The Perverse Incentive of Knowing the Truth

Garcia-Martinez, Jose A.

Universidad Miguel Hernandez

15 January 2013

The Perverse Incentive of Knowing the Truth[1]

Jose A. Garcia-Martinez

Departamento de Estudios Económicos y Financieros. Universidad Miguel Hernández

January 2013

E-mail address: Jose.Garciam@umh.es

Phone 1: +34 966658886

Address for manuscript correspondence:
Jose A. Garcia-Martinez
Departamento de Estudios Económicos y Financieros
Universidad Miguel Hernández
Avenida de la Universidad s/n, Edificio Torreblanca
E-03202, Elche, Alicante (SPAIN)

[1] I thank Barton Lipman, Dilip Mookherjee, Ascension Andina, Miguel Angel Melendez, the workshop participants at Boston University, and at the University of Malaga for helpful comments. Special thanks go to Juan Pablo Juarez for proofreading and giving constructive suggestions. I'd like to thank The Institute for Economic Development and the Department of Economics at Boston University, and the Departamento de Analisis Economico y Finanzas at Universidad Castilla-La Mancha for their hospitality. I also thank the Junta de Anadalucia for support through grant SEJ-8065. The usual disclaimers apply.

Abstract

We show that the observation by a principal of the effectiveness of an expert‘s action could induce the expert to lie, damaging the principal. A career-minded expert receives a private-informative signal about the real state of the world, and then he takes an action that can match or not the real state. If a principal observes the *consequences* of this expert's action, i.e., if the action matches or not the real state, this expert could disregard his valuable information damaging the principal: the expert plays the opposite action to that recommended by his signal and consequently decreases the probability of matching the real state. However, this expert could play the "recommended" action with positive probability if consequences are not observed. The previous literature has found that "transparency of consequence" can only improves the incentives of the expert to reveal his valuable information. The paradoxical behavior we have found can appear when the expert needs to signal with one action two different kinds of information, and there is a particular "trade-off" in the way of signaling; this "trade-off" can be affected in an unexpected way by the observation of the expert's action consequences. In this paper, we present a simple model to capture this idea, and characterize the range of the parameters where that occurs.

JEL classification: D82;C72
Keywords: Transparency, Principal-Agent, Reputation.

1 Introduction

There are many situations in which an individual (the agent) acts on behalf of another (the principal). For example, the shareholders (the principal) delegate the power to manage a company to the directors (the agent); the investor (the principal) entrusts his/her money to a fund manager (the agent); the voters (the principal) elect politicians (the agent) to run the government as set public policy. In general, the principal delegates to an informed expert (agent), who usually has valuable information for the principal, in the hopes that this expert makes better decisions than the principal would have made by herself[2]. However, the two parties have asymmetric information and usually different interests, in addition, the principal cannot directly verify that the expert is always acting in the principal's best interests. Consequently, a problem may appear: the behavior of the expert could damage the principal.

It could be expected that the more information the principal has about the expert, the better for the principal. The idea is that the more information, the more accountability, which should align the interests of the agent and the principal. However, different settings have been identified in the previous literature in which more information can damage the principal[3]. In our paper we show as a novelty, that if the principal observes the true effectiveness of the expert's action, this observation could paradoxically creates a perverse incentive that moves the expert to disregard useful private signals, which in turn makes less effective the expert's action, which finally damages the principal.

In order to reach our conclusion, we consider a simple model of career concerns for experts, where an expert receives a private-informative signal about the real state of the world; then, he chooses an action that can match or not the real state. The principal observes the expert's action, and after that she can or cannot observe the consequences of this action[4], i.e., whether the action matches or not the real state.

We identify conditions under which if the principal observes the consequences of the expert's action, this observation could induce the expert to disregard his private-informative signal and plays the opposite action to that recommended by the signal. Taking the opposite action to that of the signal, the probability of matching the real state decreases. This kind of behavior can be found when the expert needs to signal with only one action two different kinds of information, and a particular "trade-off" in the way of signaling arises.

This particular "trade-off" can arise when certain types of experts are considered. First, the principal has to believe that there exists a biased type of expert that always takes the same action. It is an extreme assumption but it is not too weird[5]. For example, consider an expert that do not want to regulate financial markets because

[2] Female pronouns for the principal and male pronouns for the expert are used in this paper.

[3] For example, Holmströn (1999); Prat (2005); Fox and Van Weelden (2012); Dewatripont et al. (1999); Crémer (1995). See Prat (2005) for a discussion.

[4] In our model, the principal can always observe the action. If the principal does not observe the action, the observation of the consequences will increase the agent's incentives to tell the truth as can be expected.

[5] This kind of agent has previously been used in the literature, e.g., in Morris (2001), it is found a similar biased agent and several examples are provided.

he "deeply believes" that it cannot be a good action regardless any kind of economic signals. It is also needed that the principal believes that there exists a good type of expert always telling the truth. It is also extreme, but we do not think either it is too weird for the principal to believes that this kind of expert can exist with some probability (possibly very low). For example an expert that will be entirely sincere because his utility function is equal to that of the principal, and any reputation concerns is overwhelmed by this fact, e.g. the principal can be a politician and the expert his advisor, and the principal and this expert belong to the same political party, they are ideologically identical in essence, and they share the same interests.

Therefore, we consider three types of experts: the good type, that receives a perfect signal and always reveals it truthfully; the bad type, who is a biased type and always takes the same action independently of the signal; the normal type, who receives an *informative signal* and has to choose whether to reveal it or not. This normal type is the only active player in the model. These three types of experts are considered so as to introduce more than one dimension in the expert's reputation concerns. Thus, a normal type, on the one hand, wants to maximize the principal's expost beliefs that he is a good type; and on the other hand he wants simultaneously to minimize the principal's expost beliefs that he is a bad type. Consequently, if the signal is equal to the action taken by the biased type, then there appears a trade-off: the normal expert can either maximize the probability of matching the real state taking the action that is indicated by the signal, or he can choose other action and clearly show that he is not a bad type. Under certain conditions, this trade-off can be affected in an unexpected way when the principal observes the consequences of the expert's action.

We found that, with a prior of the bad type of expert low enough (but not too low) and with a quality of the signal not too accurate, a particular behavior appears in equilibrium. A normal expert tells the truth with a positive probability if the probability of observing the consequences is negligible, and he lies if the consequences are going to be certainly observed. What is happening is that the distortion caused by the bad type of expert is stronger with full transparency than without it.

There are several previous works arguing that the observation of the consequences of the expert's action is never harmful, e.g. Canes-Wrone et al. (2001), Maskin and Tirole, (2004), and Prat (2005). The latter, Prat (2005), is a paper markedly significant for our work, which also makes a very good discussion of the positive and negative aspects of the role of transparency. Transparency is understood as the ability of the principal to observe how the experts behave and the consequences of expert's behavior. Specifically, Prat (2005) states "the main contribution of this paper (Prat's) is to show that, while transparency on consequences is beneficial, transparency on action can have detrimental effects." We show that introducing one more dimension in the reputation concerns could make that observing consequences might also be harmful.

We only found a paper where observing consequences damages the principal, Fox and Van Weelden (2012). However, the damage for the principal comes from a different way. In fact, in their model, as in the previous literature and unlike in our work, the observation of the consequences of expert's action induces the expert *not* *to disregard* his informative signal and tell the truth, but this can hurt the principal because of the particular payoffs structure they consider. In their model, the cost of not matching the real state of the world depends on the state, which is greater in one state than in the other. This asymmetry in the principal payoffs could make optimal playing against the signal because this action increases the expected payoffs for certain prior and cost values. Anyway, the observation of the consequences of the expert's action induces the expert to act according to his private information, telling the truth, as in the previous literature. In our model, on the contrary, the observation of the consequences can induce the expert not to tell the truth.

Therefore, in previous literature the transparency on consequences induces the expert to reveal his valuable information. However, in the present paper, it is shown that it may not be true if there is a certain kind of multidimensionality. The state and the policy space are one-dimensional, however, there is a multidimensionality in the expert utility and principal beliefs that generates a particular trade-off. This trade-off is affected by the transparency on consequences in an unexpected way. The simplest model to capture this idea is presented in the following section, and after that the model is analyzed and the range of the parameters where this occurs characterized.

2 Model

In a career concerns game, there are a principal and a privately informed expert, and the expert makes a decision on behalf of the principal. For our purposes, it is sufficient to consider a simple model in which the expert's action, signal and consequences are all binary. However, there have to be three types of experts. Thus, there are two equiprobable states of the world, $\omega \in \{R, L\}$, with $P(\omega = R) = \frac{1}{2}$. The principal does not know the state of the world. The expert observes a private signal, $s \in \{r, l\}$, of the state of the world with probability $P(s = x) = \gamma > \frac{1}{2}$ and takes an action $a \in \{\hat{r}, \hat{l}\}$. There are three types of experts (θ): the good one, the normal, and the bad ($\theta \in \{G, N, B\}$) with prior probabilities $P(\theta = G) = \lambda_G$, $P(\theta = N) = \lambda_N$ and $P(\theta = B) = \lambda_B$, and $\lambda_G + \lambda_N + \lambda_B = 1$. The good type observes a perfect signal ($\gamma = 1$) and always revels the true state $a = s = \omega$. The bad type is a biased expert that always takes action r, regardless of the signal. The normal type receives an informative signal $P(r \mid R) = P(l \mid L) = \gamma > \frac{1}{2}$ and has to choose the action to take. All that is common knowledge. The expert knows his type, but this information is not observable for the principal.

The normal type is the only active player in our model. The mixed strategy of this expert is a pair $(v, x) \in [0, 1]^2$, where v represents the probability that the expert plays action $\hat{l}$ if he receives signal r, and x represents the probability that the expert plays action $\hat{r}$ if he receives signal l, i.e., those are the probabilities of "lying" in either one of the two information sets. It is considered that the expert "tells the truth" if he takes the action that matches the signal and "lies" otherwise.

The utility of the principal is greater if the action matches the real state of the world than otherwise. The principal takes no action but updates her belief about the type of the expert based on the information she observes.

The principal always observes the action of the expert[6]. However, she observes with probability $\mu \in (0,1)$ whether or not the expert's action matches the state of the world (i.e. the consequence) before updating the beliefs about his type. The probability μ is common knowledge. When $\mu = 1$, following Prat (2005), it is said that there is full "transparency on consequences". As μ goes to zero, there is less transparency on consequences. The transparency on consequences is also called feedback.

The principal's belief that the expert's type is $\theta \in \{G, N, B\}$ is $\Lambda_\theta[a, X]$, where $[a, X]$ is the information available to the principal, with $X = \{0, R, L\}$ and $a \in \{\hat{r}, \hat{l}\}$. Without feedback, the principal only observes the action and not the consequence: $\Lambda_\theta[a, 0] = P(\theta \mid a, 0)$. With feedback, the principal observes the action and the state of the world (ex-post): $\Lambda_\theta[a, \omega] = P(\theta \mid a, \omega)$. If action a is not played in equilibrium, perfect Bayesian equilibrium imposed no restriction on $\Lambda_\theta[a, X]$.

The normal expert seeks to maximize the probability the principal place of him being the good type, and simultaneously to minimize the probability the principal place of him being the bad type, which is equivalent to maximize the probability of not being a bad type. Thus, the normal expert maximizes the function[7] $f(\Lambda_G[a, X], 1 - \Lambda_B[a, X]) = \Lambda_G[a, X] + \Lambda_{\bar{B}}[a, X]$, where $1 - \Lambda_B[a, X] = \Lambda_{\bar{B}}[a, X]$ is the probability of not being a bad type. Some of our results are obtained considering only that $f(\Lambda_G[a, X], 1 - \Lambda_B[a, X])$ is continuous and increasing in both arguments.

Given any equilibrium strategy (v^*, x^*), it said that the equilibrium is informative if $v^* \neq 1 - x^*$. We focus on *non-perverse informative equilibrium*, i.e., $v^* < 1 - x^*$.

3 Analysis

In equilibrium, the principal forms her beliefs via Bayesian updating. The principal's beliefs in terms of the prior are detailed in the appendix A in section A.1.

Before analyzing the model described above, it will be useful to consider a simplification which differs only in two features: there is not a bad type expert, and the normal expert is only concerned with being considered a good type. Thus, if there is no transparency ($\mu = 0$) and consequently the principal cannot observe whether the action matches or not the state of the world, the normal expert mimics in equilibrium the frequency of the good type's actions. The good type receives with equal probability each signal, consequently, this good type takes (exante) action $\hat{l}$ with the same probability than $\hat{r}$, i.e., $\frac{1}{2}$. Therefore, any strategy of the normal type in which actions $\hat{l}$ and $\hat{r}$ are taken with equal probability will be part of the equilibrium, i.e. $v^* = x^*$, and there will be equilibrium multiplicity. However, if there is some probability that the consequences can be observed ($\mu > 0$), the

[6] In our model, the principal can always observe the action. If the principal does not observe the action, the observation of the consequences cannot decrease the incentives to tell the truth as can be expected. In that case, the distortion the bad type causes plays a much less important role. The expert cannot clearly signal that he is not a bad type choosing other action to that of the bad type. The only way to signal the type to the principal is matching or not the state of the world, and matching always will improve the reputation of the agent (being good and not bad). The trade-off between the two reputations disappears.

[7] Remark 6, in the appendix B, shows an explanation of that payoffs function.

only strategy in equilibrium will be to tell the truth and $v^* = x^* = 0$ because this maximizes the probability of matching the state as the good type of expert does. Therefore, there are two ways to maximize the probability of being considered as a good type of expert: taking both actions with equal probability, and telling the truth in order to maximize the probability of matching the real state. This simple model is analyzed in the appendix B, and in that model the transparency on consequences has the expected effect of improving accountability, which aligns the interests of the expert with the interests of the principal.

When a biased type is also considered, a new concern arises: to be considered as a bad type of expert. Thus, the normal type has an incentive not to follow the action taken by the biased expert ($\hat{r}$), this makes the normal expert to take more often the action $\hat{l}$ than $\hat{r}$, as it is shown below. The normal expert wants to signal that he is a good type, but also that he is not a bad type. The following propositions show that the distortion caused by the bad type of expert can be stronger with full transparency on consequences than without it. Thus the expert could be more honest if there is no full transparency on consequences.

Let us now analyze the model. The following result shows the equilibrium strategy without transparency, $\mu = 0$. Let $v^0 = \frac{\lambda_B(1-\lambda_B+\lambda_G)}{(\lambda_G-\lambda_B)(1-\lambda_G-\lambda_B)}$, note that $v^0 > 0$ if and only if $\lambda_B < \lambda_G$. In addition, let $\lambda_B^{**} = \frac{1}{4}\left(2+\lambda_G-\sqrt{9\lambda_G^2-4\lambda_G+4}\right)$. It is straightforward to show that $\lambda_B^{**} < \lambda_G$, thus if $\lambda_B < \lambda_B^{**}$, then $\lambda_B < \lambda_G$.

Proposition 1 *With $\mu = 0$, in equilibrium:*

If $\lambda_B < \lambda_B^{**}$, then $v^* = v^0 + x^*$ (where $v^0 \in (0,1)$)

If $\lambda_B \geq \lambda_B^{**}$, then $(v^* = 1, x^* = 0)$.

If λ_B is too high, the normal type always takes action $\hat{l}$ in both information sets in equilibrium. The distortion caused by the bad type is too strong. In that case there is not an informative equilibrium.

However, if λ_B is low enough, then the normal type tells the truth with a positive probability in both information sets in equilibrium. The distortion that the bad type exerts on the equilibrium strategy of the normal type of expert is captured by v^0. This parameter v^0 determines how much more the expert has to lie if he receives the signal r instead of signal l. As both signals are received with the same probability, the normal type takes more often action $\hat{l}$ than action $\hat{r}$, because $v^* = v^0 + x^*$ and consequently $v^* > x^*$, unlike the case considered above in which there was not a bad type and in equilibrium $v^* = x^*$. Note that $v^0 \simeq 0$ if $\lambda_B \simeq 0$.

Therefore, if $\lambda_B < \lambda_B^{**}$, then there are multiple equilibria and one of them is always an equilibrium in which $x^* = 0$. As it is shown in the following result, if $\mu > 0$, then any equilibrium in which $x^* > 0$ vanishes.

The following proposition shows that if there is a positive probability of observing the consequences, i.e. $\mu \in (0, 1]$, the incentive of lying disappears if signal l is received. Thus, any tiny degree of transparency on consequence is enough for the normal expert to tell the truth in such information set. This proposition is proved not only for the payoff function assumed, but also for any payoff function, $f(\Lambda_G[a, X], \Lambda_{\bar{B}}[a, X])$, increasing in both arguments

Proposition 2 *With $\mu \in (0, 1]$, in equilibrium $x^* = 0$. The equilibrium strategy always has the form $(v^*, 0)$.*

If the signal received is l and there is a positive probability of observing the consequences of expert's action, the normal expert always tells the truth and takes action $\hat{l}$. The incentives created on the one hand by the bad type and on the other hand by the transparency on consequences are aligned if signal l is received. In that case, action $\hat{l}$ increases the probability of matching the real state, and the good type always matches the real state. In addition, action $\hat{l}$ is the best way to signaling that the expert is not a bad type, because a bad type never takes this action.

Therefore, there is not any equilibrium in which the normal expert lies with positive probability in both information sets. The expert could only lie in equilibrium if he receives signal r, consequently, we can only focus on this information set.

The following result shows the strategy in equilibrium if μ is close enough to zero.

Proposition 3 *Let $\mu > 0$ close enough to zero ($\mu \simeq 0$).*

If $\lambda_B < \lambda_B^{**}$, then in equilibrium always $(v^*, x^*) = (\hat{v}, 0)$, where $0 < \hat{v} < 1$, and $\hat{v}$ goes to v^0 as μ goes to zero.

If $\lambda_B \geq \lambda_B^{**}$, then in equilibrium always $(v^*, x^*) = (1, 0)$.

As in proposition 1, if the prior probability of being a bad type is greater than a threshold, the only strategy in equilibrium is $(v^*, x^*) = (1, 0)$, i.e. action $\hat{l}$ is always taken. However, if λ_B is lower than this threshold, the normal type tells the truth and takes action $\hat{r}$ with a positive probability $(1 - \hat{v})$ in equilibrium, and $\hat{v}$ goes to v^0 as μ goes to zero. If the prior of being a bad type is low, $\lambda_B \simeq 0$, then the normal type tells the truth (almost always), $\hat{v} \simeq 0$.

With $\mu \simeq 0$, the importance of the transparency on consequence in the payoff function of the normal type is negligible, i.e., to match or not to match the real state does not matter too much. Anyway, it matters enough to prevent any equilibrium in which $x^* \neq 0$: the action will always be $\hat{l}$ with signal l. However, when signal r is received, a more complex scenario arises. Taking action $\hat{r}$ makes positive the probability of being considered a bad type, however, this action also increases the probability of being considered as a good type. Note that, a good type takes action $\hat{r}$ and $\hat{l}$ with the same frequency, i.e. $\frac{1}{2}$. With a negligible probability of observing the consequences, to match or not to match the real state does not matter too much in the payoffs function of the normal expert. Thus, the best way to look like a good type is mimicking the frequency of his actions. As in the information set where signal l is received the normal expert always takes action $\hat{l}$ in equilibrium, in the information set where signal r is received the normal experts should take action $\hat{r}$, thus, the strategy takes both actions with the same frequency. However, the distortion caused by the bad type of expert induces the normal type to take action $\hat{l}$ with a positive probability. The distortion decreases as λ_B goes to zero, i.e., $\hat{v}$ goes to zero.

Therefore, with a negligible transparency on consequences, we have characterized the condition ($\lambda_B < \lambda_B^{**}$) under which in equilibrium the normal expert tells the truth with a positive probability if signal r is received. The question to be answered below is the following: Is the normal expert behavior in equilibrium less honest with full transparency on consequences if $\lambda_B < \lambda_B^{**}$?

With full transparency on consequences and $\lambda_B < \lambda_B^{**}$, the following result characterized the conditions under which the normal expert always lies when signal r is received.

Let $\lambda_B^* = \frac{\lambda_G\left(2+\lambda_G+\lambda_G^2-2\sqrt{2}\sqrt{\lambda_G(\lambda_G+1)}\right)}{(\lambda_G+2)^2}$ and $\tilde{\gamma} = \frac{\left(\lambda_B^2-\lambda_G^2\right)}{2\sqrt{2\lambda_B\lambda_G}-(2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G)}$. The proof of the following proposition in the appendix also shows that $\lambda_B^* < \lambda_B^{**}$, and that $\tilde{\gamma} > \frac{1}{2}$ if $\lambda_B \in (\lambda_B^*, \lambda_B^{**})$.

Proposition 4 *With $\mu = 1$, if $\lambda_B \in (\lambda_B^*, \lambda_B^{**})$ and $\gamma \in (\frac{1}{2}, \tilde{\gamma})$, then in equilibrium always $(v^*, x^*) = (1, 0)$*

Proposition 4 states that with full transparency on consequences, with a prior of the biased type of expert lower than λ_B^{**} but higher than λ_B^* and with a quality of the signal not too high, the normal expert disregards his informative signal (r) and lies taking action $\hat{l}$. In that case, the expert always takes action $\hat{l}$ regardless of the signal.

If the prior of the bad type is too low (below λ_B^*), the incentives to lie will be too week and the normal type of expert will tell the truth with full transparency on consequences, and if that prior were above λ_B^{**}, the normal type will always lie without full transparency on consequences. Figure 1, in the appendix, shows a subset of parameters (called Ψ) in which full transparency damages the principal.

With $\mu = 1$, the importance of the transparency on consequence in the payoff function of the normal type is maximum: the principal will observe the consequences for sure. When signal r is received and action $\hat{r}$ taken in order to maximize the probability of being considered as a good type, the cost in reputation for the normal expert is very high if he does not match the real state. On the one hand, he is revealing that he is not a good type because the state is not matched. On the other hand, he cannot exclude the possibility of being considered as a bad type. The payoff in that case is the lowest possible. The probability of obtaining this payoff depends on the signal quality, i.e., γ. Thus, a low level of the quality of the signal increases the probability of obtaining this low payoff. For this reason, if γ is not high enough the normal expert will prefer to lie and takes action $\hat{l}$, because at least he avoids being considered as a bad type, and he still can be considered as a good type with a positive probability if the action eventually matches the state (because the signal was wrong).

Therefore, it is proved that if λ_B is low enough but not too much, and γ is lower than a certain threshold, a normal expert tells the truth with a positive probability if the probability of observing the consequences is negligible, and he lies if the consequences are going to be certainly observed.

Our results compare two extreme scenarios, $\mu \simeq 0$ and $\mu = 1$, and they are enough to make our point. However, in the subset of interest where $(\lambda_G, \lambda_B) \in \Psi$ and $\gamma \in (\frac{1}{2}, \tilde{\gamma})$, a clear picture of what happens if μ is increasing from zero to one can be drawn. By proposition 2, if signal l is received the normal expert always tells the truth in equilibrium and takes action $\hat{l}$ with probability one for any $\mu > 0$. If signal r is received the normal expert will tell the truth with a positive probability with $\mu \simeq 0$ and will always lie if $\mu = 1$. Let $r(v)$ be the net utility gain to the normal type of expert of taking $\hat{l}$ (rather than $\hat{r}$) when his signal is r, that is: $r(v) = (1-\mu)r(v)^0 + \mu r(v)^1$, where $r(v)^0$ is the same net utility gain without transparency and $r(v)^1$ with transparency[8]. The function $r(v)^0$ has the following properties: it has only one root which is v^0, the function $r(v)^0$ is positive for any $v < v^0$, and it is negative for any $v > v^0$. On the other hand, the function $r(v)^1 > 0$ for any v. Therefore, for any $v < v^0$, the function $r(v)$ has to be positive, and if μ increases the normal experts never lies less than with $\mu \simeq 0$. In addition, for any given $\bar{v} \geq v^0$, as $r(\bar{v})^0 \leq 0$ and $r(\bar{v})^1 > 0$, the value of $r(\bar{v})$ increases with μ, i.e., $r(\bar{v})|_{\mu_1} < r(\bar{v})|_{\mu_2}$ if $\mu_1 < \mu_2$. As $r(v)$ is a second-degree polynomial in v, it can only have two roots at most. Consequently, if there is only one root, it will increase as μ increases, and if there are two roots, the lowest one will also increase with μ. It can be stated that the equilibrium where the normal experts tells the truth more often[9] changes as μ increases, increasing the probability of lying, i.e., v^* increases.

4 Conclusion

We have shown that the behavior of a career-minded expert could be affected in an undesirable way when the principal observes the consequences of this expert's action. The novelty of the paper is to show that transparency on consequences could induce the expert to disregard valuable information in the following way: with probability equal to one, the expert takes a different action from the one recommended by his private-informative signal, and this behavior decreases the probability of matching the real state, which eventually damages the principal. However, the expert will play according to his private-informative signal with a positive probability if the principal does not observe the consequences of the expert's action.

With three particular types of experts, the incentive structure that makes transparency on consequences harmful for the principal arises. The multidimensionality in the beliefs creates a particular trade-off between lying and not lying that can be more favorable to lie if there is more transparency on consequences than if not. The role of the biased type of expert is essential. The probability of being a biased type of expert has to be low enough. If this probability is too high, the normal expert will never take in equilibrium the action played by the biased type: taking the action of the biased type, the normal expert bears a reputation cost. The distortion caused by the biased type of expert decreases as the prior of this type of expert decreases. However, the behavior of the normal type is affected in a different way with full transparency on consequences than without it. Thus, if the prior of the biased type of expert is between two thresholds and the quality of the signal is not too accurate, the distortion caused by the biased type is stronger with full transparency than without it. Consequently, the normal type will be less honest with full transparency on consequences. This property can hold even if the priors of the good type and the bad type of expert are very low, thus, if the principal beliefs that a good type and a bad type could exist although with a very low probability and the experts knows it, the problem shown in this paper can appear.

[8] In the appendix, these functions are made explicit.

[9] In case that there are several equilibria.

The widespread lack of transparency in agency relationships, e.g., in government activities, politics in general, corporate governance, and delegate portfolio management, among others can be explained by several factors. As Prat (2005) pointed out, the inefficient arrangement that survive because of institutional inertia or resistance from settled interest can be one of them. However, as this author states, the lack of transparency on agent's action could be also optimal under certain circumstances. With our paper, we added to this topic that not only transparency on action can be detrimental to the principal. Under certain conditions, transparency on consequences can be also detrimental in a way not considered so far.

Therefore, we identify conditions under which the role of transparency on consequences is quite different to those shown in the previous literature[10], in which always this kind of transparency induces the expert to reveal his valuable information, unlike in our paper.

[10] Canes-Wrone et al. (2001), Maskin and Tirole, (2004), Prat (2005), and Fox and Van Weelden (2012).

A Appendix A

The outline of the appendix is the following. Firstly the principal's beliefs are detailed. Then, some auxiliary payoffs functions are defined. Finally, the propositions are proven.

A.1 Principal's Beliefs

Firstly we detail the principal's beliefs in terms of the priors and given an expert's strategy (v, x). There are six different kinds of events from which the principal can obtain information for evaluating the type of the expert. There are two of them in which the principal does not observe the consequence of the action. In that case , the principal only observes if the expert takes either action $\hat{r}$ or action $\hat{l}$. In the other four events the principal can also observe the consequences and they are the following: the expert takes action $\hat{r}$ and the real state of the world is R, or he takes $\hat{r}$ and the state of the world is L, or he takes $\hat{l}$ and the state of the world is L, or finally he takes $\hat{l}$ and the state of the world is R. The table on the left shows the principal's beliefs about the expert being a good type and the table on the right the expert being a bad type. The two rows show the two possible actions ($\hat{r}$,$\hat{l}$), the first column (0) means that consequences are not observable, and the other two that consequences are observable, and state is R or L:

	Good Reputation		
Action \ State	0	R	L
$\hat{r}$	$\Lambda_G[\hat{r}, 0]$	$\Lambda_G[\hat{r}, R]$	0
$\hat{l}$	$\Lambda_G[\hat{l}, 0]$	0	$\Lambda_G[\hat{l}, L]$

	Bad Reputation		
Action \ State	0	R	L
$\hat{r}$	$\Lambda_B[\hat{r}, 0]$	$\Lambda_B[\hat{r}, R]$	$\Lambda_B[\hat{r}, L]$
$\hat{l}$	0	0	0

The zeros indicate zero probability, as it is not possible that a bad type send $\hat{l}$ or a good type does not match the state of the world.

By Bayes rule these beliefs can be detailed. Let T1 be one of the three possible types of experts and T2 and T3 be the other two.

$\Lambda_{T1}[a, 0] = P(\mathrm{T1}|\ a) = \frac{P(\mathrm{T1},a)}{P(a)} = \frac{P(\mathrm{T1},a)}{P(\mathrm{T1},a)+P(\mathrm{T2},a)+P(\mathrm{T3},a)} = \frac{P(a|\mathrm{T1})P(\mathrm{T1})}{P(a|\mathrm{T1})P(\mathrm{T1})+P(a|\mathrm{T2})P(\mathrm{T2})+P(a|\mathrm{T3})P(\mathrm{T3})}$

$\Lambda_{T1}[a, \omega] = P(\mathrm{T1}|\ a, \omega) = \frac{P(\mathrm{T1},a,\omega)}{P(a,\omega)} = \frac{P(\mathrm{T1},a,\omega)}{P(\mathrm{T1},a,\omega)+P(\mathrm{T2},a,\omega)+P(\mathrm{T3},a,\omega)}$

$= \frac{P(a|\mathrm{T1},\omega)P(\omega|\mathrm{T1})P(\mathrm{T1})}{P(a|\mathrm{T1},\omega)P(\omega|\mathrm{T1})P(\mathrm{T1})+P(a|\mathrm{T2},\omega)P(\omega|\mathrm{T2})P(\mathrm{T2})+P(a|\mathrm{T3},\omega)P(\omega|\mathrm{T3})P(\mathrm{T3})}$

$= \frac{P(a|\mathrm{T1},\omega)P(\omega)P(\mathrm{T1})}{P(a|\mathrm{T1},\omega)P(\omega)P(\mathrm{T1})+P(a|\mathrm{T2},\omega)P(\omega)P(\mathrm{T2})+P(a|\mathrm{T3},\omega)P(\omega)P(\mathrm{T3})}$

For example, $\Lambda_G[\hat{r}, 0] = \frac{P(\hat{r}|G)P(G)}{P(\hat{r}|G)P(G)+P(\hat{r}|N)P(N)+P(\hat{r}|B)P(B)}$ and $\Lambda_G[\hat{r}, R] = \frac{P(\hat{r}|G,R)P(G)}{P(\hat{r}|G,R)P(G)+P(\hat{r}|N,R)P(N)+P(\hat{r}|B,R)P(B)}$.

The probability $P(\hat{r} \mid N)$ (probability that a normal type of expert takes action $\hat{r}$) can be easily calculated: $P(\hat{r} \mid N) = P(s = l \mid N)P(\hat{r} \mid s = l, N) + P(s = r \mid N)P(\hat{r} \mid s = r, N) = \frac{1}{2}x + \frac{1}{2}(1 - v)$. Note that the probability that a normal type of expert receives signal r is $P(s = r \mid N) = P(\omega = R \mid N)P(s = r \mid N, \omega = R) + P(\omega = L \mid N)P(s = r \mid N, \omega = L) = \frac{1}{2}\gamma + \frac{1}{2}(1 - \gamma) = \frac{1}{2}$. Thus, $P(s = r \mid N) = \frac{1}{2} = P(s = l \mid N)$, the ex-ante probability a normal type of expert receives signal r is $\frac{1}{2}$. However, the signal is informative.

It is straightforward to calculate the rest of the probabilities required:

$P(\hat{r} \mid N) = \frac{1}{2}x + \frac{1}{2}(1 - v)$	$P(\hat{r} \mid G) = \frac{1}{2}$	$P(\hat{r} \mid B) = 1$
$P(\hat{l} \mid N) = \frac{1}{2}(1 - x) + \frac{1}{2}v$	$P(\hat{r} \mid G) = \frac{1}{2}$	$P(\hat{r} \mid B) = 0$

, and

$P(\hat{r} \mid N, R) = (1 - \gamma)x + \gamma(1 - v)$	$P(\hat{r} \mid G, R) = 1$	$P(\hat{r} \mid B, R) = 1$
$P(\hat{r} \mid N, L) = \gamma x + (1 - \gamma)(1 - v)$	$P(\hat{r} \mid G, L) = 0$	$P(\hat{r} \mid B, L) = 1$
$P(\hat{l} \mid N, R) = (1 - \gamma)(1 - x) + \gamma v$	$P(\hat{l} \mid G, R) = 1$	$P(\hat{l} \mid B, R) = 0$
$P(\hat{l} \mid N, L) = \gamma(1 - x) + (1 - \gamma)v$	$P(\hat{l} \mid G, L) = 0$	$P(\hat{l} \mid B, L) = 0$

.

Finally, the principal's beliefs in terms of the priors and given an expert's strategy (v, x) will be:

Good Reputation			
	0	R	L
$\hat{r}$	$\Lambda_G[\hat{r},0]=\frac{\lambda_G}{2\lambda_B+\lambda_G+(1-v+x)\lambda_N}$	$\Lambda_G[\hat{r},R]=\frac{\lambda_G}{\lambda_B+\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}$	$\Lambda_G[\hat{r},L]=0$
$\hat{l}$	$\Lambda_G[\hat{l},0]=\frac{\lambda_G}{\lambda_G+(1+v-x)\lambda_N}$	$\Lambda_G[\hat{l},R]=0$	$\Lambda_G[\hat{l},L]=\frac{\lambda_G}{\lambda_G+(\gamma(1-x)+(1-\gamma)v)\lambda_N}$

Table 1

Bad Reputation			
	0	R	L
$\hat{r}$	$\Lambda_B[\hat{r},0]=\frac{2\lambda_B}{2\lambda_B+\lambda_G+(1-v+x)\lambda_N}$	$\Lambda_B[\hat{r},R]=\frac{\lambda_B}{\lambda_B+\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}$	$\Lambda_B[\hat{r},L]=\frac{\lambda_B}{\lambda_B+(\gamma x+(1-\gamma)(1-v))\lambda_N}$
$\hat{l}$	$\Lambda_B[\hat{l},0]=0$	$\Lambda_B[\hat{l},R]=0$	$\Lambda_B[\hat{l},L]=0$

Table 2

A.2 Auxiliary payoffs functions.

In this section, some auxiliary and payoffs functions are defined.

Let $\Pi_s(v,x)$ stand for the expected payoffs of the normal expert if he plays strategy (v,x) and he receives signal s.

Let $\Pi_{s,a}(v,x)$ stand for the expected payoffs of the normal expert if he plays strategy (v,x) and he receives signal s and the principal observes action a. Thus, for example, $\Pi_r(v,x)=v\Pi_{r,\hat{l}}(v,x)+(1-v)\Pi_{r,\hat{r}}(v,x)$, i.e., the probability of lying multiply by the payoff of lying plus the probability of telling the truth multiply by the payoff of telling the truth. Analogously, $\Pi_l(v,x)=(1-x)\Pi_{l,\hat{l}}(v,x)+x\Pi_{l,\hat{r}}(v,x)$.

Thus, (v^*,x^*) is an *equilibrium strategy* if v^* maximizes expected payoffs of the expert after observing signal r, and x^* maximizes it after signal l.

The following auxiliary functions will be used to calculate the equilibrium:

$$r(v,x) = \Pi_{r,\hat{l}}(v,x)-\Pi_{r,\hat{r}}(v,x) \tag{1}$$
$$l(v,x) = \Pi_{l,\hat{l}}(v,x)-\Pi_{l,\hat{r}}(v,x) \tag{2}$$

Remark 1 *Therefore, if $r(v^*,x^*)=l(v^*,x^*)=0$, then (v^*,x^*) is an equilibrium strategy. Additionally, if $r(v,x)>0(<0)$ for all v, then $v^*=1(0)$. On the other hand, if $l(v,x)>0(<0)$ for all x, then $x^*=0(1)$.*

As mention above, the principal can observe the consequences with probability μ. Thus, the payoffs of (1) and (2) can be expressed as

$$\Pi_{s,a}(v,x)=(1-\mu)\Pi^0_{s,a}(v,x)+\mu\Pi^1_{s,a}(v,x) \tag{3}$$

where the payoff if principal does not observe consequences is $\Pi^0_{s,a}(v,x)$ and if she observes consequences is $\Pi^1_{s,a}(v,x)$. Therefore, functions $r(v,x)$ and $l(v,x)$ can be written as

$$r(v,x) = (1-\mu)\Pi^0_{r,\hat{l}}(v,x)+\mu\,\Pi^1_{r,\hat{l}}(v,x)-\left((1-\mu)\Pi^0_{r,\hat{r}}(v,x)+\mu\,\Pi^1_{r,\hat{r}}(v,x)\right) \tag{4}$$
$$l(v,x) = (1-\mu)\Pi^0_{l,\hat{l}}(v,x)+\mu\,\Pi^1_{l,\hat{l}}(v,x)-\left((1-\mu)\Pi^0_{l,\hat{r}}(v,x)+\mu\,\Pi^1_{l,\hat{r}}(v,x)\right) \tag{5}$$

The functions (4) and (5) can be rewritten as

$$r(v,x) = (1-\mu)r^0(v,x)+\mu\,r^1(v,x) \tag{6}$$
$$l(v,x) = (1-\mu)l^0(v,x)+\mu\,l^1(v,x) \tag{7}$$

where

$$
\begin{aligned}
r^0(v,x) &= \Pi^0_{r,\hat{l}}(v,x) - \Pi^0_{r,\hat{r}}(v,x) && (8)\\
r^1(v,x) &= \Pi^1_{r,\hat{l}}(v,x) - \Pi^1_{r,\hat{r}}(v,x) && (9)\\
l^0(v,x) &= \Pi^0_{l,\hat{l}}(v,x) - \Pi^0_{l,\hat{r}}(v,x) && (10)\\
l^1(v,x) &= \Pi^1_{l,\hat{l}}(v,x) - \Pi^1_{l,\hat{r}}(v,x) && (11)
\end{aligned}
$$

The above payoffs are shown below in terms of the principal's beliefs, where $\Lambda_{\bar{B}}[a,X]$ is the probability of not being a bad type, i.e., $1-\Lambda_B[a,X]$:

$$
\begin{aligned}
\Pi^0_{r,\hat{l}}(v,x) &= f\left(\Lambda_G[\hat{l},0],\Lambda_{\bar{B}}[\hat{l},0]\right) = f\left(\Lambda_G[\hat{l},0],1\right) && (12)\\
\Pi^1_{r,\hat{l}}(v,x) &= \gamma f\left(\Lambda_G[\hat{l},R],\Lambda_{\bar{B}}[\hat{l},R]\right) + (1-\gamma) f\left(\Lambda_G[\hat{l},L],\Lambda_{\bar{B}}[\hat{l},L]\right) && (13)\\
&= \gamma f(0,1) + (1-\gamma) f\left(\Lambda_G[\hat{l},L],1\right) &&\\
\Pi^0_{r,\hat{r}}(v,x) &= f\left(\Lambda_G[\hat{r},0],\Lambda_{\bar{B}}[\hat{r},0]\right) && (14)\\
\Pi^1_{r,\hat{r}}(v,x) &= \gamma f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right) + (1-\gamma) f\left(\Lambda_G[\hat{r},L],\Lambda_{\bar{B}}[\hat{r},L]\right) && (15)\\
&= \gamma f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right) + (1-\gamma) f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right) &&\\
\Pi^0_{l,\hat{l}}(v,x) &= f\left(\Lambda_G[\hat{l},0],\Lambda_{\bar{B}}[\hat{l},0]\right) = f\left(\Lambda_G[\hat{l},0],1\right) && (16)\\
\Pi^1_{l,\hat{l}}(v,x) &= \gamma f\left(\Lambda_G[\hat{l},L],\Lambda_{\bar{B}}[\hat{l},L]\right) + (1-\gamma) f\left(\Lambda_G[\hat{l},R],\Lambda_{\bar{B}}[\hat{l},R]\right) && (17)\\
&= \gamma f\left(\Lambda_G[\hat{l},L],1\right) + (1-\gamma) f(0,1) &&\\
\Pi^0_{l,\hat{r}}(v,x) &= f\left(\Lambda_G[\hat{r},0],\Lambda_{\bar{B}}[\hat{r},0]\right) && (18)\\
\Pi^1_{l,\hat{r}}(v,x) &= \gamma f\left(\Lambda_G[\hat{r},L],\Lambda_{\bar{B}}[\hat{r},L]\right) + (1-\gamma) f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right) && (19)\\
&= \gamma f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right) + (1-\gamma) f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right) &&
\end{aligned}
$$

Remark 2 *It is straightforward to show from (12), (14), (16) and (18) that* $r^0(v,x) = l^0(v,x)$.

A.3 Proofs of Propositions

A.3.1 Proof of Proposition 1

From (6), (7) and remark 2, if $\mu = 0$, $r(v,x) = l(v,x) = r^0(v,x)$, thus, we can focus on $r^0(v,x)$.

The following two claims are the first steps to prove the proposition.

Claim 5 *With* $v^0 = \frac{\lambda_B(\lambda_G-\lambda_B+1)}{(\lambda_G-\lambda_B)(1-\lambda_G-\lambda_B)}$ *and* $\mu = 0$:

a) if $\lambda_G > \lambda_B$*, then* $r^0(v,x) \le 0 \iff v \ge v^0 + x$

b) if $\lambda_G < \lambda_B$*, then* $r^0(v,x) \le 0 \iff v \le v^0 + x$

c) if $\lambda_G = \lambda_B$*, then* $r^0(v,x) > 0$

Proof.

From (8), (12), (14), Table 1, and Table 2, and as $\lambda_B + \lambda_G + \lambda_N = 1$,

$$
\begin{aligned}
r^0(v,x) &= f\left(\Lambda_G[\hat{l},0],1\right) - f\left(\Lambda_G[\hat{r},0],\Lambda_{\bar{B}}[\hat{r},0]\right)\\
&= \Lambda_G[\hat{l},0] + 1 - \Lambda_G[\hat{r},0] - \Lambda_{\bar{B}}[\hat{r},0]\\
&= \tfrac{\lambda_G}{\lambda_G+(1+v-x)\lambda_N} + 1 - \tfrac{\lambda_G}{2\lambda_B+\lambda_G+(1-v+x)\lambda_N} - \left(1 - \tfrac{2\lambda_B}{2\lambda_B+\lambda_G+(1-v+x)\lambda_N}\right)\\
&= \tfrac{\lambda_G}{\lambda_G+(1+v-x)\lambda_N} + \tfrac{2\lambda_B-\lambda_G}{2\lambda_B+\lambda_G+(1-v+x)\lambda_N}\\
&= \tfrac{\lambda_G}{\lambda_G+(1+v-x)\lambda_N} + \tfrac{2\lambda_B-\lambda_G}{1+\lambda_B-(v-x)\lambda_N} \le 0\\
\iff & \tfrac{\lambda_G(1+\lambda_B-(v-x)\lambda_N)+(\lambda_G+(1+(v-x))\lambda_N)(2\lambda_B-\lambda_G)}{(\lambda_G+(1+(v-x))\lambda_N)(1+\lambda_B-v\lambda_N)} \le 0
\end{aligned}
$$

$\Longleftrightarrow \lambda_G\left(1+\lambda_B-(v-x)\,\lambda_N\right)+\left(\lambda_G+(1+(v-x))\lambda_N\right)\left(2\lambda_B-\lambda_G\right)\leq 0 \qquad$ (with $\lambda_N = 1-\lambda_G-\lambda_B$)
$\Longleftrightarrow \lambda_G\left(1+\lambda_B-(v-x)\left(1-\lambda_G-\lambda_B\right)\right)+\left(\lambda_G+(1+(v-x))\left(1-\lambda_G-\lambda_B\right)\right)\left(2\lambda_B-\lambda_G\right)\leq 0$
$\Longleftrightarrow \left(\lambda_G\left(\lambda_B+\lambda_G-1\right)+\left(\lambda_G-2\lambda_B\right)\left(\lambda_B+\lambda_G-1\right)\right)(v-x)+\left(\lambda_G\left(\lambda_B+1\right)+\left(\lambda_B-1\right)\left(\lambda_G-2\lambda_B\right)\right)\leq 0$
$\Longleftrightarrow \left(-2\lambda_B^2+2\lambda_B+2\lambda_G^2-2\lambda_G\right)(v-x)+2\lambda_B\left(\lambda_G-\lambda_B+1\right)\leq 0$
$\Longleftrightarrow \left(-\lambda_B^2+\lambda_B+\lambda_G^2-\lambda_G\right)(v-x)+\lambda_B\left(\lambda_G-\lambda_B+1\right)\leq 0$
$\Longleftrightarrow -\left(\lambda_G-\lambda_B\right)\left(1-\lambda_G-\lambda_B\right)(v-x)+\lambda_B\left(\lambda_G-\lambda_B+1\right)\leq 0$
Therefore,

$$r^0(v,x)\leq 0 \Longleftrightarrow \lambda_B\left(\lambda_G-\lambda_B+1\right)\leq\left(\lambda_G-\lambda_B\right)\left(1-\lambda_G-\lambda_B\right)(v-x) \tag{20}$$

Clearly, if $\lambda_G=\lambda_B$, then $r^0(v,x)>0$ and consequently $r(v,x)=l(v,x)>0$, therefore, $v^*=1$ and $x^*=0$ see remark 1.

Let $v^0=\frac{\lambda_B(\lambda_G-\lambda_B+1)}{(\lambda_G-\lambda_B)(1-\lambda_G-\lambda_B)}$. As $(1-\lambda_G-\lambda_B)>0$, the expression (20) is equivalent to:
$r^0(v,x)\leq 0 \Longleftrightarrow v\geq v^0+x \quad$ if $\lambda_G>\lambda_B$
and
$r^0(v,x)\leq 0 \Longleftrightarrow v\leq v^0+x \quad$ if $\lambda_G<\lambda_B$■

Claim 6 *If $\lambda_G<\lambda_B$, then $v^0<-1$. If $\lambda_G>\lambda_B$, then $v^0>0$.*

Proof
Let us prove that $v^0<-1$ with $\lambda_G<\lambda_B$.
$v^0=\frac{\lambda_B(\lambda_G-\lambda_B+1)}{(\lambda_G-\lambda_B)(1-\lambda_G-\lambda_B)}<-1$
$\Longleftrightarrow \lambda_B\left(\lambda_G-\lambda_B+1\right)>-\left(\lambda_G-\lambda_B\right)\left(1-\lambda_G-\lambda_B\right)$
$\Longleftrightarrow \left(\lambda_B-\lambda_G+1\right)>0$
The above expression always holds.
It is straightforward to show that $v^0=\frac{\lambda_B(\lambda_G-\lambda_B+1)}{(\lambda_G-\lambda_B)(1-\lambda_G-\lambda_B)}>0$ if $\lambda_G>\lambda_B$.■

By claim 5, if $\lambda_G=\lambda_B$, then $r^0(v,x)>0$ and consequently $r(v,x)=l(v,x)>0$, therefore, the only equilibrium can be $v^*=1$ and $x^*=0$, see remark 1.

Let's assume that (v^*,x^*) is a strategy in equilibrium. As $r(v,x)=l(v,x)=r^0(v,x)$ with $\mu=0$. Only three cases may occur in equilibrium:

1) $r\left(v^*,x^*\right)=l\left(v^*,x^*\right)=r^0(v^*,x^*)<0$

2) $r\left(v^*,x^*\right)=l\left(v^*,x^*\right)=r^0(v^*,x^*)>0$

3) $r\left(v^*,x^*\right)=l\left(v^*,x^*\right)=r^0(v^*,x^*)=0$

By remark 1, if 1) occurs, then necessarily $v^*=0$ and $x^*=1$. Let us see that it cannot occur. By claim 5, with $\lambda_G>\lambda_B$ if 1) occurs, then $v^*>v^0+x^*$, however it cannot be held with $v^*=0$ and $x^*=1$ because $v^0>0$ by claim 6. On the other hand, with $\lambda_G<\lambda_B$, if 1) occurs, then $v^*<v^0+x^*$ by claim 5, and it is not possible because $v^0<-1$ by claim 6.

By remark 1, if 2) occurs, then necessarily $v^*=1$ and $x^*=0$. By claim 5, if 2) occurs and $\lambda_G>\lambda_B$, then $v^*<v^0+x^*$, and it can be held if $v^0>1$. Therefore, in equilibrium $v^*=1$ and $x^*=0$ when $\lambda_G>\lambda_B$ and $v^0>1$. On the other hand, if 2) occurs and $\lambda_G<\lambda_B$, then $v^*>v^0+x^*$, and it always holds because $v^0<-1$ with $\lambda_G<\lambda_B$, see claim 6. Therefore, in equilibrium $v^*=1$ and $x^*=0$ with $\lambda_G<\lambda_B$.

Finally, by claim 5, if 3) occurs, then $v^*=v^0+x^*$. Thus, if $\lambda_G>\lambda_B$, then $v^0>0$ by claim 6, and there will be multiple equilibria with $v^0<1$. If $v^0=1$, then $v^*=1$ and $x^*=0$ is the only strategy in equilibrium. If $v^0>1$, the case 3) cannot occur. On the other hand, it is not possible that 3) occurs with $\lambda_G<\lambda_B$ because by claim 6, $v^0<-1$, and $v^*=v^0+x^*$ cannot be held $(v^0+x^*<0)$.

The following claim shows when $v^0\leq 1$.

Claim 7 *With* $\lambda_G > \lambda_B$*,* $v^0 \leq 1 \Leftrightarrow \lambda_B \leq \frac{1}{4}\left(2+\lambda_G-\sqrt{9\lambda_G^2-4\lambda_G+4}\right)$

Proof

With $\lambda_G > \lambda_B$,

$v^0 \leq 1 \Leftrightarrow \frac{\lambda_B(1-\lambda_B+\lambda_G)}{(\lambda_G-\lambda_B)(1-\lambda_G-\lambda_B)} \leq 1$

$\Leftrightarrow \lambda_B(1-\lambda_B+\lambda_G) \leq 1\,(\lambda_G-\lambda_B)\,(1-\lambda_G-\lambda_B)$

$\Leftrightarrow \lambda_B(1-\lambda_B+\lambda_G) - 1\,(\lambda_G-\lambda_B)\,(1-\lambda_G-\lambda_B) \leq 0$

$\Leftrightarrow -2\lambda_B^2 + \lambda_B\,(\lambda_G+2) + \lambda_G\,(\lambda_G-1) \leq 0$

It is concave in λ_B and the roots are $\frac{1}{4}\left(2+\lambda_G \pm \sqrt{9\lambda_G^2-4\lambda_G+4}\right)$.

As shown below, $\frac{1}{4}\left(2+\lambda_G+\sqrt{9\lambda_G^2-4\lambda_G+4}\right) > 1$. Therefore, if $\lambda_B \leq \frac{1}{4}\left(2+\lambda_G-\sqrt{9\lambda_G^2-4\lambda_G+4}\right)$, then $v^0 \leq 1$.

$\frac{1}{4}\left(2+\lambda_G+\sqrt{9\lambda_G^2-4\lambda_G+4}\right) > \frac{1}{4}\left(2+\lambda_G+\sqrt{9\lambda_G^2}-\sqrt{4\lambda_G}+\sqrt{4}\right)$

$= \frac{1}{4}\left(2+\lambda_G+3\lambda_G-2\sqrt{\lambda_G}+2\right)$

$= \frac{1}{4}\left(4+4\lambda_G-2\sqrt{\lambda_G}\right) > 1$

Where clearly $4\lambda_G > 2\sqrt{\lambda_G}$, and the expression holds.$\blacksquare$

Therefore, it has been proved that if $\lambda_G \leq \lambda_B$, then the only strategy in equilibrium is $v^* = 1$ and $x^* = 0$. If $\lambda_G > \lambda_B$, then the equilibrium depends on the threshold $\lambda_B^{**} = \frac{1}{4}\left(2+\lambda_G-\sqrt{9\lambda_G^2-4\lambda_G+4}\right)$. When $\lambda_B \geq \lambda_B^{**}$, the only strategy in equilibrium is again $v^* = 1$ and $x^* = 0$. Finally, if $\lambda_B < \lambda_B^{**}$, in equilibrium $v^* = v^0 + x^*$.

It is straightforward to show that $\lambda_B^{**} = \frac{1}{4}\left(2+\lambda_G-\sqrt{9\lambda_G^2-4\lambda_G+4}\right) < \lambda_G$, thus, if $\lambda_B < \lambda_B^{**}$, then $\lambda_G > \lambda_B$ and $v^0 \in (0,1)$. The proof is completed.$\blacksquare$

A.3.2 Proof of Proposition 2.

The proposition is proved through six lemmas. The first four are auxiliary results, which are needed for proving the proposition. After that, it is proved that there is not a possible equilibrium in which the expert lies with a positive probability in both information sets, i.e., $v^* \neq 0$ and $x^* \neq 0$. Therefore, the strategy profile in equilibrium has the form either $(v^*, 0)$ or $(0, x^*)$. It is eventually proved that $(0, x^*)$ is not possible.

Lemma 8 $\Lambda_G[\hat{l}, L] > \Lambda_G[\hat{r}, R] \iff (v-x)\lambda_N < \lambda_B$

Proof

From Table 1, $\Lambda_G[\hat{l}, L] = \frac{\lambda_G}{\lambda_G+(\gamma(1-x)+(1-\gamma)v)\lambda_N}$ and $\Lambda_G[\hat{r}, R] = \frac{\lambda_G}{\lambda_B+\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}$, then:

$\Lambda_G[\hat{l}, L] > \Lambda_G[\hat{r}, R] \iff \frac{\Lambda_G[\hat{l},L]}{\Lambda_G[\hat{r},R]} > 1 \iff \frac{\lambda_B+\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}{\lambda_G+(\gamma(1-x)+(1-\gamma)v)\lambda_N} > 1$

$\iff \lambda_B - v\lambda_N + x\lambda_N > 0 \iff (v-x)\lambda_N < \lambda_B\blacksquare$

Lemma 9 $\Lambda_G[\hat{l}, 0] > \Lambda_G[\hat{r}, 0] \iff (v-x)\lambda_N < \lambda_B$

Proof

From Table 1, $\Lambda_G[\hat{l}, 0] = \frac{\lambda_G}{\lambda_G+(1+v-x)\lambda_N}$ and $\Lambda_G[\hat{r}, 0] = \frac{\lambda_G}{2\lambda_B+\lambda_G+(1-v+x)\lambda_N}$, then:

$\Lambda_G[\hat{l}, 0] > \Lambda_G[\hat{r}, 0] \iff \frac{\Lambda_G[\hat{l},0]}{\Lambda_G[\hat{r},0]} > 1 \iff \frac{2\lambda_B+\lambda_G+(1-v+x)\lambda_N}{\lambda_G+(1+v-x)\lambda_N} > 1$

$\iff \lambda_B - v\lambda_N + x\lambda_N > 0 \iff (v-x)\lambda_N < \lambda_B\blacksquare$

Lemma 10 $\Lambda_{\bar{B}}[\hat{r}, L] < \Lambda_{\bar{B}}[\hat{r}, R]$

Proof

$\Lambda_{\bar{B}}[\hat{r}, L] < \Lambda_{\bar{B}}[\hat{r}, R] \iff 1-\Lambda_B[\hat{r}, L] > 1-\Lambda_B[\hat{r}, R] \iff \Lambda_B[\hat{r}, L] < \Lambda_B[\hat{r}, R]$

From table 2, $\Lambda_B[\hat{r}, L] = \frac{\lambda_B}{\lambda_B+(\gamma x+(1-\gamma)(1-v))\lambda_N}$ and $\Lambda_B[\hat{r}, R] = \frac{\lambda_B}{\lambda_B+\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}$, then:

$\Lambda_B[\hat{r}, L] < \Lambda_B[\hat{r}, R] \iff \frac{\Lambda_B[\hat{r},L]}{\Lambda_B[\hat{r},R]} > 1 \iff \frac{\lambda_B+\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}{\lambda_B+(\gamma x+(1-\gamma)(1-v))\lambda_N} > 1$
$\iff \lambda_G - \lambda_N + v\lambda_N + x\lambda_N + 2\gamma\lambda_N - 2v\gamma\lambda_N - 2x\gamma\lambda_N > 0$
$\iff \lambda_G + (2\gamma - 1)((1 - v) - x))\lambda_N > 0$

On the one hand $2\gamma - 1 > 0$ because $\gamma > \frac{1}{2}$. On the other hand, we had assumed that $x < (1 - v)$ for avoiding perverse equilibria, thus the above expression has to be positive.$\blacksquare$

Lemma 11 $\Pi_{r,\hat{r}}(v, x) > \Pi_{l,\hat{r}}(v, x)$ *and* $\Pi_{l,\hat{l}}(v, x) > \Pi_{r,\hat{l}}(v, x)$ *if* $\mu > 0$, *i.e., taking action* $\hat{r}$ *gives a greater expected payoff if signal is* r *than if signal is* l, *and taking action* $\hat{l}$ *gives a greater expected payoff if signal is* l *than if signal is* r.

Proof

- First, it is proved that $\Pi_{r,\hat{r}}(v, x) > \Pi_{l,\hat{r}}(v, x)$.

By (3), these functions can be written as
$\Pi_{r,\hat{r}}(v, x) = (1 - \mu)\Pi^0_{r,\hat{r}}(v, x) + \mu\Pi^1_{r,\hat{r}}(v, x)$
$\Pi_{l,\hat{r}}(v, x) = (1 - \mu)\Pi^0_{l,\hat{r}}(v, x) + \mu\Pi^1_{l,\hat{r}}(v, x)$,
and with (14), (15), (18), and (19), they can be written in terms of principal's beliefs as
$\Pi_{r,\hat{r}}(v, x) = (1 - \mu)f\left(\Lambda_G[\hat{r}, 0], \Lambda_{\bar{B}}[\hat{r}, 0]\right) + \mu\left(\gamma f\left(\Lambda_G[\hat{r}, R], \Lambda_{\bar{B}}[\hat{r}, R]\right) + (1 - \gamma)f\left(0, \Lambda_{\bar{B}}[\hat{r}, L]\right)\right)$
$\Pi_{l,\hat{r}}(v, x) = (1 - \mu)f\left(\Lambda_G[\hat{r}, 0], \Lambda_{\bar{B}}[\hat{r}, 0]\right) + \mu\left(\gamma f\left(0, \Lambda_{\bar{B}}[\hat{r}, L]\right) + (1 - \gamma)f\left(\Lambda_G[\hat{r}, R], \Lambda_{\bar{B}}[\hat{r}, R]\right)\right).$

One the one hand, the first terms (multiplied by $(1 - \mu)$) are equal. On the other hand, it is straightforward to see that the second term of $\Pi_{r,\hat{r}}(v, x)$ (multiplied by μ) is greater than that of $\Pi_{l,\hat{r}}(v, x)$. To see this, note that $f\left(\Lambda_G[\hat{r}, R], \Lambda_{\bar{B}}[\hat{r}, R]\right) > f\left(0, \Lambda_{\bar{B}}[\hat{r}, L]\right)$ because $\Lambda_G[\hat{r}, R] > 0$ and by lemma 10 $\Lambda_{\bar{B}}[\hat{r}, R] > \Lambda_{\bar{B}}[\hat{r}, L]$. In addition, f is increasing in both arguments and $\gamma > \frac{1}{2}$.

- Second, $\Pi_{l,\hat{l}}(v, x) > \Pi_{r,\hat{l}}(v, x)$ is proved.

By (3), these functions can be written as
$\Pi_{l,\hat{l}}(v, x) = (1 - \mu)\Pi^0_{l,\hat{l}}(v, x) + \mu\Pi^1_{l,\hat{l}}(v, x)$
$\Pi_{r,\hat{l}}(v, x) = (1 - \mu)\Pi^0_{r,\hat{l}}(v, x) + \mu\Pi^1_{r,\hat{l}}(v, x)$
and in terms of the principal's beliefs by (12), (13), (16), and (17),
$\Pi_{l,\hat{l}}(v, x) = (1 - \mu)f\left(\Lambda_G[\hat{l}, 0], 1\right) + \mu\left(\gamma f\left(\Lambda_G[\hat{l}, L], 1\right) + (1 - \gamma)f(0, 1)\right)$
$\Pi_{r,\hat{l}}(v, x) = (1 - \mu)f\left(\Lambda_G[\hat{l}, 0], 1\right) + \mu\left(\gamma f(0, 1) + (1 - \gamma)f\left(\Lambda_G[\hat{l}, L], 1\right)\right)$
As $\gamma > \frac{1}{2}$, it is straightforward to show that $\Pi_{l,\hat{l}}(v, x) > \Pi_{r,\hat{l}}(v, x)$$\blacksquare$

Corollary 12 $r(v, x) < l(v, x)$ *if* $\mu > 0$.

Proof
As
$l(v, x) = \Pi_{l,\hat{l}}(v, x) - \Pi_{l,\hat{r}}(v, x)$
$r(v, x) = \Pi_{r,\hat{l}}(v, x) - \Pi_{r,\hat{r}}(v, x)$
by lemma 11, $\Pi_{l,\hat{l}}(v, x) > \Pi_{r,\hat{l}}(v, x)$ and $\Pi_{l,\hat{r}}(v, x) < \Pi_{r,\hat{r}}(v, x)$. Therefore, the corollary is proved.$\blacksquare$
It is followed from the previous result that:

Remark 3
If $l(v, x) < 0$, *then* $r(v, x) < 0$
If $r(v, x) > 0$, *then* $l(v, x) > 0$

The following lemma excludes equilibria in fully mixed strategies:

Lemma 13 *There cannot exists an equilibrium in which* $v^* \neq 0$ *and* $x^* \neq 0$ *if* $\mu > 0$.

Proof

Let us assume that there exists an equilibria in which $v^* \neq 0$ and $x^* \neq 0$. Then, $r(v^*, x^*) = 0 = l(v^*, x^*)$, however, that is not possible by corollary 12 and we have a contradiction.■

Remark 4 *Therefore, the strategy in equilibrium has to be either* $(v^*, 0)$ *or* $(0, x^*)$

Lemma 14 *The strategy in equilibrium cannot be* $(0, x^*)$ *with* $x^* > 0$.

Proof

If $v^* = 0$ and $x^* > 0$, then $(v^* - x^*)\lambda_N < \lambda_B$ holds, which, by lemmas 8 and 9, implies that $\Lambda_G[\hat{l}, 0] > \Lambda_G[\hat{r}, 0]$ and $\Lambda_G[\hat{l}, L] > \Lambda_G[\hat{r}, R]$. This, in turn, implies that $l(0, x) > 0$ for all x as it is proved below. Finally, if $l(0, x) > 0$ for all x, then $x^* = 0$ (see remark 1) and the lemma is proved by contradiction.

To prove that $l(0, x) > 0$ if $\Lambda_G[\hat{l}, 0] > \Lambda_G[\hat{r}, 0]$ and $\Lambda_G[\hat{l}, L] > \Lambda_G[\hat{r}, R]$, the function $l(0, x)$ is written in terms of principal's beliefs.

$$\begin{aligned} l(0,x) &= \Pi_{l,\hat{l}}(0,x) - \Pi_{l,\hat{r}}(0,x) \\ &= (1-\mu) f\left(\Lambda_G[\hat{l},0],1\right) + \mu\left(\gamma f\left(\Lambda_G[\hat{l},L],1\right) + (1-\gamma) f\left(0,1\right)\right) \\ &\quad - \left((1-\mu) f\left(\Lambda_G[\hat{r},0],\Lambda_{\bar{B}}[\hat{r},0]\right) + \mu\left(\gamma f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right) + (1-\gamma) f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right)\right)\right) \end{aligned}$$

Clearly $f\left(\Lambda_G[\hat{l},0],1\right) > f\left(\Lambda_G[\hat{r},0],\Lambda_{\bar{B}}[\hat{r},0]\right)$ because $\Lambda_G[\hat{l},0] > \Lambda_G[\hat{r},0]$.

Let us prove that $\gamma f\left(\Lambda_G[\hat{l},L],1\right) + (1-\gamma) f\left(0,1\right) > \gamma f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right) + (1-\gamma) f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right)$. That expression can be rewritten as:

$$\gamma f\left(\Lambda_G[\hat{l},L],1\right) - \gamma f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right) > (1-\gamma) f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right) - (1-\gamma) f\left(0,1\right)$$

$$\Longleftrightarrow \tfrac{\gamma}{(1-\gamma)}\left(f\left(\Lambda_G[\hat{l},L],1\right) - f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right)\right) > f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right) - f\left(0,1\right)$$

The previous expression always holds because, on the one hand, $\frac{\gamma}{(1-\gamma)}$ is positive and increasing in γ, thus $\frac{\gamma}{(1-\gamma)}$ takes the minimum value if $\gamma = \frac{1}{2}$, which is 1. On the other hand, as it is shown above $\Lambda_G[\hat{l},L] > \Lambda_G[\hat{r},R]$, and this implies that $f\left(\Lambda_G[\hat{l},L],1\right) > f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right)$. In addition, $f\left(0,1\right) > f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right)$. Consequently, $f\left(\Lambda_G[\hat{l},L],1\right) - f\left(0,\Lambda_{\bar{B}}[\hat{r},L]\right) > f\left(\Lambda_G[\hat{r},R],\Lambda_{\bar{B}}[\hat{r},R]\right) - f\left(0,1\right)$.■

Therefore, with $\mu \in (0,1]$, in equilibrium $x^* = 0$, and this is true for any payoff function, $f(\Lambda_G[a,X],\Lambda_{\bar{B}}[a,X])$, increasing in both arguments. The equilibrium strategy always has the form $(v^*, 0)$ and proposition 2 is proved■

Remark 5 *In the information set, in which signal* l *is received, the normal expert always tells the truth and makes* $\hat{l}$ *in equilibrium if* $\mu > 0$. *Henceforth, we focus on the information set in which the expert receives signal* r *and on the auxiliary function* $r(v,x) = \Pi_{r,\hat{l}}(v,x) - \Pi_{r,\hat{r}}(v,x)$, *which can be rewritten as* $r(v) = \Pi_{r,\hat{l}}(v) - \Pi_{r,\hat{r}}(v)$. *Thus, if* $r(v^*) = 0$ *with* $v^* \in (0,1)$, *then we have* $(v^*, 0)$ *in equilibrium. If* $r(v) > 0$ *for all* v, *then* $v^* = 1$ *in equilibrium. If* $r(v) < 0$ *for all* v, *then* $v^* = 0$ *in equilibrium.*

Hereafter, the x is omitted both in auxiliary functions and payoffs functions wherever it is not confusing to do so.

A.3.3 Proof of Proposition 3.

By proposition 2, if $\mu > 0$, then in equilibrium $x^* = 0$. With $x = 0$, the function 6 can be rewritten as:

$$r(v) = (1-\mu) r^0(v) + \mu r^1(v) \tag{21}$$

Thus, if $\mu \simeq 0$ then $r(v) \simeq r^0(v)$.

From (8), (12), (14),

$$r^0(v) = \Pi^0_{r,\hat{l}}(v) - \Pi^0_{r,\hat{r}}(v) = f\left(\Lambda_G[\hat{l},0],1\right) - f\left(\Lambda_G[\hat{r},0],\Lambda_{\bar{B}}[\hat{r},0]\right)$$

As if $v = 0$ and $x = 0$, then the condition $(v - x)\lambda_N < \lambda_B$ holds and by lemma 9, $\Lambda_G[\hat{l}, 0] > \Lambda_G[\hat{r}, 0]$, so that $f\left(\Lambda_G[\hat{l}, 0], 1\right) > f\left(\Lambda_G[\hat{r}, 0], \Lambda_{\bar{B}}[\hat{r}, 0]\right)$ and $r^0(v = 0) > 0$. Consequently, if μ is close enough to zero, then $r(v = 0) > 0$, and there cannot be an equilibrium in which $v = 0$. The normal expert lies with some probability in equilibrium.

The expression (20) from the proof of proposition 1 can be rewritten with $x = 0$ as:

$$r^0(v) \leq 0 \Longleftrightarrow \lambda_B\left(\lambda_G - \lambda_B + 1\right) \leq \left(\lambda_G - \lambda_B\right)\left(1 - \lambda_G - \lambda_B\right) v$$

Therefore, if $\lambda_G \leq \lambda_B$, then $r^0(v) > 0$, and if μ is close enough to zero, then $r(v) = (1-\mu)r^0(v) + \mu r^1(v) > 0$. In that case, in equilibrium always $v^* = 1$ and there is not informative equilibrium.

However, if $\lambda_G > \lambda_B$ the function $r^0(v)$ is positive for $v < v^0 = \frac{\lambda_B(\lambda_G - \lambda_B + 1)}{(\lambda_G - \lambda_B)(1 - \lambda_G - \lambda_B)}$ and negative for $v > v^0$. If in addition $v^0 < 1$, then $r^0(v)$ has only one root in the open interval $v \in (0, 1)$. Consequently, if μ is close enough to zero, then $r(v) = (1-\mu)r^0(v) + \mu r^1(v)$ will also have only one root[11]. Let $\hat{v}$ be that root, consequently, the closer μ to 0, the closer $\hat{v}$ to v^0. Therefore, in equilibrium the normal expert will tell the truth with a certain probability ($\simeq (1 - v^0)$) if $\mu \simeq 0$, $\lambda_G > \lambda_B$ and $v^0 < 1$. In this case, the only strategy profile in equilibrium will be $(v^* = \hat{v}, x^* = 0)$.

To end the proof, Claim 7 shows that with $\lambda_G > \lambda_B$, $v^0 \leq 1 \Leftrightarrow \lambda_B \leq \lambda_B^{**} = \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right)$, and it is straightforward to show that $\lambda_B^{**} = \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right) < \lambda_G$ ■

A.3.4 Proof of Proposition 4.

By (21), if $\mu = 1$, then $r(v) = r^1(v)$, thus, $r^1(v) > 0 \iff r(v) > 0$. Therefore, if $r^1(v) > 0$, in equilibrium $v^* = 1$.

In the following, the condition $r^1(v) > 0$ is expressed in terms of the priors:

From (9), (13), (15), Table 1, and Table 2, and with $x = 0$,

$$\begin{aligned}
r^1(v) &= \Pi^1_{r,\hat{l}}(v) - \Pi^1_{r,\hat{r}}(v) \\
&= \left(\gamma f(0,1) + (1-\gamma) f\left(\Lambda_G[\hat{l}, L], 1\right)\right) - \left(\gamma f\left(\Lambda_G[\hat{r}, R], \Lambda_{\bar{B}}[\hat{r}, R]\right) + (1-\gamma) f\left(0, \Lambda_{\bar{B}}[\hat{r}, L]\right)\right) \\
&= \left(\gamma + (1-\gamma)\left(\Lambda_G[\hat{l}, L] + 1\right)\right) - \left(\gamma\left(\Lambda_G[\hat{r}, R] + \Lambda_{\bar{B}}[\hat{r}, R]\right) + (1-\gamma)\Lambda_{\bar{B}}[\hat{r}, L]\right) \\
&= \left(\gamma + (1-\gamma)\left(\tfrac{\lambda_G}{\lambda_G + (\gamma + (1-\gamma)v)\lambda_N} + 1\right)\right) \\
&\quad - \left(\gamma\left(\tfrac{\lambda_G}{\lambda_B + \lambda_G + (\gamma(1-v))\lambda_N} + \left(1 - \tfrac{\lambda_B}{\lambda_B + \lambda_G + (\gamma(1-v))\lambda_N}\right)\right) + (1-\gamma)\left(1 - \tfrac{\lambda_B}{\lambda_B + ((1-\gamma)(1-v))\lambda_N}\right)\right) \\
&= \left(1 + (1-\gamma)\left(\tfrac{\lambda_G}{\lambda_G + (\gamma + (1-\gamma)v)\lambda_N}\right)\right) \\
&\quad - \left(\gamma\left(\tfrac{\lambda_G - \lambda_B}{\lambda_B + \lambda_G + (\gamma(1-v))\lambda_N} + 1\right) + (1-\gamma)\left(1 - \tfrac{\lambda_B}{\lambda_B + ((1-\gamma)(1-v))\lambda_N}\right)\right) \\
&= \left((1-\gamma)\left(\tfrac{\lambda_G}{\lambda_G + (\gamma + (1-\gamma)v)\lambda_N}\right)\right) \\
&\quad - \left(\gamma\left(\tfrac{\lambda_G - \lambda_B}{\lambda_B + \lambda_G + (\gamma(1-v))\lambda_N}\right) - (1-\gamma)\left(\tfrac{\lambda_B}{\lambda_B + ((1-\gamma)(1-v))\lambda_N}\right)\right) \\
&= \tfrac{(1-\gamma)\lambda_G}{\lambda_G + (\gamma + (1-\gamma)v)\lambda_N} + \tfrac{\gamma(\lambda_B - \lambda_G)}{\lambda_B + \lambda_G + (\gamma(1-v))\lambda_N} + \tfrac{(1-\gamma)\lambda_B}{\lambda_B + ((1-\gamma)(1-v))\lambda_N}
\end{aligned}$$

If the above expression is greater than zero, then in equilibrium $v^* = 1$ when $\mu = 1$. This means that the normal expert lies with probability one if he receives signal r.

Clearly, the expression will be positive if $\lambda_B > \lambda_G$. In that case, the normal expert always lies ($v^* = 1$) in equilibrium if $\mu \simeq 0$. However, we search for the parameter values for which the normal expert behaves more efficiently (from the principal point of view) if $\mu \simeq 0$ than if $\mu = 1$. Therefore, we search for the parameter values that make $r^1(v) > 0$ for all v assuming that $\lambda_B < \lambda_B^{**} = \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right)$, where $\lambda_B^{**} < \lambda_G$.

Note that the denominators of the three fractions of $r^1(v)$ are greater than zero, thus,

[11] Both $r^0(v)$ and $r^1(v)$ are continuous in $v \in (0, 1)$. In addition, the function $r^1(v)$ has no more than two roots in the interval $v \in (0, 1)$ as it will be shown in the proof of proposition 4.

$$
\begin{aligned}
& r^1(v) = \tfrac{(1-\gamma)\lambda_G}{\lambda_G+(\gamma+(1-\gamma)v)\lambda_N} + \tfrac{\gamma(\lambda_B-\lambda_G)}{\lambda_B+\lambda_G+(\gamma(1-v))\lambda_N} + \tfrac{(1-\gamma)\lambda_B}{\lambda_B+((1-\gamma)(1-v))\lambda_N} > 0 \\
& \Longleftrightarrow (1-\gamma)\lambda_G \left(\lambda_B + \lambda_G + (\gamma(1-v))\,\lambda_N\right)\left(\lambda_B + ((1-\gamma)(1-v))\,\lambda_N\right) \\
& \quad +\gamma\left(\lambda_B - \lambda_G\right)\left(\lambda_G + (\gamma + (1-\gamma)v)\,\lambda_N\right)\left(\lambda_B + ((1-\gamma)(1-v))\,\lambda_N\right) \\
& \quad +(1-\gamma)\lambda_B\left(\lambda_G + (\gamma + (1-\gamma)v)\,\lambda_N\right)\left(\lambda_B + \lambda_G + (\gamma(1-v))\,\lambda_N\right) > 0
\end{aligned}
$$

The expression above is a second-degree polynomial in v and can be rewritten as:

$$
\begin{aligned}
p(v) \quad = \quad & v^2\left(2\gamma\lambda_N^2\,(1-\gamma)^2\,(\lambda_G-\lambda_B)\right) \\
& +v\left(\lambda_N\,(1-\gamma)\left(-4\gamma^2\lambda_N\lambda_B + 4\gamma^2\lambda_N\lambda_G - 4\gamma\lambda_B\lambda_G + 2\gamma\lambda_N\lambda_B + 2\gamma\lambda_G^2 - 3\gamma\lambda_N\lambda_G + \lambda_B^2 - \lambda_G^2\right)\right) \\
& +\gamma\left(\lambda_G + \gamma\lambda_N\right)\left(\lambda_B - \lambda_N\,(\gamma-1)\right)\left(\lambda_B - \lambda_G\right) - \lambda_G\,(\gamma-1)\left(\lambda_B - \lambda_N\,(\gamma-1)\right)\left(\lambda_B + \lambda_G + \gamma\lambda_N\right) \\
& \quad -\lambda_B\left(\lambda_G + \gamma\lambda_N\right)(\gamma-1)\left(\lambda_B + \lambda_G + \gamma\lambda_N\right)
\end{aligned} \tag{22}
$$

Thus, if $p(v) > 0$, then $r^1(v) > 0$. To facilitate the analysis, $p(v) = av^2 + bv + c$ stands for (22). The first derivative is $p'(v) = 2av + b$ and the second one is $p^{''}(v) = 2a$. Therefore, $p(v)$ is convex in v because $a = \left(2\gamma\lambda_N^2\,(1-\gamma)^2\,(\lambda_G-\lambda_B)\right) > 0$ (it has been assumed that $\lambda_G > \lambda_B$). The first derivative gives the minimum value of $p(v)$: $p'(v_{\min}) = 2av_{\min} + b = 0 \Leftrightarrow v_{\min} = \frac{-b}{2a}$. Consequently, $p(v_{\min}) > 0 \Rightarrow p(v) > 0 \Rightarrow r^1(v) > 0$. Let us determine when $p(v_{\min}) > 0$. Note that, $p(v_{\min}) = a(\frac{-b}{2a})^2 + b(\frac{-b}{2a}) + c = c - \frac{b^2}{4a}$.

From (22),

$$
\begin{aligned}
\frac{b^2}{4a} &= \frac{\left(\lambda_N(1-\gamma)\left(-4\gamma^2\lambda_N\lambda_B+4\gamma^2\lambda_N\lambda_G-4\gamma\lambda_B\lambda_G+2\gamma\lambda_N\lambda_B+2\gamma\lambda_G^2-3\gamma\lambda_N\lambda_G+\lambda_B^2-\lambda_G^2\right)\right)^2}{4\left(2\gamma\lambda_N^2(1-\gamma)^2(\lambda_G-\lambda_B)\right)} \\
&= \frac{\left(-4\gamma^2\lambda_N\lambda_B+4\gamma^2\lambda_N\lambda_G-4\gamma\lambda_B\lambda_G+2\gamma\lambda_N\lambda_B+2\gamma\lambda_G^2-3\gamma\lambda_N\lambda_G+\lambda_B^2-\lambda_G^2\right)^2}{8\gamma(\lambda_G-\lambda_B)}
\end{aligned}
$$

and $c - \frac{b^2}{4a}$ is:

$$
\begin{aligned}
\Big(& (\gamma(\lambda_G+\gamma\lambda_N)(\lambda_B-\lambda_N(\gamma-1))(\lambda_B-\lambda_G)-\lambda_G(\gamma-1)(\lambda_B-\lambda_N(\gamma-1))(\lambda_B+\lambda_G+\gamma\lambda_N)-\lambda_B(\lambda_G+\gamma\lambda_N)(\gamma-1)(\lambda_B+\lambda_G+\gamma\lambda_N))(8\gamma(\lambda_G-\lambda_B)) \\
& -\left(-4\gamma^2\lambda_N\lambda_B+4\gamma^2\lambda_N\lambda_G-4\gamma\lambda_B\lambda_G+2\gamma\lambda_N\lambda_B+2\gamma\lambda_G^2-3\gamma\lambda_N\lambda_G+\lambda_B^2-\lambda_G^2\right)^2 \Big) \frac{1}{8\gamma(\lambda_G-\lambda_B)}
\end{aligned}
$$

The denominator is positive because $\lambda_G > \lambda_B$. The sign of the numerator determines the sign of that expression. Although tedious, it is straightforward to expand and simplify the numerator of the above expression, the result is:

$$
\begin{aligned}
& 8\gamma^2\lambda_B^3\lambda_G - 4\gamma^2\lambda_B^2\lambda_N^2 - 8\gamma^2\lambda_B\lambda_G^3 + 4\gamma^2\lambda_B\lambda_G\lambda_N^2 - 4\gamma^2\lambda_G^4 - 4\gamma^2\lambda_G^3\lambda_N - \gamma^2\lambda_G^2\lambda_N^2 - 8\gamma\lambda_B^3\lambda_G - 4\gamma\lambda_B^3\lambda_N \\
& -4\gamma\lambda_B^2\lambda_G^2 - 2\gamma\lambda_B^2\lambda_G\lambda_N + 8\gamma\lambda_B\lambda_G^3 + 4\gamma\lambda_B\lambda_G^2\lambda_N + 4\gamma\lambda_G^4 + 2\gamma\lambda_G^3\lambda_N - \lambda_B^4 + 2\lambda_B^2\lambda_G^2 - \lambda_G^4
\end{aligned}
$$

It can be rewritten as a polynomial in γ:

$$
\begin{aligned}
pol(\gamma) = {} & \gamma^2\left(8\lambda_B^3\lambda_G - 4\lambda_B^2\lambda_N^2 - 8\lambda_B\lambda_G^3 + 4\lambda_B\lambda_G\lambda_N^2 - 4\lambda_G^4 - 4\lambda_G^3\lambda_N - \lambda_G^2\lambda_N^2\right) \\
& +\gamma\left(8\lambda_B\lambda_G^3 - 4\lambda_N\lambda_B^3 - 4\lambda_B^2\lambda_G^2 - 2\lambda_N\lambda_B^2\lambda_G - 8\lambda_B^3\lambda_G + 4\lambda_N\lambda_B\lambda_G^2 + 4\lambda_G^4 + 2\lambda_N\lambda_G^3\right) \\
& +\left(2\lambda_B^2\lambda_G^2 - \lambda_B^4 - \lambda_G^4\right)
\end{aligned}
$$

Simplifying the coefficients of the polynomial,

$$
\begin{aligned}
pol(\gamma) \quad = \quad & \gamma^2\left(8\lambda_B^3\lambda_G - 4\lambda_B^2\lambda_N^2 - 8\lambda_B\lambda_G^3 + 4\lambda_B\lambda_G\lambda_N^2 - 4\lambda_G^4 - 4\lambda_G^3\lambda_N - \lambda_G^2\lambda_N^2\right) \\
& +\gamma\left(2\left(\lambda_G - \lambda_B\right)\left(2\lambda_B + \lambda_G\right)\left(\lambda_B + \lambda_G\right)\left(2\lambda_G + \lambda_N\right)\right) \\
& -\left(\lambda_B^2 - \lambda_G^2\right)^2
\end{aligned} \tag{23}
$$

Thus, $pol(\gamma) > 0 \Leftrightarrow p(v_{\min}) > 0 \Rightarrow p(v) > 0 \Leftrightarrow r^1(v) > 0 \Leftrightarrow r(v) > 0$. Let us determine when $pol(\gamma)$ is greater than zero.

The polynomial $pol(\gamma)$ is concave in γ because the coefficient of γ^2 is negative as it is proved below:

$$
\begin{aligned}
& 8\lambda_B^3\lambda_G + 4\lambda_B\lambda_G\lambda_N^2 - 4\lambda_G^3\lambda_N - 4\lambda_B^2\lambda_N^2 - 8\lambda_B\lambda_G^3 - 4\lambda_G^4 - \lambda_G^2\lambda_N^2 \\
& = \lambda_N^2\left(4\lambda_B\lambda_G - 4\lambda_B^2 - \lambda_G^2\right) - 4\lambda_G^3\lambda_N + 8\lambda_B^3\lambda_G - 8\lambda_B\lambda_G^3 - 4\lambda_G^4 \\
& = \lambda_N^2\left(-\left(2\lambda_B - \lambda_G\right)^2\right) - 4\lambda_G^3\lambda_N + 4\lambda_G\left(2\lambda_B^3 - 2\lambda_B\lambda_G^2 - \lambda_G^2\right) \\
& = -\lambda_N^2\left(2\lambda_B - \lambda_G\right)^2 - 4\lambda_G^3\lambda_N + 4\lambda_G\left(2\lambda_B^3 - \lambda_G^2\left(2\lambda_B + 1\right)\right) < 0 \\
& \Longleftarrow 2\lambda_B^3 - \lambda_G^2\left(2\lambda_B + 1\right) < 0 \\
& \Longleftrightarrow \lambda_B^2 2\lambda_B - \lambda_G^2\left(2\lambda_B + 1\right) < 0
\end{aligned}
$$

Clearly, the expression above is negative if $\lambda_G > \lambda_B$, consequently $pol(\gamma)$ is concave if $\lambda_G > \lambda_B$.

On the other hand, the first derivative of $pol(\gamma)$ in $\gamma = \frac{1}{2}$ is negative:
$$\left.\frac{dpol(\gamma)}{d\gamma}\right|_{\gamma=\frac{1}{2}} = \left(8\lambda_B^3\lambda_G - 4\lambda_B^2\lambda_N^2 - 8\lambda_B\lambda_G^3 + 4\lambda_B\lambda_G\lambda_N^2 - 4\lambda_G^4 - 4\lambda_G^3\lambda_N - \lambda_G^2\lambda_N^2\right)$$
$$+ \left(2\left(\lambda_G - \lambda_B\right)\left(2\lambda_B + \lambda_G\right)\left(\lambda_B + \lambda_G\right)\left(2\lambda_G + \lambda_N\right)\right)$$
$$= -4\lambda_B^3\lambda_N - 4\lambda_B^2\lambda_G^2 - 2\lambda_B^2\lambda_G\lambda_N - 4\lambda_B^2\lambda_N^2 + 4\lambda_B\lambda_G^2\lambda_N + 4\lambda_B\lambda_G\lambda_N^2 - 2\lambda_G^3\lambda_N - \lambda_G^2\lambda_N^2$$
$$= -4\lambda_B^2\lambda_G^2 - 2\left(2\lambda_B^3 + \lambda_B^2\lambda_G - 2\lambda_B\lambda_G^2 + \lambda_G^3\right)\lambda_N - \left(-2\lambda_B + \lambda_G\right)^2\lambda_N^2 < 0$$
$$\Longleftarrow\ 2\lambda_B^3 + \lambda_B^2\lambda_G - 2\lambda_B\lambda_G^2 + \lambda_G^3 > 0$$
$$\Longleftarrow\ 2\lambda_B^3 + \lambda_G\left(\lambda_B^2 - 2\lambda_B\lambda_G + \lambda_G^2\right) > 0$$
$$\Longleftarrow\ 2\lambda_B^3 + \lambda_G\left(\lambda_B - \lambda_G\right)^2 > 0$$
Clearly, the last expression is greater than zero and consequently $\left.\frac{dpol(\gamma)}{d\gamma}\right|_{\gamma=\frac{1}{2}} < 0$. Thus, as $pol(\gamma)$ is concave for $\gamma \in (\frac{1}{2}, 1)$ and decreasing in $\gamma = \frac{1}{2}$, The polynomial $pol(\gamma)$ is necessarily decreasing from $\gamma = \frac{1}{2}$ to $\gamma = 1$. Therefore, there are only two possibilities. One is that $pol(\gamma)$ is negative for $\gamma \in \left(\frac{1}{2}, 1\right)$. The second is that $pol(\gamma)$ is positive for $\gamma \in (\frac{1}{2}, \tilde{\gamma})$ and negative if $\gamma > \tilde{\gamma}$, where $\tilde{\gamma}$ is the greater real root of $pol(\gamma) = 0$ (note that $pol(\gamma)$ is a second-degree polynomial).

Let us calculate that $\tilde{\gamma}$. Let $pol(\gamma) = A\gamma^2 + B\gamma + C$ stands for (23) As proved above, $A < 0$. Clearly, $B > 0$ because $\lambda_G > \lambda_B$ and finally $C < 0$. Therefore, the roots are $\frac{-B+\sqrt{B^2-4AC}}{2A} < \frac{-B-\sqrt{B^2-4AC}}{2A}$, and $\tilde{\gamma}$ has to be the greater root because $pol(\gamma)$ is decreasing in $\tilde{\gamma}$, thus, $\tilde{\gamma} = \frac{-B-\sqrt{B^2-4AC}}{2A}$.

It is first calculated the discriminant of that root,
$$B^2 - 4AC$$
$$= \left(2\left(\lambda_G - \lambda_B\right)\left(2\lambda_B + \lambda_G\right)\left(\lambda_B + \lambda_G\right)\left(2\lambda_G + \lambda_N\right)\right)^2$$
$$-4\left(8\lambda_B^3\lambda_G - 4\lambda_B^2\lambda_N^2 - 8\lambda_B\lambda_G^3 + 4\lambda_B\lambda_G\lambda_N^2 - 4\lambda_G^4 - 4\lambda_G^3\lambda_N - \lambda_G^2\lambda_N^2\right)\left(-\left(\lambda_B^2 - \lambda_G^2\right)^2\right)$$
$$= 32\lambda_B^7\lambda_G + 64\lambda_B^6\lambda_G^2 + 64\lambda_B^6\lambda_G\lambda_N - 32\lambda_B^5\lambda_G^3 + 64\lambda_B^5\lambda_G^2\lambda_N + 32\lambda_B^5\lambda_G\lambda_N^2 - 128\lambda_B^4\lambda_G^4$$
$$-128\lambda_B^4\lambda_G^3\lambda_N - 32\lambda_B^3\lambda_G^5 - 128\lambda_B^3\lambda_G^4\lambda_N - 64\lambda_B^3\lambda_G^3\lambda_N^2 + 64\lambda_B^2\lambda_G^6 + 64\lambda_B^2\lambda_G^5\lambda_N$$
$$+32\lambda_B\lambda_G^7 + 64\lambda_B\lambda_G^6\lambda_N + 32\lambda_B\lambda_G^5\lambda_N^2$$
$$= 32\lambda_B\lambda_G\left(\lambda_B^2 - \lambda_G^2\right)^2\left(\lambda_B + \lambda_G + \lambda_N\right)^2$$
$$= 32\lambda_B\lambda_G\left(\lambda_B^2 - \lambda_G^2\right)^2$$
The discriminant is not negative, so there are always real roots.

As $\lambda_G > \lambda_B$, the expression $\sqrt{B^2 - 4AC} = \sqrt{32\lambda_B\lambda_G\left(\lambda_B^2 - \lambda_G^2\right)^2} = -\left(\lambda_B^2 - \lambda_G^2\right)4\sqrt{2\lambda_B\lambda_G}$

Therefore,
$$\tilde{\gamma} = \frac{-B-\sqrt{B^2-4AC}}{2A}$$
$$= \frac{-(2(\lambda_G-\lambda_B)(2\lambda_B+\lambda_G)(\lambda_B+\lambda_G)(2\lambda_G+\lambda_N)) - \left(-\left(\lambda_B^2-\lambda_G^2\right)4\sqrt{2\lambda_B\lambda_G}\right)}{2\left(8\lambda_B^3\lambda_G-4\lambda_B^2\lambda_N^2-8\lambda_B\lambda_G^3+4\lambda_B\lambda_G\lambda_N^2-4\lambda_G^4-4\lambda_G^3\lambda_N-\lambda_G^2\lambda_N^2\right)}$$
$$= \frac{\left(\lambda_B^2-\lambda_G^2\right)\left(2\sqrt{2\lambda_B\lambda_G}+(2\lambda_B+\lambda_G)(2\lambda_G+\lambda_N)\right)}{\left(8\lambda_B^3\lambda_G-4\lambda_B^2\lambda_N^2-8\lambda_B\lambda_G^3+4\lambda_B\lambda_G\lambda_N^2-4\lambda_G^4-4\lambda_G^3\lambda_N-\lambda_G^2\lambda_N^2\right)}$$
Clearly, $\tilde{\gamma} > 0$ with $\lambda_G > \lambda_B$ because the numerator is negative and the denominator is equal to the coefficient A of $pol(\gamma)$ and it has been proved that it is also negative.

Let's simplify the expression,
$$\tilde{\gamma} = \frac{\left(\lambda_B^2-\lambda_G^2\right)\left(2\sqrt{2\lambda_B\lambda_G}+(2\lambda_B+\lambda_G)(2\lambda_G+\lambda_N)\right)}{\left(8\lambda_B^3\lambda_G-4\lambda_B^2\lambda_N^2-8\lambda_B\lambda_G^3+4\lambda_B\lambda_G\lambda_N^2-4\lambda_G^4-4\lambda_G^3\lambda_N-\lambda_G^2\lambda_N^2\right)}$$
$$= \frac{\left(\lambda_B^2-\lambda_G^2\right)\left(2\sqrt{2\lambda_B\lambda_G}+(2\lambda_B+\lambda_G)(2\lambda_G+\lambda_N)\right)}{\lambda_N^2\left(4\lambda_B\lambda_G-4\lambda_B^2-\lambda_G^2\right)-4\lambda_G^3\lambda_N-8\lambda_B\lambda_G^3+8\lambda_B^3\lambda_G-4\lambda_G^4}$$
As $\lambda_N = (1 - \lambda_B - \lambda_G)$,
$$\tilde{\gamma} = \frac{\left(\lambda_B^2-\lambda_G^2\right)\left((2\lambda_B+\lambda_G)(2\lambda_G+(1-\lambda_B-\lambda_G))+2\sqrt{2\lambda_B\lambda_G}\right)}{(1-\lambda_B-\lambda_G)^2\left(4\lambda_B\lambda_G-4\lambda_B^2-\lambda_G^2\right)-4\lambda_G^3(1-\lambda_B-\lambda_G)-8\lambda_B\lambda_G^3+8\lambda_B^3\lambda_G-4\lambda_G^4}$$
The denominator can be rewritten as:
$$(1 - \lambda_B - \lambda_G)^2\left(4\lambda_B\lambda_G - 4\lambda_B^2 - \lambda_G^2\right) - 4\lambda_G^3\left(1 - \lambda_B - \lambda_G\right) - 8\lambda_B\lambda_G^3 + 8\lambda_B^3\lambda_G - 4\lambda_G^4$$
$$= -4\lambda_B^4 + 4\lambda_B^3\lambda_G + 8\lambda_B^3 + 3\lambda_B^2\lambda_G^2 - 4\lambda_B^2 - 2\lambda_B\lambda_G^3 - 6\lambda_B\lambda_G^2 + 4\lambda_B\lambda_G - \lambda_G^4 - 2\lambda_G^3 - \lambda_G^2$$
$$= \left(2\sqrt{2\lambda_B\lambda_G}\right)^2 - \left(\left(2\lambda_B + \lambda_G\right)\left(1 - \lambda_B + \lambda_G\right)\right)^2$$
Therefore,

$\tilde{\gamma} = \left(\lambda_B^2 - \lambda_G^2\right) \frac{2\sqrt{2\lambda_B\lambda_G} - (2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G)}{\left(2\sqrt{2\lambda_B\lambda_G}\right)^2 - ((2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G))^2}$
$= \left(\lambda_B^2 - \lambda_G^2\right) \frac{2\sqrt{2\lambda_B\lambda_G} - (2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G)}{\left(2\sqrt{2\lambda_B\lambda_G} - (2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G)\right)\left(2\sqrt{2\lambda_B\lambda_G} + (2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G)\right)}$
$= \frac{\left(\lambda_B^2 - \lambda_G^2\right)}{2\sqrt{2\lambda_B\lambda_G} - (2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G)}$

Therefore, if $\gamma < \tilde{\gamma}$, then $pol(\gamma) > 0$ which implies that $r^1(v) > 0$, which implies in turn that $r(v) > 0$, which implies finally that in equilibrium $v^* = 1$.

The parameter values for which $\tilde{\gamma}$ is greater than $\frac{1}{2}$ are determined:

$\tilde{\gamma} = \frac{\left(\lambda_B^2 - \lambda_G^2\right)}{2\sqrt{2\lambda_B\lambda_G} - (2\lambda_B+\lambda_G)(1-\lambda_B+\lambda_G)} > \frac{1}{2}$

$\iff \left(\lambda_B^2 - \lambda_G^2\right) - \left(2\sqrt{2\lambda_B\lambda_G} - (2\lambda_B + \lambda_G)(1 - \lambda_B + \lambda_G)\right)\frac{1}{2} < 0$
$\iff 2\lambda_B + \lambda_G - \lambda_G^2 + \lambda_B\lambda_G < 2\sqrt{2}\sqrt{\lambda_B\lambda_G}$
$\iff \left(2\lambda_B + \lambda_G - \lambda_G^2 + \lambda_B\lambda_G\right)^2 < 8\lambda_B\lambda_G$
$\iff \left(2\lambda_B + \lambda_G - \lambda_G^2 + \lambda_B\lambda_G\right)^2 - 8\lambda_B\lambda_G < 0$
$\iff \lambda_B^2\left(\lambda_G^2 + 4\lambda_G + 4\right) + \lambda_B\left(-2\lambda_G^3 - 2\lambda_G^2 - 4\lambda_G\right) + \left(\lambda_G^4 - 2\lambda_G^3 + \lambda_G^2\right) < 0$

As the expression above is convex in λ_B because $\left(\lambda_G^2 + 4\lambda_G + 4\right) > 0$,
and the roots are $\frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 \pm 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2}$,
then $\tilde{\gamma} > \frac{1}{2}$ if $\lambda_B \in \left(\frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 - 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2}, \frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 + 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2}\right)$

The following claims prove that
$\frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 - 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2} < \lambda_B^{**} = \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right) < \frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 + 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2}$.

Claim 15 *For any* λ_G, $\frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 + 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2} > \lambda_B^{**} = \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right)$.

Proof

$\frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 + 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2} > \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right)$
$\iff \frac{1}{4}\left(\frac{8\lambda_G + 4\lambda_G^2 + 4\lambda_G^3 + 8\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3} - (\lambda_G+2)^2(2+\lambda_G) + (\lambda_G+2)^2\sqrt{9\lambda_G^2 - 4\lambda_G + 4}}{(\lambda_G+2)^2}\right) > 0$
$\iff 8\lambda_G + 4\lambda_G^2 + 4\lambda_G^3 - (\lambda_G + 2)^2(2 + \lambda_G) + 8\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3} + (\lambda_G + 2)^2\sqrt{9\lambda_G^2 - 4\lambda_G + 4} > 0$
$\iff 3\lambda_G^3 - 2\lambda_G^2 - 4\lambda_G - 8 + 8\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3} + (\lambda_G + 2)^2\sqrt{9\lambda_G^2 - 4\lambda_G + 4} > 0$
$\Longleftarrow -2\lambda_G^2 - 4\lambda_G - 8 + (\lambda_G + 2)^2\sqrt{9\lambda_G^2 - 4\lambda_G + 4} > 0$
$\iff \sqrt{9\lambda_G^2 - 4\lambda_G + 4} > \frac{2\lambda_G^2 + 4\lambda_G + 8}{(\lambda_G+2)^2}$
$\iff 9\lambda_G^2 - 4\lambda_G + 4 - \left(\frac{2\lambda_G^2 + 4\lambda_G + 8}{(\lambda_G+2)^2}\right)^2 > 0$
$\iff \frac{9\lambda_G^6 + 68\lambda_G^5 + 184\lambda_G^4 + 208\lambda_G^3 + 64\lambda_G^2}{(\lambda_G+2)^4} > 0 \blacksquare$

Claim 16 *For any* λ_G, $\frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 - 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2} < \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right)$.

Proof

$\frac{2\lambda_G + \lambda_G^2 + \lambda_G^3 - 2\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3}}{(\lambda_G+2)^2} < \frac{1}{4}\left(2 + \lambda_G - \sqrt{9\lambda_G^2 - 4\lambda_G + 4}\right)$
$\iff \frac{1}{4}\left(\frac{8\lambda_G + 4\lambda_G^2 + 4\lambda_G^3 - 8\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3} - (\lambda_G+2)^2(2+\lambda_G) + (\lambda_G+2)^2\sqrt{9\lambda_G^2 - 4\lambda_G + 4}}{(\lambda_G+2)^2}\right) < 0$
$\iff 8\lambda_G + 4\lambda_G^2 + 4\lambda_G^3 - (\lambda_G + 2)^2(2 + \lambda_G) - 8\sqrt{2}\sqrt{\lambda_G^4 + \lambda_G^3} + (\lambda_G + 2)^2\sqrt{9\lambda_G^2 - 4\lambda_G + 4} < 0$
$\iff 3\lambda_G^3 - 2\lambda_G^2 - 4\lambda_G - 8 - \sqrt{\left(8\sqrt{2}\right)^2\left(\lambda_G^4 + \lambda_G^3\right)} + \sqrt{(\lambda_G + 2)\left(9\lambda_G^2 - 4\lambda_G + 4\right)} < 0$
$\iff 3\lambda_G^3 - 2\lambda_G^2 - 4\lambda_G - 8 - \sqrt{128\lambda_G^4 + 128\lambda_G^3} + \sqrt{9\lambda_G^3 + 14\lambda_G^2 - 4\lambda_G + 8} < 0$
$\Longleftarrow 3\lambda_G^3 - 2\lambda_G^2 - 4\lambda_G - 8 - \sqrt{128\lambda_G^4 + 128\lambda_G^3} + \left(9\lambda_G^3 + 14\lambda_G^2 - 4\lambda_G + 8\right) < 0$

$$\iff 3\lambda_G^3 - 2\lambda_G^2 - 4\lambda_G - 8 + 9\lambda_G^3 + 14\lambda_G^2 - 4\lambda_G + 8 < \sqrt{128\lambda_G^4 + 128\lambda_G^3}$$
$$\iff 4\lambda_G\left(3\lambda_G + 3\lambda_G^2 - 2\right) < \sqrt{128\lambda_G^4 + 128\lambda_G^3}$$
$$\iff 3\lambda_G + 3\lambda_G^2 - 2 < \sqrt{\tfrac{128\lambda_G^4 + 128\lambda_G^3}{(4\lambda_G)^2}}$$
$$\iff 3\lambda_G + 3\lambda_G^2 - 2 < \sqrt{8\lambda_G^2 + 8\lambda_G}$$

It is straightforward to show that the left side of the inequality is convex in λ_G and the right side is concave. In addition, the left side is lower than the right side in $\lambda_G = 0$, and in $\lambda_G = 1$, they are equal. Therefore, the inequality holds.■

Therefore, with $\mu = 1$, if $\lambda_B \in (\lambda_B^*, \lambda_B^{**})$ and $\gamma \in (\frac{1}{2}, \tilde{\gamma})$, then in equilibrium always $v^* = 1$.■

A.4 Figure

In figure 1, the different areas and thresholds are shown. In the area $\Psi = \{(\lambda_G, \lambda_B) / \lambda_B \in (\lambda_B^*, \lambda_B^{**})\}$, full transparency damages the principal with $\gamma \in (\frac{1}{2}, \tilde{\gamma})$.

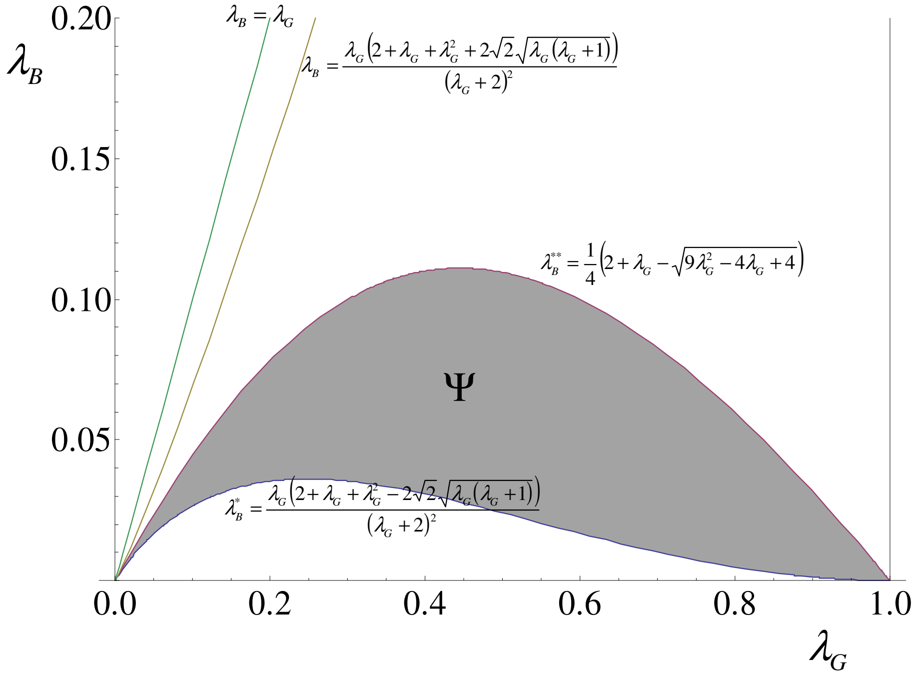

Figure 1: The grey area is the set Ψ.

B Appendix B

In this appendix, first the expert utility function considered is motivated in the following remark, and after that, a model as the previously considered but without a bad type of expert is analyzed.

Remark 6 *Let $u(\theta)$ the utility the expert obtains if principal believes that expert's type is θ. It is assumed that $u(G) > u(N) > u(B) = 0$, i.e. the utility of being considered a bad type is normalized to zero. The expected payoff is,*

$$\Lambda_G[a, X]u(G) + \Lambda_N[a, X]u(N) + \Lambda_B[a, X]u(B)$$
$$= \Lambda_G[a, X]u(G) + \Lambda_N[a, X]u(N)$$
$$= \Lambda_G[a, X]u(G) + (1 - \Lambda_G[a, X] - \Lambda_B[a, X])u(N)$$
$$= \Lambda_G[a, X]\left(u(G) - u(N)\right) + (1 - \Lambda_B[a, X])u(N)$$

$= \Lambda_G[a,X]\,(u(G)-u(N)) + \Lambda_{\bar{B}}[a,X]u(N)$

Let $\alpha = (u(G)-u(N))$ *and* $\beta = u(N)$. *The expression above will be:*

$\alpha\Lambda_G[a,X] + \beta\Lambda_{\bar{B}}[a,X]$

The expected payoff is a linear combination of the principal belief that the expert is good and is not bad. To make the analysis simpler it is considered $\alpha = \beta$, *which means that* $u(G) = 2u(N)$.

B.1 A model without a bad type of expert

There are only a good type and a normal type of expert with the same characteristics as above. Thus, the normal expert seeks to maximize the function $f(\Lambda_G[a,X]) = \Lambda_G[a,X]$, i.e., the probability the principal place of him being the good type. The beliefs of the principal will be as the table 1 but with $\lambda_B = 0$:

Good Reputation			
	0	R	L
$\hat{r}$	$\Lambda_G[\hat{r},0] = \frac{\lambda_G}{\lambda_G+(1-v+x)\lambda_N}$	$\Lambda_G[\hat{r},R] = \frac{\lambda_G}{\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}$	$\Lambda_G[\hat{r},L] = 0$
$\hat{l}$	$\Lambda_G[\hat{l},0] = \frac{\lambda_G}{\lambda_G+(1+v-x)\lambda_N}$	$\Lambda_G[\hat{l},R] = 0$	$\Lambda_G[\hat{l},L] = \frac{\lambda_G}{\lambda_G+(\gamma(1-x)+(1-\gamma)v)\lambda_N}$

The auxiliary functions for this case are

$$\begin{aligned} r(v,x) &= (1-\mu)\left(\Lambda_G[\hat{l},0] - \Lambda_G[\hat{r},0]\right) + \mu\left((1-\gamma)\Lambda_G[\hat{l},L] - \gamma\Lambda_G[\hat{r},R]\right) \\ l(v,x) &= (1-\mu)\left(\Lambda_G[\hat{l},0] - \Lambda_G[\hat{r},0]\right) + \mu\left(\gamma\Lambda_G[\hat{l},L] - (1-\gamma)\Lambda_G[\hat{r},R]\right) \end{aligned}$$

Proposition 17 *If* $\mu = 0$, *then in equilibrium always* $v^* = x^*$. *If* $\mu > 0$, *then in equilibrium always* $v^* = x^* = 0$.

Proof

It is straightforward to prove that

$\left(\Lambda_G[\hat{l},0] = \frac{\lambda_G}{\lambda_G+(1+v-x)\lambda_N} > \Lambda_G[\hat{r},0] = \frac{\lambda_G}{\lambda_G+(1-v+x)\lambda_N}\right) \iff x > v$

$\left(\Lambda_G[\hat{l},L] = \frac{\lambda_G}{\lambda_G+(\gamma(1-x)+(1-\gamma)v)\lambda_N} > \Lambda_G[\hat{r},R] = \frac{\lambda_G}{\lambda_G+((1-\gamma)x+\gamma(1-v))\lambda_N}\right) \iff x > v$

With $\mu = 0$, clearly,

$r(v,x) = l(v,x) = \left(\Lambda_G[\hat{l},0] - \Lambda_G[\hat{r},0]\right)$

In equilibrium only $v^* = x^*$ can occur. Let us argue by contradiction

$x^* > v^* \iff \Lambda^*_G[\hat{l},0] > \Lambda^*_G[\hat{r},0] \iff r(v^*,x^*) = l(v^*,x^*) > 0 \iff x^* = 0$ and $v^* = 1$, a contradiction.

$x^* < v \iff \Lambda^*_G[\hat{l},0] < \Lambda^*_G[\hat{r},0] \iff r(v^*,x^*) = l(v^*,x^*) < 0 \iff x^* = 1$ and $v^* = 0$, a contradiction.

On the other hand, if $\mu > 0$, then in equilibrium only $v^* = x^* = 0$ can occur. Let us argue by contradiction.

$$\begin{aligned} x^* > v^* &\iff \left(\Lambda^*_G[\hat{l},0] > \Lambda^*_G[\hat{r},0] \text{ and } \Lambda^*_G[\hat{l},L] > \Lambda^*_G[\hat{r},R]\right) \\ &\iff l(x^*,v^*) > 0 \iff x^* = 0 \text{ a contradiction because } v^* \geq 0. \\ x^* < v^* &\iff \left(\Lambda^*_G[\hat{l},0] < \Lambda^*_G[\hat{r},0] \text{ and } \Lambda^*_G[\hat{l},L] < \Lambda^*_G[\hat{r},R]\right) \\ &\iff r(x^*,v^*) < 0 \iff v^* = 0 \text{ a contradiction because } x^* \geq 0. \end{aligned}$$

Therefore, v^* has to be equal to x^*. That $v^* = x^* = 0$, it is now proved.

If $v^* = x^*$, then $\Lambda^*_G[\hat{l},0] = \Lambda^*_G[\hat{r},0]$ and $\Lambda^*_G[\hat{l},L] = \Lambda^*_G[\hat{r},R]$. Consequently,

$$\begin{aligned} r(v^*,x^*) &= (1-\mu)\left(\Lambda^*_G[\hat{l},0] - \Lambda^*_G[\hat{r},0]\right) + \mu\left((1-\gamma)\Lambda^*_G[\hat{l},L] - \gamma\Lambda^*_G[\hat{r},R]\right) \\ &= \mu\left((1-\gamma)\Lambda^*_G[\hat{l},L] - \gamma\Lambda^*_G[\hat{r},R]\right) < 0 \text{ because } \gamma > \tfrac{1}{2}, \text{ thus } v^* \text{ has to be equal to } 0. \blacksquare \end{aligned}$$

REFERENCES

Canes-Wrone, Brandice, Michael Herron, and Kenneth Shotts (2001), "Leadership and pandering: a theory of executive policymaking." *American Journal of Political Science*, 45 (3), 532–550.

Cremer, Jacques (1995), "Arm's Length Relationships."*Quarterly Journal of Economics*, 110(2), 275-95.

Dewatripont, Mathias, Ian Jewitt and Jean Tirole (1999), "The Economics of Career Concerns, Part I: Comparing Information Structures." *Review of Economic Studies*, 66(1), 183-98.

Fox, Justin and Richard Van Weelden (2012), "Costly Transparency." *Journal of Public Economics*, 96 (1): 142-150.

Holmstrom, Bengt (1999), "Managerial Incentive Problems: A Dynamic Perspective." *Review of Economic Studies*, 66(1), 169-82.

Maskin, Eric and Jean Tirole (2004), "The politician and the judge." *The American Economic Review,* 94 (4), 1034–1054.

Morris, Stephen (2001)," Political Correctness." *Journal of Political Economy*, 109(2), 231-65.

Prat, Andrea (2005), "The wrong kind of transparency". *The American Economic Review,* 95 (3), 862–877.