# Keynes's missing axioms

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# Keynes's Missing Axioms

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**Abstract**

Between Keynes's verbalized theory and its formal basis persists a lacuna. The conceptual groundwork is too small and not general. The quest for a comprehensive formal basis is guided by the question: what is the minimum set of foundational propositions for a consistent reconstruction of the money economy? We start with three structural axioms. The claim of generality entails that it should be possible to prove that Keynes's formalism is a subset of the structural axiom set. The axioms are applied to a central part of the *General Theory* in order to achieve consistency and generality.

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The Keynesian Revolution was intended as both, a radical change of economic policy and a groundbreaking paradigm shift. Keynes left no doubt about the scientific scope of the *General Theory*:

> The classical theorists resemble Euclidean geometers in a non-Euclidean world .... Yet, in truth, there is no remedy except to throw over the axiom of parallels and to work out a non-Euclidean geometry. Something similar is required to-day in economics. (Keynes, 1973, p. 16)

While the political impact of Keynes's ideas surpassed that of his precursors by several magnitudes, the policy proposals themselves had already been popular in the economic literature of the 1930s (Laidler, 1999, p. 10). The ratification of Keynes's scientific claims therefore depends on the question whether he was successful in formulating some kind of non-Euclidean economic theory. By invoking Euclid, Keynes committed himself to the methodological consensus since Adam Smith (Hollander, 1977) and Senior:

> It [the axiomatic method] was introduced to economics in A.D. 1836 by Nassau William Senior in his *Outline of the Science of Political Economy* and is today more or less consciously adopted by most economic theorists as the way of theorizing in economics. (Stigum, 1991, p. 4)

Euclid's path runs through the classical school (Halévy, 1960, p. 494) the neoclassical school (Jevons, 1911, p. 21), to reach a new level of Walrasian abstraction in the 1960s (Debreu, 1959, p. x). The salient point of axiomatization is also recognized by some Post Keynesians:

> ..., before accepting the conclusions of any economist's model as applicable to the real world, the careful student should always examine and be prepared to criticize the applicability of the fundamental postulates of the model; for, in the absence of any mistake in logic, the axioms of the model determine its conclusions. (Davidson, 2002, p. 41), see also (1996, p. 49), (1998, p. 68), (2005, p. 402)

Euclid's spark, though, does not seem to have ignited Keynesianism. But one cannot *not* axiomatize. J. S. Mill clearly enunciated the question that stands at the beginning of any and every scientific inquiry:

> What are the propositions which may reasonably be received without proof? That there must be some such propositions all are agreed, since there cannot be an infinite series of proof, a chain suspended from nothing. But to determine what these propositions are, is the *opus magnum* of the more recondite mental philosophy. (Mill, 2006, p. 746), original emphasis

Keynes's critique of orthodox economics therefore rightly aimed at the premises:

> For if orthodox economics is at fault, the error is to be found not in the superstructure, which has been erected with great care for logical consistency, but in a lack of clearness and of generality in the premises. (Keynes, 1973, p. xxi)

Hence the question arises: why did Keynes not heed his own appeal and in earnest worked out the required non-Euclidean formal basis? Not the least advantage of axiomatization is that it serves efficiency and in Keynes's case it would have precluded the question 'what Keynes really meant'. There can be no conclusive answer because 'Keynes, too, sometimes gave the impression of not having fully grasped the logic of his own system' (Laidler, 1999, p. 281).

Keynes's conceptual groundwork consists in the main of two equations ($Y=C+I$ and $S=Y-C$, ergo $I=S$, Keynes, 1973, p. 63). That formal basis is too small and contains quite a number of tacit assumptions. The conjunction between the income and saving equation to, for example, wage rate, price, output, profit, or money is formally opaque (Heilbroner and Milberg, 1995, p. 52). That is the specific thesis with regard to Keynes's approach.

The general thesis says that human behavior does not yield to the axiomatic method (cf. Hudík, 2011; Rosenberg, 1980), yet the axiomatization of the money economy's fundamental structure is feasible. By choosing objective structural relationships as axioms behavioral hypotheses are not ruled out. On the contrary, the structural axiom set is open to *any* behavioral assumption and not restricted to the optimization calculus (for details see 2011b).

The objective is to establish a formalism of maximum structural simplicity. We start with an axiom set that is free of behavioral specifications and subsequently approach the complexity of the real world by a process of consistent differentiation. The claim of generality entails that it should be possible to prove that Keynes's basic formalism is a subset of the structural axiom set.

We proceed as follows. The formal ground is systematically prepared in Sections 1 to 3. The structural axiom set represents the pure consumption economy. In Sections 4 to 6 the structural employment equation is derived and the full employment conditions are established. After the introduction of the 4th axiom Keynes's intermediate situation is modeled as an elementary random economy with employment dependent, for a start, on the varying market configurations of wage rate and price. Money, too, follows in direct lineage from the axiom set. In Sections 7 to 10 the interrelations of the three aspects: stock of money, quantity of money, and transaction money are identified. In Sections 11 to 14 the definitions of profit and saving are introduced. The distinction between profit and distributed profit on the one hand and the relation between retained profit and saving on the other is crucial for the analysis of the functioning of the money economy. Standard profit theory is known to be incoherent, hence a new conceptual approach is in order. The structural axiom set is then applied to consistently establish the relation between investment and saving. In the final part, Sections 15 to 20, Keynes's formal flaws, which are still with us, are untangled. Section 21 concludes.

## 1 Axioms

The first three axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product. Quantitative and qualitative differentiation is obviously the next logical step after having worked out the implications of the following three axioms (for details see 2011d, pp. 5-7).

Total income of the household sector $Y$ in period $t$ is the sum of wage income, i.e. the product of wage rate $W$ and working hours $L$, and distributed profit, i.e. the product of dividend $D$ and the number of shares $N$.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector $O$ is the product of productivity $R$ and working hours.

$$O = RL \quad |t \tag{2}$$

Consumption expenditures $C$ of the household sector is the product of price $P$ and quantity bought $X$.

$$C = PX \quad |t \tag{3}$$

A set of axioms cannot be assessed *ex ante*, because the full range of implications is not immediately transparent (Klant, 1984, p. 10). Self-evidence is neither necessary nor sufficient (Popper, 1980, pp. 71-72). Therefore, a set of axioms is either agreed upon as a *tentative* formal starting point or prematurely rejected out of hand. The assessment of axioms comes at the second stage with the interpretation of the logical implications of the formal world and the comparison with selected data and phenomena of the real world.

Axioms should have an intuitive economic interpretation (von Neumann and Morgenstern, 2007, p. 25), (Chick, 1998, pp. 1860-1861). The economic meaning is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. Profit and distributed profit have to be thoroughly kept apart. All structural axiomatic variables are measurable in principle.

## 2 Definitions

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms (Boylan and O'Gorman, 2007, p. 431). With (4) wage income $Y_W$ and distributed profit income $Y_D$ is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

With (5) the expenditure ratio $\rho_E$, the sales ratio $\rho_X$, the distributed profit ratio $\rho_D$, and the factor cost ratio $\rho_F$ is defined:

$$\rho_E \equiv \frac{C}{Y} \qquad \rho_X \equiv \frac{X}{O} \qquad \rho_D \equiv \frac{Y_D}{Y_W} \qquad \rho_F \equiv \frac{W}{PR} \quad |t. \tag{5}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

## 3 Nothing simpler than that

The axioms and definitions are consolidated to one single equation:

$$\frac{\rho_F \rho_E \,(1+\rho_D)}{\rho_X} = 1 \quad |t. \tag{6}$$

The *period core* (6) as the absolute formal minimum determines the interdependencies of the measurable *key ratios* for each period. The period core is purely structural, i.e. free of any behavioral assumptions, unit-free because all real and nominal dimensions cancel out,[1] and contingent. Contingency means that it is open until explicitly stated which of the variables are independent and which is dependent. The form of (6) precludes any notion of causality; it simply states the *interdependence* of the key ratios. The period core represents the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other state activity.

The factor cost ratio $\rho_F$ summarizes the internal conditions of the firm. A value of $\rho_F < 1$ signifies that the real wage is lower than the productivity or, in other words, that unit wage costs are lower than the price, or in still other words, that the value of output exceeds the value of input. In this case the profit per unit is positive. Then we have the conditions in the product market. An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures are equal to income and a value of $\rho_X = 1$ of the sales ratio means that the quantities produced and sold are equal in period $t$ or, in other words, that the product market is cleared. In the special case $\rho_E = 1$ *and* $\rho_X = 1$ with market clearing *and* budget balancing the profit per unit is determined solely by the distributed profit ratio $\rho_D > 0$. In one sentence: the period core covers the key ratios about the firm, the market, and the income distribution and determines their mutual interdependencies.

[1] "This procedure is in accordance with the principle of objectivity requiring that the whole theory and its interpretations have to be independent of the choice of the units of measurement. And this requirement is met, if the theory is unit-free, the necessary condition stated in Buckingham's Π-theorem." (Schmiechen, 2009, p. 176).

## 4 Employment

The first markedly Keynesian relation that follows from the period core (6) is the structural employment equation:

$$L = \frac{Y_D}{PR\dfrac{\rho_X}{\rho_E} - W} \quad |t. \tag{7}$$

As a purely formal relationship the period core must hold in *each* period. Its new form now implies the *additional* assumption that employment as dependent variable is determined by the rest of the system. This is an assumption about the direction of dependency in a system with complex and mutual interrelations and this *add-on* assumption is not implied in the axiom set which is clearly open to various dependency interpretations. Dependency is conceptually different from causality. The structural employment equation states – with the other variables unaltered in each case:

(i) An increase of the wage rate leads to *higher* employment, i.e. to a lower unemployment rate.

(ii) A price increase is conductive to *lower* employment.

(iii) Provided that wage rate, price and distributed profit all change with the same rate ($\ddot{W} = \ddot{P} = \ddot{Y}_D$ see Section 6) there is no effect on employment.

(iv) If the configuration of price and wage rate changes is such that the denominator remains unchanged then employment stays where it is, no matter how large wage rate and price changes are. In this case perfect wage-price flexibility has no impact on employment (cf. Hahn and Solow, 1997, p. 134).

(v) An increase of the expenditure ratio $\rho_E$ leads to higher employment. An expenditure ratio $\rho_E > 1$ presupposes the existence of a banking system (see Section 7).

(vi) A productivity increase leads to lower employment.

(vii) As the difference in the denominator approaches zero employment goes (formally) off to infinity. This singularity is an implicit property of the economy as given by the structural axiom set (see Section 10).

(viii) Distributed profits exert a positive influence on employment.

Statements (i) to (viii) follow without regress to any behavioral assumptions from the axiom set and the 'laws of algebra' (Shaik, 1980, p. 83). When the axioms

capture reality the logical implications are observable. Equation (7) contains the *original* Phillips curve as special case.[2]

With regard to the process of adaptation of employment to changes of the independent variables (7) implies that the independent variables have to be fixed at the beginning of the period under consideration. Since the period length is arbitrary no great distortions arise from this idealization if the length is conveniently chosen.

## 5 Full employment conditions

The standard key variable for the establishment of full employment is the real wage $\frac{W}{P}$ which has to fall (Keynes, 1973, p. 17). The structural axiomatic approach asserts that in the consumption economy employment is determined by the expenditure ratio $\rho_E$ and the factor cost ratio $\rho_F = \frac{W}{PR}$ of which the real wage is a constituent. This follows from (7) under the conditions that the product market is cleared, i.e. $\rho_X = 1$, and that the relation of dividend to wage rate $\rho_V$ is held constant:

$$L = \frac{\frac{DN}{PR}}{\frac{\rho_X}{\rho_E} - \frac{W}{PR}} = \frac{\rho_V N}{\frac{\rho_X}{\rho_E \rho_F} - 1} = \frac{\rho_V N}{\frac{1}{\rho_E \rho_F} - 1} \tag{8}$$

$$\text{if} \quad \rho_X = 1;\ \rho_V \equiv \frac{D}{W} \quad |t.$$

Employment depends in the pure consumption economy on the relation of consumption expenditures to income $\rho_E$, i.e. on the axiomatic version of Keynes' effective demand (Keynes, 1973, pp. 23-24),[3] (Kaldor, 1988, p. 153) and the outcome of the market price mechanism, i.e. the relation of wage rate, price, and productivity $\rho_F$.

Under the conditions that the product market is cleared, i.e. $\rho_X = 1$, and the household sector's budget is balanced, i.e. $\rho_E = 1$, a higher factor cost ratio $\rho_F$ means higher employment as shown in Figure 1. The curve entails that there is no such thing as a natural rate of unemployment.[4]

There exists a unique factor cost ratio $\rho_F^\star$, and by consequence a unique real wage, that is consistent with full employment (however defined). From (8) follows as desideratum that condition (9) is satisfied:

[2] It is noteworthy that Phillips "had not made an explicit link between inflation and unemployment" (Ormerod, 1994, p. 120). It was the Samuelson–Solow version of the 'Phillips' curve that ultimately failed, and (7) explains why.

[3] The explicit inclusion of the consumption function determines the expenditure ratio as follows: $\rho_E = \frac{a}{Y} + b$.

[4] "It is not news that NAIRU theory is a failure." (Hall, 2011, p. 446).

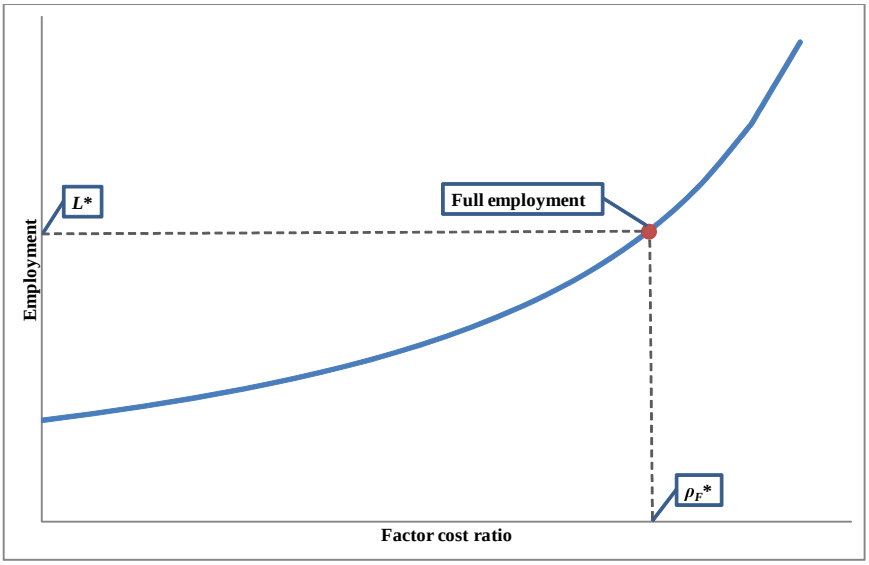



**Figure 1:** Structural relationship between factor cost ratio and employment ($\rho_E = 1$)

$$\rho_F^* = \frac{1}{\frac{\rho_V N}{L^\star} + 1} \quad \text{or} \quad \left(\frac{W}{P}\right)^\star = \frac{R}{\frac{\rho_V N}{L^\star} + 1} \quad \text{if} \quad \rho_X = 1; \rho_E = 1 \quad |t. \tag{9}$$

The numerical value of $L^\star$ depends on the actual definition of full employment. If (9) is satisfied the product and the labor market is cleared and the budget is balanced. Since this result follows without regress to behavioral assumptions directly from the axioms it would be conceptually inappropriate to refer to this configuration as full employment equilibrium. Equilibrium would in addition require some economic mechanism which guarantees that $\rho_F$ speedily approaches $\rho_F^\star$. No such mechanism is known.

The point to emphasize is: since the structure that is given by the axiom set does not adapt to behavior, behavior has to adapt to structure. For the economy as a whole the behavioral real-wage/marginal-productivity condition is inapplicable and has to give way to (9).

In the general case, the expenditure ratio $\rho_E$ is different from unity and the condition for full employment reads:

$$\rho_F \rho_E = \frac{1}{\frac{\rho_V N}{L^\star} + 1} \quad \text{if} \quad \rho_X = 1 \quad |t. \tag{10}$$

Full employment, then, can be realized with *any combination* of the expenditure ratio and the factor cost ratio that satisfies (10)[5] which in turn entails both, Keynes's principle of effective demand and the outcome of the market price mechanism.

In order to establish full employment, business has to accept a lower profit ratio $\rho_Q$. This ratio is inverse to the factor cost ratio $\rho_F$ and follows from (24) as:

$$\rho_Q \equiv \frac{\Delta Q_{fi}}{WL} \quad \Rightarrow \quad \rho_Q \equiv \frac{1}{\rho_F} - 1 \quad \text{if} \quad \rho_X = 1 \quad |t. \tag{11}$$

It can be said, then, that full employment is not prevented by a 'high' wage rate $W$ or a 'high' real wage $\frac{W}{P}$ but by a 'high' profit ratio $\rho_Q$. It is the profit ratio that has to fall as long as there is unemployment in the pure consumption economy.

An increase of the wage rate lowers the profit ratio and thus necessitates an employment expansion to realize the same *absolute* amount of profit. The general relationship between total profit and the factor cost ratio follows from (24) in combination with the employment equation (7) and is given by:

$$\Delta Q_{fi} \equiv \frac{1-\rho_F}{\dfrac{1}{\rho_E} - \rho_F} Y_D \quad \text{if} \quad \rho_X = 1 \quad |t. \tag{12}$$

If the expenditure ratio $\rho_E$ is *unity* then the effects of a higher factor cost ratio $\rho_F$ (lower profit ratio $\rho_Q$) are always exactly compensated for by a higher employment and the overall impact on total profit is nil if distributed profits remain constant. With regard to total profit business could in this case be *indifferent* between different employment levels. If the *relation* between dividend and wage rate $\rho_V$ is kept constant, as in (8), then both distributed profit and profit rise and fall with the wage rate, i.e. $Y_D = (\rho_V N) W$. A constant $\rho_V$ simply amplifies the wage rate effect of (7).

From the accustomed perspective[6] it seems to be counter-intuitive that a wage rate reduction, which lowers the real wage and raises the profit ratio, coincides with lower employment. This dissonance between standard behavioral *assumptions* and structural *fact* explains why the usual recipe for more employment does not succeed in getting the economy out of a slump (cf. Leijonhufvud, 1967, p. 402). The microeconomic optimization calculus and Marshall's pair of demand–supply scissors simply do not apply to the economy as a whole. When behavioral and structural logic are at odds, behavioral logic is conductive to frustrated plans and expectations. Neoclassical prescriptions deteriorate a underemployment situation.

[5] If $\rho_X = 1$, $\rho_F = 1$, and $\rho_E = 1$ then $\rho_D = 0$, i.e. $Y_D = 0$, according to (6). In this limiting case employment is indeterminate.

[6] "It is a well-known generalisation of theoretical Economics that a wage which is held above the equilibrium level necessarily involves unemployment .... This is one of the most elementary deductions from the theory of economic equilibrium." (Robbins, 1935, p. 146), for a commentary see (Weintraub, 1978).

"If the first classical postulate where correct, then we would expect real wages ... to move counter-cyclically. However, Dunlop and Tarshis found that product-wages were, if anything, procyclical." (Tobin, 1997, p. 7)

## 6 The intermediate situation

The period values of the variables are connected formally by the familiar growth equation, which is added to the structural set as the 4th axiom:

$$Z_t = Z_{t-1}\,(1 + \dddot{Z}) \quad Z\,|\,W, P, R, \rho_E \tag{13}$$

The path of the representative variable $Z_t$, which stands here for wage rate, price, productivity, and the expenditure ratio, is then determined by the initial value $Z_0$ and the rates of change $\dddot{Z}_t$ for each period:

$$Z_t = Z_0\,(1 + \dddot{Z}_1)\,(1 + \dddot{Z}_2)\ldots(1 + \dddot{Z}_t) = Z_0 \prod_{t=1}^{t} (1 + \dddot{Z}_t) \tag{14}$$

Equation (14) describes the paths of the variables with the *rates of change* as unknowns. These unknowns are in need of determination and explanation. Since we do not wish to get involved into speculations about human behavior at this stage (for details see 2011g), we have to choose the random hypothesis because:

> The simplest hypothesis is that variation is random until the contrary is shown, the onus of the proof resting on the advocate of the more complicated hypothesis ... (Kreuzenkamp and McAleer, 1995, p. 12)

By feeding the employment equation with random rates of change for wage rate and price (1.000 changes between 0% and 0.4%) employment in this simple random economy develops over time as shown in Figure 2.[7] Since all other variables are kept constant employment changes depend alone on changes of the real wage. Real wage and employment are *positively* related (cf. Hahn and Solow, 1997, p. 136).

In the selected simulation employment remains within a corridor with the lower bound defined as intolerable unemployment and the upper bound defined as capacity limit. Full employment is somewhere in between. Keynes characterized the situation as follows:

> In particular, it is an outstanding characteristic of the economic system in which we live that, whilst it is subject to severe fluctuations in respect of output and employment, it is not violently unstable. ... Fluctuations may start briskly but seem to wear themselves out before they have proceeded to great extremes, and an intermediate situation which is neither desperate nor satisfactory is our normal lot. (Keynes, 1973, pp. 249-250)

In structural axiomatic terms our normal lot is explained by the probability that employment stays within the corridor. Yet this probability is not unity. There

[7] The term random economy has been introduced for the equilibrium analysis of pure exchange economies (Föllmer, 1974). It is adopted in the present paper without this specific connotation. For a full account of the pure structural random economy see (2011c).


**Figure 2:** Keynes's intermediate situation (with no singularities)

is a positive probability for a singularity, that is, employment may *formally* go off to infinity and *actually* press against the capacity limit for a longer time span. A situation that is prone to inflation (see Section 10). And there is a positive probability that employment falls below the tolerable level of unemployment (in whatever sense). The probability for the intermediate situation therefore depends on the width of the corridor and the fluctuations of the real wage, that is, on the *relative* magnitudes of the random rates of change of wage rate and price (Leijonhufvud, 2009, p. 750).

The invisible hand takes effect trough the law of large numbers and there is no such thing as a full employment equilibrium. There is no disequilibrium either. The intermediate situation becomes more complex, of course, when all independent variables of the employment equation vary at random. But this does not alter the fundamental structural fact that the probability for the intermediate situation is *below* unity. This in turn implies that the economy cannot *always* left to itself.

## 7 Money

The money economy is the *real* economy. The dichotomization of the real and the monetary sphere is the central point of Keynes's methodological critique of orthodox economics:

> The division of economics between the theory of value and distribution on the one hand and the theory of money on the other hand is, I think, a false division. (Keynes, 1973, p. 293)

Therefore, the first task is to show how money consistently follows from the given axiom set (for details see 2011e).

If income is higher than consumption expenditures the household sector's stock of money increases. It decreases when the expenditure ratio $\rho_E$ is greater than unity. The change of the household sector's stock of money in period $t$ is defined as:

$$\Delta M_{\mathbf{H}} \equiv Y - C \equiv Y\,(1-\rho_E) \quad |t. \tag{15}$$

The stock of money at the end of an arbitrary number of periods is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$M_{\mathbf{H}} \equiv \sum_{t=1}^{t} \Delta M_{\mathbf{H}t} + M_{\mathbf{H}0}. \tag{16}$$

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta M_{\mathbf{B}} \equiv C - Y \equiv Y\,(\rho_E - 1) \quad |t. \tag{17}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$M_{\mathbf{B}} \equiv \sum_{t=1}^{t} \Delta M_{\mathbf{B}t} + M_{\mathbf{B}0}. \tag{18}$$

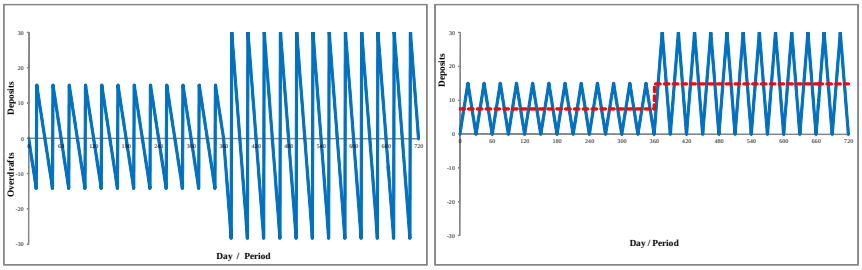
To simplify matters here it is supposed that all financial transactions are carried out without costs by the central bank. The stock of money then takes the form of current deposits or current overdrafts (Wicksell, 1936, p. 70). Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (16) the current overdrafts of the business sector are of equal amount according to (18) and vice versa if the business sector owns current deposits. Money and credit are symmetrical; the stock of money of each sector can be either positive or negative. The current assets and liabilities of the central bank are equal by construction. From its perspective the quantity of money at the end of an arbitrary number of periods is given by the absolute value either from (16) or (18):

$$M_t \equiv \left| \sum_{t=1}^{t} \Delta M_t \right| \quad \text{if} \quad M_0 = 0. \tag{19}$$

The quantity of money is always $\geq 0$. Equation (19) implies for a start that the central bank plays an accommodative role. Thus it is not necessary for the firms and households to resort to funds that have been accumulated before period$_1$ and we can postpone the question of how the firms finance their operations (cf. Lavoie, 1992, p. 153). The central bank provides elastic currency roughly in accordance with the definition of the Federal Reserve Act: 'Currency that can, by the actions of the central monetary authority, expand or contract in amount warranted by economic conditions.'

## 8 Endogenous and neutral

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 3 results (cf. Schmitt, 1996, p. 134). At the end of each subperiod the stock of money is zero. For the expenditure ratio in period$_1$ $\rho_E = 1$ holds. In period$_2$ the wage rate, the dividend and the price is doubled. Since no cash balances are carried forward from one period to the next, there results *no* real balance effect provided the doubling takes place exactly at the beginning of period$_2$.




**(a)** Transaction pattern over two periods **(b)** Average stock of transaction money $\hat{M}_T$

**Figure 3:** Graphical derivation of the average stock of transaction money from elementary transactions

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore the pattern of Figure 3 translates into an average amount of current deposits. This average stock of transaction money depends on income according to the transaction equation

$$M_T \equiv \kappa Y \quad |t \tag{20}$$

which resembles Pigou's Cambridge equation (the underlying theory is thereby not adopted).

For the transaction pattern that is here assumed as an idealization the index is $\frac{1}{48}$. Different transaction patterns are characterized by different numerical values of the transaction pattern index.

Taking the definitions of the sales ratio $\rho_X$ and the expenditure ratio $\rho_E$ from (5) one gets the explicit transaction equation:

$$\text{(i) } M_T \equiv \kappa \frac{\rho_X}{\rho_E} RLP \quad \text{(ii) } \frac{M_T}{P} = \kappa O \quad \text{if} \quad \rho_X = 1; \rho_E = 1 \quad |t. \tag{21}$$

We are now in the position to substantiate the notions of elastic currency and accommodation as a money-growth formula. According to (i) the central bank enables the average stock of transaction money to expand or contract with the development of productivity, employment, and price. In other words, the real average stock of transaction money, which is a statistical artifact and not a physical stock, is proportional to output (ii) if the transaction index is given and if the ratios

$\rho_E$ and $\rho_X$ are unity. Under these *initial* conditions money is endogenous (Desai, 1989, p. 150), (Nell, 1991, p. 187) and neutral (Patinkin, 1989a) in the structural axiomatic context. Money emerges from *autonomous* market transactions and has three aspects: stock of money ($M_{\mathbf{H}}$, $M_{\mathbf{B}}$), quantity of money (here $M = 0$ at period beginning and end; cf. Graziani, 1996, p. 143) and average stock of transaction money (here $M_T > 0$). The quantity of money changes as soon as $\rho_E \neq 1$, i.e. with saving or dissaving. Then, the function of a store of value is activated.

## 9 Transaction money

The average stock of transaction money is given by (21). Taking the employment equation (7) into account, the definition of the average stock of transaction money boils down to what may be referred to as augmented transaction equation:

$$M_T = \kappa \frac{\rho_V N}{\frac{1}{W} - \frac{\rho_E}{PR}} = \frac{(\kappa \rho_V N) W}{1 - \rho_E \rho_F} \quad \text{if} \quad \rho_X = 1 \quad |t. \tag{22}$$

From this relation follows – with all other variables fixed in each case:

(i) An increase of the expenditure ratio $\rho_E$ leads according to (8) to higher employment and exacts a higher average stock of transaction money $M_T$ according to (22).

(ii) When the rates of change of wage rate and price are identical employment stays where it is and $M_T$ rises. Both, employment and the average transaction balance remain unaltered if the rate of change of wage rate and price is zero.

(iii) A wage increase is conductive to higher employment and exacts a higher $M_T$.

(iv) A price increase leads to a drop of employment and exacts a lower $M_T$. Under the condition of budget balancing, i.e. $\rho_E = 1$, and market clearing, i.e. $\rho_X = 1$, the varying configuration of *W, P, R,* i.e. of $\rho_F$, determines the development of the average stock of transaction money.

It is, in principle, possible to have a stable price, a rising stock of transaction money, wage increases marginally above productivity increases, and increasing employment. It is equally possible to have a stagflation if the price rises faster than the wage rate.

## 10 The singularity

There is, though, a pitfall in augmented transaction equation which is shown in Figure 4. What hits the eye is that there is a point of discontinuity where the average

stock of transaction money goes off to infinity. A glance at (22) reveals that this happens when the inverse of the expenditure ratio $\frac{1}{\rho_E}$ is equal to the factor cost ratio $\rho_F$. Since both ratios vary independently this point moves unpredictably. The singularity is the formal point of entry of system immanent risk and rather the opposite of equilibrium.

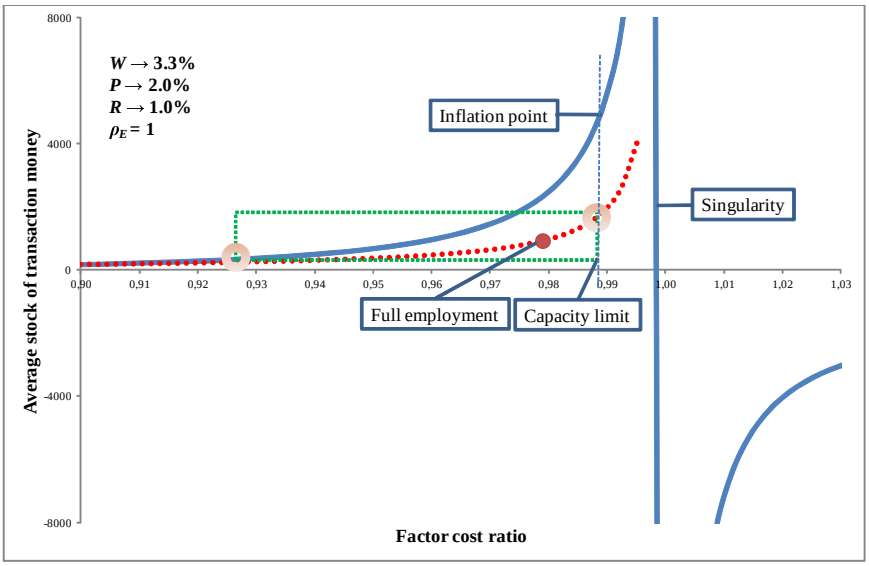



**Figure 4:** Structural singularity and goal compatible corridor

While the growth of the average stock of transaction money could go a long way, the coextensive employment expansion first reaches full employment and eventually runs against the capacity limit (if the factor cost ratio is increased *continuously*, which of course does not occur in the random economy or in the real world). The augmented transaction equation cannot tell us more about what then happens. A new phenomenon must emerge. The circumstances suggest that the new phenomenon could be inflation.

What follows, then, for stabilization policy? Granted that the axiom set truly represents the elementary structure of the money economy, one has to face the fact that there are two holes in the floor: at the one end of the corridor intolerable unemployment and at the other a high risk of inflation. Therefore, given enough random trials, the economy will eventually hit the one hole or the other. This state of the world requires and justifies discretionary economic policy as soon as the economy tends to leave the goal compatible corridor. To effectively steer the pure consumption economy away from both holes it would be necessary to fine-tune the relation of expenditure ratio, wage rate, and price.

## 11 Profit

The business sector's financial profit in period $t$ is defined with (23) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditures $C$ – and costs – here identical with wage income $Y_W$:

$$\Delta Q_{fi} \equiv C - Y_W \quad |t. \tag{23}$$

In explicit form, after the substitution of (3) and (4), this definition is identical with that of the theory of the firm:[8]

$$\Delta Q_{fi} \equiv PX - WL \quad |t. \tag{24}$$

Using the first axiom (1) and the definitions (4) and (5) one gets:

$$\Delta Q_{fi} \equiv C - Y + Y_D \quad \text{or} \quad \Delta Q_{fi} \equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \quad |t. \tag{25}$$

In the pure consumption economy profit is greater than zero if the expenditure ratio $\rho_E$ is $> 1$ or the distributed profit ratio $\rho_D$ is $> 0$, or both. If distributed profit $Y_D$ is set to zero, then profit or loss of the business sector is determined solely by the expenditure ratio. For the business sector as a whole to make a profit consumption expenditures $C$ have in the simplest case to be greater than wage income $Y_W$. So that profit comes into existence in the pure consumption economy the household sector must run a deficit at least in one period. This in turn makes the inclusion of the financial sector mandatory. A theory that does not include at least one bank that supports the concomitant credit expansion, which is covered by (16), cannot capture the essential features of the market economy (Keynes, 1973, p. 85).[9]

It needs hardly emphasis that in the investment economy the process of profit generation appears more complex (for details see 2011f). This does not affect the nature of profit but simply removes the *formal* necessity that the *households* have to incur a deficit to get the economy going. This is then done by the investing business sector. It is not advisable, though, to tackle the complexities of the investment economy before the pure consumption economy is *fully* understood. Mention should be made that neither neoclassicals nor Keynesians ever came to grips with profit (Desai, 2008, p. 10), (Tómasson and Bezemer, 2010).

## 12 A cognitive dissonance – but no contradiction

The determinants of profit look essentially different depending on the perspective. For the firm price $P$, quantity $X$, wage rate $W$, and employment $L$ in (24) appear to

[8] Nonfinancial profits are neglected here, i.e. $\rho_X = 1$ throughout. For details see (2011a).

[9] The purchase of all long lived consumption goods, e.g. houses, has to be subsumed under consumption expenditures. With regard to collateral there arises no problem for the banking industry and a sound credit expansion may proceed – in principle – for an indefinite time in the pure consumption economy.

be all important; under the broader perspective of (25) these variables play no role at all. The profit definition provokes a cognitive dissonance between the micro and the macro view.

It is of utmost importance that profit $\Delta Q_{fi}$ and distributed profit $Y_D$ is clearly distinguished. The latter is a flow of income from the business to the household sector analogous to wage income. By contrast, profit is the difference of flows within the business sector (Keynes, 1973, p. 23). Profit is not connected to a factor input. So far, we have labor input as the sole factor of production and wage income as the corresponding factor remuneration. Since the factor capital is nonexistent in the pure consumption economy, profit cannot be assigned to it in functional terms. And since profit cannot be counted as factor income (cf. Knight, 2006, pp. 308-309, Schumpeter, 2008, p. 153), there is no place for it in the theory of income distribution. This would plainly be a category mistake (for details see 2012).

The individual firm is blind to the structural relationship given by (25). On the firm's level profit is therefore subjectively interpreted as a reward for innovation or superior management skills or higher efficiency or toughness on wages or for risk taking or capitalizing on market imperfections or as the result of monopolistic practices. These factors play a role when it comes to the *distribution* of profits between firms and these phenomena become visible when similar firms of an industry are compared. Business does not 'make' profit, it redistributes profit. The case is perfectly clear when there is only one firm. It is a matter of indifference whether the firm's management thinks that it needs profit to cover risks or to finance growth or whether it realizes the profit maximum or not. If the expenditure ratio is unity and the distributed profit ratio is zero, profit will invariably be zero. The existence and magnitude of total profit is not explicable by the marginal principle.

Because of this, it is not wise to take the considerations of the individual firm's management as analytical starting-point and then to generalize. The microeconomic approach is inherently prone to the fallacy of composition. The profit definition entails a cognitive dissonance between micro and macro, but no logical contradiction. *Ab origine* total profit is a factor-independent residual (Ellerman, 1986, pp. 61-65). This distinction is crucial.

> We know from the history of science that entrenched classificatory schemes and misleading descriptive vocabularies have impeded scientific advance as much or more than the complexities and observational inaccessibility of the subject matter. (Rosenberg, 1980, p. 114)

Under the condition $\rho_E = 1$ profit $\Delta Q_{fi}$ must, as a corollary of (25), be equal to distributed profit $Y_D$. The fundamental difference between the two variables is not an issue in this *limiting* case. The equality of profit and distributed profit is an implicit feature of equilibrium models (Godley and Shaikh, 2002, p. 425), (Patinkin, 1989b, p. 329), (Buiter, 1980, pp. 3, 7). These have *no* counterpart in reality.

The barter-economic notion of surplus stands in no relation to profit as determined with definition (23). Neither is the neoclassical equilibrium condition, profit rate = marginal productivity of capital, applicable in the pure consumption economy

because we have profit but no capital. And, since profit and capital must not be treated like Siamese Twins, as they have by the classics, the tendency of the profit rate to fall is also in need of a thorough revision (for details see 2011f, pp. 18-20).

The question of whether in equilibrium profit is zero or not – Walras's 'ni bénéfice ni perte' – is of no concern within the structural axiomatic framework because the notion of simultaneous equilibrium is *no* constituent part of it (cf. Kaldor, 1985, p. 12). In the general case, profit or loss depends on consumer spending and profit distribution. If in the limiting case distributed profit in (25) is zero, then any loss of the business sector must be equal to the saving of the household sector as specified by (28). Since saving is – in the absence of distributed profits – the exact complement of loss, it must be overcompensated by dissaving within a short time interval, i.e. $\rho_E > 1$, otherwise the economy faces major challenges. So the real question is not about the existence of a zero-profit equilibrium, but how the market economy can, and in fact does, avoid this predicament over a longer time span (Keynes, 1973, pp. 158-159), (Rotheim, 1981, p. 581).

The definition of profit (23) has another important implication. There is no real residual that corresponds to the nominal residual profit. Real (*O, X*) and nominal (*Y, C*) flows are to some degree independent. Profit belongs *entirely to the nominal sphere*, in a real model it cannot exist. This is the defining characteristic of what Keynes termed the entrepreneur economy (Rotheim, 1981, pp. 575, 577, 579).

## 13 Retained profit

Profits can either be distributed or retained. If nothing is distributed, then profit adds entirely to the financial wealth of the firm. Retained profit $\Delta Q_{re}$ is defined for the business sector as a whole as the difference between profit and distributed profit in period $t$:

$$\Delta Q_{re} \equiv \Delta Q_{fi} - Y_D \quad |t. \tag{26}$$

Using (25) and (17) it follows:

$$\Delta Q_{re} \equiv^n C - Y \equiv^m \Delta M_{\mathbf{B}} \quad |t. \tag{27}$$

Retained profit $\Delta Q_{re}$ is the residual $C - Y$ as it appears at the firm; the same residual appears at the central bank as a change of the business sector's stock of money $\Delta M_{\mathbf{B}}$. The *two aspects* are kept apart by the notation $\equiv^n$ and $\equiv^m$, respectively. It follows immediately that the development of the business sector's stock of money, which may carry a positive or negative sign, is given by (17).

## 14 Saving

Financial saving is given by (28) as the difference of income and consumption expenditures. This definition is identical with Keynes's, i.e. $\Delta S_{fi}$ equates to the Keynesian $S$. In combination with (15) this yields the straightforward relation:

$$\Delta S_{fi} \equiv Y - C \quad \Rightarrow \quad \Delta S_{fi} \equiv^{n} Y - C \equiv^{m} \Delta M_{\mathbf{H}}. \tag{28}$$

Saving and the change of the household sector's stock of money are *two aspects* of the same flow residual. It follows immediately that the development of the household sector's stock of money is thus given by (16).

Financial saving (28) and retained profit (27) always move in opposite directions, i.e. $\Delta Q_{re} \equiv -\Delta S_{fi}$. Let us call this the complementarity corollary because it follows directly from the definitions themselves. The corollary asserts that the complementary notion to saving is *not* investment but negative retained profit. Positive retained profit is the complementary of dissaving. Since there is no investment in the pure consumption economy the IS-equality-identity-equilibrium cannot hold. The complementarity corollary entails that the plans of households and firms are in the general case *not* mutually compatible.

## 15 Allais is general

Having clarified the structural properties of the pure consumption economy we are now ready to assess the relation between the axiomatic and the Keynesian approach in still more detail. Based on the differentiated formalism it is assumed that the investment goods industry, which consists of one firm, produces $O_{\mathbf{I}} = X_{\mathbf{I}}$ units of an investment good, which is bought by the consumption goods industry to be used for the production of consumption goods in future periods. The households buy but the output of the consumption goods industry (for details see 2011f). From (24) then follows for the financial profit of the consumption and investment goods industry, respectively:

$$\Delta Q_{fi\mathbf{C}} \equiv C - Y_{W\mathbf{C}} \qquad \Delta Q_{fi\mathbf{I}} \equiv I - Y_{W\mathbf{I}} \qquad Y_W \equiv Y_{W\mathbf{C}} + Y_{W\mathbf{I}} \quad |t. \tag{29}$$

Total financial profit, defined as the sum of both industries, is then given by the sum of consumption expenditures and investment expenditures minus wage income which is here expressed as the difference of total income minus distributed profit:

$$\Delta Q_{fi} \equiv C + I - (Y - Y_D) \quad |t. \tag{30}$$

From this and the definition of financial saving (28) follows:

$$\Delta Q_{fi} \equiv I - \Delta S_{fi} + Y_D \quad |t. \tag{31}$$

Higher total financial profits on the one side demand as a corollary, i.e. as a logical implication of the definition itself, higher investment expenditures and

distributed profits and lower saving on the other side and vice versa. By finally applying the definition of retained profit (26) the Allais-Identity follows:

$$\Delta Q_{re} \equiv I - \Delta S_{fi} \quad |t. \tag{32}$$

> Autrement dit l'investissement n'est pas égal à l'épargne spontanée, mais à l'épargne spontanée augmenté du revenue non distribué des entreprises .... (Allais, 1993, p. 69), see also (Robinson, 1956, p. 402), (Lavoie, 1992, p. 159 eq. (4.3)), (Godley and Lavoie, 2007, p. 37 fn 9)

If retained profit is zero, that is, if profit and distributed profit happen to be equal in (26), then, as a corollary, investment expenditures and household saving in (32) must be equal too. Vice versa, if it happens that household saving is equal to investment expenditure then, as a corollary, profit and distributed profit must be equal too. In reality, though, profit and distributed profit are virtually *never* equal and correspondingly household saving and investment are not equal either. The fact that retained profit is different from zero in each period can be taken as an *empirical proof* of the logically equivalent inequality of household saving and business investment. Allais has definitively settled the IS-debate of the 1930s in 1993. Since then, all models – including IS-LM – that have been built and are still being built on the arguments of (Hicks, 1939, pp. 181-184), (Ohlin, 1937), (Lutz, 1938), (Lerner, 1938), (Keynes, 1973, p. 63), (Kalecki, 1987, p. 138) and others have to be regarded either as limiting cases or as formally deficient.

## 16 *Treatise* and *General Theory* as limiting cases

When the profit definition for the pure consumption economy (i) in (33) and the investment economy (ii) is compared

$$\begin{array}{ll} \text{(i)} & \Delta Q_{fi} \equiv Y_D - \Delta S_{fi} \\ \text{(ii)} & \Delta Q_{fi} \equiv I + Y_D - \Delta S_{fi} \end{array} \tag{33}$$

the first point to emphasize is that definition (i) is consistently replaced by the broader definition (ii). The inclusion of the investment process significantly changes the scope of profit generation. This change, though, is opaque to the agents, which can perceive scarcely more than their firm's sales revenues and factor costs. For definition (ii) the corollary (34) holds: if it happens that investment expenditures are zero then it must be the case that financial profit is equal to the difference of distributed profit and household saving, and vice versa. The corollary (34) replaces definition (i) in (33) and now applies to the pure consumption economy as a limiting case:

$$I = 0 \Leftrightarrow \Delta Q_{fi} = Y_D - \Delta S_{fi} \quad |t. \tag{34}$$

For definition (ii) a second corollary (35) holds: if it happens that distributed profit is zero then financial profit must be equal to the difference of investment expenditures and household sector's saving:

$$Y_D = 0 \Leftrightarrow \Delta Q_{fi} = I - \Delta S_{fi} \quad |t. \tag{35}$$

This implication of (ii) is well known as one of Keynes's 'fundamental equations for the value of money' (Keynes, 1971, pp. 124, 136). This means that, although Keynes was closer to the axiomatic formalism in his *Treatise* than in his *General Theory* he nonetheless was not general there either (cf. Hicks, 1939, p. 184). The reason is that he, in accordance with orthodox economic theory, did not accurately discriminate between profit and distributed profit and by consequence failed to take into account the process of profit distribution that is crucial for the functioning of the market system. Structural axiomatization ultimately boils down to the rejection of Keynes's definition:

> Thus the factor cost and the entrepreneur's profit make up, between them, what we shall define as the *total income* resulting from the employment given by the entrepreneur. (Keynes, 1973, p. 23), original emphasis

Total income consists in the simplest case of wage income and *distributed profits*.

> Toutes ses [Keynes's] deductions, à notre avis, manquent absolument de rigeur. ... L'intuition de Keynes lui a fait sentir où se trouvaient les difficultés, mais son insuffisance logique ne lui a pas permis de résoudre les problèmes que son intuition lui avait fait entrevoir. (Allais, 1993, p. 70)

## 17 Delicate distinctions

The present formalism is composed of axioms and definitions. In a strictly formal sense the definitions are dispensable. Any new symbol (definiendum) that is introduced with a definition is an abbreviation for a longer expression (definiens) that is composed of the variables of the axiom set and the familiar mathematical operators. So, when the word processor is instructed to replace one definiendum after another by its definiens then the equations become longer yet nothing else changes. No variables other than those of the axiom set remain.

Since it is true that everybody is free to define whatever appears to be appropriate it seems that a definition could not pose any real problem. This, indeed, is *not* true because the full freedom of definition holds but for the first definition. As Georgescu-Roegen put it:

> In fact, the history of every science, including that of economics, teaches us that the elementary is the hotbed of the errors that count

most. (Georgescu-Roegen, 1970, p. 9), see also (Boland, 2003, p. 87), (Hahn, 1984, p. 40).

Let us suppose somebody looks at the Allais-Identity (32), which states that retained profit for the economy as a whole is equal to the difference of the business sector's investment expenditure and the household sector's financial saving, and proposes to refer to the *sum* of saving and retained profit as *total private saving* $\Sigma$ because retained profit may, after all, well be regarded as saving of the business sector (e.g. Lavoie, 1992, p. 159). Thereby a new definition, (i) in (36), would be added to the already existing formalism. Together with the Allais-Identity (ii) this gives (iii) which states that *total private saving* $\Sigma$ (and *not* household saving $\Delta S_{fi}$ respectively $S$ in Keynes's notation) "equals" investment:

$$\text{(i) } \Sigma \equiv \Delta S_{fi} + \Delta Q_{re} \quad \text{(ii) } \Delta Q_{re} \equiv I - \Delta S_{fi} \quad \Rightarrow \quad \text{(iii) } \Sigma \equiv I \quad |t. \tag{36}$$

We thus arrive at an implicit definition that is no proper definition at all:

> For a definition to be valid it must meet several conditions: (1) it must be *dispensable*, that is, the scientist must be able to do without it; and (2) it must be *noncreative*, that is, the scientist cannot use the definition to establish formulas that do not contain the defined term, unless these formulas can be proved without using the definition. (Stigum, 1991, pp. 35-36), original emphasis

Equation (36) (iii) is no dispensable abbreviation but simply permits the arbitrary *permutation* of the symbols $\Sigma$ and $I$. While the Allais-Identity contains valuable information, $\Sigma \equiv I \equiv S$ is a homespun muddle. To define $\Sigma$ and then to place $S$ for $\Sigma$ is an elementary formal mistake.

But, and this makes things a bit complicated, if it happens that retained profit is zero in (i) then, as a corollary, it must hold that total private saving $\Sigma$ and household saving $\Delta S_{fi}$ are *equal*, i.e. $\Sigma = S_{fi}$. From (ii) then results as a corollary $I = \Delta S_{fi}$ or in plain words: household sector's saving equals investment – *if retained profit is zero*, which *never* happens. In contrast, (iii) states that total private saving $\Sigma$ is identical with investment $I$ by definition (cf. Samuelson and Nordhaus, 1998, p. 204 and p. 194 for corporate saving[10]).

A *complete* resolution of this formally unacceptable state of affairs requires that the wrong turnoff (i) in (36) is *not* taken. This definition implicitly leads to (iii) which signals redundancy. Redundancy calls for Occam's razor.

Under the purely formal perspective the salient point is: in a system of equations $x = y$ signifies a condition that is satisfied by certain values of the unknowns; in a system of definitions $x \equiv y$ signifies a dead end. The latter expression allows replacing the word apple wherever it appears by the word orange and vice versa. From this, no profound insights are to be expected.

[10] From the 1948 edition onwards, Samuelson never came to grips with profits (Tómasson and Bezemer, 2010, p. 16-17). "I often wonder whether other subjects suffer as much from textbook writers." (Hahn, 1980, p. 127)

## 18 A look at the ledger

Under the conceptual perspective the salient point is: saving as the complement of consumption expenditures refers *exclusively* to the household sector.

> It is true, of course, that neoclassical economists also consider *total private saving*, defined as the sum of personal and business saving, since the distinction between households and firms is often treated as a veil and individual agents are assumed to optimize total private (rather than merely household) saving. (Gordon, 1995, p. 62), original emphasis

There is *no* such thing as saving of the business sector. Ultimately, the saving-equals-investment formula results in superficial empirical studies (Gordon, 1995, pp. 60-62) and unacceptable bookkeeping conventions in national accounting (cf. Eisner, 1995, p. 109; Godley and Lavoie, 2007, pp. 260-263). To demonstrate this, Figure 5 reconstructs the steps from pure transaction recording to the formally indefensible and ultimately futile collapsing of the business sector's retained profit and the household sector's saving (cf. Boulding, 1950, pp. 248-252, Levy and Levy, 1983, pp. 44-48).

Collapsing is futile because it just annihilates what has been gained by differentiation and because the result is predictable: all surpluses and deficits between economic units and all credit relations vanish. The very essence of economics evaporates.

Conceptual consistency *prohibits* the application of the notion of saving to the business sector. The compelling reason for rejecting the definition of total private saving $\Sigma$ in (36), and *everything* that follows from it, boils down to that it is conceptually inadmissible, implicitly leads to $\Sigma \equiv I$, which signifies redundancy, and for certain conditions to $I = \Delta S_{fi}$, which is a limiting case of the Allais-Identity with *no* real world correspondence.

## 19 Never *ex ante*, never *ex post*

Needless to emphasize that it did not got lost in the discussion that in fact investment expenditures might not be equal to household saving and this was explained with the perfect reconcilability of an *ex ante* disequilibrium with the *ex post* bookkeeping truism $I \equiv S$ (Myrdal, 1939, p. 47), which in turn is different from the equilibrium condition $I = S$. This rationalization is beside the point for the simple reason that a meticulous recording of all transactions during one period arrives at the Allais-Identity. Only after applying the indefensible definition of total private saving $\Sigma$ the national accountant will arrive at $I \equiv \Sigma$ (with $\Sigma$ being *different* from $S$). These extra entries are formally redundant. The *ex ante–ex post* interpretation, or, for that matter, the designed–undesigned interpretation (Heilbroner, 1942, p. 828) fits the prevailing mode of 'loose verbal reasoning' (Dennis, 1982, p. 698) that cares

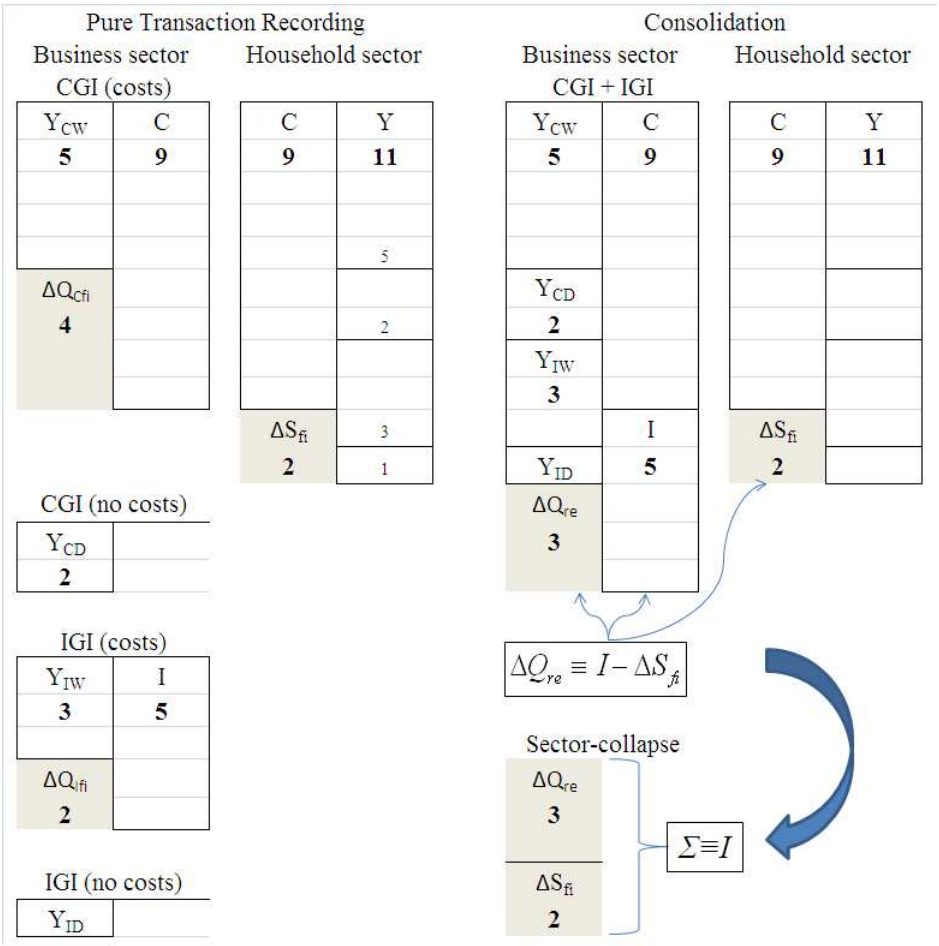



**Figure 5:** How the accountant produces valuable information before collapsing it away (CGI consumption goods industry, IGI investment goods industry)

not much for conceptual consistency. All that is necessary, then, is to add up the available numbers and to abstain from redundant definitions.

## 20 Set and subset

Keynes's characterization of the 'nature of economic thinking' (Keynes, 1973, p. 297) may be rhetorically summed up to: better vaguely right (ordinary discourse) than precisely wrong (blind manipulation of symbols). This alternative does not exist, at least not in science. Keynes recognized that without formal principles of thought 'we shall be lost in the wood' and struggled in Book II with fundamental definitions and ideas. He finally came up with equation (i⋆), which follows from (30) as a limiting case:

$$
\begin{array}{cc|cc}
\multicolumn{2}{c|}{\text{Axioms}} & \multicolumn{2}{c}{\text{Definitions}} \\
\text{(i)} & Y = WL + DN & \text{(iv)} & \Delta Q_{fi} \equiv PX - WL \\
\text{(ii)} & O = RL & \text{(v)} & \Delta S_{fi} \equiv Y - C \\
\text{(iii)} & C = PX & & \\
(\text{i}^{\star}) & Y = C + I & & \\
 & \text{if } Y_D = \Delta Q_{fi} & &
\end{array} \quad |t. \tag{37}
$$

The structural axiomatic approach rests on the three axioms (i)-(iii) that capture the elementary facts of a money economy and two definitions. It formally reduces to Keynes's limiting case (i$^{\star}$) and (v) if profit is exactly equal to distributed profit which, obviously, does not happen in the real world.

Keynes's main concern in the *General Theory* was not market or policy failure but theory failure. By consequence he envisioned nothing less than a paradigm shift (Coddington, 1976) and called for a 'complete theory of a monetary economy' (Keynes, 1973, p. 293), see also (Dillard, 2010). While perfectly aware that this at the same time required a consistent set of some kind of non-Euclidean axioms, Keynes had no desire that the particular forms of his 'comparatively simple fundamental ideas ... should be crystallized at the present state of the debate' (cited in Rotheim, 1981, p. 571). Hahn's balanced view, though, might be closer to the mark:

> I consider that Keynes had no real grasp of formal economic theorizing (and also disliked it), and that he consequently left many gaping holes in his theory. I none the less hold that his insights were several orders more profound and realistic than those of his recent critics. (Hahn, 1982, pp. x-xi)

From all this follows:

> We are not time-locked by the particular (and provisional) choice Keynes made in expositing his ideas in 1936. (O'Donnell, 1997, p. 158)

## 21 Conclusions

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. The present paper suggests three non-behavioral axioms as groundwork for the formal reconstruction of the evolving money economy.

The analytical priority claim of the structural axiomatic approach rests on the simple fact that, since the structure that is given by the axiom set does not adapt to behavior, behavior has to adapt to structure. When behavioral and structural logic are at odds, behavioral logic is conductive to frustrated plans and expectations. That is the normal state of economic affairs. The main results of the inquiry are:

- The expenditure-income asymmetry is the indispensable prerequisite for favorable business conditions and prolonged growth. This holds for the elementary consumption economy and the complex investment economy in equal measure.
- The key variables for the attainment of full employment are the expenditure ratio, i.e. the axiomatic version of Keynes' effective demand, and the factor cost ratio, i.e. the configuration of wage rate, price, and productivity as outcome of the market price mechanism.
- There is no structural trade-off between higher price inflation and lower unemployment.
- The employment effect depends on the *relative* magnitude of wage rate and price changes.
- Higher employment is compatible with a higher real wage, a lower unit profit ratio and unaltered profit for the business sector as a whole.
- Models that are based on the collapsed definition total income $\equiv$ wages + profits are erroneous because profit and distributed profit is not the same thing.
- The structural axiom set implies that it is possible to have a stable price, a rising stock of transaction money, wage increases marginally above productivity increases, and rising employment.
- There is no such thing as a natural rate of unemployment and it is not a 'high' nominal or real wage that prevents full employment but a 'high' profit ratio.
- The structural axiom set implies a singularity. A singularity is the point of entry of systemic risk and rather the opposite of equilibrium.
- Keynes proposed to 'throw over' the axioms of the orthodox theorists which 'resemble Euclidean geometers in a non-Euclidean world', but failed to heed his own appeal. His own formal basis is too small, contains too many tacit assumptions, and is not general.
- The Keynesian formalism is a subset of the structural axiom set. The general Allais-Identity is confirmed. With regard to all $I = S$ or $I \equiv S$ models it asserts that household saving is virtually never equal to investment expenditures, neither *ex ante* nor *ex post*. The standard *ex ante–ex post* explanation consists of multiple logical errors that support one another.

The structural axiomatic approach provides Keynes's missing axioms and fits the Keynesian approach consistently into a general context.

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