The wine maker's business and the logical origin of interest in the monetary economy

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Abstract

Any serious alternative to the standard approach requires a distinct axiomatic foundation. The crucial point is not axiomatization per se but the real world content of axioms. The purpose of the present paper is to make the implications of the objective structural axiom set concerning the relation of profit, money, the nominal/real rate of interest, and the time structure of production explicit and to contrast them with the familiar conceptions.

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Each theory starts from a small set of foundational 'hypotheses or axioms or postulates or assumptions or even principles' (Schumpeter, 1994, p. 15). Standard economics rests on a set of *behavioral* axioms (Arrow and Hahn, 1991, p. v).

> ... there is more agreement on the defects of orthodox theory than there is on what theory is to replace it: but all agreed that the point of the criticism is to clear the ground for construction. (Nell, 1980, p. 1)

Boland aptly reminded critics what the real point is:

> ... if you think you can do better with a non-neoclassical model ..., then you are quite welcome to try. (Boland, 1992, p. 19)

This challenge is taken up. The main thesis of the present paper is that human behavior does not yield to the axiomatic method, yet the axiomatization of the money economy's fundamental structure is feasible. The crucial point is not axiomatization *per se* but the real world content of axioms (cf. Nell, 1984). The objective is to make the implications of the *structural* axiom set concerning profit and interest explicit and to contrast them with the familiar conceptions.

> And similarly in economic theory, certain results ... may be known already. Yet it is of interest to derive them again from an exact theory. The same could and should be said concerning practically all established economic theorems. (von Neumann and Morgenstern, 2007, p. 6)

Accordingly, the formal frame that constitutes the pure consumption economy is set up in Section 1. In Section 2 financial profit is defined. Then the interrelation of the real and the monetary sphere is at first established in Section 3 for one single firm, the bread maker. The introduction of the wine maker's firm, that is characterized by a different time structure of production, gives rise to a reallocation of labor and constitutes the familiar choice situation between bread an wine. Relative prices are determined by applying the zero profit condition. In Section 4 it is shown how the time consuming process of wine making gives rise to a change of the monetary transaction pattern which in turn creates the demand for credit. Credit is produced by the banking unit of the central bank. How the reallocation of resources and the production conditions of the banking unit determine the rate of interest is shown in Section 5. In Section 6 the production process of wine is lengthened. The effects of more roundaboutness on productivity and on the nominal/real rate of interest are elaborated and compared with the results of the standard approach of Fisher. In Section 7 the rate of interest is determined under the condition of profit ratio equalization. To close the circle the classical capitalist is, in Section 8, finally fitted into the structural axiomatic framework. It turns out that interest is not an elementary income category and has to be replaced by distributed profit. Section 9 concludes.

1 Axioms

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of objective premises.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other government activity.

2 Profit

The business sector's financial profit in period t is defined with (4) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditures C – and costs – here identical with wage income Y_W:

$$Q_{fi} \equiv C - Y_W \quad |t. \tag{4}$$

In explicit form, after the substitution of (3), this definition is identical with that of the theory of the firm:

$$Q_{fi} \equiv PX - WL \quad \leftarrow \quad Y_W \equiv WL \quad |t. \tag{5}$$

With (6) the expenditure ratio ρ_E, the sales ratio ρ_X, and the distributed profit ratio ρ_D is added for formal convenience as:

$$\rho_E \equiv \frac{C}{Y} \qquad \rho_X \equiv \frac{X}{O} \qquad \rho_D \equiv \frac{Y_D}{Y_W} \quad \leftarrow \quad Y_D \equiv DN \quad |t. \tag{6}$$

An expenditure ratio $\rho_E = 1$ indicates that total consumption expenditures are equal to total income, or, in other words, that the household sector's budget is balanced; a value of $\rho_X = 1$ of the sales ratio means that the quantities produced and sold are equal in period t or, in other words, that the product market is cleared. If profits are distributed the distributed profit ratio ρ_D is > 0. Definitions add no new content to the set of axioms but determine the logical context of concepts (Stigum, 1991, pp. 35-36).

Using the 1st axiom in combination with (6) one gets from (4) the relation between financial profit and the key ratios:

$$Q_{fi} \equiv C - Y + Y_D \quad \Rightarrow \quad Q_{fi} \equiv \left(\rho_E - \frac{1}{1 + \rho_D} \right) Y \quad |t. \tag{7}$$

In the pure consumption economy financial profit is greater than zero if the expenditure ratio ρ_E is > 1 or the distributed profit ratio ρ_D is > 0, or both. If distributed profit Y_D is set to zero, then profit or loss of the business sector is determined solely by the expenditure ratio. For the business sector as a whole to make a profit consumption expenditures C have in the simplest case to be greater than wage income Y_W. So that financial profit comes into existence in the pure consumption economy the household sector must run a deficit at least in one period. This in turn makes the inclusion of the financial sector mandatory.

The determinants of profit look essentially different depending on the perspective. For the firm price P, quantity X, wage rate W, and employment L in (5) seem to be all important; under the broader perspective of (7) these variables play no role at all. Both views are formally equivalent.

3 Bread and wine

The wine maker's business is 'a copybook example rightly favoured by economist' (Wicksell, 1949, p. 172) to elucidate the nature and significance of interest.

For a clear-cut point of departure it is assumed that the output of the business sector consists at first solely of bread, produced by firm A. Total employment L is given and remains constant for the time being. Distributed profit is set to zero. In period$_1$ the axioms then take the simplest form:

$$Y = W_A L \qquad O_A = R_A L \qquad C = P_A X_A \quad |1. \qquad (8)$$

The bread is produced continuously and sold to the households. Wages, which include the wages of management, are paid monthly at mid-month, that is, one twelfth of total income Y. The wage rate W is by implication an average for the firm. Under the condition that money consists of current deposits and current overdrafts at the central bank this yields the perfectly symmetric transaction pattern of Figure 1. All transaction are carried out by the transaction unit of the central bank which is a firm like any other that pays wages and sells its services at a certain transaction price to the households. The transaction unit is neglected in the following (for details see 2011b, Sec. 4).
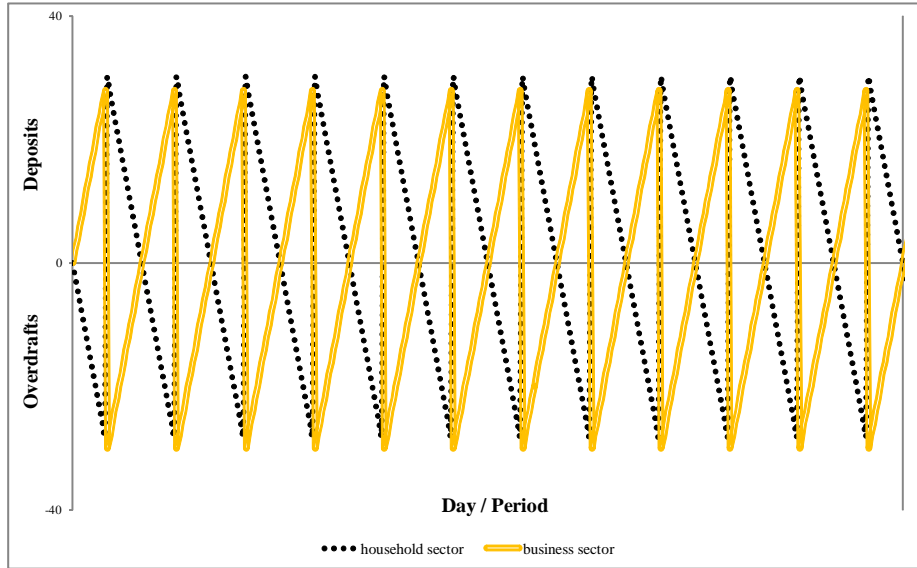

Figure 1: Monthly wage payments to the household sector at mid-month and continuous consumption expenditures during period$_1$

In period$_2$ the wine maker takes up production. Total employment L splits up into:

$$L \equiv L_A + L_B \quad |2. \qquad (9)$$

Labor input L_B moves from firm A to firm B. This entails that the production of bread shrinks and the production of wine expands. Total income Y as given by (1) is now composed of:

$$Y = \underbrace{W_A}_{W} L_A + \underbrace{W_B}_{W} L_B + \underbrace{(D_A N_A + D_B N_B)}_{Y_D = 0} \quad |2. \qquad (10)$$

5

To bar distracting secondary effects the wage rates are set equal for all firms and distributed profits are excluded. Total income therefore does not change with the reallocation of labor input.

The household sector apportions its consumption expenditures C as given by (3) between the purchases of bread and wine:

$$C = P_A X_A + P_B X_B \quad |2. \tag{11}$$

Total consumption expenditures are equal to total income, i.e. $\rho_E = 1$. The household sector as a whole neither saves nor dissaves.

According to (5) the profits of the two firms are given by:

$$\begin{aligned} Q_{fiA} &\equiv P_A X_A - W L_A \\ Q_{fiB} &\equiv P_B X_B - W L_B \end{aligned} \quad |2. \tag{12}$$

Under the condition that both markets are cleared, i.e. $\rho_X = 1$, this can be rewritten as:

$$\begin{aligned} Q_{fiA} &= P_A R_A L_A \left(1 - \frac{W}{P_A R_A} \right) \quad \text{if} \quad \rho_{XA} = 1 \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad |2. \\ Q_{fiB} &= P_B R_B L_B \left(1 - \frac{W}{P_B R_B} \right) \quad \text{if} \quad \rho_{XB} = 1 \end{aligned} \tag{13}$$

Overall profits are zero according to (7) because of $\rho_E = 1$ and $\rho_D = 0$. The zero profit condition for a single firm reads $\frac{W}{PR} = 1$. Under this conditions follows from (13) that prices are equal to unit wage costs:

$$P_A = \frac{W}{R_A} \qquad P_B = \frac{W}{R_B} \quad |2. \tag{14}$$

In sum: relative prices $\frac{P_A}{P_B}$ are inverse to the productivity ratio $\frac{R_A}{R_B}$. Both markets are cleared, the household sector's budget is balanced and profits are zero for both firms.

The production-possibility frontier (Samuelson and Nordhaus, 1998, pp. 19-22) follows from the 2nd axiom (2) in combination with (9) and reads:

$$O_A = R_A L - \frac{R_A}{R_B} O_B \quad |t. \tag{15}$$

Figure 2 shows the hypothetical choice between bread and wine that is open to the household sector in period$_2$. This choice determines the allocation of labor input. The period view, though, is too coarse. The choice consists not only in

a combination of output quantities O_A and O_B. While the bread comes daily the wine's date of delivery is by assumption New Year's Eve because of the specific production conditions of wine. Figure 2 refers to one single period but to different time patterns of delivery *within* the period. In the undifferentiated period treatment the time dimension of production is not present, that is, the choice situation is not fully specified in the familiar graphical representation. The households choice is not between *x* units of bread and *y* units of wine but between *x* continuously delivered units of bread and *y* units of wine delivered at New Year's Eve.
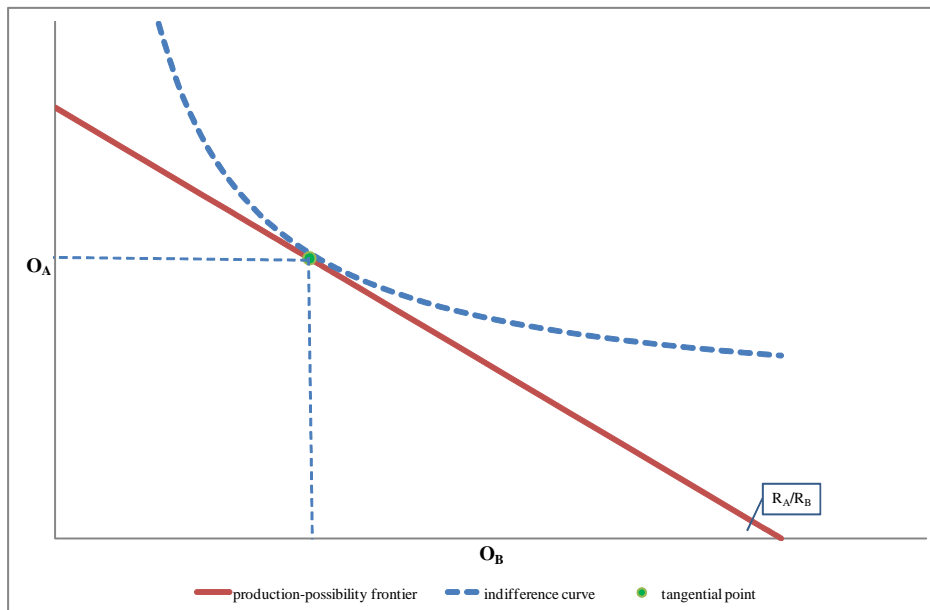

Figure 2: Alternative combinations of bread and wine open to the household sector in period$_2$

The household sector chooses one point on the production-possibility frontier, but we cannot say which one. The adduction of a set of indifference curves does not really help much. It merely tells us that the marginal rate of substitution, which is unknown, must be equal to the productivity ratio $\frac{R_A}{R_B}$ which in turn is inverse to the price ratio $\frac{P_A}{P_B}$ as we know from (14). It is therefore gratuitous to characterize the selected point as consumer optimum. *Any* point can be characterized as an optimum. All depends on how the unknown indifference curves are painted. This is a matter of fantasy, not of analysis.

With the arbitrary choice of the output combination all real and nominal variables are fixed under the conditions of market clearing, budget balancing, and zero profit for all firms. This leaves only the monetary side to be determined.

4 Transaction patterns and the need for credit

The transaction pattern in Figure 1 is quite simple because the households buy only bread. When they buy also wine, conditions change. It is at first assumed that the households make part payments for the wine that are evenly spread over $period_2$. Total consumption expenditures remain unaltered, they are now only distributed between firms *A* and *B*. Then the transaction pattern looks the same as in $period_1$. The wine maker is in the position to pay the monthly wages exactly as the bread maker does.

When the households pay their wine on delivery, though, the new transaction pattern looks like Figure 3. The current deposits of the households accumulate during $period_2$ because the former part payments to the wine maker are now saved. By consequence, the current overdrafts of the wine maker increase on account of the monthly wage payments. The time consuming production of the wine is at first financed by the transaction unit of the central bank. At New Year's Eve the households pay the wine in one amount and thereby the wine maker's overdrafts return to zero.
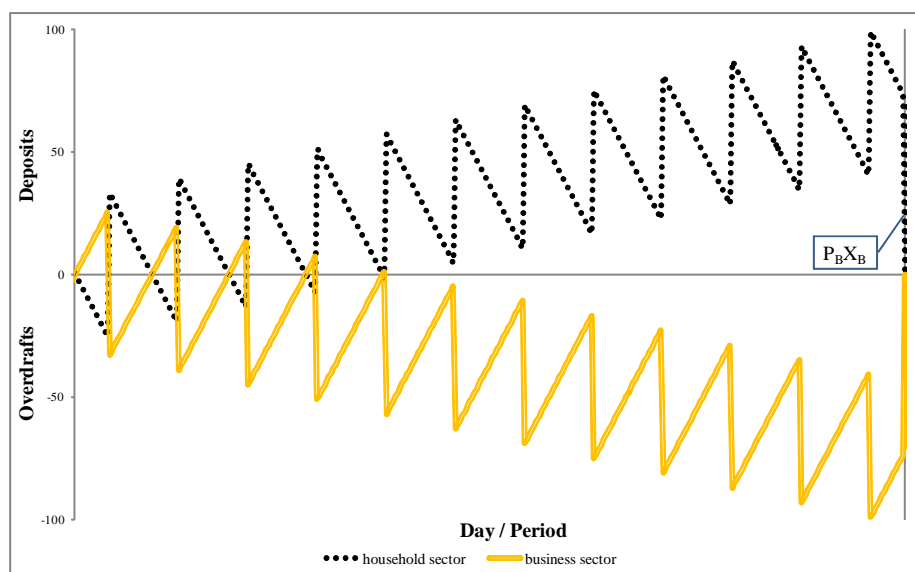

Figure 3: The households pay the wine on delivery at New Year's Eve

The saving of the households *during* $period_2$ necessitates the financing of the wine maker. Saving therefore uno actu *creates* the demand for credit of exactly equal amount. Hence demand and supply are *not* independent as they are assumed to be in partial analysis. The credit can take quite different forms. One rather unsophisticated alternative to overdraft financing is that the wine maker takes up a one-period loan at the central bank. The transaction pattern then changes as shown in Figure 4.
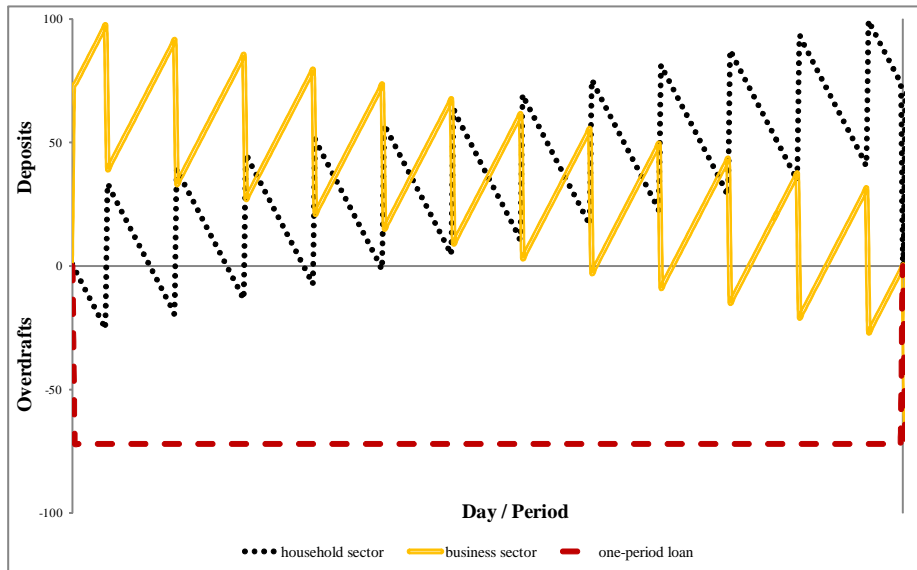
Figure 4: The wine maker's one-period loan is taken up at the beginning of period$_2$ and repaid when the households purchase the wine at New Year's Eve

To handle loans, a separate unit is established at the central bank. It has to be emphasized that only the central bank is capable of providing the one-period loan at the beginning of period$_2$. No classical capitalist stands ready to finance the wine maker's production process because none has as yet emerged from the pure consumption economy. Therefore, the loan has to be *created* and only the central bank can do this. The capitalist cannot be taken as historically given, he has to be analytically brought into being from scratch.

5 The banking unit and the rate of interest

The inclusion of the banking unit in period$_2$ entails that the given resources of the business sector L have at first to be reallocated:

$$L \equiv L_A + L_B + L_C \quad |2. \tag{16}$$

As a consequence total income (1) differentiates to:

$$Y = \underbrace{W_A}_{W} L_A + \underbrace{W_B}_{W} L_B + \underbrace{W_C}_{W} L_C + \underbrace{(D_A N_A + D_B N_B + D_C N_C)}_{Y_D = 0} \quad |2. \tag{17}$$

Total income does not change. It is assumed that labor input moves from firm A to firm C, hence the bread output diminishes. The concomitant increase in output

9

consists of loan processing services. Being not storable, there can be no difference between services produced O by the banking unit and services bought X, hence $O_C = X_C$. The banking unit sells a quantity of loan services, which is related to the stock of loans, at an interest rate l_C to firm B. The banking unit's profit follows from (5) as:

$$Q_{fiC} \equiv P_C X_C - W L_C \quad \Rightarrow \quad Q_{fiC} \equiv l_C \bar{A}_C - W L_C \quad |2. \tag{18}$$

The output O_C and the quantity bought from the banking unit X_C have to be adapted to the specifics of the banking business and can be set equal to the average loan stock $\bar{A}$ (for details see 2011a, Sec. 5). Default risk, collateral and other peculiarities of the lending business are neglected.

Firm B pays for the financial services, therefore its profit equation changes from (5) to:

$$Q_{fiB} \equiv P_B X_B - l_C \bar{A}_C - W L_B \quad |2. \tag{19}$$

By adding up (18) and (19) to total profits of the business sector interest cancels out. Changes of the interest rate therefore do not affect total profit but only the distribution of profits between firm A and B. From this follows that interest cannot be interpreted as a part of profit or basically the same thing as profit. This, though, was exactly what the classics did (Schumpeter, 1994, p. 925).

The reallocation of labor input is neutral with regard to the price of bread. When labor input L_C moves from firm A to C bread output falls. Since productivity and wage rate remain unaltered the bread price P_A remains steady according to (14).

The price of wine P_B goes up compared to (14). This follows from (19) under the condition that the profit of the wine maker is again zero.

$$P_B = \frac{W}{R_B}\left(\frac{L_C}{L_B} + 1\right) \quad \text{if} \quad l_C \bar{A}_C = W L_C \quad |2. \tag{20}$$

The wine price now depends also on the relation of labor inputs in the banking unit and the winery. When the banking business is small compared to the wine maker's business the wine price is close to unit wage costs. The labor input in the banking unit L_C depends on the volume of processed loans which is taken to be numerically equal with the stock of loans $\bar{A}_C$.

Consumption expenditures, which remain unchanged since $\rho_E = 1$ and income is constant, are redirected away from purchases of bread to purchases of the higher priced wine:

$$C = P_A X_A + P_B X_B \quad |2. \tag{21}$$

The quantity of bread X_A diminishes. The quantity of wine X_B is here kept constant and the wine price P_B increases. Total output of consumption goods is lower and the price level is higher. The households buy indirectly the financial services of the banking unit in the form of a higher price for the wine. Demand changes, expressed by the shift of consumption expenditures, and supply changes, expressed by the reallocation of labor, are perfectly symmetrical under the given conditions.

Compared to (13) the profit equations for the firms as given by (12), (19), and (18) become more differentiated:

$$Q_{fiA} = P_A R_A L_A \left(1 - \frac{W}{P_A R_A} \right) \quad \text{if} \quad \rho_{XA} = 1$$

$$Q_{fiB} = P_B R_B L_B \left(1 - \frac{W}{P_B R_B} \left(\frac{L_C}{L_B} + 1 \right) \right) \quad \text{if} \quad \rho_{XB} = 1 \quad |2. \quad (22)$$

$$Q_{fiC} = \mathsf{l}_C \bar{A}_C \left(1 - \frac{W L_C}{\mathsf{l}_C \bar{A}_C} \right) \quad \text{if} \quad \rho_{XC} = 1$$

The zero profit condition demands that the expressions in the brackets be zero. This then determines the relations of bread price, wine price, and rate of interest. The inclusion of the banking unit and the appearance of the rate of interest on loans results in a reallocation of demand and resources. The loan interest rate is determined by the production conditions of the banking unit and given by a modified form of unit labor costs:

$$\mathsf{l}_C = \frac{W}{\dfrac{\bar{A}}{L_C}} \quad \text{this compares to} \quad \mathsf{l}_C = \frac{W}{R_C^*} \quad |2. \quad (23)$$

The banking unit is not different from any other firm and the rate of interest is not different from any other price except for the dimension (for details see 2011a, Sec. 5). The rate of interest remains unaltered if labor input and the average stock of loans move in lockstep, in other words, if the loan processing productivity remains unaltered.

The production-possibility frontier follows from the 2nd axiom (2) and (16):

$$O_A = R_A (L - L_C) - \frac{R_A}{R_B} O_B \quad |2. \quad (24)$$

Compared to Figure 2 the inclusion of the banking unit amounts to a parallel shift of the frontier toward the origin as shown in Figure 5.

Because of the zero profit condition firm B does not bear the interest. Ultimately the households pay it in the form of a higher wine price. In real terms they pay
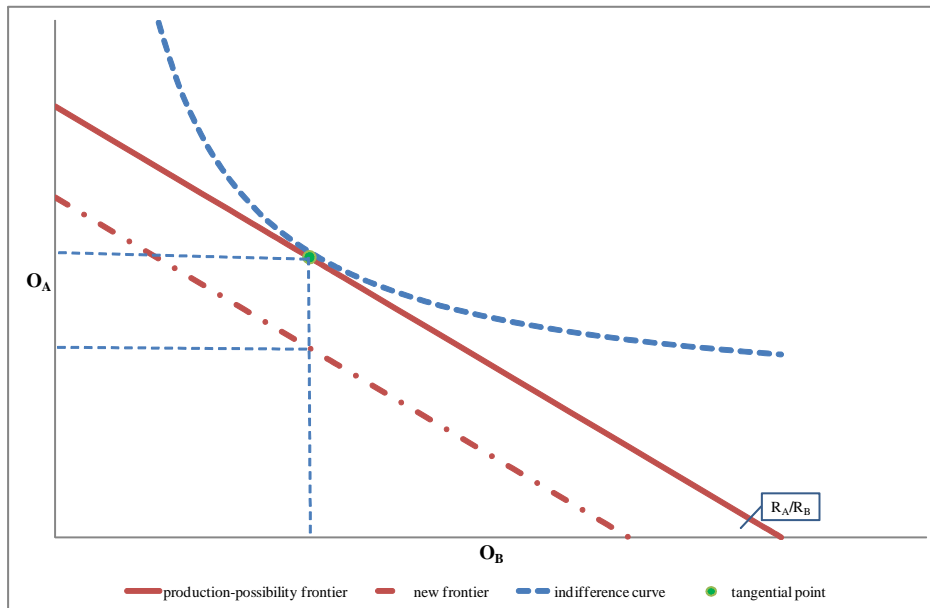
11

Figure 5: Forgone bread–wine output due to the reallocation of labor input to the banking unit

here in the form of less bread. Compared to the initial case of continuous part payments households are worse off. It can be taken for granted that the households cannot perceive the underlying connection between their transaction pattern, the rate of interest and the fall of bread output due to the reallocation of labor. The emergence of interest is in the last instance not effected by the time consuming production process of wine but by the divergence of wage payments and sales proceeds. These time divergences determine the financing requirement of the wine maker. The volume of loans processed per period in turn determines the labor input of the banking unit. The requisite diversion of labor input to the banking unit shifts the production-possibility frontier of bread and wine toward the origin. In Figure 5 only bread is reduced but it is obvious that any other combination of bread and wine on the production-possibility frontier is open to the household sector.

In sum: the rate of interest has not much to do with the production conditions of the wine maker yet with the transaction pattern of the household sector and the production conditions of the banking unit. Interest is, in the first instance, no compensation for waiting or abstinence but a compensation for the loan processing services of the banking unit. Under the conditions of market clearing, budget balancing and zero profit for all firms the rate of interest is determined by the productivity of the banking unit and it moves, given the productivity, with the wage rate. The same is true according to (22) for the prices of bread and wine. Relative prices are determined by the productivity ratios.

6 Lengthening the production process

In the foregoing it has been assumed that the households buy the wine output at the end of $period_2$. As an alternative the case is now considered that the households postpone their purchases until the end of $period_3$. The resulting new transaction pattern is depicted in Figure 6.
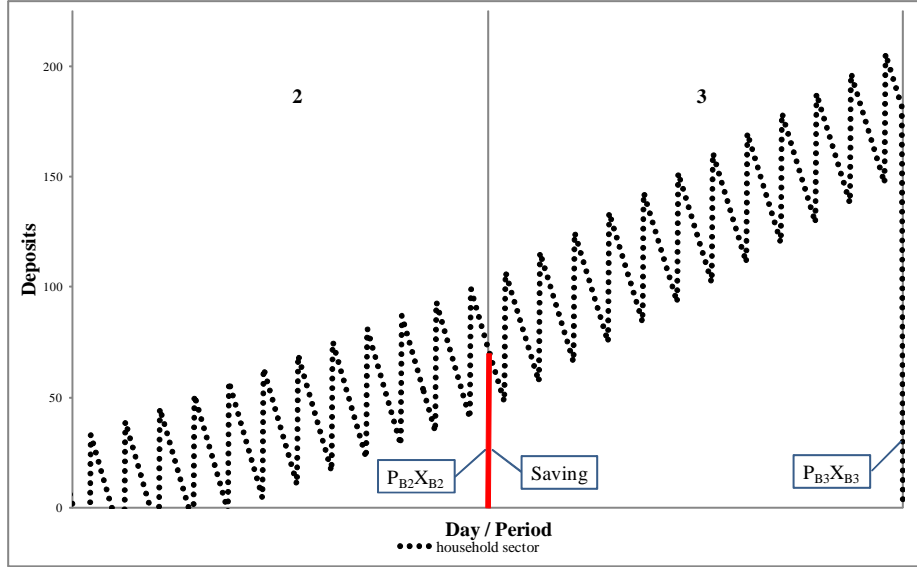

Figure 6: New transaction pattern in accordance with the lengthening of the production process

Not spending the whole period income means that the expenditure ratio ρ_E in (7) is less than unity in $period_2$. Financial saving is defined as the difference of income and consumption expenditures:

$$S_{fi} \equiv Y - C \equiv (1 - \rho_E)Y \quad \Rightarrow \quad S_{fi} = P_{B2}X_{B2} \quad |2. \tag{25}$$

Saving is here equal to the amount that has been initially earmarked for the wine purchase. Saving and the change of the household sector's stock of money are two aspects of the same flow residual (for details see 2012, Sec. 4).

With regard to profit the set of axioms is extended because additional variables have to be introduced (for details see 2012, Sec. 6). The 5th axiom states that total profit has a financial and nonfinancial component:

$$Q = Q_{fi} + Q_{nf} \quad |t. \tag{26}$$

Firm B's financial profit follows from (19) and it is negative:

13

$$Q_{fiB} = -\mathsf{I}_C \bar{\mathsf{A}}_C - WL_B \quad |2. \tag{27}$$

Nonfinancial profit consists in the value of the unsold stock of wine:

$$Q_{nf} = P_B X_B \quad |2. \tag{28}$$

Hence total profit as the sum of (27) and (28) is zero according to (19).

The winery makes no loss but faces a liquidity shortage. It is neither possible to redeem the one-period loan nor to pay the interest. The one-period loan has not only to be renewed but ramped-up by interest. In total the wine maker now has to finance $P_{B2}X_{B2}$ plus the wage payments in period$_3$, i.e. WL_{B3}.

The higher loan amount affords more labor input in the banking unit. It is again taken from firm A. The price of bread remains constant but total expenditures for bread $P_A X_A$ decline. The rate of interest remains unaltered because the productivity and the wage rate in the banking unit stay where they are.

Taken all effects together, the price of wine at the end of period$_3$ must be higher compared to the price at the end of period$_2$. This follows from (19) under the condition of zero profit. The price increase is given by:

$$P_{B3} - P_{B2} = \frac{\mathsf{I}_C \left(\bar{\mathsf{A}}_{C3} - \bar{\mathsf{A}}_{C2} \right)}{X_{B3}} \tag{29}$$

The higher wine price is due to the higher loan amount which includes the accrued interest of period$_2$ and therefore entails compound interest.[1] The required financing in the two periods is given by:

$$\bar{\mathsf{A}}_{C2} = WL_{B2}$$

$$\bar{\mathsf{A}}_{C3} = WL_{B2} \left(1 + \mathsf{I}_C \right) + WL_{B3} \tag{30}$$

The difference in (29) is then given by:

$$\bar{\mathsf{A}}_{C3} - \bar{\mathsf{A}}_{C2} = \mathsf{I}_C WL_{C2} + WL_{C3} \tag{31}$$

The zero-profit conditions of (22) apply also to period$_3$.

The households' decision for point (A) induces the wine maker to lengthen the production period in order to produce the two-period vintage. Thereby the quantity of the wine remains by assumption unchanged only the quality changes. The higher

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[1] Wicksell derived the rate of interest from the higher prices of older vintages (Wicksell, 1949, pp. 174-176). This amounts to a petitio principii.

price is necessary to recoup the wages of two periods and the interest. Interest in turn is equal to the wages in the banking unit under the condition of zero profit. In the final analysis interest resolves itself completely into wage income.

The household sector's decision at the beginning of period_3 is between wine of actual production and future production as shown in the nominal and real part of Figure 7.
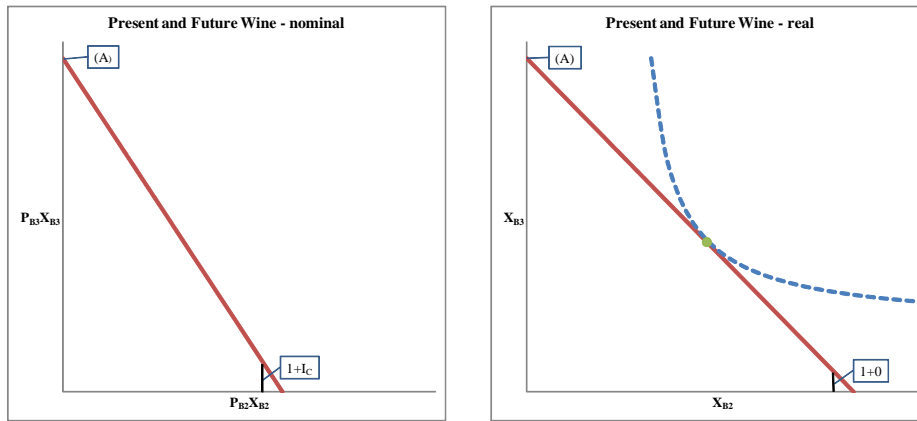
(a) The nominal rate of interest, here I_C, is given by the slope of the nominal intertemporal transformation line segment

(b) The implicit real rate of interest, here zero, is given by the slope of the real intertemporal transformation line segment

Figure 7: Comparison of the nominal and implicit real rate of interest in the case of constant productivity over two periods

The left part, 7a, shows the nominal trade-off between present and future wine. The slope of the transformation line represents the nominal rate of interest which can be derived from the respective values at the different points in time as:

$$I^{wine} \equiv \frac{P_{B3} X_{B3}}{P_{B2} X_{B2}} - 1 \equiv \frac{I_C \bar{A}_{C3} + W L_{B3}}{I_C \bar{A}_{C2} + W L_{B2}} - 1 \quad \Rightarrow \quad I^{wine} = I_C \quad |3. \quad (32)$$

The nominal rate of interest that is derived from the time indexed values of the wine is equal to the banking unit's rate of interest.

Since the quantity of wine does not change the real trade-off between X_{B2} and X_{B3} is represented by the 45° angle in Figure 7b. The implicit real rate of interest is therefore zero. An indifference curve has been adduced to establish the connection to Fisher's explanation of the rate of interest.

> Fisher completed the analysis by expressing the interplay of the three factors [future income, time preference, diminishing returns] in terms of a modern general equilibrium framework. Eighty years later, his solution may still be called definitive. (Niehans, 1994, p. 275)

Figure 7b is basically identical with Fisher's graphical representation (Niehans, 1994, p. 275, Figure 23.1) and only differs with regard to the inclusion of diminishing returns. Fisher's production frontier is convex but this additional assumption is of no consequence for the origination of the rate of interest. In the foregoing analysis it has been assumed that point (A) is realized but actually any point on the intertemporal line segment is possible.

Indifference curves are usually assumed to be asymmetrical with respect to a 45° line through the origin and this entails for time preference:

> A larger quantity today and a smaller quantity tomorrow is preferred to the smaller quantity today and the larger tomorrow. (Niehans, 1994, p. 276)

By taking all assumptions together one arrives at the conclusion:

> In pure logic it is easy, as Fisher shows, to construct situations in which the rate of interest is zero or even negative, but the asymmetry of both the indifference curves and the production frontier will almost invariably give rise to a positive rate of interest even under stationary conditions. (Niehans, 1994, p. 277)

It will be recalled that we have *assumed* that the quantity of wine X_B remains constant during period$_3$ and that only the quality changes. It is of course possible that the quantity diminishes somewhat in the process of fermentation. It this case the implicit real rate of interest would be negative. This, though, has no effect on the nominal rate of interest which is determined by (32). This rate is greater or at least zero but cannot become negative. It is vital to keep the concepts of nominal and implicit real interest strictly apart. The implicit real rate depends on the productivity in the winery R_{B3} whereas the nominal rate depends on the productivity of loan processing R_{C3} in the banking unit.

At the end of period$_3$ the financial profit of firm B is positive because of the dissaving of households, i.e. $\rho_E > 1$. Nonfinancial profit on the other hand is negative and consists of the nominal depreciation of the hitherto owned stock of wine. Total profit is again zero. Firm B is in the position to settle all its liabilities to the banking unit in one amount.

To allow a closer look at the role of productivity wine is finally replaced by firewood. Up to the beginning of period$_3$ there is no great difference to the production of wine. Only the output quantities and prices are diverse. This changes now insofar as it is assumed that the quality of firewood remains the same but the quantity doubles in the 3rd period. Under the conditions of market clearing, budget balancing and zero profit for all firms this productivity increase effects a lower price of firewood at period end. In nominal terms, though, there is no difference between Figure 8a and

7a. In real terms the household sector's hypothetical choice is between firewood now or the double quantity at next New Year's Eve or something in between as depicted in Figure 8b. The implicit real rate of interest is now greater than zero while it was exactly zero for the wine in Figure 7b.
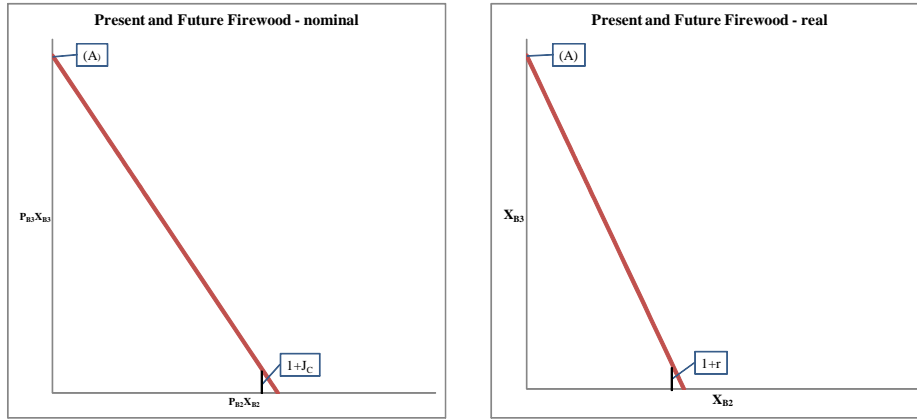
(a) The nominal rate of interest, here I_C, is given by the slope of the nominal intertemporal transformation line segment

(b) The implicit real rate of interest, here greater than zero, is given by the slope of the real intertemporal transformation line segment

Figure 8: Comparison of the nominal and implicit real rate of interest in the case of increasing productivity over two periods

The implicit real rate of interest as defined by (33) has obviously nothing in common with the loan rate of interest. The lengthening of the production process increases output due to the higher productivity in period$_3$:

$$ I^{real} \equiv \frac{X_{B3}}{X_{B2}} - 1 \quad \Rightarrow \quad I^{real} = \frac{R_{B3}}{R_{B2}} - 1 \quad \text{if} \quad L_{B3} = L_{B2}. \tag{33} $$

The nominal value of the output, though, is the same as in the case of wine. The productivity effect leads to a lower price of firewood in view of the given conditions. From this follows that the one-period vintage vanishes from the market after period$_3$ because its price is higher than that of the two-period vintage while the quality is the same. Firm *B* will reorganize its production schedule such that it is in the position to bring a two-period vintage to market in each period. Hence there is no longer any question of time preference for the households. However, for firm *B* the necessity to finance the production process does not vanish and therefore the loan rate of interest will stay above zero. The interest is embodied in the price of firewood as it was formerly embodied in the price of wine. The productivity effect of the lengthening of the production process affects the market price but not the rate of interest which is the same in Figures 8a and 7a. From the point of conceptual consistency it is important to note that the loan interest rate is a price for the services of the banking unit while definition (33) is about a natural growth rate. The formal similarity of

both rates has no deeper meaning. The relation between the real rate of interest and roundaboutness is the fulcrum of the 'Austrian habit of thought' (Laidler, 1999, p. 33-36) which, from the structural axiomatic viewpoint, misses the crucial point.

7 The rate of interest in the general case

Hitherto profit has been set to zero for all firms. This made it easy to determine the prices and the rate of interest. A positive profit for the economy as a whole leads to indeterminacy and opens the opportunity for profit redistribution among firms. Therefore, in order to eliminate all subjective elements and to determine the rate of interest objectively an additional assumption is required. The most suitable condition is profit ratio equalization.

The profit ratio of the banking unit follows from (18) and is defined as:

$$\rho_{QC} \equiv \frac{\mathsf{I}_C \bar{\mathsf{A}}_C}{W_C L_C} - 1 \quad \text{cond.} \quad \rho_{XC} = 1 \quad |t. \tag{34}$$

The profit ratio of the wine maker follows from (19) and is defined as:

$$\rho_{QB} \equiv \frac{P_B X_B}{\mathsf{I}_C \bar{\mathsf{A}}_C + W_B L_B} - 1 \quad \text{cond.} \quad \rho_{XB} = 1 \quad |t. \tag{35}$$

From the equalization of profit ratios (34)=(35) then follows the rate of interest:

$$\mathsf{I}_C = \frac{1}{\bar{\mathsf{A}}_C} \left(\sqrt{P_B R_B L_B W_C L_C + \left(\frac{1}{2} W_B L_B\right)^2} - \frac{1}{2} W_B L_B \right) \quad |t. \tag{36}$$

The rate of interest depends on the price of the final product P_B and on variables that refer to labor costs in both firms. Given these variables the rate of interest moves with the square root of the price of the final product under the conditions of market clearing and equal profit ratios.

What remains to be determined is the price for the final product. To eliminate the interdependencies that arise from profit ratio equalization among three firms the analysis is focused on the interdependencies between the wine maker and the banking unit. Consumption expenditures therefore are no longer split up between two firms but go entirely to the wine maker:

$$C = \underbrace{P_A X_A}_{0} + P_B X_B \quad |t. \tag{37}$$

From the axiom set and (6) then follows under the condition of market clearing:

18

$$P_B = \rho_E \left(1 + \rho_D\right) \frac{W}{R_B} \frac{L}{L_B}$$

$$(38)$$

$$L \equiv L_B + L_C \qquad W \equiv \frac{1}{L} \left(W_B L_B + W_C L_C\right) \qquad \rho_{XB} = 1, \rho_{XC} = 1 \quad |t.$$

The price of the final product depends on the expenditure ratio ρ_E, the distributed profit ratio ρ_D, unit factor costs and the relative size of firm B. The expenditure ratio is here equal for wage income and distributed profit. Inserted in (36) this finally yields for the rate of interest under the condition of equal profit ratios:

$$I_C = \frac{1}{\bar{A}_C} \left(\sqrt{\rho_E \left(1 + \rho_D\right) W L W_C L_C + \left(\frac{1}{2} W_B L_B\right)^2} - \frac{1}{2} W_B L_B \right) \quad |t. \qquad (39)$$

In the general case the rate of interest depends on the demand for the final product, which is determined by the expenditure ratio and the distributed profit ratio, and on variables that refer to labor costs in both firms. Given these variables and the distributed profit ratio the rate of interest decreases with saving, i.e. $\rho_E < 1$. The case of $\rho_E > 1$ has to be excluded because dissaving of the household sector means that the business sector makes a profit, which facilitates self-financing, and that the household sector becomes the borrower. Household sector borrowing is analytically different from business sector borrowing (for details see 2011a, Sec. 5). Since the relation between higher saving and a lower rate of interest appears at first sight commonsensical it is worth emphasizing that it follows without reference to the time-honored conception of supply and demand in the market for loanable funds (Mill, 2006, p. 647). The interrelation is here established by the condition of profit ratio equalization. In the limiting case of overall zero-profit, i.e. $\rho_E = 1$ and $\rho_D = 0$, the rate of interest depends alone on the labor costs of both firms.

Profit ratio equalization, though, is not a 'law' but at best a tendency. When this tendency is not very strong more household saving translates one-to-one into a higher demand of the business sector for loans. If it is satisfied profits are redistributed in favor of the banking unit with the result that its profit ratio exceeds that of the wine maker. From (34) and (35) the lower and upper bounds for the rate of interest are derived as:

$$I^{min} = \frac{W_C L_C}{\bar{A}_C} \qquad I^{max} = \frac{P_B X_B - W_B L_B}{\bar{A}_C} \quad |t. \qquad (40)$$

Reality will be found somewhere between this concrete values.[2]

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[2] "The economists have embarked on a fishing expedition in the hyperspace of possible worlds. The trouble is that they have not caught any fish with the theoretical line. The activity works as science only when it gets actual numbers to fish in. But economic speculation does not use actual numbers. It makes qualitative arguments, such as existence theorems." (McCloskey, 1994, p. 141)

Since an expenditure ratio $\rho_E < 1$ implicates a financial loss for the business sector as a whole, stability in the pure consumption economy requires an expenditure ratio of unity. In this case total profit is determined by the distributed profit ratio. Under the conditions of budget balancing and profit ratio equalization the rate of interest moves, given the production conditions in both firms, with the square root of the distributed profit ratio within the boundaries given by (40).

8   The classical capitalist and the notion of factor income

The banking unit's profit is in no way different from the profit of any other firm. This profit is either distributed to the household sector or retained. Let us assume for the moment full profit distribution then the households receive from the banking unit wage income in their capacity as workers and distributed profits in their capacity as owners. The same applies to the winery. The interest that the wine maker pays to the banking unit is in this limiting case completely resolved into wages and *distributed* profits (cf. Smith, 2008, p. 57). The same applies to the revenues of the winery. Interest is not an elementary income category.

It has been assumed hitherto that the wine maker's operations were completely financed by the banking unit of the central bank, hence the classical capitalist was nonexistent. It is no problem, though, to fit this checkered economic character into the given analytical frame. First of all the capitalist's roles have to be separated. The operation of lending entails a certain amount of administrative work that gives rise to wage income. The role of the worker has to be separated from the role of the pure capitalist. The capitalist proper is the owner of a firm that lends a certain amount of money for a definite term. This firm makes a profit or loss like any other firm. The distributed profit of the firm is the income of the pure capitalist. There is no difference to the ownership of any other firm. The capitalist's income does not consist of interest but of distributed profit. Hence there is no need to provide a justification for interest, not more in any case than to provide one for the revenues of the winery. The rationale of distributed profits of the banking unit or the analytically identical case of the pure capitalist is the same as for any other firm; it consists of the ownership of the firm, however legally defined (Ellerman, 1986). Interest, therefore, is no factor income (cf. Godley and Lavoie, 2007, p. 264). In the limiting case of zero profit for each firm the rate of interest is positive and interest is equal to the wage income of the banking unit. In this case profit is zero and by consequence distributed profit too, that is, the income of the pure capitalist is zero. The existence of a positive rate of interest can in this case obviously not be explained by psychological factors like waiting or abstinence. The rate of interest is entirely determined by structural conditions. Interest has to be deleted from the list of factor incomes.

The profit ratio is the general concept because it is also applicable to a pure consumption economy without credit or capital. Defining the profit ratios (34) and

(35) meant relating profit to costs. The capitalist, though, may calculate differently. Assumed that profits are fully distributed he may relate distributed profit to the amount of money that has been lent and that is here equal to the firm's equity. This profit rate is therefore conceptually different from the profit ratio. Profit ratio equalization, by consequence, is not the same thing as profit rate equalization. While the firm's equity remains unchanged the expected dividends and an apposite discount rate determine, in principle, the price of the firm's share. Profit rate equalization, understood as the relation of dividend to share price, is therefore effected by the adaptation of the share price.

According to (39) the interest rate depends on distributed profit. Yet distributed profit is on the other hand the income of capitalists. Since the expenditure ratio is unity in the viable pure consumption economy the capitalists fully spend all distributed profit income. Their capital remains constant. In the final analysis the intricate circular relationship between interest and distributed profit as well as between profit and distributed profit is self-supporting at any level. For the economy as a whole there exists no relation between profit, the rate of interest and the productivity effect of a lengthening of the production process. For the individual firm this relation exists but it is a logical mistake to jump from the conditions of the individual firm to the economy as a whole.

9   Conclusions

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. The present paper suggests three non-behavioral axioms as groundwork for the clarification of the logical origin of the rate of interest. The main results of the structural axiomatic analysis are:

- The rate of interest has not much to do with the production conditions of the wine maker yet with the transaction pattern of the household sector and the production conditions of the banking unit. Interest is no compensation for waiting or abstinence but a compensation for the loan processing services of the banking unit.

- Under the conditions of market clearing, budget balancing and zero profit for all firms the rate of interest is determined by the productivity of the banking unit.

- The implicit real rate of interest can be negative. This, though, has no effect on the nominal rate of interest which is greater or at least zero. The implicit real rate depends on the productivity of the consumption goods producing firm whereas the nominal rate depends on the productivity of loan processing

in the banking unit. The implicit real rate of interest has nothing in common with the loan rate of interest.

- The productivity effect of the lengthening of the production process affects the market price of the final product but not the rate of interest.

- In the general case with non-zero profit for the economy as a whole the rate of interest depends on the demand for the final product, which is determined by the expenditure ratio and the distributed profit ratio, and on variables that refer to labor costs in both firms.

- Under the conditions of budget balancing and profit ratio equalization the rate of interest moves, given the production conditions in both firms, with the square root of the distributed profit ratio.

- Interest is not an elementary income category. The classical capitalist's income does not consist of interest but of distributed profit.

Both, the classical and neoclassical theories of interest are incomplete. The structural axiomatic approach enables an comprehensive analysis that covers the real, the nominal, and the monetary aspects and provides actual numbers for the lower and upper bound of the rate of interest.

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