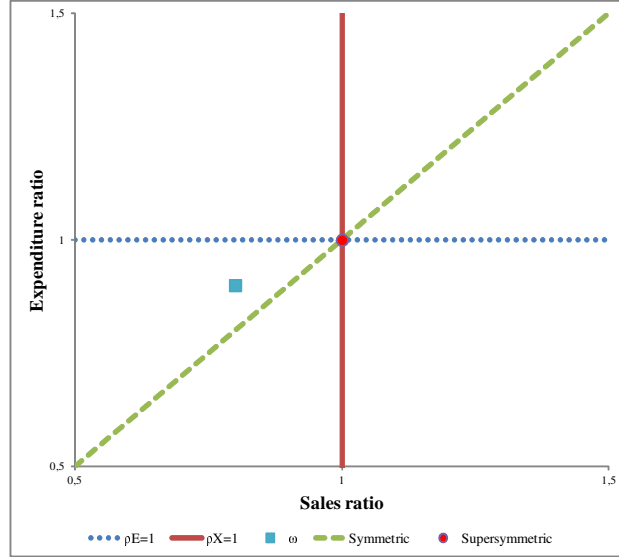


Figure 2: Configurations of the sales ratio and the expenditure ratio with supersymmetry as a limiting case among an infinity of possible random configurations of the outcome ratio ω

To determine the supersymmetric price is beyond the capacity of myopic agents. They neither know the actual random changes in period t nor the true model of the economy which is given with the set of structural axioms. Supersymmetry has no behavioral connotation. The idea that some forces move the economy towards this unique configuration is therefore utterly misleading. Equilibrium and supersymmetry are entirely different concepts. Alone rational economic man – a quixotic figure, indeed – believes in equilibrium.

2.2 Perfect price adaptation to stationary random changes

With (9) the woolly phrase that the price is determined by demand and supply acquires a concrete meaning. Ultimately, demand boils down to the wage rate W , and supply to the productivity R . The supersymmetric product price results from the interaction of a nominal and a real variable. The exchange relation between two consumption goods can be determined with two market prices (for details see 2011b). Here we focus on the one-good case.

The path of the supersymmetric price follows from (9) and (5) as:

$$P_t = \frac{W_0 \Pi \ddot{W}_t}{R_0 \Pi \ddot{R}_t} \quad (12)$$

For the simulation the computer generated random variates that are uniformly distributed on the closed interval $[0; 1]$ are transformed into the set of possible outcomes:

$$[0; 1] \rightarrow \{ \ddot{z}_l \leq \ddot{Z}_t \leq \ddot{z}_u \} \quad (13)$$

The lower \ddot{z}_l and upper \ddot{z}_u values of the transformed random variates are fixed in accordance with observation, plausibility and convenience. For the description of a stationary random path of productivity a range of about ± 3 percent is taken as plausible in the absence of extraordinary exogenous shocks. Then, given symmetric, identical and independent sets of possible outcomes of productivity changes $\{ \ddot{R}_t \}$ and wage rate changes $\{ \ddot{W}_t \}$ in period t , eqs. (12) and (13) produce the simulation outcome that has been selected as Figure 3. Of the infinite number of possible random paths the special feature of the selected path is that the price returns to its initial value within the time span of observation. Most simulated paths, to be sure, do not show such a neat cyclical pattern. It is worth to recall that employment is irrelevant for price determination under the stated conditions, only productivity and wage rate count.

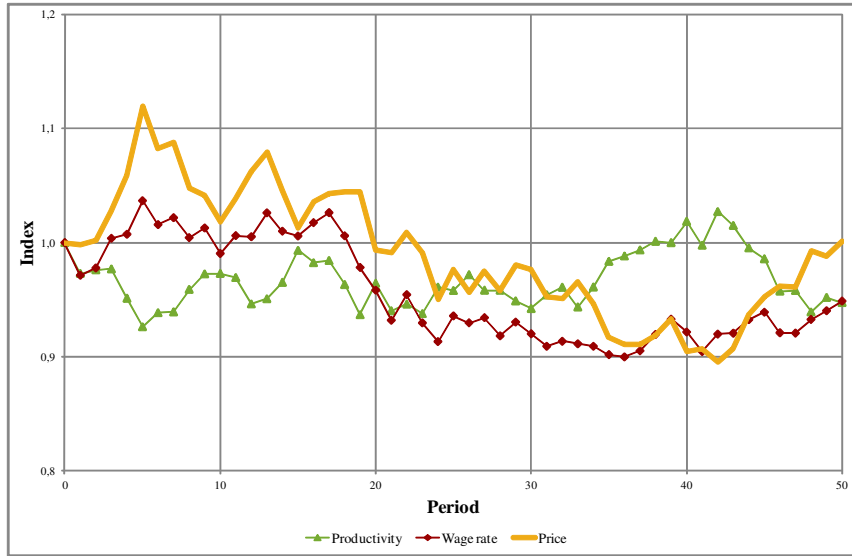


Figure 3: Determination of the market price in the pure consumption economy over 50 periods under the conditions $X = O$, $C = Y$, $Y_D = 0$ and symmetric random changes of productivity respectively wage rate in each period

It has been observed by Slutsky (1937, p. 114) that the summation of random causes may produce cyclic processes in the economy. This seems also to be the case for the price in Figure 3 although the cyclic pattern does not result from a simple summation but from the economic structure that is given with the axioms and the conditions of market clearing and budget balancing. Stochastic cycles invite

explanations in terms of the agents' behavior. This, though, is not much different from interpreting an ink blot. The sine-like fluctuation of the supersymmetric price is fully explained by the random paths of productivity and wage rate in the structural axiomatic setting (cf. Kyun, 1988, pp. 1, 60, 62).

We normally characterize a couple of periods with rising prices as inflationary. Figure 3 makes it clear that it might occasionally be useful to take a second reference point into consideration. With regard to the price in the initial period the price increase in the rightmost panel is better characterized as reflation.

Informally speaking, in the very long run, i.e. $t \rightarrow \infty$, the market clearing and budget balancing price is close to the initial price if the random distributions (13) of the rates of change of productivity and wage rate are symmetric, identical and independent. That is, inflationary and deflationary phases cancel out eventually. This is not the result of goal-directed human behavior, inter-temporal optimization, or some occult equilibrating forces but ultimately of the law of large numbers. It would be beside the point to interpret this result as 'equilibrium in the long run.'

The price-quantity correlation in Figure 4 that shows just another aspect of the happenings in Figure 3 has some superficial similarity with a demand schedule. It should be obvious, however, that Figure 4 keeps record of a well-defined stochastic process and that, on the other hand, the demand schedule is a hypothetical construct that is derived from untenable behavioral assumptions. The visual similarity has no deeper meaning. It may be the case, though, that empirical tests that seem to confirm a demand schedule *de facto* confirm a structural axiomatic random process.

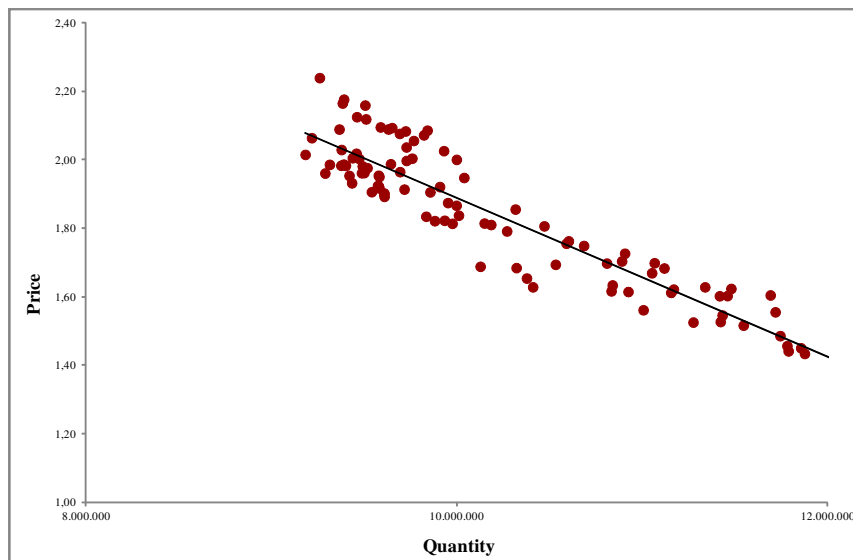


Figure 4: Negatively sloped price-quantity correlation as produced by the stationary random process of Figure 3

The perfect price adaption to changes of productivity and wage rate follows as a logical necessity from the supersymmetry conditions. The price setting is

entirely free of assumptions about human behavior. As a matter of fact, it is hard to imagine how real world agents could practically attain supersymmetry. In order to calculate the market clearing and budget balancing price an agent must know, first, the concrete values of the random changes and, second, the true model of the economy. Strictly speaking, the price setting should occur simultaneously with the random changes of productivity and wage rate. This, clearly, involves a contradiction. Randomness, human action, and simultaneity are mutually exclusive.

Nevertheless, let us suppose for a moment that the agents are endowed with all required faculties and ask: could they achieve perfect price stability? In other words, could they iron out the inflationary and deflationary phases in Figure 3? Perfect price stability requires that the random distributions are not only equal, i.e. $\{\ddot{W}_t\} = \{\ddot{R}_t\}$, but that they collapse to $\ddot{W}_t = \ddot{R}_t$. That is, the agents have to make it happen that the wage rate changes are in each period exactly equal to the realized random changes of productivity: if productivity increases by 2.39 percent at the beginning of period t the wage rate must increase by 2.39 percent in order to keep the supersymmetric price unchanged and vice versa if productivity declines. Agents – there is no need to loose much words about it – lack all the required faculties to achieve this. From the theoretical limiting case of Figure 3 the rule of thumb can be derived that wage rate changes should follow random productivity changes, as well as larger random shocks, as closely as possible.

The classical theory of the price level consisted of an uneasy blend of the cost of production theory of value and the quantity theory of money (Laidler, 1993, p. 51). This led in the course of time to the explanation of price level variations by underlying changes in the demand for and supply of money (Hoover, 1984). Obviously, this mode of explanation is not supported by the structural axiomatic analysis. Neither are explanations that are based on individual optimizing behavior. The structural axiomatic explanation of long-run, i.e. $t \rightarrow \infty$, price stability in the stationary consumption economy consists of two elements: identical symmetric random distributions of productivity and wage rate changes and the law of large numbers. The invisible hand plays dice (cf. Weintraub, 1970, p. 382).

2.3 Profit

The business sector's financial profit in period t is defined with (14) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W :

$$\Delta \bar{Q}_{fi} \equiv C - Y_W \quad |t. \quad (14)$$

Because of (3) and (6) this is identical with:

$$\Delta \bar{Q}_{fi} \equiv PX - WL \quad |t. \quad (15)$$

With the supersymmetric price (9) inserted this gives zero profit for the business sector as a whole independently of the configurations of productivity, wage rate, and

employment. Note that a productivity increase has no effect on profit but only on the product market price. The same holds for changes of the wage rate. Profit will not appear before the invisible hand makes $\rho_E > 1$ and/or $Y_D > 0$ in (9). This follows with logical necessity from the set of axioms. In contrast, no useful conclusion about profit of the business sector as a whole can be derived from the behavioral hypothesis of profit maximization. Neither neoclassicals nor Keynesians ever came to grips with profit (Desai, 2008, p. 10), (Tómasson and Bezemer, 2010).

We first consider the case with distributed profit greater zero. The supersymmetric price follows from (3) and (1) as:

$$P = \frac{W}{R} \left(1 + \frac{DN}{WL} \right) \quad (16)$$

$$\text{if } \rho_X = 1, \rho_E = 1 \quad |t.$$

The market price P is higher than unit wage costs $\frac{W}{R}$ in the case of market clearing and budget balancing if distributed profit is greater than zero. Given the amount of distributed profit Y_D as well as wage rate W and productivity R the price varies with employment L . With increasing employment the market price falls (all other variables constant, of course).

With symmetric, identical and independent sets of possible random outcomes (13) for all variables in (16) the exemplary process of Figure 5a results.

For profit follows from (15) in combination with (16):

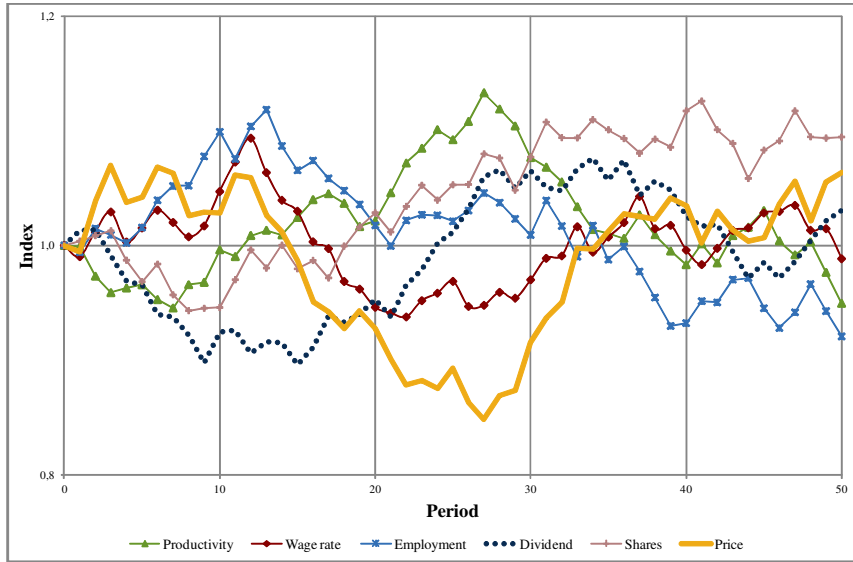
$$\Delta \bar{Q}_{fi} = Y_D \quad (17)$$

$$\text{if } \rho_X = 1, \rho_E = 1 \quad |t.$$

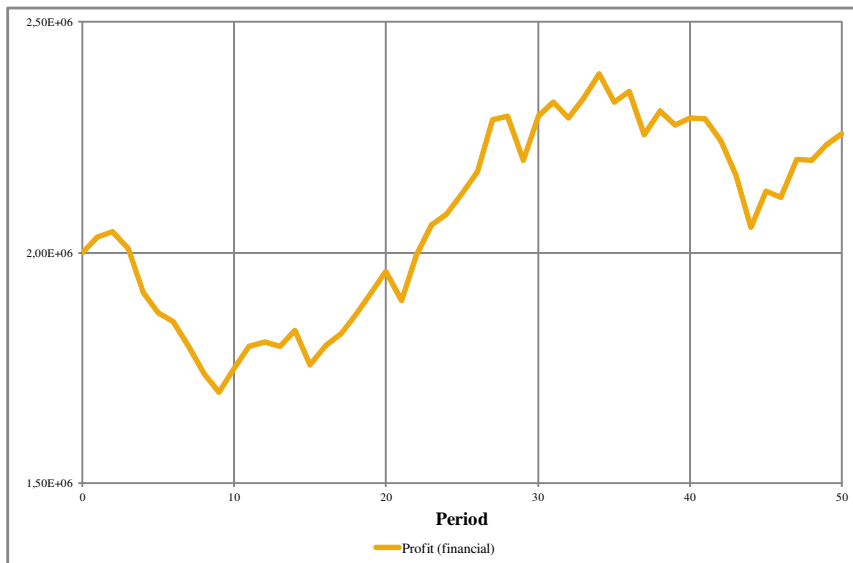
Financial profit is equal to distributed profit under the condition of supersymmetry. The equality of profit and distributed profit is an implicit feature of equilibrium models (Godley and Lavoie, 2007, p. 35), (Godley and Shaikh, 2002, p. 425), (Patinkin, 1989, p. 329), (Buiters, 1980, pp. 3, 7). Figure 5b shows the fluctuations of profit in the pure consumption economy as determined by the random development of dividend and number of shares in Figure 5a.

By observing a single firm one arrives at the commonsensical conclusion that the normal sequence is that profit comes first and then comes profit distribution. This does not hold for the economy as a whole. For the business sector profit distribution in period t is itself one of the two sources of profit in period t . To apply the microeconomic logic to the whole economy is the standard fallacy of composition. There is nothing paradoxical in the assertion that profit distribution generates profits. To the contrary, to overlook this fact is one of the worst analytical blunders of standard economics.

Profits can either be distributed or retained. If nothing is distributed, then profit adds entirely to the financial wealth of the firm. Retained profit $\Delta \bar{Q}_{re}$ is defined for the business sector as a whole as the difference between profit and distributed profit in period t :



(a) The supersymmetric market price depends, in addition to productivity and wage rate, now also on distributed profit and employment



(b) Financial profit is equal to distributed profit under the condition of continuous market clearing and budget balancing (refers to Figure 5a)

Figure 5

$$\Delta\bar{Q}_{re} \equiv \Delta\bar{Q}_{fi} - Y_D \Rightarrow \Delta\bar{Q}_{re} \equiv C - Y \quad |t. \quad (18)$$

Retained profit is, because of (14) and (1), equal to the difference of consumption expenditure C and total income Y . Under the condition of budget balancing, i.e. $C = Y$ or $\rho_E = 1$, retained profit is always zero. This, of course, is an analytical limiting case. The normal real world case is that profit is different from distributed profit and that retained profit is therefore different from zero. From this observation follows, conversely, that in the general case the household sector's budget is not balanced.

2.4 Growth and long run price stability

The stationarity of the foregoing processes is due to the symmetry of positive and negative rates of change in (13). To switch to a growth regime is therefore quite simple. We have only to adapt the random distribution, that is, to confine it to positive variates:

$$[0; 1] \rightarrow \left\{ 0 \leq \ddot{Z}_t \leq \ddot{z}_u \right\} \quad (19)$$

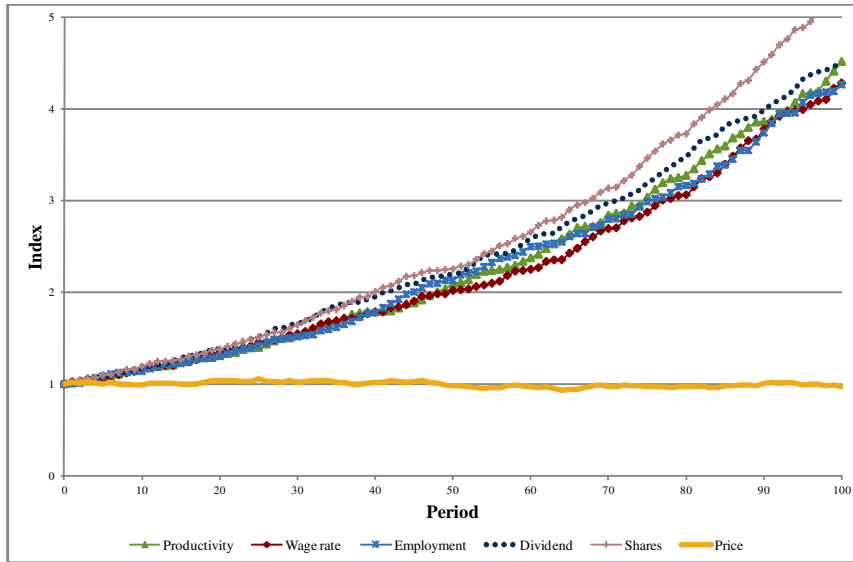
The path of the supersymmetric market price in the pure consumption economy with profit distribution (16) is given by:

$$P_t = \frac{W_0 \Pi \ddot{W}_t}{R_0 \Pi \ddot{R}_t} \left(1 + \frac{D_0 \Pi \ddot{D}_t N_0 \Pi \ddot{N}_t}{W_0 \Pi \ddot{W}_t L_0 \Pi \ddot{L}_t} \right) \quad (20)$$

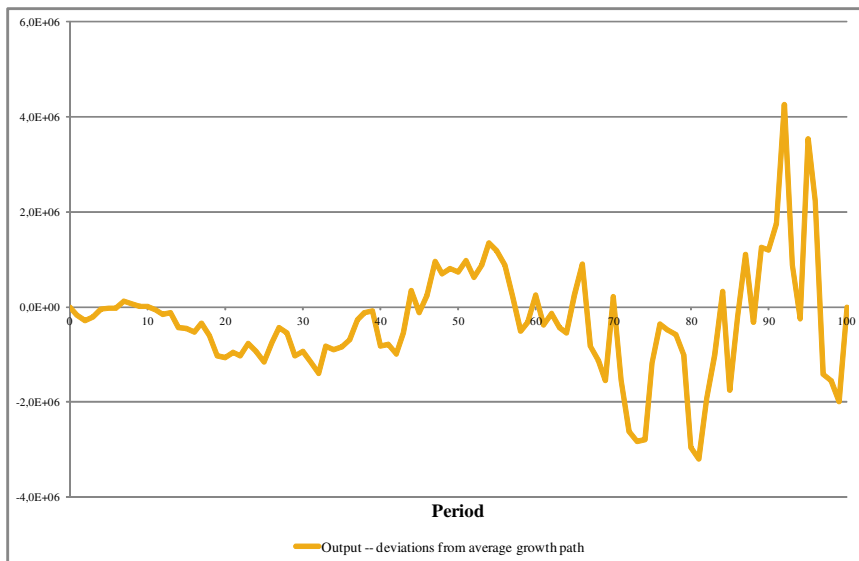
It follows immediately that, if the rates of change $\ddot{Z} \rightarrow \ddot{W}, \ddot{R}, \ddot{D}; \ddot{N}, \ddot{L}$ were exactly equal in each period, then the price would remain constant. Correspondingly, with identical independent random distributions (19) the flat price path in Figure 6a results from a simulation run of (20).

More specifically it follows from (20) as a rule of thumb that, in order to achieve stochastic price stability, the set of random changes of the dividend $\{\ddot{D}\}$ should be equal to the set of wage rate changes $\{\ddot{W}\}$; and likewise for the number of shares $\{\ddot{N}\}$ and employment $\{\ddot{L}\}$. This correspondence rule holds in addition to $\{\ddot{W}\} = \{\ddot{R}\}$ of Section 2.2. Note again that price stability is then produced by the system itself. If human behavior had any systematic influence this should be clearly visible as departure from the pure random paths. In the last instance it is the formal structure of eq. (20) in combination with the assumed probability distributions that channels the development of the supersymmetric price. If one of the probability distributions violates the correspondence rule we will see either inflationary or deflationary fluctuations of the supersymmetric price.

Output as given with (2) grows because of employment and productivity increases. The 2nd axiom contains only real variables. These interact via the other members of the structural axiom set with nominal variables. This in turn implicates that the product price is perfectly flexible otherwise the conditions of market clearing and budget balancing could not be realized and the output could not be sold.



(a) Long run price stability follows from an identical independent set of random outcomes for the changes of the axiomatic variables, i.e. for the random distribution $\{0 \leq \bar{Z} \leq 3\%\}$



(b) The real business cycle consists of stochastic fluctuations of output around the average growth path and is produced by the interaction of the random paths of Figure 6a

Figure 6

Formally, output is given with the 2nd axiom, in a broader sense it is determined as one element of the axiom set. The output path follows from (2) and (5):

$$O_t = R_0 \Pi \ddot{R}_t L_0 \Pi \ddot{L}_t. \quad (21)$$

The deviations of actual output from the average growth path over 100 periods are given by $O_t - O_{avg}$. As shown in Figure 6b output fluctuates in a rather sophisticated pattern around the average growth path. Structural Slutsky-cycles, though, are not in need of any behavioral explanation. The exemplary real business cycle is produced by (21) and the random distribution (19) as an integral part of the structural axiomatic consumption economy of Figure 6a. Note that this economy is supersymmetric, i.e. the product market is always cleared and the household sector's budget is always balanced. With a perfectly flexible price in the product market phenomena like overproduction or underconsumption cannot turn up in this economy. Whether the wage rate is flexible or not is irrelevant.

Due to the supersymmetry condition the structural axiomatic framework reproduces, in the first round, the classical results for the product market. But what about the labor market?

3 Formal properties of Say's regime

In a rather casual formulation it can be said that Say's law asserts that all markets clear but that some markets can be in a temporary disequilibrium. The classics, except Malthus, took this tenet for granted without formal proof (Laidler, 1993, p. 25). We have achieved market clearing in the product market by assumption. The supersymmetric price is an algebraic concept and not a behavioral concept.

Supersymmetry is compatible with any level of employment and any wage rate. Say's law implies, of course, also the clearing of the labor market which is to say that the economy always operates at full employment. The next task is to formally integrate the labor market and to give a comprehensive description of Say's regime in structural axiomatic terms.

In order to compactify the formalism the distributed profit ratio is defined as:

$$\rho_D \equiv \frac{DN}{WL} \equiv \frac{Y_D}{Y_W} \quad |t. \quad (22)$$

The ratio is a measure of the income distribution that follows quite naturally from (16).

The profit ratio is defined as:

$$\rho_Q \equiv \frac{\Delta \bar{Q}_{fi}}{WL} \quad |t. \quad (23)$$

Note that there is no capital in the pure consumption economy. Hence profit cannot be attributed to capital and there can be no profit rate. In combination with

(17) it follows that the profit ratio is, under the conditions of supersymmetry, equal to the distributed profit ratio:

$$\rho_Q = \rho_D \quad (24)$$

if $\rho_X = 1, \rho_E = 1 \quad |t.$

In order to neutralize the effect of employment on the income distribution and ultimately on the market price we at first define the ratio of number of shares to employment as:

$$\rho_N \equiv \frac{N}{L} \quad |t. \quad (25)$$

Under the condition $\rho_N = \text{const.}$ the number of shares moves always with employment.

In order to neutralize the effect of dividend on the income distribution and ultimately on the market price we next define the ratio of dividend to wage rate as:

$$\rho_V \equiv \frac{D}{W} \quad |t. \quad (26)$$

Under the condition $\rho_V = \text{const.}$ the dividend moves always with the wage rate. With the conditions (25) and (26) the profit ratio in (24) is constant, no matter what happens to employment and the wage rate. Say's regime implies the conservation of the initially given income distribution which is expressed by ρ_D . Distributional effects are analytically a quite separate matter and are therefore put aside here.

Eq. (16) can now be reformulated as:

$$P = \frac{W}{R} (1 + \rho_D) \quad (27)$$

if $\rho_X = 1, \rho_E = 1 \quad |t.$

The supersymmetric price depends on unit wage costs $\frac{W}{R}$ and the income distribution which is held constant by assumption. Thus, the profit ratio is always the same according to (24). The single firm that represents the business sector can remain completely indifferent between various employment levels. For the single firm unemployment is as good as full employment. To break the indifference and to tip the balance in the right direction we need a behavioral assumption. It is assumed that, given the profit ratio ρ_Q , the firm prefers a greater absolute profit $\Delta\bar{Q}_{fi}$. In other words, the firm prefers to be larger than smaller if the profit ratio is equal. With regard to employment this implies that the firm grows:

$$\ddot{L}_t > 0 \quad \text{as long as} \quad u > 0 \quad \text{and vice versa.} \quad (28)$$

The firm hires workers at the going wage rate until the labor market is cleared, that is, until there is no more labor supply $L^\theta - L$ at the going wage rate or, in still other words, until the unemployment rate u is zero. L^θ denotes the desired number

of total working hours. The unemployment rate is defined as (cf. Blanchard, 2000, p. 118):

$$u \equiv \frac{L^\theta - L}{L^\theta} \quad |t. \quad (29)$$

Condition (28) guarantees full employment in Say's regime. This condition is more general than profit maximization. The latter presupposes decreasing returns. Whether this is the case in the real world is an open question. Decreasing returns can by no means be taken for granted in a general theory. This would be an elementary methodological mistake.

It is assumed that desired employment L^θ grows randomly between 0 and 1 percent. The business sector, on the other hand, adapts employment L with a random rate between 0 and 2 percent. Figure (7) shows the outcome of the selected simulation run.

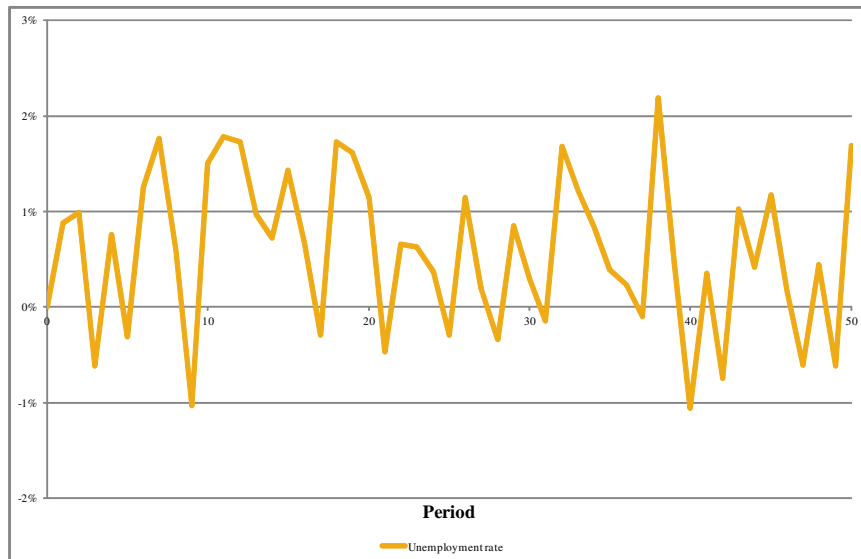


Figure 7: Unemployment rate in Say's regime

Because of the random changes full employment can only be achieved on the average. The unemployment rate hovers closely around zero. For all practical purposes the supersymmetric consumption economy is as close as possible to full employment. If the business sector could exactly foresee the random changes of L^θ full employment would be possible as a limiting case in each period.

The wage rate has no effect on the business sector's profit ratio but only on the supersymmetric price. From (27) and the condition of a constant income distribution follows that the real wage depends alone on the productivity which may rise or fall with increasing employment (cf. Gamber and Joutz, 1997, p. 277). If the productivity is assumed to be constant in the relevant range then movements to a higher employment level are at least indifferent for the business as well as the

household sector, no matter how the wage rate is determined in the labor market. This is the simplest case to start with. With changes of the income distribution and the productivity on the way to full employment things become a bit more complicated. Nevertheless, there is no analytical hindrance to the realization of Say's regime. The most important practical hindrance is that the agents cannot foresee the relevant random changes and have no true model of the economy in the back of their minds. This leads to adaptation cycles in the labor market and to inventory cycles in the product market. To these we turn next.

4 Inventory cycles

Hitherto, the supersymmetric price P has been the dependent variable. Now the price becomes an independent variable and the sales ratio ρ_X is the dependent variable. That means, not the anonymous market or fictitious auctioneer but some identifiable agent in the business sector acts as price setter. Since no agent has a precise idea of what the market clearing price is, the condition of market clearing can only be realized approximately. Rational expectations do not help much because the rational agent has, by definition, a general equilibrium model in the back of his mind. Since GE theory is vacuous (Ackerman and Nadal, 2004) the rational agent is inescapably behind the curve.

There is no change in the axiomatic formalism only in the direction of dependency. From the set of axioms and the definitions follows:

$$\rho_X = \rho_E \frac{W}{PR} \left(1 + \frac{DN}{WL} \right) \quad (30)$$

All other things equal, a price increase lowers the sales ratio. A lower sales ratio translates into an increase of the stock of products:

$$\Delta \bar{O} \equiv O - X \equiv O(1 - \rho_X) \quad (31)$$

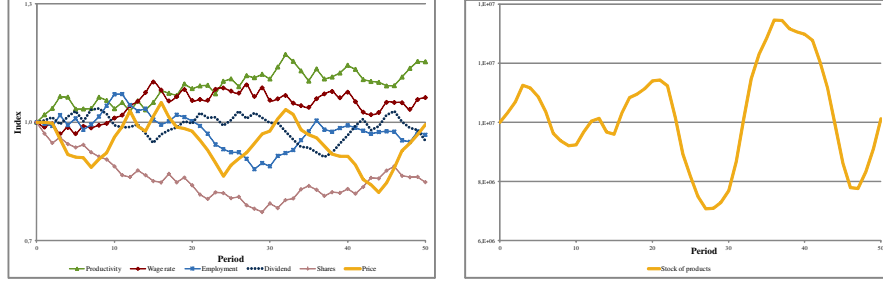
The stock of products \bar{O} at the end \bar{t} of an arbitrary number of periods is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{O} \equiv \sum_{t=1}^{\bar{t}} O(1 - \rho_{Xt}) + O_0 \quad |\bar{t}. \quad (32)$$

It is assumed now that the business sector lowers the price if the actual stock is above the desired stock \bar{O}^θ . In the opposite case the agent in charge increases the price. The rate of change is assumed to be a random variate between 0 and 10 percent:

$$\ddot{P}_t < 0 \quad \text{as long as} \quad \bar{O}_{t-1} > \bar{O}_{t-1}^\theta \quad \text{and vice versa.} \quad (33)$$

The interaction of (30) with $\rho_E = 1$, (32) and (33) produces the interdependent price and inventory cycles that are exemplified in Figure 8.



(a) Price setting by the business sector in dependence of the stock of products (b) Stock of products as result of the business sector's price setting

Figure 8

Is it possible to eliminate the inventory cycle? Only if the agent sets the price in each period exactly at:

$$P_t = \rho_{E0} \Pi \ddot{p}_{Et} \frac{W_0 \Pi \ddot{w}_t}{R_0 \Pi \ddot{r}_t} \left(1 + \frac{D_0 \Pi \ddot{d}_t N_0 \Pi \ddot{n}_t}{W_0 \Pi \ddot{w}_t L_0 \Pi \ddot{l}_t} \right) \quad (34)$$

The price setting task is trivial in a stationary environment if all independent rates of change are zero, otherwise it is a mission impossible. The existence of inventory cycles is the empirical proof that the agents have not solved eq. (34), for whatever reasons.

5 Money, credit, and transactions

Money as the defining characteristic of the economy cannot be added as an appendix to a real model but has to be consistently derived from the axiom set. There are real variables but no such thing as a real economy. Corn models are obsolete since Ricardo invented them (for details see 2011c).

If income is higher than consumption expenditure the household sector's stock of money increases. The change in period t is defined as:

$$\Delta \bar{M}_{\mathbf{H}} \equiv {}^m Y - C \equiv {}^m Y (1 - \rho_E) \quad |t. \quad (35)$$

The identity sign's superscript m indicates that the definition refers to the monetary sphere.

The stock of money $\bar{M}_{\mathbf{H}}$ at the end \bar{t} of an arbitrary number of periods is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{M}_{\mathbf{H}} \equiv \sum_{t=1}^{\bar{t}} \Delta \bar{M}_{\mathbf{H}t} + \bar{M}_{\mathbf{H}0} \quad |\bar{t}. \quad (36)$$

The changes of the business sector's stock of money are symmetrical to those of the household sector:

$$\Delta \bar{M}_B \equiv {}^m C - Y \quad |t. \quad (37)$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{M}_B \equiv \sum_{t=1}^t \Delta \bar{M}_{Bt} + \bar{M}_{B0} \quad |\bar{t}. \quad (38)$$

In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions are carried out by the central bank. The stock of money then takes the form of current deposits or current overdrafts. Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (36) the current overdrafts of the business sector are of equal amount according to (38), and vice versa. As it happens, each sector's *stock* of money is either positive (= deposits) or negative (= overdrafts). Money and credit are at first symmetrical. From the central bank's perspective the quantity of money at the end of an arbitrary number of periods is then given by the absolute value either from (36) or (38):

$$\bar{M}_t \equiv \left| \sum_{t=1}^t \Delta \bar{M}_{Ht;Bt} \right| \quad \text{if} \quad \bar{M}_{H0;B0} = 0. \quad (39)$$

The *quantity* of money is always ≥ 0 and follows directly from the axioms. It is assumed at first that the central bank plays an *accommodative* role and simply supports the autonomous market transactions between the household and the business sector. For the time being, the quantity of money is the *dependent* variable.

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 9a results. At the end of each subperiod, and therefore also at the end of the year, both the stock of money and the quantity of money is zero. Money is present and absent depending on the time frame of observation.

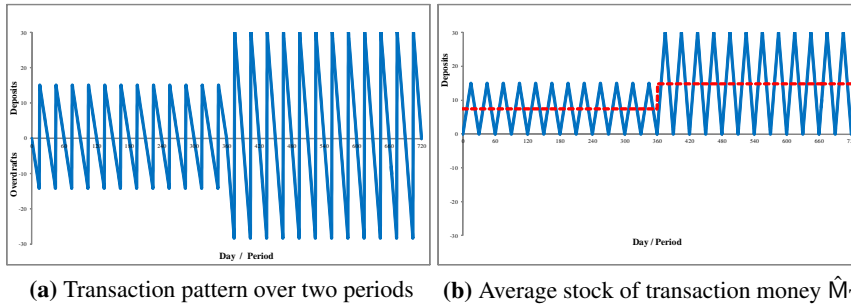


Figure 9: Household sector's transaction pattern for different nominal incomes in two periods; the business sector's pattern is perfectly symmetrical

In period₂ the wage rate, the dividend and the price is doubled. Since no cash balances are carried forward from one period to the next, there results *no* real balance effect provided the doubling takes place exactly at the beginning of period₂.

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore, the pattern of Figure 9a translates into the average amount of current deposits in Figure 9b. The average stock of transaction money depends on income according to the transaction equation:

$$\hat{M}_T \equiv \kappa Y \quad |t. \quad (40)$$

For the regular transaction pattern that is here assumed as an idealization the index is $\kappa = \frac{1}{48}$. Different transaction patterns are characterized by different numerical values of the transaction pattern index.

Taking (40) and (7) and (8) together one gets the explicit transaction equation for the limiting case of market clearing and budget balancing:

$$(i) \hat{M}_T \equiv \kappa \frac{\rho_X}{\rho_E} RLP \quad (ii) \frac{\hat{M}_T}{P} \equiv \kappa O \quad \text{if } \rho_X = 1, \rho_E = 1 \quad |t. \quad (41)$$

We are now in the position to substantiate the notion of accommodation as a money-growth formula. According to (i) the central bank enables the average stock of transaction money to expand or contract with the development of productivity, employment, and price. In other words, the real average stock of transaction money, which is a statistical artifact and not a physical stock, is proportional to output (ii) if the transaction index is given and if the ratios ρ_E and ρ_X are unity (cf. King and Plosser, 1984, p. 374). Under these *initial* conditions money is endogenous and neutral in the structural axiomatic context. Money emerges from *autonomous* market transactions and has three aspects: stock of money (\bar{M}_H, \bar{M}_B), quantity of money (here $\bar{M} = 0$ at period start and end because of $\rho_E = 1$) and average stock of transaction money (here $\hat{M}_T > 0$).

6 Employment fluctuations

6.1 In the pure consumption economy

In Say's regime market clearing is approximately realized in the product and labor market. The crucial alteration vis-à-vis Say's regime consists in making the price P the independent variable. Now L becomes the dependent variable. From the axioms and definitions follows:

$$L = \frac{\frac{D}{W}N}{\frac{\rho_X}{\rho_E} \frac{PR}{W} - 1} \quad |t. \quad (42)$$

The factor cost ratio is defined as:

$$\rho_F \equiv \frac{W}{PR} \quad (43)$$

In combination with (26) and (43) the employment equation finally reads:

$$L = \frac{\rho_V N}{\frac{\rho_X}{\rho_E \rho_F} - 1} \quad |t. \quad (44)$$

The structural axiomatic employment equation is testable in principle. For the pure consumption economy eq. (44) yields the exact result if we measure the ratios and the number of shares on the right hand side exactly.

One configuration of (44) is obviously critical: if the denominator becomes zero employment is not defined. It is assumed for the moment that the economy moves in safe distance around this singularity. What has to be kept in mind, though, is that there is nothing like an equilibrium in the pure consumption economy. Instead, there is a singularity.

In the supersymmetric case, i.e. $\rho_X = 1$ and $\rho_E = 1$, employment depends alone on ρ_F . The dimensionless numerical value of the factor cost ratio in turn depends on the configuration of wage rate, price and productivity according to (43). The random changes of these three variables determine in the supersymmetric case the employment fluctuations, the unemployment rate (29) and output (2). It is important to note that these real magnitudes depend not only on the productivity, which is determined by physical production conditions, but substantially on the configuration of the two nominal variables wage rate and price. From this follows that the whole idea of a real business cycle is definitively a nonstarter (Summers, 1986, p. 24), (Blaug, 2002, pp. 41-44), (Quiggin, 2010, pp. 99-101). Employment in (44) depends on the configuration of nominal variables and is inexplicable without it.

From (44) follows that employment increases if the factor cost ratio increases and vice versa. In order to achieve full employment things cannot be left to chance. Independent random changes of productivity, price and wage rate may send the factor cost ratio and by consequence employment in any direction. Under the condition that productivity varies at random with rates greater than zero and that the price remains unaltered it follows from (43) that the wage rate must increase according to the formula:

$$W_t = \rho_{F0} \Pi \ddot{\rho}_{Ft} P_0 R_0 \Pi \ddot{R}_t. \quad (45)$$

$$\ddot{\rho}_{Ft} > 0, \ddot{R}_t > 0$$

Figure 10 summarizes the interrelations. Employment increases according to (44) while the supersymmetric price remains perfectly stable. Productivity, wage rate and dividend pursue similar paths while output and the stock of transaction money pursue identical paths according to (41). The quantity of money is zero because ρ_E is unity throughout.

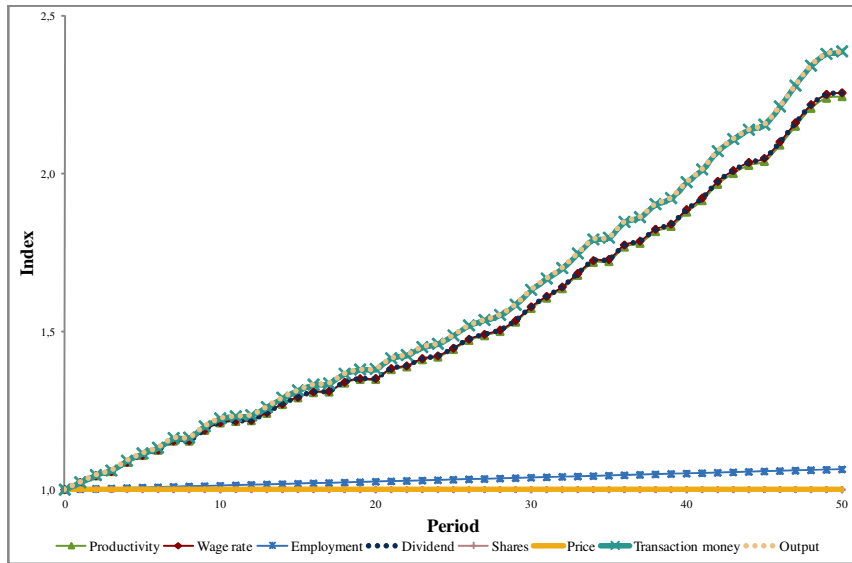


Figure 10: Price stability on the way to full employment

The expenditure ratio is the second determinant of employment in the general equation (44). Since $\rho_E \neq 1$ involves saving and dissaving this branch of analysis leads to the credit cycle models of business fluctuations (Laidler, 1993, p. 41). If the expenditure ratio is unity intertemporal optimization of the household sector is ruled out. The structural axiomatic interaction of credit and employment has been dealt with at length elsewhere (for details see 2012).

For small random changes of the expenditure ratio around unity follows the Ur-cycle from (42) as:

$$L = \frac{Y_D}{\frac{PR}{\rho_E} - W} \quad (46)$$

$$\text{if } \rho_X = 1 \quad |t.$$

Since an expenditure ratio of unity is an analytical limiting case that can never be realized exactly it has to be assumed as a first approximation that the fluctuations of ρ_E are small and symmetric around unity. Hence they cancel out over a longer time span. If Y_D, P, R, W are fix in (46) employment increases if the expenditure ratio is above unity and vice versa. That is, employment moves in parallel with the fluctuations of consumption expenditures that are determined by total income and the expenditure ratio in each period. Now, the latter is also one of the two key determinants of overall profit.

Profit follows from (14), (8), (6) and (1) as:

$$\Delta \bar{Q}_{fi} \equiv (\rho_{Et} - 1) Y_W + \rho_E Y_D \quad |t. \quad (47)$$

The fluctuations of profit depend alone on the fluctuations of the expenditure ratio around unity if wage income and distributed profit are taken as constants. In this case, both profit and employment depend on the expenditure ratio and therefore move in step. Yet, distributed profit is not a constant.

It is assumed that profit distribution in period t depends on profit in the foregoing period. Both magnitudes are formally linked by the payout factor:

$$Y_{Dt} = \varphi_D \Delta \bar{Q}_{fit-1}. \quad (48)$$

The substitution of (48) in (47) finally establishes a positive feedback loop for profit:

$$\Delta \bar{Q}_{fit} \equiv (\rho_{Et} - 1) Y_W + \rho_E \Delta \bar{Q}_{fit-1} \quad (49)$$

if $\varphi_D = 1$.

If $\rho_E > 1$ profit increases over time and vice versa if the expenditure ratio is below one under the condition that profits are fully distributed. A payout factor $\varphi_D < 1$ dampens the self-amplifier. The interaction of the expenditure ratio and profit distribution that is determined with (46) is visualized in Figure 11.

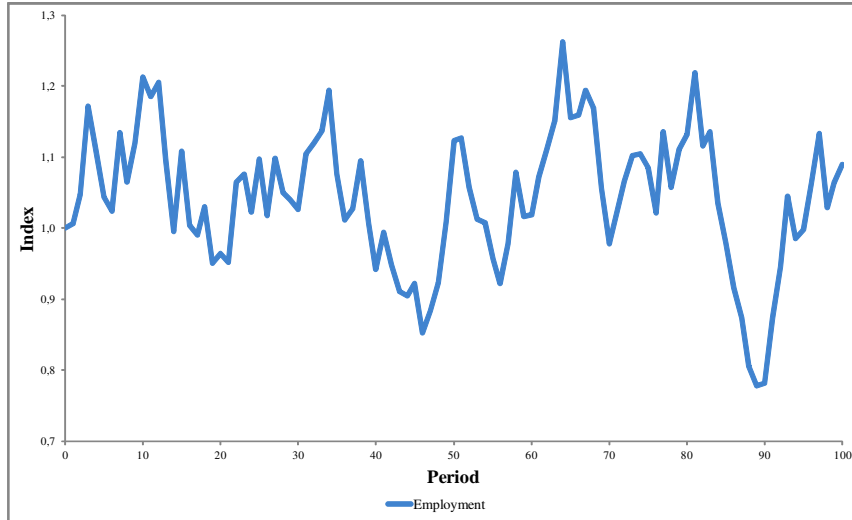


Figure 11: The Ur-cycle: employment is determined by the interaction of a symmetric random expenditure ratio and full profit distribution

The Ur-cycle is inescapable because perfect budget balancing in each period is impossible. In the simplest case the deviations of the expenditure ratio from unity are small and symmetric. This produces a rather stable cyclical pattern of employment and output. However, the changes of the expenditure ratio also affect profit and this establishes a positive feedback loop via profit distribution. This loop is potentially destabilizing. Whether this happens or not depends on the random

sequences of the expenditure ratio and the payout factor. There is an inherent structural instability in the pure consumption economy (cf. Haberler, 1964, p. 10).

6.2 In the investment economy

In order to include investment as the second important component of effective demand the axioms and definitions have first to be differentiated for two industries. The differentiated structural axiom set follows quite naturally from (1) to (3) and reads:

$$Y = W_C L_C + W_I L_I + \underbrace{D_C N_C + D_I N_I}_{\equiv Y_D} \quad |t. \quad (50)$$

$$\begin{aligned} O_C &= R_C L_C \\ O_I &= R_I L_I \end{aligned} \quad |t. \quad (51)$$

$$\begin{aligned} C &= P_C X_C \\ I &= P_I X_I \end{aligned} \quad |t. \quad (52)$$

Total employment with two industries is given by $L = L_C + L_I$. From the differentiated formalism follows the structural employment function under the condition of cleared product markets as:

$$L = \frac{1}{1 - \rho_E \rho_{FC}} \left(\rho_{FI} \frac{I}{W} + \rho_E \rho_{FC} \rho_V N \right) \quad (53)$$

$$\text{if } W_C = W_I = W, \rho_{XC} = 1, \rho_{XI} = 1 \quad |t.$$

Total employment depends on effective demand, i.e. on ρ_E and investment expenditure I , as well as on the respective configurations of wage rate, price, and productivity, i.e. on the factor cost ratios ρ_{FC} and ρ_{FI} . If independent consumption expenditure cycles are precluded by the condition $\rho_E = 1$ the two fluctuation producing variables are investment expenditure and the respective factor cost ratios. If real investment follows a random cycle then employment too performs a cycle that is magnified by the multiplier $\frac{1}{1 - \rho_{FC}}$ or in the general case by $\frac{1}{1 - \rho_E \rho_{FC}}$. Note that the Keynesian multiplier does not account for the key variables wage rate, price and productivity and is therefore inoperative.

What we have ascertained about the effects of the factor cost ratio in Section 6.1 applies *mutatis mutandis* to the investment economy. The strategic variables for employment fluctuations are, quite independent from the two demand components C and I , wage rate, price and productivity in both industries. All other things equal, the move to full employment presupposes a *rising* wage rate. This is a testable proposition that follows in direct lineage from the set of structural axioms and provides the perfect opportunity to refute the settled Marshallian sticky-wages tenet (Laidler, 1993, 96). It is the relation of the nominal variables wage rate and price that, for any given level of effective demand and productivity, drives the real business cycle.

7 Conclusion

Since real models of the business cycle lack a correct profit theory they are at best useless. To develop a correct profit theory requires a complete formal reset. This reset consists in the move from behavioral axioms to structural axioms. From this secure formal foundation the multitude of phenomena that make the business cycle can be consistently derived. The main results of the structural axiomatic analysis are:

- In the pure consumption economy with market clearing and budget balancing the inflationary and deflationary cycles of the market price are determined by the random paths of wage rate and productivity.
- The structural axiomatic explanation of long-run price stability in the stationary consumption economy consists of two elements: identical symmetric random distributions of productivity and wage rate changes and the law of large numbers.
- The fluctuations of financial profit in the supersymmetric consumption economy are determined by the random paths of dividend and number of shares.
- Long run price stability in a growing consumption economy follows from identical and independent sets of random distributions for the rates of change of the structural axiomatic variables.
- A complete formal description of Say's regime including the income distribution can be given in structural axiomatic terms. The most important hindrance for the realization of perfect market clearing is that the agents cannot foresee the relevant random changes and have no true model of the economy in the back of their minds. This gives rise to employment and inventory cycles.
- The attainment of full employment presupposes a rising factor cost ratio which is defined as quotient of wage rate, price and productivity W/PR .
- In the investment economy the cycle of real investment is amplified by the structural axiomatic multiplier which consists of the expenditure ratio and the factor cost ratio.
- It is the configuration of the nominal variables wage rate and price that, for any given level of effective demand and productivity, drives the real business cycle.

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