# A Study on the Dynamics of Interest Rate

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*Abstract*— **By integrating the fiat money into the structural growth model in [1], this paper presents a dynamic model for the simulation study of interest rate. And the model is illustrated with a numerical example. The equilibria of the numerical example are also computed by the method in [2]. The monetary policies of controlling the interest rate and controlling the money supply are simulated.**

***Keywords-smulation; interest rate; money supply; general equilibrium; price***

## I. INTRODUCTION

The structural growth model in [1] is a multi-agent, multi-commodity dynamic economic model, and money is not yet included in it. This paper presents a method of integrating money into the model so that some monetary problems can be studied by simulation with the model.

## II. A MONETARY STRUCTURAL GROWTH MODEL

To integrate money into the structural growth model in [1], let's assume that each agent needs money to maintain its production or consumption. The money-owners lend money to earn interest in each period and spend all the interest for consumption, just like land-owners lease land to earn land-rent. And the other consumers and firms also spend all their income for consumption or reproduction. An agent's demand for money is assumed to be proportional to the value of the commodity bundle it purchases, hence the total demand for money depends on the price level and the activity level in the economy. For now the model excludes the mechanism that equalizes the profit rate and the interest rate as discussed in [3]. That is, in an equilibrium the interest rate is allowed to be far away from the profit rate of firms.

The model is as follows:

$$\mathbf{p}^{(t+1)} = \widehat{\mathbf{e} - \theta\mu(\mathbf{u}^{(t)})\mathbf{e} + \theta\mathbf{u}^{(t)}}\mathbf{p}^{(t)}, \tag{1}$$

$$\mathbf{S}^{(t+1)} = \mathbf{B}\left(\mathbf{p}^{(t)}\right)\widehat{\mathbf{z}^{(t)}} + \widehat{\mathbf{e} - \mathbf{u}^{(t)}}\mathbf{S}^{(t)}, \tag{2}$$

$$\left(\mathbf{u}^{(t+1)}, \mathbf{z}^{(t+1)}\right) = Z\left(\mathbf{A}\left(\mathbf{p}^{(t+1)}\right), \mathbf{p}^{(t+1)}, \mathbf{S}^{(t+1)}\right). \tag{3}$$

Here $\mathbf{p}$ is the price vector, $\mathbf{S}$ is the supply matrix, $\mathbf{u}$ is the sales rate vector, $\mathbf{z}$ is the exchange vector and activity level vector. $\mathbf{A}(\mathbf{p})$ is the input coefficient matrix and $\mathbf{B}(\mathbf{p})$ is the output coefficient matrix. $\mathbf{e}$ denotes the all-ones vector, $\theta\,(0 \le \theta \le 1)$ denotes the velocity of price adjustment, and $\mu(\mathbf{u}^{(t)})$ denotes the mean value of all sales rates. $\hat{\mathbf{x}}$ denote the diagonal matrix with the vector $\mathbf{x}$ as the main diagonal.

Equation (1) means that the price of a commodity will rise if its sales rate is higher than the average sales rate, otherwise it will drop or keep unchanged. Unlike the relative price vector in the original model in [1], here the prices are measured in terms of money (say, dollar).

Equation (2) means that outputs and inventories in the preceding period constitute the supplies of the current period. The supplies of some commodities may be exogenous (e.g. labor, land and money).

Equation (3) stands for the exchange process (see [1]). Linear homogeneity is assumed for both production functions and utility functions. The input coefficient matrix indicates the demand structures of agents (i.e. consumers and firms). More precisely, the *i*th column of the input coefficient matrix stands for the commodity bundle needed by the *i*th agent at a unit level of activity (i.e. producing a unit of product or yielding a unit of utility).

## III. A NUMERICAL EXAMPLE

In this section we give a numerical example of a monetary economy and compute its equilibria, and its dynamics will be simulated in the next section.

Suppose there are three commodities (namely wheat, labor and money) and three representative agents (namely a firm producing wheat, a laborer and a money-owner). The production function of the firm and the utility functions of the two consumers are all assumed to be $x_1^{0.5}x_2^{0.5}$, wherein $x_1$ denotes the amount of wheat and $x_2$ the amount of labor. In each period the laborer supplies 100 units of labor. The money-owner owns $\xi$ dollars and intends to lend it out to earn interest in each period. Under a price vector $\mathbf{p}$ (consisting of the wheat price, wage rate and interest rate), the input coefficient matrix $\mathbf{A}(\mathbf{p})$ is as follows and the output coefficient matrix $\mathbf{B}$ is $\widehat{[1 \quad 100 \quad \xi]}$:

$$\mathbf{A}(\mathbf{p}) = \left[\mathbf{a}(\mathbf{p}) \quad \mathbf{a}(\mathbf{p}) \quad \mathbf{a}(\mathbf{p})\right], \tag{4}$$

wherein

$$\mathbf{a}(\mathbf{p}) = \left[ \left( p_2 / p_1 \right)^{0.5} \quad \left( p_1 / p_2 \right)^{0.5} \quad 2\left( p_1 p_2 \right)^{0.5} \right]^T . \tag{5}$$

Let $\mathbf{p}^*$ denote an equilibrium price vector and $\boldsymbol{\upsilon}^*$ denote an equilibrium utility vector (consisting of the utility levels of the laborer and the money-owner), the equilibrium input coefficient matrix $\mathbf{A}(\mathbf{p}^*, \boldsymbol{\upsilon}^*)$ is defined as $\widehat{\mathbf{A}(\mathbf{p}^*)[1 \quad \upsilon_1^* \quad \upsilon_2^*]}$. By the method in [2] the equilibrium price vector is computed to be $\mathbf{p}^* = \left((1+p_3^*)^2 \xi / 50, \xi / 200, p_3^*\right)^T$, wherein the equilibrium interest rate $p_3^*$ may assume any nonnegative real number. The equilibrium output amount of wheat is $25/(1+p_3^*)^2$, which is unrelated to the money supply $\xi$. And the equilibrium output value of wheat is $\xi / 2$. The equilibrium utility levels of the laborer and the money-owner are $25/(1+p_3^*)^2$ and $50 p_3^* / (1+p_3^*)^2$ respectively. Note that in this economy both the equilibrium profit rate and the equilibrium growth rate are 0. If we assume that the equilibrium interest rate equals the equilibrium profit rate, the equilibrium price vector will be $\mathbf{p}^* = \left(\xi / 50, \xi / 200, 0\right)^T$.

Note that when the interest rate rises, the wheat output and the utility level of the laborer will decrease. Hence to avoid a too low wheat output and to maintain the subsistence of the laborer, the interest rate cannot be too high.

When $\xi = 100$ and the interest rate is 0.25, the equilibrium price vector is $\mathbf{p}^* = (3.125, 0.5, 0.25)^T$, and the equilibrium production and consumption processes are as shown in Table 1.

In each period, the firm borrows 40 dollars to purchase its inputs for production, and its total output value is 50 dollars. After selling its output, the firm repays the loan together with 10-dollar interest. The laborer borrows 40 dollar to buy 6.4 units of wheat and 40 units of labor (i.e. service), and after receiving her wage, the laborer repays the loan together with 10-dollar interest. The money-owner borrows 20 dollars to buy wheat and service, and after receiving her 25-dollar interest, money-owner repays the loan together with 5-dollar interest.

## IV. Simulations

Let's set the initial values of $\mathbf{p}^{(0)}$, $\mathbf{z}^{(0)}$ and $\mathbf{u}^{(0)}$ to $(1, 1, 1)^T$. The velocity of price adjustment $\theta$ is set to 0.4.

First let's set the supply amounts of both labor and money in each period to 100. Fig. 1 shows the dynamics of the economy in period 1 to 100, which starts with a disequilibrium initial state and converges to an equilibrium eventually. Since there are multiple equilibria, the equilibrium to which the system converges depends on the initial state. In Fig. 1 the interest rate converges to 0.2375 and the output of wheat converges to 16.33 units.

Recall that when the money supply is specified the equilibrium price vector and the equilibrium wheat output in this economy depend only on the interest rate. If the interest rate is set to an exogenous level without adjusting the money supply (as the monetary policy discussed in [4]), the economy may converge to a specified equilibrium. For instance, if the money-owner can control the interest rate, the interest rate will be set to 1 to maximize her utility. If the laborer can control the interest rate, the interest rate will be set to 0. Fig. 2 shows the dynamics of the economy with an exogenous interest rate 1, wherein the money supply is 100 dollars all the time.

The target interest rate can also be attained by controlling the supply of money instead of controlling the interest rate directly. The (3, 3) element of the supply matrix (i.e. $S_{33}$) denotes the exogenous supply of money. Let's assume the money policy $S_{33}^{(t+1)} = (0.9 + 0.1 p_3^{(t+1)}) S_{33}^{(t)}$, that is, if the current interest rate $p_3^{(t+1)}$ is higher than 1 the money supply will be increased, and if the current interest rate is lower than 1 the money supply will be reduced. Fig. 3 shows that under the money policy the interest rate also converges to 1. The money supply eventually converges to 23.63. Hence the equilibrium wage rate and the wheat price are much lower than those in Fig. 2.

Though in the simulations above the economy converges to equilibrium eventually, it's not always the case. The model may exhibit unattenuated business cycles in some cases, especially when the production and utility functions are Leontief-type.

TABLE I. EQUILIBRIUM PRODUCTION AND CONSUMPTION PROCESSES

| | **Wheat Producer** | **Laborer** | **Money-owner** | **Total Input** |
|---|---|---|---|---|
| **Wheat Input (Value)** | 6.4 (20$) | 6.4 (20$) | 3.2 (10$) | 16 (50$) |
| **Labor Input (Value)** | 40 (20$) | 40 (20$) | 20 (10$) | 100 (50$) |
| **Money in Use (Interest)** | 40$ (10$) | 40$ (10$) | 20$ (5$) | 100$ (25$) |
| **Output (Value or Interest) [Utility Level]** | 16 (50$) | 100 (50$) [16] | 100$ (25$) [8] | |

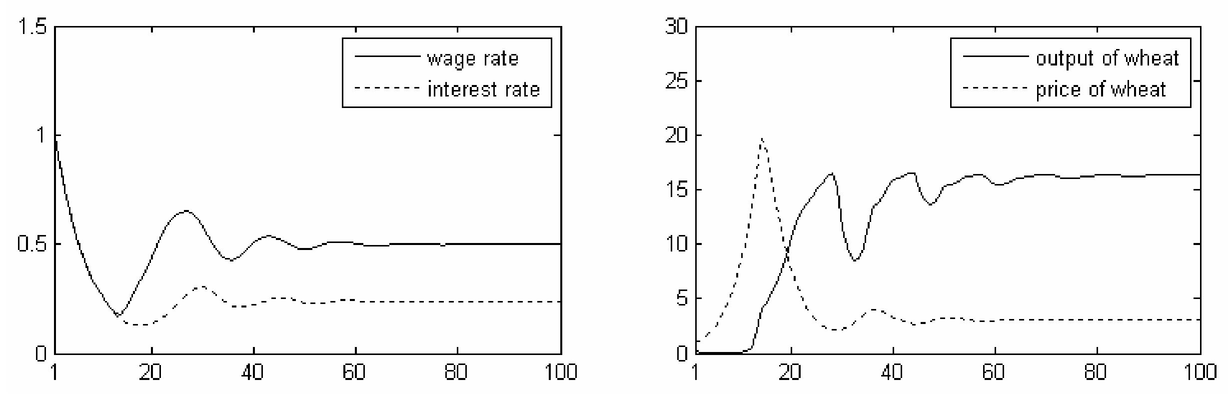

Figure 1. An economy with endogenous interest rates

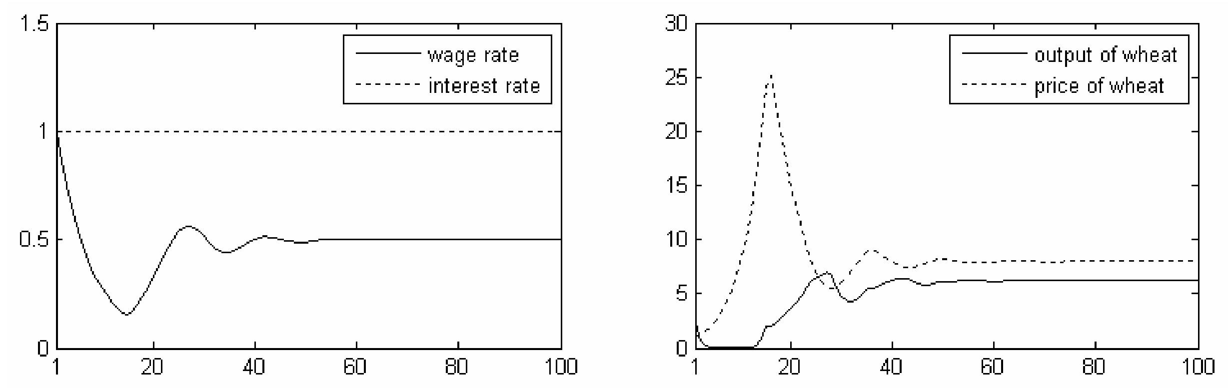

Figure 2. An economy with an exogenous interest rate 1

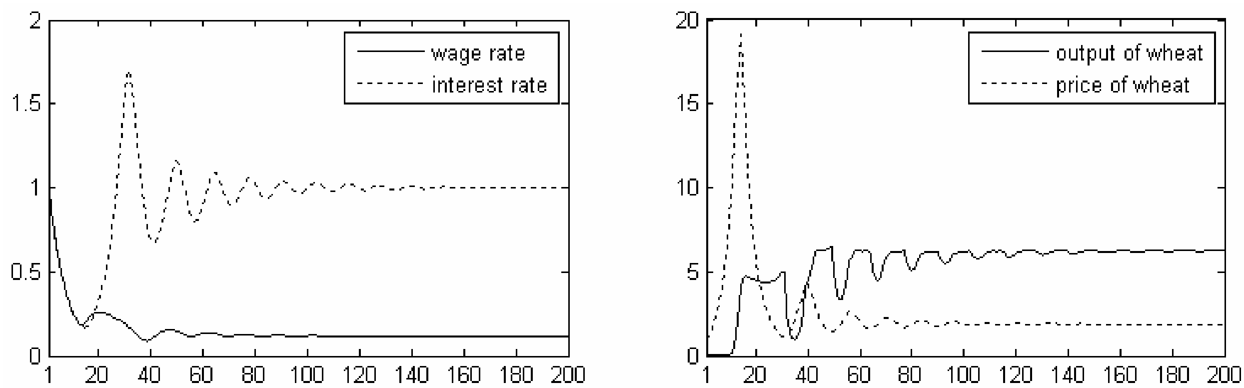

Figure 3. Controlling money supply and targeting interest rate 1

## V. Concluding Remarks

In the model presented in this paper there is only a money market, and the capital market is excluded. Since there is no saving in the model, an agent cannot expand (or contract) the scale of its production or consumption by long-term credit, and the interest rate is no longer the price of (long-term) credit as discussed in [5]. That is, in this paper the money is assumed to serve only as the medium of exchange, although the model doesn't contain an explicit monetary exchange process, and the role of money as the medium of (long-term) credit in capital market is excluded.

In the future study a capital market should be integrated into the model and both roles of money aforementioned should be embodied, thereby the equality of the equilibrium interest rate with the equilibrium profit rate will be guaranteed.

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