General formal foundations of the virtuous deficit–profit symmetry and the vicious debt deflation

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General Formal Foundations of the Virtuous Deficit–Profit Symmetry and the Vicious Debt Deflation

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Abstract

A comprehensive dynamic model of the monetary economy that produces the key characteristics of a debt deflation has been presented recently by Steve Keen as an alternative to conventional approaches. His model is based on a double-entry bookkeeping methodology but lacks an acceptable profit theory. In this respect it is not different from familiar approaches. Clearly, a deficient profit theory prevents a proper understanding of how the real world economy works. The present paper takes an entirely different route and places the core of Fisher's debt deflation theory into the context of the consistent structural axiomatic approach.

JEL E12, E31, E50

Keywords new framework of concepts; structure-centric; axiom set; income; profit; distributed profit; quantity of money; credit expansion; maximum debt/income ratio; annuity; positive feedback; built-in instability

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The latest financial crisis tells us that something went wrong, but not exactly what. If it is taken as an empirical proof, what does it prove: systemic failure, market failure, policy failure, institutional failure, moral failure, theory failure? If it is theory failure which variant of theoretical economics has been definitively falsified? Or, even more important for future research, has any of the alternatives to the ruling paradigm been corroborated?

Among serious scholars it has been known long before the financial crisis that general equilibrium theory as the core of the orthodox research program has at best a metaphorical relation to the real world (e.g. Hahn, 1981; Kirman, 1989; Ackerman, 2004). The crucial weakness of orthodox economics is that it has no proper place for money and credit. Keynes, for one, realized this clearly and proposed an alternative approach (Minsky, 2008, pp. 8-11). Hence, for deeper methodological reasons, orthodox economics was never a promising candidate to predict the latest financial crisis (cf. Keen, 2011, pp. 203-204; Bezemer, 2009, p. 3).

Not all methodologists are agreed that predictive capacity is the ultimate quality criterion of theories. Complementary or alternative demands refer to consistency, realism, testability, or the explanatory power of a theory. The actual state is: the core of orthodox economics is formally consistent but has no counterpart in the real world. Applied economics consists of a multitude of models that are either superficially related to the core (Quiggin, 2010, pp. 94-111) or stand unrelated on their own feet. This guarantees that each economic phenomenon has multiple explanations which seems to be somewhat unsatisfactory unless one is strong-willed to see everywhere 'virtue in diversity' (Dow, 2006, p. 3).

The more or less official explanation of the Great Depression is by now that it was seriously aggravated, if not caused, by contractive measures of the monetary authorities (Bernanke, 2000). Irving Fisher, famous for his optimistic outlook just before the Dow Jones's precipitous fall, gave an alternative and more convincing explanation (1933). His debt deflation theory was developed further by Minsky (1982), Koo (2009), and Keen (2011, pp. 370-377), among others.

Fisher's explanation was purely verbal. In an environment that prefers formal expression this was not exactly a recommendation, quite independent from the fact that there was no strong demand for this type of theory as long as the major economies performed satisfactorily on the average. In this respect, the financial crisis of 2007 marks a turning point that affects the acceptance of theories and boosts the rating of approaches that were hitherto regarded as heterodox.

A comprehensive model of the monetary economy that produces the key characteristics of a debt deflation has been developed by Steve Keen. Unfortunately, his model lacks a correct profit theory. This finds its formal expression in an indefensible definition of total income (Keen, 2011, pp. 366-368). The present paper takes an entirely different route and places the core of Fisher's debt deflation theory into the context of the structural axiomatic approach, in other words, it reconstructs it from general and consistent formal foundations. The paper focuses on household sector debt. Other forms, notably government and business sector debt, require separate studies.

Theories have a logical architecture consisting of premises and conclusions or, in a purely formal context, of axioms and theorems. Each theory starts from a small set of foundational 'hypotheses or axioms or postulates or assumptions or even principles' (Schumpeter, 1994, p. 15). The first task of theoretical economics has been clearly defined by J. S. Mill:

> What are the propositions which may reasonably be received without proof? That there must be some such propositions all are agreed, since there cannot be an infinite series of proof, a chain suspended from nothing. But to determine what these propositions are, is the *opus magnum* of the more recondite mental philosophy. (Mill, 2006, p. 746), original emphasis

In the language of contemporary methodology:

> If we are going to say anything useful at all about the economy, we have to make assumptions. There is no getting round that. But which assumptions? (Dow, 2006, p. 12)

General equilibrium theory rests on a set of *behavioral* axioms (Arrow and Hahn, 1991, p. v), (Weintraub, 1985, p. 109). This formal point of departure is in the present paper abolished and replaced by *structural* axioms. By choosing objective structural relationships as axioms the familiar behavioral hypotheses are not ruled out but at first relegated to the periphery. Structural axiomatization provides the correct profit theory. This, in turn, is the prior condition for the explanation of how the monetary economy works.

The methodological case for structural axiomatization has been made elsewhere (2012c). With the basic understanding that a general and consistent formal foundation is highly desirable the minimalistic structural frame is set up in Sections 1 to 3. The shortest possible description of the most elementary economic configuration includes money, credit, debt, profit, distributed profit and the market clearing price at any level of employment. In Section 4 the stylized three-period credit cycle is expounded. By logical necessity the economy is in one of the three states: credit expansion, constant debt, or credit contraction. For the respective configurations the market clearing price, profit, and the nominal/real rate of interest is in direct lineage derived from the structural axiom set. In Section 5 the different dynamics of debt deflation that crucially depend on the destabilizing feedback loop between profit and distributed profit are systematically established. Section 6 concludes.

1 The shortest possible description of the most elementary economic configuration

1.1 Axioms

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. The period length is conveniently assumed to be

the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. All quantitative and temporal extensions have to be deferred until the implications of the most elementary economic configuration are perfectly understood. Axiomatization is about the *minimum* number of basic propositions. A set of axioms, then, is the common formal core of more or less comprehensive models that embody the interdependencies of measurable variables which produce outcomes that have, in principle, a counterpart in the real world. A model consists of axioms and additional assumptions, conditions or specifications. There is no upper limit for the coverage of a model but there is a lower limit which is given with the following axiom set.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other activity of the government sector. All axiomatic variables are measurable in principle. No nonempirical concepts like utility, equilibrium, rationality, decreasing returns or perfect competition are put into the premises.

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (4) wage income Y_W and distributed profit income Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

1.2 At one glance

The four quadrant positive rational diagram, 4QPR-diagram for short, makes the pure consumption economy immediately comprehensible. The four axes in Figure 1 represent the positive rational values of the variables employment L, income Y, consumption expenditures C, quantity bought X and output O, respectively. The bisecting line in the northwestern quadrant mirrors income from the horizontal to

the vertical axis. Negative or real values of the axiomatic variables are excluded from the 4*QPR*-diagram. The quadrants are numbered according to the axioms they enclose.

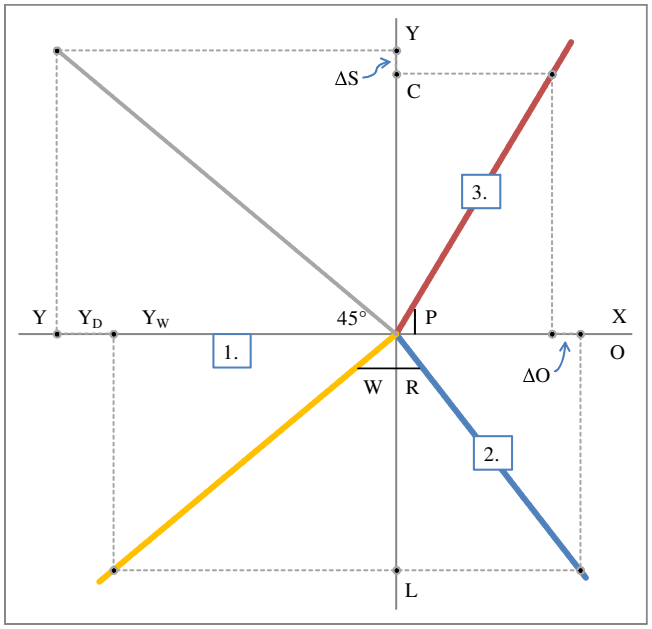


Figure 1: The 4*QPR*-diagram gives a view of the minimalistic – one world, one, firm, one product, one period – pure consumption economy as formalized by the structural axiom set

In the 1^{st} quadrant total income Y is given as product of wage rate W and working hours L plus distributed profits Y_D. The wage rate is equal to the tangent function of the ray's angle at $L = 1$. Graphical multiplication with total employment L delivers wage income Y_W. What deserves mention is that profit and distributed profit are quite different things that have to be kept apart. Profit is defined later with (10).

In the 2^{nd} quadrant output O is given as product of productivity R and working hours L. The productivity is determined by the underlying production process and may vary with labor input. The ray which represents the 2^{nd} axiom (2) should therefore not be interpreted as a linear production function. The functional relationship between productivity and employment, i.e. decreasing, increasing or constant returns, is not an *a priori* property of the axiom set but has to be taken as a contingent input from the real world. The 2^{nd} axiom can track any production function.

In the 3^{nd} quadrant consumption expenditures C is given as product of price P and quantity bought X.

Since the quantity produced O is here larger than the quantity bought X the firm that at the moment represents the entire business sector has an unsold quantity ΔO left over at period end which has to be taken into stocks. The change of inventory in period t is defined as:

$$\Delta O \equiv O - X \quad |t. \tag{5}$$

In the period under consideration the product market is not cleared. We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{6}$$

An alternative form of stating that the market is not cleared in period t is $\rho_X \neq 1$.

Consumption expenditures C in the 1^{st} quadrant is less than income Y, that is, the households save. Financial saving is defined as:[1]

$$\Delta S_{fi} \equiv Y - C \quad |t. \tag{7}$$

In the period under consideration the household sector's budget is not balanced. We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \tag{8}$$

An alternative form of stating that the household sector's budget is not balanced in period t is $\rho_E \neq 1$.

1.3 Market clearing and budget balancing

Up to this point the $4QPR$-diagram makes just the relations of the axiomatic variables transparent. Strictly speaking, Figure 1 represents the formal skeleton of a model but not yet a model. A model consists of necessary axioms and contingent assumptions. These assumptions are now added.

After the implementation of the conditions of market clearing $\rho_X = 1$ and budget balancing $\rho_E = 1$ the pure consumption economy looks as shown in Figure 2.

The market clearing price follows from (3) and (1) as:

$$\begin{gathered} P = \frac{W}{R} + \frac{Y_D}{RL} \\ \text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t. \end{gathered} \tag{9}$$

The market clearing price is higher than unit wage costs $\frac{W}{R}$ in the case of market clearing and budget balancing if distributed profit is greater than zero. Given the amount of distributed profit Y_D as well as wage rate W and productivity R the price varies with employment L. With increasing employment the market clearing price falls. If distributed profits are zero, i.e. $Y_D = 0$, then the market clearing price is equal to unit wage costs. In this limiting case profit per unit of output is zero and therefore overall profit is zero. The firm sells its period output completely and fully recoups its wage costs – not more, not less.

[1] For the treatment of nonfinancial saving see (2011c, Sec. 4.2).

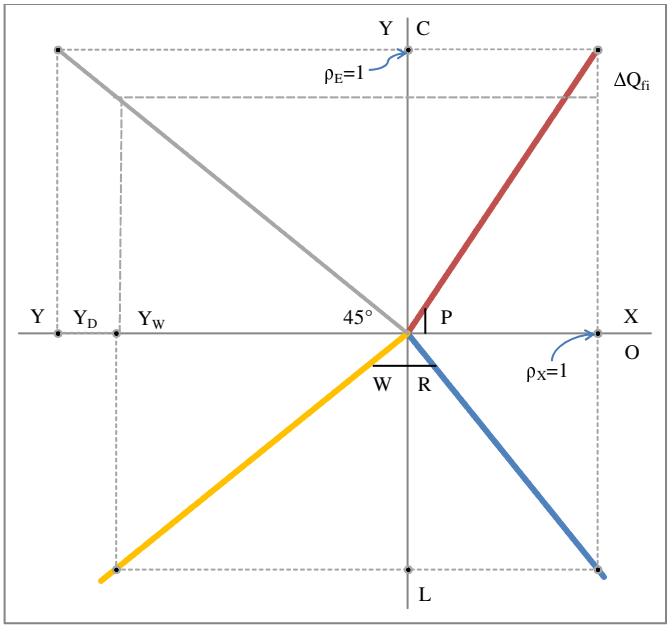


Figure 2: Implementation of the conditions of product market clearing and budget balancing, that is, of $\rho_X = 1$ and $\rho_E = 1$, and determination of the resulting market clearing price

It is worth emphasizing that the market clearing price is unequivocally determined by the three axioms and the conditions of market clearing and budget balancing. Therefore it is impossible to add behavioral demand and supply functions. This would amount to formal over-determination. The objective 4*QPR*-diagram supplants the familiar single market demand and supply schedules. In a comparison of tools the 4*QPR*-diagram refers to the economy as a whole while demand–supply schedules refer to a single market which is disconnected from the rest of the economy by *ceteris paribus*. What makes the familiar schedules dispensable in the first place, though, is that they depend on vacuous behavioral assumptions.

Due to the interdependence of markets, the market clearing price in the product market P depends inter alia on the current wage rate W in the labor market. Whether the economy is at full employment or not is a matter of indifference. The price determining equation (9) holds in any case. Note that the quantity of money is *not* among the price determinants.

In sum: the configuration of Figure 2 is – in principle – reproducible for an indefinite time span. All variations of employment, wage rate, productivity or distributed profit are transformed via (9) into a new market clearing price.

1.4 Profit and profit ratio

The business sector's financial profit in period t is defined with (10) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:[2]

$$\Delta Q_{fi} \equiv C - Y_W \quad |t. \tag{10}$$

In explicit form, after the substitution of (3) and (4), this definition is identical with that of the theory of the firm:

$$\Delta Q_{fi} \equiv PX - WL \quad |t. \tag{11}$$

Using the first axiom (1) and the definitions (4) one gets:

$$\Delta Q_{fi} \equiv C - Y + Y_D \quad |t. \tag{12}$$

The three definitions are formally equivalent, that is, profit can be looked at under three different perspectives that together render the whole picture.

If distributed profit Y_D is set to zero in the 1^{st} axiom, then profit or loss of the business sector ΔQ_{fi} is determined solely by expenditures and wage income as shown in Figure 3.

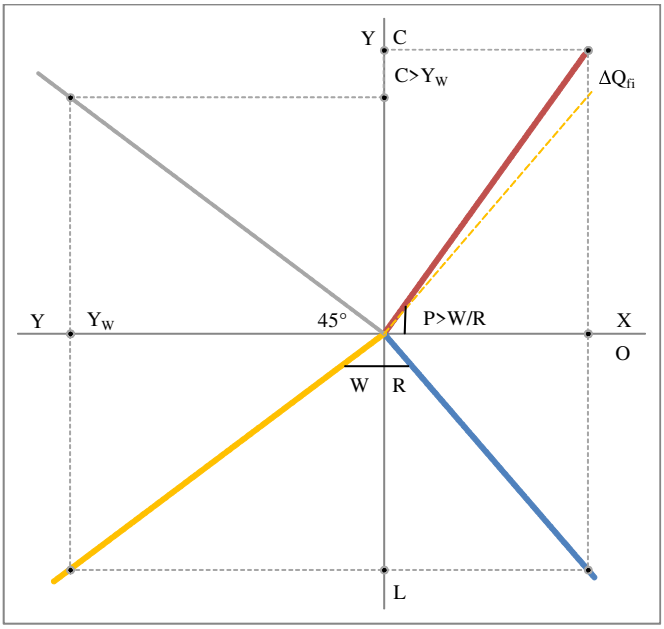


Figure 3: The emergence of financial profit ΔQ_{fi} in the simplest possible case

[2] Profits from changes in the value of nonfinancial assets are neglected here, i.e. the condition of market clearing $\rho_X = 1$ holds throughout. For details about changes of inventory see (2011g, Sec. 1). Nonfinancial profit is treated at length in (2011c).

For the business sector as a whole to make a profit consumption expenditures C have in the simplest case, i.e. $Y_D = 0$, to be greater than wage income Y_W. So that profit comes into existence in the pure consumption economy the household sector must run a deficit at least in one period. This in turn makes the inclusion of the financial sector mandatory. A theory that does not include at least one bank that supports the concomitant credit expansion cannot capture the essential features of the market economy.[3] Mention should be made that neither the neoclassical nor the Keynesian school ever came to grips with profit (Desai, 2008), (Tómasson and Bezemer, 2010).[4]

The distributed profit ratio is defined as:

$$\rho_D \equiv \frac{Y_D}{Y_W} \quad |t. \tag{13}$$

In the general case $Y_D > 0$, hence $\rho_D > 0$. In Figure 2 holds $\rho_D > 0$; in Figure 3 holds $\rho_D = 0$.

From (12), the first axiom (1), and definitions (8) and (13) one gets for total profit:

$$\Delta Q_{fi} \equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \quad |t. \tag{14}$$

Total profit in period t is positive if $\rho_E > 1$ or $\rho_D > 0$, or both. The limiting case of budget balancing $\rho_E = 1$ is shown in Figure 2. Under this condition profit ΔQ_{fi} is equal to distributed profit Y_D according to (14), respectively (12).

To get rid of all absolute magnitudes, the profit ratio ρ_Q is defined as:[5]

$$\rho_Q \equiv \frac{\Delta Q_{fi}}{Y_W} \quad |t. \tag{15}$$

Together with (14) this gives a succinct summary of the *structural* interrelations of the profit ratio, the expenditure ratio, and the distributed profit ratio for the business sector as a whole:

$$\rho_Q \equiv \rho_E (1+\rho_D) - 1 \quad |t. \tag{16}$$

The overall profit ratio ρ_Q is positive if the expenditure ratio ρ_E is > 1 or the distributed profit ratio ρ_D is > 0, or both. In the case of budget balancing $\rho_E = 1$

[3] If the purchase of all long lived consumption goods, e.g. houses, is correctly subsumed under consumption expenditures there arises no practical problem with regard to collateral for the banking industry and a sound credit expansion may proceed – in principle – for an indefinite time span in the pure consumption economy.

[4] Figure 3 says in plain words: the value of output is greater than the sum of factor incomes. The fundamental error of value theory is to start from the premise that the value of output is always equal to the sum of factor incomes (cf. Samuelson and Nordhaus 1998, pp. 391-392). For the finer points see (2012a).

[5] The profit ratio is more general than the profit rate which presupposes a capital stock.

the profit ratio ρ_Q is equal to the distributed profit ratio ρ_D. If $\rho_D = 0$ then profit depends alone on the expenditure ratio ρ_E as shown in Figure 3.

Eq. (16) is the most concise description of the business sector's profit situation and embodies the structural axiomatic profit theory (for a detailed account of the far reaching implications see 2011e, Sec. 5). The profit ratio, to repeat, is objectively given. This saves us a lot of pointless filibustering about maximizing behavior.

2 Money and credit

> The introduction of money at the very origin of market coordination would call into question the abstraction used to establish the theory of value. But this remains to be done. (Benetti and Cartelier, 1997, p. 216)

There can be no dichotomization of the real and the monetary sphere. The first task, then, is to demonstrate how money follows consistently from the axiom set.

If income is higher than consumption expenditures the household sector's stock of money increases. The change in period t is defined as:

$$\Delta\bar{\mathsf{M}}_{\mathbf{H}} \equiv^{m} Y - C \quad |t. \tag{17}$$

The identity sign's superscript m indicates that the definition refers to the monetary sphere.

The stock of money $\bar{\mathsf{M}}_H$ at the end $\bar{t}$ of an arbitrary number of periods is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{\mathsf{M}}_{\mathbf{H}} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{H}t} + \bar{\mathsf{M}}_{\mathbf{H}0} \quad |\bar{t}. \tag{18}$$

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta\bar{\mathsf{M}}_{\mathbf{B}} \equiv^{m} C - Y \quad |t. \tag{19}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{\mathsf{M}}_{\mathbf{B}} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{B}t} + \bar{\mathsf{M}}_{\mathbf{B}0} \quad |\bar{t}. \tag{20}$$

In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions are carried out by the central bank. The stock of money then takes the form of current deposits or current overdrafts. Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (18) the current overdrafts of the business sector are of equal amount according to (20), and vice versa. As it happens, each sector's *stock* of money is either positive

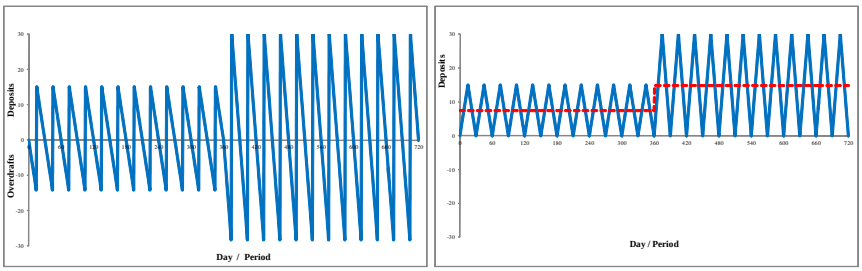
(= deposits) or negative (= overdrafts). Money and credit are at first symmetrical. From the central bank's perspective the quantity of money at the end of an arbitrary number of periods is then given by the absolute value either from (18) or (20):

$$\bar{\mathsf{M}}_{\bar{t}} \equiv \left| \sum_{t=1}^{t} \Delta \bar{\mathsf{M}}_{\mathbf{H}t;\mathbf{B}t} \right| \quad \text{if} \quad \bar{\mathsf{M}}_{\mathbf{H}0;\mathbf{B}0} = 0. \tag{21}$$

The *quantity* of money is always ≥ 0 and follows directly from the axioms. It is assumed at first that the central bank plays an *accommodative* role and simply supports the autonomous market transactions between the household and the business sector. For the time being, the quantity of money is the *dependent* variable.

3 Transaction money

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 4a results. To give an example, it is assumed that the monthly income $\frac{Y}{12}$ is paid out at mid-month. In the first half of the month the daily spending of $\frac{Y}{360}$ increases the current overdrafts of the households. At mid-month the households change to the positive side and have current deposits of $\frac{Y}{24}$ at their disposal. This amount reduces continuously towards the end of the month. This pattern is exactly repeated over the rest of the year. At the end of each subperiod, and therefore also at the end of the year, both the stock of money and the quantity of money is zero. Money is present and absent depending on the time frame of observation.



(a) Transaction pattern over two periods **(b)** Average stock of transaction money $\hat{M}_T$

Figure 4: Household sector's transaction pattern for different nominal incomes in two periods; the business sector's pattern is exactly symmetrical

In period$_2$ the wage rate, the dividend and the price is doubled. Since no cash balances are carried forward from one period to the next, there results *no* real balance effect provided the doubling takes place exactly at the beginning of period$_2$.

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore, the pattern of Figure 4a translates into the average amount of current deposits in Figure 4b. This

average stock of transaction money depends on income according to the transaction equation:

$$\hat{\mathsf{M}}_T \equiv \kappa Y \quad |t. \tag{22}$$

For the regular transaction pattern that is here assumed as an idealization the index is $\kappa = \frac{1}{48}$. Different transaction patterns are characterized by different numerical values of the transaction pattern index.

Taking (22) and (6) and (8) together one gets the explicit transaction equation for the limiting case of market clearing *and* budget balancing:

$$\text{(i) } \hat{\mathsf{M}}_T \equiv \kappa \frac{\rho_X}{\rho_E} RLP \quad \text{(ii) } \frac{\hat{\mathsf{M}}_T}{P} \equiv \kappa O \quad \text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t. \tag{23}$$

We are now in the position to substantiate the notion of accommodation as a money-growth formula. According to (i) the central bank enables the average stock of transaction money to expand or contract with the development of productivity, employment, and price. In other words, the real average stock of transaction money, which is a statistical artifact and not a physical stock, is proportional to output (ii) if the transaction index is given and if the ratios ρ_E and ρ_X are unity. Under these *initial* conditions money is endogenous and neutral in the structural axiomatic context. Money emerges from *autonomous* market transactions and has three aspects: stock of money ($\bar{\mathsf{M}}_{\mathbf{H}}$, $\bar{\mathsf{M}}_{\mathbf{B}}$), quantity of money (here $\bar{\mathsf{M}} = 0$ at period start and end because of $\rho_E = 1$) and average stock of transaction money (here $\hat{\mathsf{M}}_T > 0$).

As long as the expenditure ratio is unity money performs only the function of a medium of transaction. Not before the expenditure ratio is different from unity the function of a store of value is activated. Both functions are implied in (21), (22) and (23). With money in all its forms consistently derived from the axiom set our picture of the pure consumption economy is now complete.

4 The stylized three-period credit cycle

The business sector is, trivially, populated by firms and the household sector by households. The sectors $\mathscr{E}_{\mathbf{B}}$ respectively $\mathscr{E}_{\mathbf{H}}$ have to be differentiated. The differentiated structure of the pure consumption economy is, to begin with, rather simple. The business sector consists of the consumption goods producing firm $\mathscr{E}_{\mathbf{B}A}$ and the central bank. The latter handles all monetary and financial transactions. Accordingly, the central bank consists of a transaction unit $\mathscr{E}_{\mathbf{B}B}$ and a banking unit $\mathscr{E}_{\mathbf{B}C}$. The transaction unit is here ignored (for details about the transaction business see 2011f, Sec. 4). The 1^{st} axiom is now differentiated for the two firms A and C:

$$Y = \underbrace{W_A}_{W} L_A + \underbrace{W_C}_{W} L_C + \underbrace{D_A N_A + D_C N_C}_{Y_D=0} \quad |t. \tag{24}$$

To simplify matters, the wage rates are set equal for both firms and distributed profits are set to zero. Total employment is taken as constant over all periods:

$$L \equiv L_A + L_C \quad |t. \tag{25}$$

The banking unit of the central bank makes its appearance only in the middle period of the credit cycle as shown in Figure 5. This implies a reallocation of labor input from the consumption goods producing firm $\mathscr{E}_{\mathbf{B}A}$ to the credit unit $\mathscr{E}_{\mathbf{B}C}$, that is, L_A goes down and L_C goes up, with L remaining constant. This shift of labor input has no effect on wage income because the wage rates are, by assumption, equal in both firms. Total income Y remains unchanged over time since the wage rate W is, by assumption, fixed.

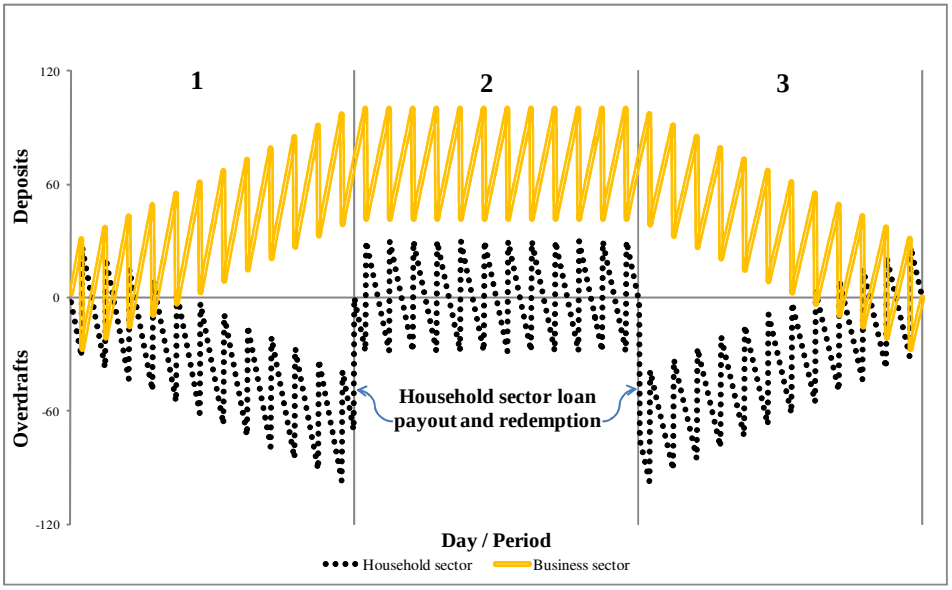


Figure 5: Household sector dissaving in period$_1$ and saving in period$_3$ with a consolidation of current overdrafts by the sum of perfectly synchronous individual one-period loans in period$_2$

4.1 Period$_1$

The household sector as a whole is supposed to dissave in the first period, i.e. $\rho_{E1} > 1$. This implies that we have two groups of households, group $\mathscr{E}_{\mathbf{H}F}$ who dissaves, i.e. $\rho^F_{E1} > 1$, and those households $\mathscr{E}_{\mathbf{H}G}$ who spend just their period income, i.e. $\rho^G_{E1} = 1$. There are no savers in period$_1$ (for savers as complementary group see 2011f, Sec. 5). The average expenditure ratio of the two groups is then given by:

$$\rho_{E1} \equiv \rho^F_{E1}\frac{Y_F}{Y} + \rho^G_{E1}\frac{Y_G}{Y}$$
$$\text{with} \quad Y \equiv Y_F + Y_G. \tag{26}$$

Since savers are excluded the average expenditure ratio ρ_{E1} is greater than unity or, as a limiting case, equal to unity if the dissaver group shrinks to zero. As shown in Figure 5, the household sector's overdrafts, respectively the business sectors deposits as mirror image, increase in period$_1$ according to (17) respectively (19). The new spending pattern in period$_1$ implies a higher transaction index κ and by consequence a higher average stock of transaction money according to (22). Accommodation means in the given circumstances that the central bank enables the autonomous increase of the household sector's overdrafts.

Under the condition of market clearing, i.e. $\rho_X = 1$, the price is higher compared to the initial period. The market clearing price in period$_1$ can be derived as dependent variable from the 3rd axiom (3) and definition (8):

$$P_{A1} = \rho_{E1} \frac{W}{R} \quad \text{if} \quad \rho_X = 1, \rho_D = 0. \tag{27}$$

The market clearing price depends ultimately on the expenditure ratio (demand side in the familiar parlance) and unit wage costs. The higher price effects a redistribution of current output $O = X$ within the household sector. The dissavers absorb a greater part of output than in the initial period because their purchasing power is strengthened by overdrafts. This distributional effects, however, can be excluded by assuming that all households dissave in proportion to their individual income, i.e. have the same expenditure ratio. In this limiting case each household gets exactly the same share of the unaltered output as before with higher consumption expenditures at a higher price.

The financial profit of the business sector is now given by:

$$\Delta Q_{fiA1} \equiv P_{A1} X_{A0} - W L_{A0} \quad \text{with} \quad P_{A1} > P_{A0} \ . \tag{28}$$

Since wage costs WL and the quantity bought X remain unchanged profit increases with the rising market clearing price. Seen from a wider perspective and alternatively expressed with the help of (14) profit depends ultimately on the expenditure ratio:

$$\Delta Q_{fiA1} \equiv (\rho_{E1} - 1) Y \quad \text{with} \quad \rho_D = 0. \tag{29}$$

Profit is greater than zero if the average expenditure ratio is greater than unity.

Profits can either be distributed to the household sector or retained. If nothing is distributed, then profit adds entirely to the financial wealth of the firm. Retained profit ΔQ_{re} is defined for the business sector as a whole as the difference between profit and distributed profit in period t:

$$\Delta Q_{re} \equiv \Delta Q_{fi} - Y_D \quad \Rightarrow \quad \Delta Q_{re} \equiv C - Y \quad |t. \tag{30}$$

Retained profit is, due to (12), ultimately equal to the difference of consumption expenditures and total income. In period$_1$ there is no profit distribution, hence $Y_D = 0$ and retained profit ΔQ_{re} is equal to profit ΔQ_{fi}.

The monetary side has been defined with (19) as:

$$\Delta\bar{\mathsf{M}}_{\mathbf{B}} \equiv^{m} C - Y \quad |t. \tag{31}$$

The comparison of the two definitions (30) and (31) shows that the first refers to the nominal sphere and the second to the monetary sphere. The difference of flow variables on the right hand side of the $\equiv$-sign is identical. Retained profit and the increase of the business sector's current deposits are only two aspects of the same thing. At the end of period$_1$ the business sector's stock of money is $\Delta\bar{\mathsf{M}}_{\mathbf{B}}$ and equals retained profit as shown in Figure 5. The business sector keeps its retained profits in the form of deposits (that bear no interest) at the central bank. Mere cash-accumulation is, of course, a simplifying assumption.

4.2 Period$_2$

The expenditure ratio is again unity, i.e. $\rho_{E2} = 1$. Credit expansion in the form of rising overdrafts stops. Deposits and overdrafts at period end are the same as at period beginning. There is no further growth of household sector's debt and business sector's deposits.

Total profit is zero in period$_2$ according to (29). Profit can be greater than zero under the given conditions only if the household sector's debt grows. This is not the case in period$_2$.

The market clearing price of the consumption good as given with (27) falls compared to period$_1$. Price and profit move in step with the expenditure ratio.

The household sector takes up a one period loan in order to reduce current overdrafts. The individual households' loans are produced by the banking unit $\mathscr{E}_{\mathbf{B}C}$ and sum up to $\bar{\mathsf{A}}_C$. The 3^{rd} axiom has to be adapted with regard to the specifics of the banking business:

$$C_C = P_C X_C \quad \Rightarrow \quad C_C = \mathsf{l}_C \bar{\mathsf{A}}_C \quad |t. \tag{32}$$

The household sector's interest payments to the banking unit are subsumed under consumption expenditures. The rate of interest l thereby takes the role of the price. Whether the households buy consumption goods or the credit services of the banking unit is a matter of indifference with regard to the average expenditure ratio.

The output of the banking unit O_C consists of credit services. Under the condition of market clearing $O_C = X_C$ and the identification of P_C with the rate of interest l_C follows from the 2^{nd} axiom a reinterpretation of the banking unit's productivity (for details see 2011e, Sec. 6):

$$R_C = \frac{\bar{\mathsf{A}}_C}{L_C} \quad |t. \tag{33}$$

The productivity of the banking unit is high if a huge stock of loans $\bar{\mathsf{A}}_C$ is processed in a given period with an small number of working hours L_C.

The household sector apportions its consumption expenditures between the purchase of consumption goods C_A and interest payments C_C. Total consumption

expenditures are equal to income, i.e. $\rho_{E2} = 1$, and spent on the output of both the consumption good producing firm $\mathscr{E}_{\mathbf{B}A}$ and the banking unit $\mathscr{E}_{\mathbf{B}C}$:

$$C = P_A X_A + \mathsf{I}_C \bar{\mathsf{A}}_C \qquad |2. \tag{34}$$

Since the expenditure ratio is unity and the distributed profit ratio is zero the profit of the business sector as a whole is zero according to (14). Under the condition that both markets are cleared the profit for each firm follows from (11) and is, respectively, given by:

$$\begin{aligned} \Delta Q_{fiA} &\equiv P_A R_A L_A \left(1 - \frac{W}{P_A R_A}\right) \quad \rho_{XA} = 1 \\ \Delta Q_{fiC} &\equiv \mathsf{I}_C R_C L_C \left(1 - \frac{W}{\mathsf{I}_C \dfrac{\bar{\mathsf{A}}_C}{L_C}}\right) \quad \rho_{XC} = 1 \end{aligned} \qquad |2. \tag{35}$$

If the expressions in brackets are zero then profits are zero. With the zero profit condition the market clearing prices for both firms are determined as follows:

$$\begin{aligned} P_{A2} &= \frac{W}{R_{A2}} \qquad \text{price} \\ \mathsf{I}_{C2} &= \frac{W}{\dfrac{\bar{\mathsf{A}}_{C2}}{L_{C2}}} \quad \text{rate of interest.} \end{aligned} \tag{36}$$

Relative prices P_A/I_C are in this limiting case determined by the respective productivities in both firms. The market clearing rate of interest I_C does not, in this simple case, cover the risk of a statistical average of nonperforming loans; it covers, for the beginning, only the wage costs.

The processing of loans consumes resources. Therefore, labor input has to be reallocated. Since total employment L is given, L_A goes down as L_C goes up in period$_2$. By consequence, the output of the consumption good O_A is reduced. From this output reduction in comparison to the foregoing period a real interest rate can be calculated:

$$\mathsf{I}_{C2}^{real} \equiv \frac{\Delta O_A}{O_{A1}} \equiv \frac{R_A \left(L_{A1} - L_{A2}\right)}{R_A L_{A1}} \equiv 1 - \frac{L_{A2}}{L_{A1}} \equiv \frac{L_{C2}}{L_{A1}}. \tag{37}$$

If the labor input of the consumption good producing firm in period$_2$ is, for example, 95 percent of that in period$_1$ then the real rate of interest is 5 percent. The reallocation of labor input and the temporary reduction of the consumption goods output is, for simplicity, confined to period$_2$. The real rate of interest depends on the production conditions of the banking unit. The more efficient the banking unit is, the lower is the real interest rate. The connection between the real (37) and nominal

(36) rate of interest is established via the wage rate. Interest payments cannot be characterized as transfer payments or unearned income (cf. Hudson and Bezemer, 2012, p. 6). Credit is produced like any other good or service. In the limiting zero-profit case interest is equal to the wage income of the banking unit.

The price of the consumption good is the same as in the initial period. It depends on unit wage costs and the expenditure ratio, which are the same, and not on labor input. The price is lower than in period$_1$ but this has nothing to do with the shift of demand and of labor input from firm $\mathscr{E}_{\mathbf{B}A}$ to $\mathscr{E}_{\mathbf{B}C}$. The market clearing price is higher during the temporary credit expansion and returns to its initial level once the growth of the household sector's debt ends, provided that wage rate and productivity are kept constant. This is assumed here in order to forestall secondary effects.

4.3 Period$_3$

The household sector now saves and completely pays off its debt. With an expenditure ratio $\rho_{E3} < 1$ the market clearing price must be lower according to (27). This effects a redistribution of current output within the household sector. The savers' share of output shrinks.

Financial profit is, according to (29), now negative. The loss diminishes the business sector's current deposits as shown in Figure 5. When profits are cumulated over all three periods the total is zero. Hence we have a perfect symmetry over the complete cycle between credit expansion→profit and credit contraction→loss. This overall relationship is beyond the horizon of individual agents.

The market clearing price is, according to (27), lower in period$_3$ than in period$_2$ or, for that matter, in the initial period. This, of course, holds under the condition that wage rate and productivity are kept constant.

With regard to the quantity theory one can observe a *higher* price in period$_1$ and this correlates with a *larger* quantity of money $\bar{\mathsf{M}}$ and a higher average stock of transaction money $\hat{\mathsf{M}}_T$. In period$_3$ it is just the opposite. These observations are in harmony with what should be expected according to the commonplace quantity theory. It is important to note, however, that the market clearing price (27) does not depend on the quantity of money.

With regard to the debt deflation theory one can observe a correlation between 'inflation' and the growth of the household sector's debt and 'deflation' and the reduction of debt (cf. Keen, 2011, pp. 349-353). The inflationary or deflationary effects, however, may be so weak in the concrete case that is would be inappropriate to apply the rather strong terms inflation and deflation. Putting terminological subtleties aside the conclusion of the structural axiomatic analysis is that debt deflation is a normal occurrence in the credit cycle. Normal means that it does not presuppose excessive or speculative credit expansion, the violation of time-tested banking rules or outright sham. These accessory exaggerations put the underlying structural symmetry of 'credit inflation' and 'debt deflation' only under a magnifying glass. The symmetry follows in direct deductive lineage from the structural axiom

set and does not depend on *ad hoc* assumptions about the misbehavior of economic agents.

It is, in principle, not too difficult to avoid deflationary effects. All that is necessary according to (27) is to compensate the lower expenditure ratio with a higher wage rate. This keeps the price constant under the condition of market clearing. It is improbable, though, that businessmen or politicians find this idea convincing in a time of overall weak business conditions and widespread losses that are characteristic of period_3. It is, given the agents' myopic mind-set, more probable that the deflationary effect of the debt payoff is amplified by wage cuts. Under the condition of market clearing, wage cuts only effect a fall of the market clearing price. The whole exercise is pointless with regard to the real wage and the profit ratio but it increases the household sector's debt income ratio.

Let us, counter-factually, assume for the moment that the price can be stabilized. This, though, does not solve the real problem which consists in the overall loss of the business sector. And this loss is, as we know from (29), ultimately due to the fact that the expenditure ratio is less than unity in period_3. By consequence, the only way to stop losses is to stop the reduction of the household sector's existing debt. A reflation that leaves the expenditure ratio unaffected does not really help with losses.

Seen over all three periods, what happens in real terms is a redistribution of the current output among the wage earning households in period_1 and in period_3, respectively. This redistribution is anonymously effected by the market clearing price. The wage earning households absorb the whole output in all three periods. Profit and loss affect only the cash balances, i.e. current deposits, of the business sector. Profit and loss have no real counterpart in the form of a piece of the output cake. In other words, profit and surplus are quite different things. Profit as defined with (10) cannot exist at all in real models (for details see 2011h, Sec. 3).

It has to be noted in passing that the dissaving–saving cycle in the pure consumption economy has nothing to do with investment (for details see 2011b, Sec. 15). It is also worth noting that the dissaving-saving sequence cannot be reversed. If the household sector started with saving the overall losses of the business sector would prevent that firms live long enough to see profits eventually to appear. The economy has to be kick-started with household sector deficits.

5 Expansion, contraction, and the structural conditions of breakdown

The pure consumption economy is invariably in one of the three states that have been summarized in Figure 5. With regard to the real world, though, it is more appropriate to speak of three phases with each phase running over a couple of periods.

The period values of the axiomatic variables are connected formally by the familiar growth equation, which is added to the structural set as the 4^{th} axiom:

$$Z_t = Z_{t-1}\left(1+\dddot{Z}_t\right). \tag{38}$$

The path of the representative variable Z_t, which stands for the axiomatic variables, is then determined by the initial value Z_0 and the rates of change $\dddot{Z}_t$ for each period:

$$Z_t = Z_0\,(1+\dddot{Z}_1)\,(1+\dddot{Z}_2)\ldots(1+\dddot{Z}_t) = Z_0\prod_{t=1}^{t}(1+\dddot{Z}_t). \tag{39}$$

Given convenient initial values, eq. (39) describes the paths of the variables with the *rates of change* $\dddot{Z}_t$ as unknowns. These unknowns are in need of determination and explanation. The explanation of the rates of change is, in principle, to be found between the liming cases of perfect determinism and perfect randomness (for details see 2011d, Sec. 2). In the following the respective rates of change are, for a start, fixed by assumption.

5.1 Inclusion of distributed profit

As a first step toward the full generalization of the elementary consumption economy distributed profits are allowed to be greater than zero:

$$Y = \underbrace{W_A}_{W} L_A + \underbrace{W_C}_{W} L_C + \underbrace{D_A N_A + D_C N_C}_{Y_D>0} \quad |t. \tag{40}$$

Profit distribution depends, among other things, on the development of profit in previous periods. This obvious dependency is ignored for the moment. The absolute amount of distributed profit Y_D is *fixed*, such that the initial distributed profit ratio ρ_D is 10 percent for the economy as a whole. This is a convenient assumption to start with. The profit ratio ρ_Q in the initial period is then given by:

$$\begin{gathered}\rho_Q \equiv \rho_E\,(1+\rho_D) - 1 \\ \rho_{E0} = 1,\, \rho_{D0} = 0.1.\end{gathered} \tag{41}$$

If the initial expenditure ratio ρ_{E0} is unity the initial profit ratio ρ_{Q0} is equal to the distributed profit ratio ρ_{D0}, that is, to 10 percent. It is assumed that the respective profit ratios of firm $\mathscr{E}_{\mathbf{B}A}$ and $\mathscr{E}_{\mathbf{B}C}$ are equal to the overall profit ratio in the initial period (for details see 2011a, Sec. 9). With regard to the behavior of the consumption good producing firm $\mathscr{E}_{\mathbf{B}A}$ the assumption is supplemented that the target profit ratio is fixed at 10 percent. This ratio is realized in the initial period. Note that this behavioral assumption is more general than profit maximization because it does not – inadmissibly – presuppose a well-behaved production function (cf. Hudson, 2010, p. 11). The firm may fix its target profit ratio wherever it pleases under the condition that it is at least $\rho_{QA} \geq 0$.

5.2 The debt/income ratio

The period change of the household sector's deposits or overdrafts (17) can be rewritten with the help of definition (8) as:

$$\Delta\bar{\mathsf{M}}_{\mathbf{H}} \equiv^{m} (1-\rho_E)\,Y \quad |t. \tag{42}$$

If the expenditure ratio ρ_E is > 1 over all or most periods the household sector's overdrafts at the end of period $\bar{t}$ follow from (18). They sum up to:

$$\bar{\mathsf{M}}^{o}_{\mathbf{H}\bar{t}} \equiv \sum_{t=1}^{t} (1-\rho_{Et})\,Y_t \quad \text{with} \quad \bar{\mathsf{M}}_{\mathbf{H}0} = 0,\ \rho_{Et} > 1. \tag{43}$$

The household sector's overdrafts are a stock. This stock does not change in the period under consideration if the expression in the bracket of (43) is zero. Hence $(1-\rho_E)$ takes the role of the first derivative in continuous analysis (which is inapplicable in economics because the axiomatic variables are defined on the rationals $\mathbb{Q}$ and not the reals $\mathbb{R}$). If the difference is zero this indicates a minimum, maximum or an inflection 'point'. In other words, the expenditure ratio plays the role of an accelerator or decelerator with regard to the households sector's overdrafts (or deposits as the case may be).

Taking the debt from (43), the debt/income ratio ρ_M at the end of period $\bar{t}$ is then defined as:

$$\rho_{M\bar{t}} \equiv \frac{\bar{\mathsf{M}}^{o}_{\mathbf{H}\bar{t}}}{Y_t} \tag{44}$$

The debt/income ratio follows in direct lineage from the structural axiom set and depends on the period values of ρ_E and total income Y. Debt consists at the moment solely of overdrafts. It may take other forms, for example, long term loans. The exact composition of the household sector's debt, though, is not of much interest at the moment. A switch from overdrafts to a one-period loan does not alter the debt/income ratio if all forms of debt are assembled in the numerator of (44). Overdrafts are merely the first form of household sector debt.

Credit may expand for some periods faster than income. There exists, however, a structural limit. For the maximum amount of debt we have as a rule of thumb that only the free part of income can be employed to pay the annuity. The free part is calculated for the economy as a whole as the difference between total period income and the minimum amount of consumption expenditures that is roughly defined by average living standards:

$$\underbrace{Y - C^{min}}_{\text{free part}} = \underbrace{\bar{\mathsf{M}}^{o\,max}_{\mathbf{H}}(\mathsf{I}+\mathsf{R})}_{\text{maximum annuity}} \quad |t. \tag{45}$$

The maximum annuity is given as the sum of individual loans multiplied with the sum of the current interest rate I and the current repayment rate R. From this

follows the maximum amount the household sector is able to service at given rates as:

$$\bar{\mathsf{M}}_{\mathbf{H}}^{omax} = \frac{Y - C^{min}}{\mathsf{I}+\mathsf{R}} \quad |\bar{t}. \tag{46}$$

To simplify matters it is assumed that the relation of minimum consumption expenditures to total income is fix. The maximum debt/income ratio ρ_M^{max} then depends alone on the current interest and repayment rates:

$$\frac{\bar{\mathsf{M}}_{\mathbf{H}}^{omax}}{Y} = \frac{1 - \overbrace{\frac{C^{min}}{Y}}^{\text{fix}}}{\mathsf{I}+\mathsf{R}} \quad |\bar{t}. \tag{47}$$

For any period a maximum debt/income ratio ρ_M^{max} can be calculated with the currently prevailing rates of interest and repayment. $\mathsf{M}_{\mathbf{H}}^{omax}$ is the maximum amount of debt the household sector could service at the actual rates with the given total period income Y. A falling interest rate I boosts the maximum debt/income ratio.

The calculated maximum annuity $\mathsf{M}_{\mathbf{H}}^{omax}(\mathsf{I}+\mathsf{R})$ is different from the actual annuity in the period under consideration. The latter is the aggregate of the product of the actual amount of individual debt $\mathsf{M}_{\mathbf{H}i}^{octr}$ and the sum of the contractual interest and repayment rates which are normally different from current rates. Only when debt is completely rolled over within rather short intervals the contractual rates are close to the current rates. Otherwise they may reflect the history of interest rates of two or three decades. The actual annuity, i.e. the total contractual payment obligations, should be lower than the maximum annuity, which is given by (45), otherwise some of the households face a problem:

$$\begin{gathered} \bar{\mathsf{M}}_{\mathbf{H}}^{octr}(\mathsf{I}^{ctr}+\mathsf{R}^{ctr}) \leq \bar{\mathsf{M}}_{\mathbf{H}}^{omax}(\mathsf{I}+\mathsf{R}) \\ \text{or} \\ \rho_M^{ctr}(\mathsf{I}^{ctr}+\mathsf{R}^{ctr}) \leq \rho_M^{max}(\mathsf{I}+\mathsf{R}) \quad |t. \end{gathered} \tag{48}$$

The difference between maximum annuity and actual annuity defines the margin of a feasible credit expansion at the current rates. The households may not wish to exploit this margin to the full. On the other hand, there may exist banking rules or technical limits that prevent the expansion of credit up to the calculated maximum. All those kinds of restrictions have no bearing on the main line of the argument. What has to be emphasized is that there exists a structural upper limit for household sector debt that depends on the growth of total income.

While the maximum annuity (45) is fixed by rather stable structural parameters, the maximum amount of household sector debt $\bar{\mathsf{M}}_{\mathbf{H}}^{omax}$ is not carved in stone. Depending on the interest and repayment rates that are applied in (48) the calculated upper limit of household sector debt may be characterized in Minsky's terms as hedge (e.g. $\mathsf{I}=4\%,\ \mathsf{R}=2\%$), speculative (e.g. $\mathsf{I}=4\%,\ \mathsf{R}=0\%$), or Ponzi (e.g. $\mathsf{I}+\mathsf{R}<4\%$) (Minsky, 1982, pp. 22-23).

5.3 Increasing credit, profit, employment and price

We proceed now on the assumption that the maximum debt/income ratio ρ_M^{max} is given by (47). As long as the interest weighted actual debt/income ratio is below the maximum in (48) the household sector expands credit, that is, the expenditure ratio is greater than unity. The ratio is here fixed for simplicity at $\rho_E = 1.01$. In other words, the household sector increases the expenditure ratio and exploits the debt limit to the full. Not much depends on whether the exemplary values are deterministic or stochastic; the gist of the matter is basically the same as long as the expenditure ratio is > 1, only speed and timing are different.

According to (41) the higher expenditure ratio involves a higher overall profit ratio ρ_Q as shown in Figure 6a. If the profit ratio is higher than 10 percent, i.e. $\rho_Q > 0.1$, then employment increases with a predetermined rate, here $\dddot{L} = 2\%$ by assumption. If $\rho_Q < 0.1$ then $\ddot{L} = -1.96\%$. In other words, the business sector's behavior is formally defined by the symmetric deterministic function:

$$\begin{array}{lll|c|c|c|} \text{if} & \rho_{Qt-1} & > 0.1 & = 0.1 & < 0.1 \\ \hline \text{then} & \dddot{L}_t & +2\% & 0 & -1.96\% \\ \hline \end{array} \tag{49}$$

Since the overall profit ratio declines with increasing employment and vice versa according to the explicit form of (16)

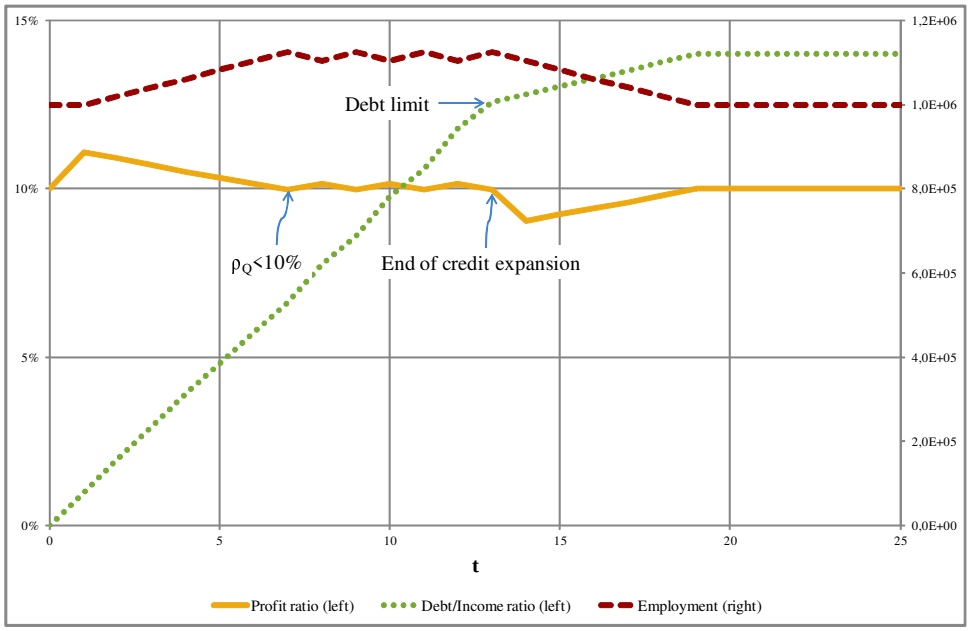
$$\rho_Q \equiv \rho_E \left(1 + \frac{Y_D^{fix}}{WL} \right) - 1 \tag{50}$$

eqs. (49) and (50) constitute a self-stabilizing negative feedback loop that is clearly visible in Figure 6a. The overall profit ratio is entirely independent from the productivity in general and from decreasing returns in particular.

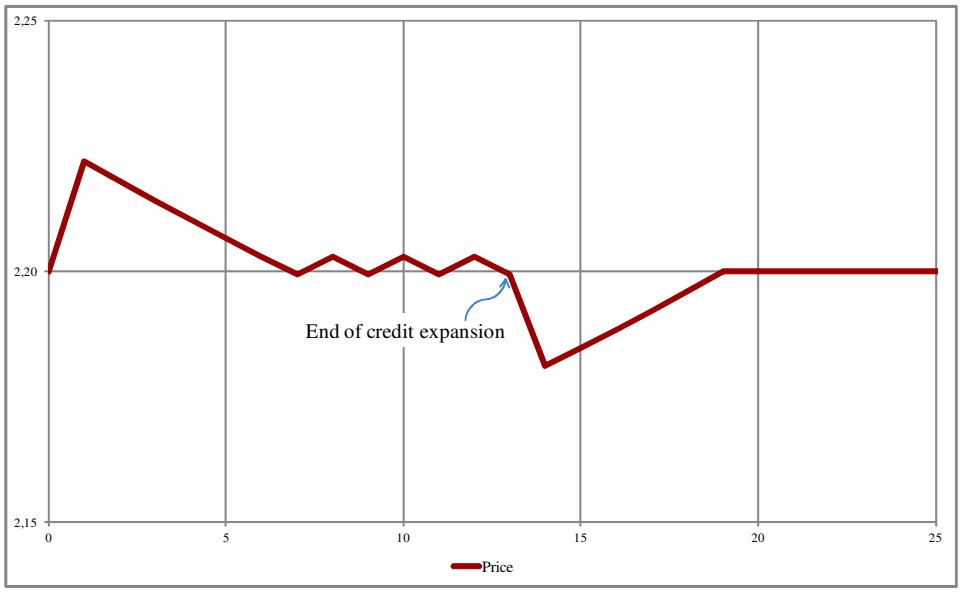
Employment rises with the household sector's credit expansion and stays high until the calculated debt limit is reached. Then the credit expansions stops. The expenditure ratio ρ_E drops from 1.01 to 1.00. According to (50) the profit ratio declines. By consequence, employment and income falls.

Although the debt/income ratio has already hit the calculated maximum value it grows further because employment and therefore income declines in (47). With falling employment L the profit ratio ρ_Q increases according to (50). The speed of adaptation depends on the respective rates of change. The employment adaptation ends as soon as the profit ratio returns to the target value of the business sector, i.e. to $\rho_Q = 0.1$. Thereafter the consumption economy is back at the initial configuration of employment and profit ratio and becomes stationary. If employment ceases to grow then the maximum of household sector debt is given by (47).

The path of the market clearing price that corresponds to Figure 6a is, since $\rho_E \equiv \rho_{EA} + \rho_{EC}$, given by



(a) A higher profit ratio and higher employment correlate with the household sector's credit expansion



(b) Fall of the market clearing price below the long run average after the termination of the credit expansion

Figure 6: Stylized interdependence of credit expansion, profit ratio, employment and market clearing price

$$P = \rho_{EA}\left(\frac{W}{R} + \frac{Y_D^{fix}}{RL}\right) \quad \text{if} \quad \rho_{XA} = 1$$
$$\rho_{EA} \cong \rho_E \quad \text{if} \quad \rho_{EC} \cong 0 \quad |t. \tag{51}$$

and is depicted in Figure 6b. The price stays above the long run average until the end of the credit expansion; subsequently it falls temporarily below the average. Needless to emphasize that interest movements may at any time superimpose the rather straightforward credit induced half-cycle of Figure 6. It is obvious that we can shift to a higher gear at any time by lowering the rate of interest in order to boost the debt/income ratio and by increasing the expenditure ratio as well as employment.

To produce something like the Great Moderation in the pure consumption economy is not terribly difficult. A gradual lowering of the interest rate boosts the debt/income ratio in (47) and makes room for a credit expansion. If the households respond, as it happened in the United States (cf. Keen, 2011, pp. 348-349), consumption expenditures, profit and employment increase. The price increase that is determined with (51) can be checked by setting the wage increase equal to productivity growth. There is no deterministic connection between credit expansion and inflation. If wage rate increases remain behind productivity growth the credit expansion that is determined by the value of the expenditure ratio may even coincide with a slight deflation. Note that (51) refers to the product market. It goes without saying that a credit-driven increase of nominal demand that is directed to the asset market instead of the product market produces quite different effects. The asset market is here kept out of the picture (for details see 2012b).

5.4 The perils of redemption

Up to this point the pure consumption economy has visited the states 1 and 2 of Figure 5 for a couple of periods, but not state 3. We have seen growing and stationary debt but no redemption. In Figure 7 the credit cycle is completed. The households start to pay off their debt beginning with period_{30}. This implies an expenditure ratio of $\rho_E < 1$.

From (50) follows that the profit ratio declines and this leads to an employment reduction according to (49). Unemployment increases and then stays at the lower level until the household sector's debt is fully redeemed. From (51) follows that the market clearing price falls temporarily, i.e. as long as $\rho_E < 1$. The price reduction, though, may easily turn into a downward spiral if the wage rate in (51) is reduced (cf. Dimand, 1994, pp. 100-101). Wage and price flexibility does not help to end a debt deflation.

The development from period_{30} onwards bears all the characteristics of a debt deflation. The employment effects of the redemption are worse than those of a mere cessation of credit growth. The market clearing price and the profit ratio fall. The

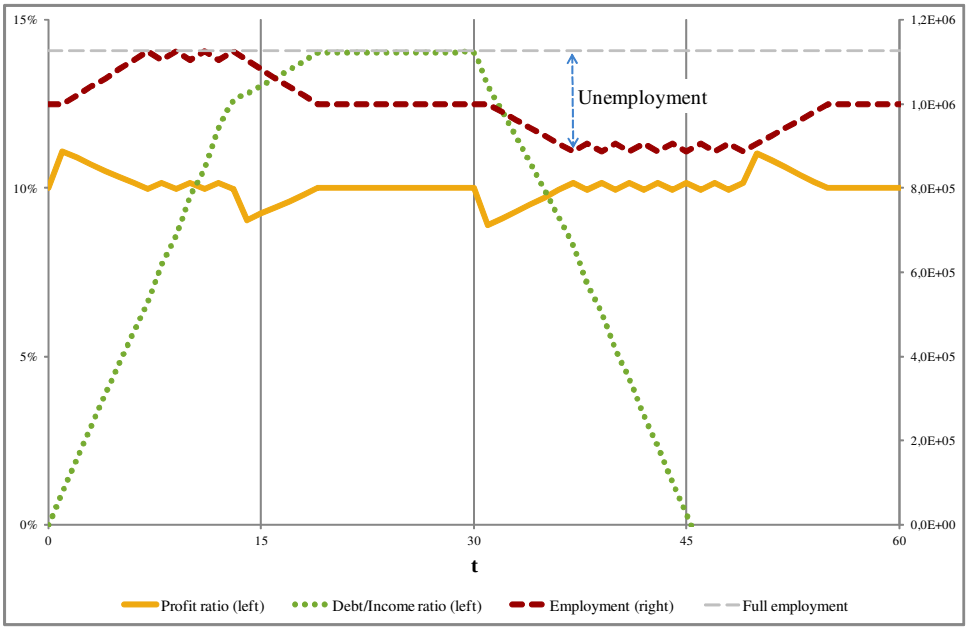


Figure 7: The effects of household sector's build-up and redemption of debt on the profit ratio and employment

crucial point is whether overall profit only shrinks or turns into a loss. Absolute profit for the business sector as a whole follows from (12) as:

$$\begin{gathered} \Delta Q_{fi} \equiv (\rho_E - 1)\, Y + Y_D \\ \text{critical value} \quad \rho_E = \frac{1}{1+\rho_D} \quad \Rightarrow \quad \Delta Q_{fi} = 0 \quad |t. \end{gathered} \tag{52}$$

As long as the expenditure ratio ρ_E stays above the critical value that is given with (52) absolute profit is lower than in the pre-repayment phase but still positive. If the expenditure ratio is less than the critical value the business sector as a whole makes a loss. In this case the scenario of Figure 7 becomes obsolete. Firm $\mathscr{E}_{\mathbf{B}A}$ does not reduce employment as a reaction to a lower profit ratio but vanishes completely. That is, the economy cannot follow the benign course of Figure 7. The first condition therefore is that the household sector does not cut consumption expenditures too drastically in order to reduce its debt. To avoid overall losses the expenditure ratio must, first of all, be kept above the critical value that is given with (52).

5.5 From bad to worse with positive feedback

What is more, the stable scenario of Figure 7 depends on the assumption that distributed profits Y_D are fix over the whole time span. Only under the initial condition of Section 5.1 is the pure consumption economy well-behaved and eventually returns to the initial configuration. The economy becomes stationary as soon as the

expenditure ratio returns to unity and the profit ratio returns to the initial value of 10 percent. According to (50) the profit ratio in turn depends on the absolute amount of distributed profit. What makes the situation unstable is the fact that profit and distributed profit is positively related. Unfortunately we have, at the heart of the pure consumption economy, a positive feedback loop. It works as follows in eq. (52): expenditure ratio ρ_E down (but still above the critical value) $\rightarrow$ profit ΔQ_{fi} down $\rightarrow$ distributed profit Y_D in the next period down $\rightarrow$ profit ΔQ_{fit+1} down, etcetera. This concatenation is, of course, not a deterministic law. Profit distribution depends on a host of other factors. This, though, does not change the basic fact that a debt deflation is prone to tilt over into a downward spiral that takes employment with it. This spiral ends, with the given behavioral assumptions, not before the business sector lowers its target profit ratio well below the actual ratio. This helps to stabilize employment but is not enough to bring the pure consumption economy back to the initial employment level.

The point to emphasize is that a debt deflation is not a Black Swan event that depends on an unfortunate and extremely improbable configuration of exogenous events (cf. Roubini, 2011, p. 16). Neither does it depend on an asset bubble. Rather, it is a latent property of the monetary economy. There is no such thing as a market force that moves the economy towards some ideal state. To the contrary, the built-in loop that is given with the two-way interrelation of profit and distributed profit is vicious. It is sufficient to install the straightforward intertemporal coupling $Y_{Dt} = \Delta Q_{fit-1}$ to bring profits speedily down to zero during a debt deflation (cf. Keen, 1995, p. 625).

Note that we have treated the central bank as a passive player. The process of expansion and contraction has been delineated under the simplifying condition that the rates of interest and repayment are held constant and that the central bank accommodates the autonomous transactions of the household and business sector. Needless to emphasize that the central bank may ease or aggravate the situation by setting the rate of interest in (47) lower or higher. Government, too, has been left out. It is, however, pretty obvious that the expansion and contraction of government debt has the same effects on the business sector as changes in household sector debt. A rising public debt may therefore compensate for a declining private debt. Things are entirely different if the business sector becomes the borrower and the household sector the lender. This variant requires its own analysis.

Let us return to the pure consumption economy and assume that it is possible to break the vicious circle and to stabilize distributed profits Y_D at some convenient level as in Figure 7. From (30) and (52) follows for retained profit:

$$\Delta Q_{re} \equiv (\rho_E - 1)\, Y \quad |t. \tag{53}$$

Since the expenditure ratio is less than unity during the debt repayment retained profit is invariably negative in each period. That is to say, the business sector's cash balances dwindle successively and become zero as a mirror image of the household sector's reduced debt. Deposits and overdrafts or, what amounts to the same in the

simplest case, money and debt are equal. Seen from the business sector as a whole, the full redemption of household sector debt amounts to a full pay-out of business sector's accumulated deposits according to (20) and is therefore a mixed blessing, to say the least. It is certainly a good thing that there is no loss, but profits are below distributed profits as long as the redemption goes on. It is not very probable that this cash drain, which amounts to a reduction of the quantity of money, extends over a longer time span. The obvious remedy from the business sector's perspective is to reduce distributed profit. This does not work because of the positive feedback that is given with (52). Hence, the only way out of a debt deflation in the pure consumption economy is to return to an expenditure ratio of unity, that is, to end the redemption of household sector debt. It is by no means in the interest of the business sector as a whole that the households pay off debt. The positive net worth of the business sector is, in the pure consumption economy, at any point in time the exact mirror image of the household sector's debt. The agents, though, cannot see the overall picture and cannot do other than to act on their micro-logic which turns out to be in opposition to the structural logic. This only prolongs the crisis.

6 Conclusion

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. The present paper excludes psychologism and suggests four nonbehavioral axioms as groundwork for the consistent reconstruction of the evolving money economy. The focus is on the effects of the household sector's credit expansion and contraction. The main results of the structural axiomatic reconstruction of the debt deflation theory are:

- The model that consists of structural axioms and straightforward behavioral assumptions produces a positive correlation between growth of household sector debt, profit, employment, and price. The end of the credit expansion is followed by a kind of mild deflation with profit, employment and price collectively falling.
- The deflationary dip is not the result of speculative excesses of either the household sector or the banking unit. It is simply due to the fact that the credit expansion necessarily reaches a structural limit if debt grows a bit faster than income for any given configuration of the rates of interest and repayment. As soon as the household sector's debt ceases to grow a more or less severe debt deflation begins.
- Wage and price flexibility does not help to end a debt deflation.
- The employment effects of the redemption of household sector debt are worse than those of a mere cessation of credit growth.

- After a debt deflation has started the crucial point is whether profit merely shrinks or turns into a loss. To avoid losses the expenditure ratio has to stay above the critical value that in turn depends on the distributed profit ratio.
- A debt deflation is not an accident but a latent property of the monetary economy.
- It is not in the interest of the business sector as a whole that the households pay off debt. The positive net-worth of the business sector is, in the pure consumption economy, at any point in time the exact mirror image of the household sector's debt.

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