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# Silence is Golden: Communication, Silence, And Cartel Stability\*

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## Abstract

This paper studies how cartel stability is influenced by asymmetric information and communication about demand. Firms in a cartel face fluctuating demand in a repeated game framework. In each period, one randomly chosen firm knows current demand. In this context we consider two different equilibria – one where the informed firm communicates its information to its partners and another where it does not. We show that cartels are extremely unstable when the informed firm communicates with the uninformed firms. However, when the informed firm does not communicate with the uninformed firms cartels can be as stable as when there are no demand fluctuations at all.

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In fact, this least controversial area of antitrust may well be the one for which economists have the least satisfactory theoretical models of how illegal activity – talking about prices (and “reaching an agreement”) matters. – Michael D. Whinston

## 1 Introduction

In markets not subject to random shocks, collusion turns out to be very easy for cartels to maintain. It therefore seems that random shocks, such as demand fluctuations, are necessary to model realistic levels of cartel instability (Rotemberg and Saloner, 1986). However, if information is symmetric, cartels still turn out to be stable, unless such fluctuations are very large. Thus, some additional challenges to coordination, such as asymmetric information, may be necessary to model plausible levels of cartel instability.

It is natural to suppose that coordination in the face of such asymmetric information would be facilitated by communication. But is it? In this paper we explore the role of communication in facilitating collusion when there is asymmetric information about demand. We model an infinitely repeated Cournot game with  $n$  firms and asymmetric information about market conditions. Demand fluctuates randomly from period to period. In each period one firm, chosen randomly, knows more about the state of demand than the others. The firms must then decide whether or not they should communicate to coordinate production decisions.

We first consider an equilibrium where the informed firm communicates with the uninformed firms through a trade association, say (Vives 1990).<sup>1</sup> The objective of this communication is to let uninformed firms know the current state of demand, so the firms can divide up the market evenly each period. In this situation we show that asymmetric information significantly amplifies the effect

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<sup>1</sup>This is consistent with Hay and Kelley (1974) and Fraas and Greer (1977), who find that collusion is often facilitated by information exchange, through a trade association, or some other channel (see e.g. Hay and Kelley p. 21).

of demand fluctuations in increasing cartel instability since, in high-demand states, the informed firm can lie as well as cheat.

Next we consider an equilibrium where the informed firm does not communicate with the other firms. In this case, since the informed firm can no longer lie to the uninformed firms, cartels become more stable. In fact, cartels turn out to be not only as stable as when there is no asymmetric information, they actually become as stable as when there are no demand fluctuations at all. Thus, we show that information asymmetry may increase cartel instability if firms communicate, but may actually cancel out the effects of demand fluctuations themselves if firms do not communicate. Intuitively, when there is no communication then, when the informed firm is most tempted to cheat – when demand is high – its output is high enough to cancel out this higher temptation to cheat.

Note, however, that in this paper communication is not actually necessary for the cartel to achieve efficiency, since the informed firm can adjust its own output to maximize cartel profits. It would therefore be interesting to consider other environments where communication is more important. For example, if more than one firm is informed, communication may be necessary to allow informed firms to coordinate with each other to maximize cartel profits. The results of this paper then suggest that such information structures may make cartels more unstable.

This paper contributes to a small but growing literature on the challenges faced by cartels which use communication to help them coordinate a collusive agreement in the face of asymmetric information. Major previous results in this literature include folk theorems in general repeated games with communication (Compte, 1998, Kandori and Matsushima, 1998; see also the symposium on repeated games with asymmetric information in the January 2002 issue of

the *Journal of Econometric Theory*). However, folk theorems focus on agents whose discount factors approach one. They therefore do not allow us to study the effect of asymmetric information on collusion between firms which are very patient, but whose discount factors are bounded away from one. Folk theorems are therefore an important, but blunt, instrument for measuring the effects of asymmetric information on cartel instability.

Papers focusing on repeated game collusion between asymmetrically informed firms include Aoyagi (2002), Athey and Bagwell (2001), and Athey, Bagwell, and Sanchirico (2004). Aoyagi (2002) considers a Bertrand-like model, where firms observe private demand signals after choosing their own prices. Thus, communication is not used to adjust production to fluctuations in demand, but only to help distinguish random demand fluctuations from shifts due to cheating by collusive partners. In Athey and Bagwell (2001), communication helps cartel members coordinate their responses to cost fluctuations observed before choosing prices, so production can be allocated to low cost firms. Athey, Bagwell and Sanchirico (2004) also focus on cost fluctuations, though they briefly consider publicly observed demand fluctuations as well. In addition, this later paper does not allow communication.

Thus, none of these papers consider the problem of coordinating collusive agreements in the face of asymmetric information about demand fluctuations. They also do not consider the possibility that the act of communication itself may be crucial to cartel stability.

Moreover, the above papers focus primarily on the problem of hard-to-detect “on schedule” deviations, in which one type of player simply pretends to be a different type of player. Thus, a player’s private information never becomes public in these models. While this assumption is plausible for the cost shocks considered by Athey and Bagwell (2001) and Athey Bagwell and Sanchirico

(2004), they may be less plausible for demand shocks. In fact, the *demand* shocks considered by Athey, Bagwell, and Sanchirico (2004) are fully public.

This paper, by contrast, focuses on coordination in the face of demand fluctuations where information is initially private but eventually becomes public. This allows us to focus on the implications of asymmetric information in a context where the only deviations cartel members must worry about are easier to detect “off schedule” deviations. Thus we can avoid the significant technical difficulties involved in the imperfect private monitoring literature. In particular, this simplification allows us to measure the *quantitative* effect of asymmetric information and communication on cartel instability among firms with discount factors bounded away from one.

Note also that the information sharing in this paper is different from that in, e.g., Vives (1984) and related papers. In that literature, information is verifiable, whereas we are assuming that information in our model is not verifiable until the next period.<sup>2</sup> Thus, the only reason why an informed firm would tell the truth in our model is the hope of future cooperation. On the other hand, the possibility of lying enhances the incentive to cheat. Of course, in the equilibrium where firms do not communicate, the issue of verifiability is irrelevant.

There is an ongoing debate about the empirical plausibility of the Rotemberg-Saloner framework, which is the starting point of our analysis.<sup>3</sup> Some of this debate concerns the behavior of cartels over the business cycle. However, this debate ignores the role of asymmetric information or communication in cartel stability, choosing instead to focus entirely on correlations between prices and business cycles. Since the focus of our paper is on asymmetric information

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<sup>2</sup>As suggested by Vives (1990, p. 414), this other literature must assume verifiability since otherwise informed firms will always give in to the temptation to lie. In our model, firms trade off the temptation to lie against the gains from being trusted in the future.

<sup>3</sup>This literature includes Porter (1983), Rotemberg and Saloner (1986), Domowitz, Hubbard and Petersen (1986), Hajivassiliou (1989), Town (1991), Rotemberg and Woodford (1992), Ellison (1994), Borenstein and Shepard (1996), Chevalier and Scharfstein (1996), and Suslow (1998) among others.

about demand fluctuations, and since firms are likely to be equally informed about the macroeconomy, we are primarily concerned with individual market fluctuations uncorrelated with business cycles. Thus, the above debate regarding the Rotemberg and Saloner framework may not be particularly relevant for our paper.

Section 2 describes the basic game. It also examines the collusive trigger strategy equilibrium when the informed firm communicates with the uninformed firms, including the critical interest rate above which collusion breaks down. Section 3 shows that, when firms communicate, asymmetric information generally explains much more cartel instability than demand fluctuations alone. Numerical simulations also show that this effect can be quite dramatic. Section 4 looks at the game described in Section 2, but modifies the trigger strategy by assuming that firms no longer communicate. We show that if firms do not communicate then cartels become as stable as when there are no demand fluctuations at all. Section 5 concludes.

Before proceeding, we should emphasize that, to facilitate computation we make very specific assumptions about functional forms – specifically linear demand and total cost curves, and a two point distribution of the demand shock. This is consistent with our goal of computing ball-park estimates of the quantitative effects of asymmetric information and communication. However, it should be noted that our most striking result – that in the absence of communication cartels are as stable as if there are no demand fluctuations at all – will not hold exactly in more general settings. It should, however, still be approximately true. Future work should investigate how sensitive this result is to the exact functional forms we consider. Indeed, it would be very interesting if, for some functional forms, asymmetric information, without communication, turned out to make cartels more stable than the no-fluctuations case!

## 2 Payoffs and Credible Trigger Strategies When Information is Shared

Consider an infinitely repeated  $n$ -firm Cournot oligopoly, where market demand is either high or low, with demand fluctuating independently across periods. Let the inverse market demand function be

$$p(Q) = a - Q$$

where  $a$  is a random variable. In the high-demand state the intercept term is  $a_H$  and in the low-demand state the intercept term is  $a_L$ , with  $a_H > a_L$ . The demand curves are linear for simplicity. All firms know that the probability of high demand is  $\phi$  and the probability of low demand is  $1 - \phi$ . In each period one firm is informed about  $a$  and  $n - 1$  firms are uninformed. However, all firms know the parameter values  $a_H$  and  $a_L$ , though uninformed firms do not know the current state of demand. The identity of the informed firm fluctuates independently from period to period, with each firm equally likely to be chosen as that period's informed firm. The informed firm is told current demand but not future demand. The other  $n - 1$  firms know the identity of the current informed firm, but do not know current or future demand.<sup>4</sup> As part of a collusive agreement, the firm that happens to be informed in a period may convey information about the state of demand to the other firms. However, the informed firm may also lie to the other firms. Nevertheless, all firms learn previous demand, so any lying by the informed firm can be detected with a one period lag.

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<sup>4</sup>This information structure is chosen to allow for the possibility that different firms are informed in different periods. Thus there is no "leading" firm that is always informed. Having said that, the fundamental results of this paper do not change if there is a leading firm that is always informed.

However, note that the information structure is restrictive in other ways. For example, to the extent that demand fluctuations follow the business cycle, information about these fluctuations may tend to be more symmetric. It would be interesting to see if any new insights could be gained from considering more complicated information structures.



Next, to reduce notation, assume that production of the good is costless. With constant marginal cost  $c$ , we could replace  $a_H$  with  $a_H - c$  and  $a_L$  with  $a_L - c$  in the equations below. Finally, assume that all firms discount using the discount factor  $1/(1+r)$ , where  $r$  is the interest rate.

Consider a collusive agreement in which the informed firm in each period informs the other firms about the state of demand – through a trade association, say. Specifically consider the following strategy profile. In each period the informed firm truthfully reveals the state of demand to the other firms. The firms then divide the monopoly output equally among themselves. If any firm deviates in any period, all firms revert to the non-cooperative one-shot equilibrium for all periods thereafter. The question is then, for what values of  $r$  is the strategy profile an equilibrium. The cartel is then unstable if the strategy profile is only an equilibrium if firms are very patient, i.e. if  $r$  is low.

Section 2.1 below determines the expected per-period payoff to each of the firms if they collude. The expected payoffs during the non-cooperative punishment phase are derived in Section 2.2. Section 2.3 derives the one-period payoff to the informed firm given that it lies and cheats on the collusive agreement at the expense of the others when demand is high. Section 2.4 derives the discounted expected payoff to each firm over time and the critical interest rate above which full collusion is not possible.

## 2.1 Expected Payoffs Per Period if the Firms Collude

If the firms collude, the informed firm in each period tells the uninformed firms the state of demand and together they divide the monopoly output and profits equally among themselves. Thus, each firm produces  $a_H/2n$  in the high demand state and  $a_L/2n$  in the low demand state, so the payoff to each firm is  $a_H^2/4n$  in the high demand state and  $a_L^2/4n$  in the low demand state. Expected per-period

payoff to any firm if the collusive agreement holds is therefore

$$\pi^{COLL} = \phi \frac{a_H^2}{4n} + (1 - \phi) \frac{a_L^2}{4n} \quad (1)$$

where the superscript COLL stands for “collusive”.

## 2.2 Strategies and Expected Per Period Payoffs in the Punishment Phase

If any firm deviates from the collusive agreement the industry reverts to a permanent non-cooperative phase.<sup>5</sup> In the punishment phase the firms do not communicate since the uninformed firms no longer trust the informed firms. Thus, uninformed firms do not know demand in a particular period, though they do know the probability of high or low demand in any period. This yields a Bayes-Nash equilibrium in each period. In this equilibrium the quantity produced by each uninformed firm is given by

$$q^{NC,U} = \frac{\phi(a_H - a_L) + a_L}{n + 1} \quad (2)$$

where the superscript NC stands for “noncooperative,” and U stands for “uninformed.” This output is independent of whether demand is high or low, since uninformed firms do not know the state of demand.

When demand is high the informed firm produces

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<sup>5</sup>Note that we do not use optimal punishment strategies of the sort discussed in Abreu (1986) or Abreu, Pierce, and Stachetti (1986). It would be interesting to extend the analysis of optimal punishment strategies to environments such as this one, with asymmetric information about game payoff functions. However, this would add considerable complexity to our analysis. Also, it would probably not change the qualitative results much. When firms communicate, our results are driven by the increased temptation to cheat in situations where the cheater can also lie about the state of the world, and this effect would remain. When firms do not communicate, our results are driven by the fact that under collusion, the informed firm meets all of the additional demand in high-demand states. In this case too, it should not matter much whether punishment is optimal or not. Note that Athey and Bagwell (2001) assume Nash reversion punishments in their numerical examples. Aoyagi (2002) also assumes Nash reversion punishment strategies.

$$q_H^{NC,I} = \frac{(n+1)a_H - (n-1)(\phi(a_H - a_L) + a_L)}{2(n+1)} \quad (3)$$

where the superscript I stands for “informed.” Finally, when demand is low the informed firm produces

$$q_L^{NC,I} = \frac{(n+1)a_L - (n-1)(\phi(a_H - a_L) + a_L)}{2(n+1)}. \quad (4)$$

Using  $p_H = a_H - (n-1)q^{NC,U} - q_H^{NC,I}$  and  $p_L = a_L - (n-1)q^{NC,U} - q_L^{NC,I}$  shows that the market price in the punishment phase with high demand is  $p_H = q_H^{NC,I}$ , and with low demand,  $p_L = q_L^{NC,I}$ . The expected payoff to an uninformed firm in a given period of the punishment phase is therefore given by

$$\pi^{NC,U} = \phi q^{NC,U} q_H^{NC,I} + (1 - \phi) q^{NC,U} q_L^{NC,I} \quad (5)$$

and the expected profit to a firm if it is informed is given by

$$\pi^{NC,I} = \phi (q_H^{NC,I})^2 + (1 - \phi) (q_L^{NC,I})^2. \quad (6)$$

Now, in any period, there is a  $1/n$  chance of a firm being informed and an  $(n-1)/n$  chance of it being uninformed. Therefore, the ex ante expected payoff to the firm in the punishment phase, before it knows whether it is informed, is given by

$$\pi^{NC} = \frac{1}{n} \pi^{NC,I} + \frac{n-1}{n} \pi^{NC,U}. \quad (7)$$

### 2.3 A Lying Cheating Informed Firm

In each period that the collusive agreement is supposed to be in place (including the one in which cheating occurs), the informed firm makes a statement to the

uninformed firms about the state of the market. The informed firm may tell the truth or lie, but the uninformed firms believe the informed firm unless the agreement has been broken previously.

With low demand, the informed firm has less incentive to lie, and so less incentive to cheat. We therefore focus on the high demand situation. If demand is high, but the informed firm cheats, it will also tell the other firms that demand is low. Thus, the uninformed firms produce their share of the low demand monopoly output. This is  $a_L/2n$  per firm, or  $(n-1)a_L/2n$  in the aggregate.

The informed firm's problem is to maximize profits given this output of the uninformed firms. Thus, the cheating informed firm produces

$$q_H^{CH,I} = \frac{2na_H - (n-1)a_L}{4n} \quad (8)$$

where CH stands for "cheating." The payoff to the cheating informed firm is then given by

$$\pi_H^{CH,I} = \left( \frac{2na_H - (n-1)a_L}{4n} \right)^2. \quad (9)$$

Of course, the uninformed firms can cheat too. However, since they cannot lie or take advantage of high demand, their temptation to cheat is lower than the informed firm's. Thus they do not affect the critical point at which collusion becomes unstable.

## 2.4 The Decision to Cooperate and the Critical Interest Rate.

We have modeled a situation where the informed firm might lie and cheat on a collusive agreement. In the equilibrium we are considering, if a firm cheats in one period, then, starting in the next period, all firms forever enter a non-

cooperative phase. In this subsection we find the maximum (critical) interest rate consistent with a credible trigger strategy that maintains full collusion. In other words we calculate the rate of return on investment in collusion for an informed firm in a high demand state.

Let  $r$  be the interest rate firms use to calculate the present value of future profits. Thus  $r$  measures the patience of a firm in terms of its willingness to wait for future profits. The higher the rate of interest, the less important is the future expected stream of collusive profits and thus the greater the relative allure of cheating today.

The expected payoff to each firm if collusion is maintained in this and all future periods, given that current demand is high, is

$$\frac{a_H^2}{4n} + \frac{1}{r}\pi^{COLL} \quad (10)$$

where  $\pi^{COLL}$  is defined in (1). Expected present and future payoff to a lying, cheating, informed firm, given that current demand is high, is

$$\pi_H^{CH,I} + \frac{1}{r}\pi^{NC} \quad (11)$$

where  $\pi_H^{CH,I}$  and  $\pi^{NC}$  are defined in (9) and (7) respectively. Thus, the informed firm is willing to supply truthful information and cooperate if and only if

$$\frac{a_H^2}{4n} + \frac{1}{r}\pi^{COLL} \geq \pi_H^{CH,I} + \frac{1}{r}\pi^{NC}. \quad (12)$$

This inequality reflects the fact that, for a trigger strategy to be credible, the expected present discounted payoff from colluding must be greater than or equal to that from cheating. It follows that full collusion is possible through this equilibrium if and only if

$$r \leq r_{asym}^* = \frac{\pi^{COLL} - \pi^{NC}}{\pi_H^{CH,I} - \frac{a_H^2}{4n}} \quad (13)$$

where  $\pi^{COLL}$ ,  $\pi^{NC}$ , and  $\pi_H^{CH,I}$  are defined in (1), (7), and (9), respectively. Proposition 1 yields a formula for the critical interest rate,  $r_{asym}^*$ . The formal proof is in the Appendix.

**Proposition 1** *The maximum interest rate consistent with the above trigger strategy, with asymmetric information and communication is*

$$r_{asym}^* = \frac{4n(n-1)(\phi(a_H - a_L) + a_L)^2}{(n+1)^2(n(2a_H - a_L)^2 - a_L^2)}. \quad (14)$$

This is the maximum value of  $r$  that will allow our trigger strategy to be credible. In other words,  $r_{asym}^*$  is the rate of return on full cooperation for the situation when cheating is most profitable (i.e., when the cheating firm is informed and demand is high). Any higher value of  $r$  will make cheating relatively more attractive by discounting the expected future profits from collusion too much. This would weaken the threat of a trigger strategy. Thus, if  $r > r_{asym}^*$ , the firms cannot maintain the symmetric, joint-profit maximizing level of collusion. Note that setting  $a_H = a_L$  in Proposition 1 simplifies (14) to  $r_{asym}^* = r_0^* = 4n/(n+1)^2$ . This, as expected, is the interest rate consistent with a credible trigger strategy if there are no demand fluctuations and consequently no information asymmetry.

### 3 The Quantitative Effects of Information Asymmetry on Cartel Stability Under Communication.

In our model there are two factors that lead to greater cartel instability. The first factor is randomly fluctuating demand (Rotemberg and Saloner, 1986), and the second is the temptation to lie to take advantage of asymmetric information. Together, these factors reduce the return to cooperation much more than do demand fluctuations alone, as shown in Proposition 2 and Table 1 below. Proposition 2 focuses on the limiting case of small fluctuations. The numerical simulations in Table 1 then illustrate the result in Proposition 2. The proof of Proposition 2 is in the Appendix.

**Proposition 2** *As  $a_H$  approaches  $a_L$ , the fraction of the fall in  $r_{asym}^*$  attributable to asymmetric information approaches the proportion*

$$\lim_{a_H \rightarrow a_L} \frac{r_{sym}^* - r_{asym}^*}{r_0^* - r_{asym}^*} = \frac{n + 1}{2n - (n - 1)\phi} \quad (15)$$

where  $r_0^*$  is the critical interest rate in the absence of demand fluctuations,  $r_{sym}^*$  is the critical interest rate with symmetric (full) information about current demand, and  $r_{asym}^*$  is the critical interest rate when the information structure is asymmetric.

The denominator,  $r_0^* - r_{asym}^*$ , in Proposition 2 measures the total fall in the critical interest rate due to demand fluctuations and asymmetric information. The numerator,  $r_{sym}^* - r_{asym}^*$ , measures the fall in this critical interest rate due to asymmetric information alone. The ratio thus gives the proportion of the total fall in this critical rate due to asymmetric information.

Table 1 shows this limiting proportion for a wide range of parameter values. First, for most values of  $\phi$  and  $n$ , information asymmetry explains much more cartel instability than demand fluctuations alone. For example, for two firms and a 50-50 chance of high versus low demand, asymmetric information explains over 85% of the total reduction in cartel stability, with demand fluctuations alone explaining less than 15%. Thus, while uncertainty alone might not explain much cartel instability, uncertainty plus asymmetric information may go a long way towards explaining why cartels are not uniformly successful.

$\phi$	$n = 2$	$n = 5$	$n = 10$
0.1	76.92%	62.5%	57.59%
0.2	78.95%	65.22%	60.44%
0.5	85.71%	75%	70.97%
0.8	93.75%	88.24%	85.94%
1	100%	100%	100%

Table 1: Limiting Proportion of the Fall in the Critical Interest Rate Due to Asymmetric Information

Notice that as  $\phi$  rises, the proportion of the fall in the critical interest rate due to asymmetric information rises. In fact, according to Proposition 2, for  $\phi \approx 1$ , asymmetric information is almost the only reason  $r_{asym}^*$  falls.

We noted earlier that there were two reasons why greater demand fluctuations lead to a fall in the critical interest rate  $r_{asym}^*$ . First, the benefit from cheating in high demand states today rises compared to the expected benefits of future cooperation, even when there is no informational asymmetry. This is the point made by Rotemberg and Saloner (1986). Second, with asymmetric information, the informed firm is tempted, in the high demand state, to lie to the uninformed firms, and tell them that demand is low. Now, as  $\phi$  rises, the high demand state becomes more likely in the future, so the expected reward to future cooperation rises relative to cheating profits. This reduces the first



reason to cheat. Thus, as  $\phi$  rises, less of the fall in  $r_{asym}^*$  is attributable to the first, Rotemberg and Saloner, reason for cheating. This leaves only the second reason, i.e. the informed firm's ability to lie under asymmetric information. Ultimately, if high demand is almost certain, then the only reason why  $r_{asym}^*$  falls below  $r_0^*$  is because the informed firm can lie. Thus lying can be a powerful source of cartel instability when informed firms are expected to share their information. In other words, asymmetric information about demand may help a lot in explaining cartel instability.

However, asymmetric information loses some of its power to explain cartel instability as the number of firms in the cartel rise. Nevertheless, even when  $n$  is very large, asymmetric information always explains at least the fraction  $1/(2 - \phi)$  of the fall in  $r^*$ , according to the approximation in Proposition 2. Note that this fraction exceeds one half for all  $\phi > 0$ . Thus, even in extreme cases, asymmetric information explains at least half of the fall in  $r_{asym}^*$  in the limit. In other words, asymmetric information always contributes significantly to the effect of demand fluctuations on cartel instability.

We next illustrate this effect for the general case where  $a_H$  does not approach  $a_L$ . Recall from Section 2 that the  $a$  parameters can be interpreted as  $a_H - c$  and  $a_L - c$ , for the case in which marginal cost is a non-zero constant  $c$ . A useful measure of demand fluctuations is then  $(a_H - c)/(a_L - c)$ , which we call the "fluctuation ratio." To get a sense of what this fluctuation ratio means, note that, for a demand intercept  $a$ , and marginal cost  $c$ , the markup of price over marginal cost is  $(a - c)/2$ . Thus, when the fluctuation ratio is 2.00, this means that, if we move from the low-demand state to the high-demand state, the monopoly markup over marginal cost doubles. Tables 2 and 3 below treat the cases of symmetric and asymmetric information, respectively, and report the critical interest rate for various values of the fluctuation ratio, and various

values for the total number of firms,  $n$ . We assume in these tables that the probability,  $\phi$ , of the high demand state is  $\phi = 0.50$ .<sup>6</sup>

$\frac{a_H - c}{a_L - c}$	$n = 2$	$n = 5$	$n = 10$
1	88.9%	55.6%	33%
1.2	75.3%	47.1%	28%
1.4	67.1%	41.9%	25%
1.6	61.8%	38.6%	23%
1.8	58.2%	36.3%	21.6%
2	35.6%	34.7%	20.7%

Table 2: Critical Interest Rate (Symmetric Information)

$\frac{a_H - c}{a_L - c}$	$n = 2$	$n = 5$	$n = 10$
1	88.9%	55.6%	33%
1.2	36.8%	30.6%	19.6%
1.4	23.4%	21%	13.6%
1.6	17.3%	16.2%	10%*
1.8	13.9%	13.3%	8.9%*
2	11.8%	11.4%	7.7%*

Table 3: Critical Interest Rate (Asymmetric Information)

To interpret these tables, note that, whether the interest rate is annual or semiannual, say, depends on the production period and/or seasonal demand patterns. Thus, for industries where demand has a steep seasonal pattern, due to high Christmas sales, for example, then we can interpret our periods as years. For other industries, a period might be six months or three months or whatever. In these cases a given  $r$  from the table might represent an annual interest rate

<sup>6</sup>There is one problem here. If the number of firms and the fluctuation ratios are both large, then the non-cooperative punishment strategy derived in Section 2.2 above requires the informed firm to produce a negative quantity when demand is low. When this happens, we solve the model again subject to the constraint that the informed firm cannot produce negative amounts. The calculations are omitted, since they add no new insights, though they are available upon request. The critical interest rates marked with an asterisk have been calculated with this constraint binding.

which is roughly two times larger (for six month periods) or four times larger (for three month periods) and so on.

Now, the first rows of Tables 2 and 3 show that, in the absence of demand fluctuations ( $a_H = a_L$ ), even relatively large cartels are *extremely* stable. Even with 10 firms in the industry, for example, full collusion is possible with an interest rate of 33%. Thus, the question ceases to be “how are cartels possible” and becomes, “why aren’t more industries cartelized?”

The other rows of Table 2 provide a possible partial explanation for cartel instability. Fluctuations in demand put pressure on collusive agreements even when information is symmetric. That is, demand fluctuations lower the critical interest rate, and so, make collusion somewhat more difficult. For a two-firm cartel, a fluctuation ratio of 2 yields a critical interest rate of about 35.6%, a drop of 53.3 percentage points from the critical interest rate of 88.9% corresponding to no fluctuations. When there are ten firms in the industry, the critical rate falls less dramatically, from 33.0% to 20.7%, or 12.3 percentage points. Thus, even with large fluctuations in demand, relatively large cartels remain surprisingly stable. Demand fluctuations by themselves are therefore incapable of explaining plausible levels of cartel instability.

Introducing asymmetric information, however, as in Table 3, dramatically increases cartel instability, as we would expect from Proposition 2 and Table 1. Thus, with a fluctuation ratio of 2 and symmetric information, a five-firm cartel can maintain a collusive agreement with an interest rate of 35%. However, if we add asymmetric information, with the same fluctuation ratio, the maximum interest rate consistent with full collusion falls to 11.4%. This underscores the potential of asymmetric information to explain cartel instability. Once we move to asymmetric information, demand fluctuations have the potential to dramatically increase cartel instability. This happens because the temptation to cheat

is greatest when demand is high, and in this situation, the cheating firm can also lie to mislead the other firms into thinking that demand is low.

Of course, these calculations assume very specific forms for the demand curves, costs, and information structure. In addition, it would be interesting to compare these fluctuation ratios to demand fluctuations in various actual markets. Finally, the demand fluctuations here must be understood as those which the uninformed firms cannot predict. The unpredictable components of demand fluctuations will generally be smaller than the total fluctuations in demand. Nevertheless, even with smaller fluctuation ratios of 1.4 and 1.6, asymmetric information has a significant potential to explain why successful cartels are not a universal phenomenon, as is clear from Tables 2 and 3.

Communication, however, plays a critical role in these results. So the obvious next question is, do the same results hold when firms do not communicate? That is, are cartels more or less stable when firms do not communicate? To answer this question we next focus on an equilibrium with no communication.

## **4 Cartel Stability When the Informed Firm Does Not Communicate**

Recall that in our game a cartel faces demand fluctuations and exactly one randomly chosen member of the cartel actually knows the state of demand. In Sections 2 and 3 we focused on an equilibrium trigger strategy where the informed firm communicates the current state of demand to the uninformed firms. We found that cartels become less stable for two reasons. First, when demand is high, the informed firm knows that the benefit from cheating today is large compared to the expected rewards of future cooperation. Second, asymmetric information gives the informed firm an incentive to lie. Thus, communication

may actually *intensify* the destabilizing effect of demand fluctuations because it gives the informed firm an opportunity to lie. The question then becomes what happens if firms do not communicate? In this section we focus on an equilibrium trigger strategy where firms do not communicate.

Recall that in our game one firm knows the state of demand in any given period while the other firms do not. Moreover, the identity of the informed firm changes independently from period to period, with each firm being equally likely to be chosen as that period's informed firm. This section considers an equilibrium where the informed firm does not communicate. Thus the key difference between the current equilibrium and the equilibrium derived in Section 2 is the lack of communication between firms. While colluding, an uninformed firm produces a quantity ( $q^U$ ) that depends only on expected demand, while the informed firm produces a level of output based on actual demand. Note that, since informed firms do not communicate, they cannot lie. This removes one source of cartel instability.

Moreover, when demand is high, the informed firm gets to increase its own output a great deal in response to the increase in demand, even if it does not cheat, since the uninformed firms do not increase their output in response to the high demand. This further reduces its incentive to cheat. In fact this means that demand fluctuations no longer cause *any* cartel instability. It therefore turns out that, when firms do not communicate, cartels are not only as stable as when there is no asymmetric information, but they are as stable as if there are no demand fluctuations at all.

In the sections below we derive the critical interest rate,  $r_H^I(q^U)$  below, at which an informed firm is willing to fully collude in the high-demand state. Note that this critical interest rate depends on the quantity,  $q^U$ , that the typical uninformed firm is supposed to produce in equilibrium. We also derive the

critical interest rate  $r_L^I(q^U)$  for the informed firm in the low-demand state, and the critical interest rate  $r^U(q^U)$  for the typical uninformed firm. We then show that all three functions,  $r_H^I(q^U)$ ,  $r_L^I(q^U)$ , and  $r^U(q^U)$ , actually cross at the critical interest rate  $r_0^*$  from the no fluctuation case. This allows us to prove our key result, Proposition 3 below.

Section 4.1 below determines the current and expected future collusive payoffs to both types of firm. Section 4.2 determines the cheating payoff to the informed firm, both when current demand is high and when it is low. We also derive the cheating profits for an uninformed firm in this section. Section 4.3 reiterates the punishment payoffs to the informed and uninformed firms from Section 2 above. In Section 4.4 we derive the discounted expected payoffs from cheating and colluding for the informed and uninformed firms. We then use these payoffs to obtain the critical interest rate, as a function of  $q^U$ , at which each type of firm, in each possible situation, is willing to follow the trigger strategy. This derivation leads us to Proposition 3, which proves that, without communication, cartels are as stable as when there are no demand fluctuations at all. Unless otherwise stated, all notation here follows the conventions introduced earlier. However, to distinguish this equilibrium from the equilibrium in Section 2, we also include a subscript NT (no talking or communication) wherever relevant.

#### 4.1 Expected Per Period Payoffs When Firms Collude

If the firms collude then the informed firm makes a production decision based on its knowledge of the state of demand, while each uninformed firm produces  $q^U$ . Thus, when current demand is high the informed firm maximizes industry profits by producing  $a_H/2 - (n-1)q^U$ . The market price in this case is  $a_H/2$ . Thus, the payoff to the informed firm given high demand is

$$\pi_{H,NT}^{COLL,I} = \left( \frac{a_H}{2} - (n-1)q^U \right) \frac{a_H}{2}. \quad (16)$$

Similarly, in the low demand case the informed firm produces  $a_L/2 - (n-1)q^U$  and the market price is  $a_L/2$ . Thus the payoff to the informed firm from colluding when demand is low is

$$\pi_{L,NT}^{COLL,I} = \left( \frac{a_L}{2} - (n-1)q^U \right) \frac{a_L}{2}. \quad (17)$$

Equations (16) and (17) imply that the expected per period payoff to the informed firm from colluding is

$$\pi_{NT}^{COLL,I} = \phi \pi_{H,NT}^{COLL,I} + (1-\phi) \pi_{L,NT}^{COLL,I}. \quad (18)$$

The uninformed colluding firm does not know if demand is high or low. When colluding it merely agrees to produce  $q^U$ . The expected per period payoff to the uninformed firm is then

$$\pi_{NT}^{COLL,U} = \phi q^U \frac{a_H}{2} + (1-\phi) q^U \frac{a_L}{2}. \quad (19)$$

Note here that firms do not communicate as part of the collusive equilibrium. Thus in any given period there is only a  $1/n$  chance that a firm knows demand, even if firms collude. Thus, in any future period the expected collusive profits are

$$\pi_{NT}^{COLL} = \frac{1}{n} \pi_{NT}^{COLL,I} + \frac{n-1}{n} \pi_{NT}^{COLL,U}. \quad (20)$$

## 4.2 Cheating Payoffs

When cheating, the informed firm maximizes current profits given that the uninformed firms continue to produce  $q_j^U$ . When demand is high the informed firm produces  $\frac{a_H - (n-1)q_j^U}{2}$ . The market price then is also equal to this quantity. Thus the cheating payoff to the informed firm is

$$\pi_{H,NT}^{CH,I} = \left( \frac{a_H - (n-1)q^U}{2} \right)^2. \quad (21)$$

Similarly, if demand is low then the cheating payoff to the informed firm will be

$$\pi_{L,NT}^{CH,I} = \left( \frac{a_L - (n-1)q^U}{2} \right)^2. \quad (22)$$

An uninformed firm may cheat as well. However, since this uninformed firm does not know the state of demand, it maximizes expected profits given that demand may be high or low. Moreover, its optimizing decision is made given that the other uninformed firms continue to produce  $q^U$ , and the one informed firm continues to produce its collusive output. Thus a cheating uninformed firm turns out to produce  $\frac{\phi(a_h - a_l) + a_l + 2q^U}{4}$ . Its expected market price is also equal to this quantity. Thus the cheating payoff to the uninformed firm is

$$\pi_{NT}^{CH,U} = \left( \frac{\phi(a_h - a_l) + a_l + 2q^U}{4} \right)^2. \quad (23)$$

## 4.3 Punishment Payoffs

In the punishment phase both types of firm revert to the asymmetric information Cournot equilibrium. Thus the payoffs in the punishment phase are the same as the payoffs presented in Section 2.2.



#### 4.4 The Trigger Strategy That Ensures Cooperation

We continue to consider an equilibrium where, if a firm cheats in one period, then, all firms produce the non-cooperative outputs forever afterwards. In this section we calculate the rate of return from collusion for an informed firm that knows that current demand is high. The calculations for the other cases are very similar.

Proposition 3 below then yields a formula for  $r_{NT}^*$ , the maximum interest rate consistent with a credible trigger strategy that maintains full collusion in the absence of communication.

An informed firm's expected discounted payoff from colluding, given that it knows that current demand is high, is

$$\pi_{H,NT}^{COLL,I} + \frac{1}{r}\pi_{NT}^{COLL} \quad (24)$$

where  $\pi_{H,NT}^{COLL,I}$ , and  $\pi_{NT}^{COLL}$  are defined in equations (16) and (20) respectively.

The expected present and future payoffs to a cheating informed firm is

$$\pi_{H,NT}^{CH,I} + \frac{1}{r}\pi^{NC}, \quad (25)$$

where  $\pi_{H,NT}^{CH,I}$  and  $\pi^{NC}$  are defined in equations (21) and (7) respectively. The informed firm will then cooperate only if

$$r \leq r_H^I(q^U) = \frac{\pi_{NT}^{COLL} - \pi^{NC}}{\pi_{H,NT}^{CH,I} - \pi_{H,NT}^{COLL,I}}. \quad (26)$$

Notice that  $r_H^I$  is a function of  $q^U$ . Similarly we can derive  $r_L^I(q^U)$  and  $r^U(q^U)$  as functions of  $q^U$  as well. In Appendix A.3.1 we prove that  $r_H^I(q^U)$  and  $r_L^I(q^U)$  are identical. Thus, calling their common value  $r^I(q^U)$  we have

$$r_H^I(q^U) = r_L^I(q^U) = r^I(q^U). \quad (27)$$

That is, the incentive to cheat is independent of whether current demand is high or low. We use these derivations to prove Proposition 3.

**Proposition 3** *Given asymmetric information about current demand, when firms do not communicate, the maximum interest rate consistent with a credible trigger strategy is  $r_{NT}^* = r_0^* = \frac{4n}{(n+1)^2}$ .*

**Proof.** In Appendix A.3.1 we first prove that  $r_H^I(q^U) = r_L^I(q^U)$  for all  $q^U$ . Thus, we represent both functions by  $r^I(q^U)$ , so  $r_H^I(q^U) = r_L^I(q^U) = r^I(q^U)$ .<sup>7</sup> Now, the informed firm will cheat for any interest rate higher than  $r^I(q^U)$ . Similarly, the uninformed firms will cheat for any interest rate higher than  $r^U(q^U)$ . This means that if the market interest rate is above  $r^U(q^U)$  but below  $r^I(q^U)$  then the uninformed firm will cheat and the collusion will fail. Similarly if the market interest rate is above  $r^I(q^U)$  but below  $r^U(q^U)$  then the informed firm will cheat and collusion will fail. Thus, the function  $r(q^U) = \min(r^I(q^U), r^U(q^U))$  gives the maximum interest rates for which both uninformed and informed firms will collude, as a function of  $r(q^U)$ . In other words, firms in the cartel are willing to collude if and only if the market interest rate lies below  $r(q^U)$ . Now, Appendix A.3.1 also proves that  $r^I(q^U)$  decreases as  $q^U$  increases, and Appendix A.3.2 shows that  $r^U(q^U)$  increases as  $q^U$  increases. Thus, the maximum market interest rate at which full collusion is stable,  $r_{NT}$ , must be where  $r^I(q^U) = r^U(q^U)$ . In Appendix A.3.3 we show that  $r^I(q^U) = r^U(q^U) = r_0^*$  for  $q^U = \frac{\phi(a_H - a_L) - a_L}{2n}$ . Thus, the maximum value of  $r_{NT}$  for which all firms in the cartel

<sup>7</sup>The difference between current cheating profits for the informed firm and its current collusive profits is the same in the high and low demand cases. Thus, the informed firm's incentive to cheat stays the same when current demand is high or low. This precise result seems to be an artefact of our assumptions of constant marginal costs and linear demand curves.

will collude is determined by setting  $q^U = \frac{\phi(a_H - a_L) - a_L}{2n}$  in  $r^I(q^U)$  and  $r^U(q^U)$ . This substitution gives us  $r_{NT}^* = r_0^* = \frac{4n}{(n+1)^2}$ .<sup>8</sup> QED. ■

Intuitively, part of the reason cartels with asymmetric information and communication are unstable is because the informed firm has an incentive to lie about the state of demand to the other, uninformed firms. However, if firms do not communicate they cannot lie, which increases cartel stability. Of course, the informed firm's incentive to cheat is also higher in the high demand state. However, if firms do not communicate then the informed firm will, under the collusive agreement, produce much more in the high demand state, while the uninformed firms produce a constant quantity based on average demand. As a consequence, the informed firm is able to counter its increased incentive to cheat by producing more when demand is high. Thus, even when demand is high, the informed firm's increased incentive to cheat is exactly countered by an increased reward from collusion. These two opposing effects therefore lead to a critical interest rate that is the same as when  $a_H = a_L$ , i.e., the same as when there are no demand fluctuations at all. Thus, demand fluctuations have no impact on cartel stability when firms do not communicate.

The main result here, that demand fluctuations have no impact on cartel stability when firms do not communicate, is of course in part a consequence of the specific assumptions we have made, e.g., linear demand curves. However, the general point – that collusion is very stable when firms do not communicate – should be quite robust. Future research may determine how sensitive this result is to different specifications of demand, information structures, and other aspects of the model.

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<sup>8</sup>Proof for this statement is provided in appendix A.3.3.

## 5 Conclusion

In this paper we show that asymmetric information decreases cartel stability when firms communicate, but may actually increase cartel stability when they do not. This is a surprising result in an area that has hitherto not received much attention. Whinston (2006 p. 26), for example, acknowledges that most economists have not cared to investigate how communication affects cartel stability “because they believe (as I do) that direct communication...often will matter for achieving cooperation.” Our results, by contrast, suggest that cartels may, in fact, often prefer *not* to communicate.

The results here, apart from shedding some light on a gap in the literature, may also have significant policy implications. For example, antitrust enforcement would have to take into account the possibility that collusion may not depend on formal communication, even in stochastic environments with asymmetric information, where one would expect that communication is needed to achieve coordination. This in turn might affect the types of legal investigations which should be made by antitrust authorities. For example, the Justice Department should expect collusive communication to occur primarily in markets where communication actually *is* needed for coordination.

Our paper may also have implications for how empiricists look at data. Thus empirical work that looks at the underlying factors that help or hinder collusion should focus on the information structure of a suspected cartel in order to determine whether collusion is likely to depend on communication between cartel members.

Of course several caveats to our argument are in order. First, our model is very simple. We assume very simple functional forms, and a very simple information structure. It may be useful, for example, to compare our model’s information structure to the information structure in Green and Porter (1984).

Second, the punishment strategy we assume is also very simple. If one allowed for optimal punishment strategies (see Abreu, 1986 and Abreu, et al., 1986), then cartels would presumably be more stable. The issue of asymmetric punishments (Segerstrom, 1985) also needs to be considered. Third, iid demand shocks might drive some of our results. If current strong demand implies stronger demand in the next period (Haltiwanger and Harrington, 1991) then the informed firm's willingness to lie and cheat may be offset by higher collusive profits tomorrow.

Finally, it would be interesting to develop models where firms *must* communicate in order to coordinate efficiently. For example, if firms have upward sloping marginal cost curves, then optimal collusion will require informed and uninformed firms to all produce similar quantities. This would require communication, which would reduce the cartel's stability. Similarly, if more than one firm is informed then our paper suggests that cartels might be more unstable if these informed firms need to communicate to determine who knows what. Investigating different information structures such as this would be an important avenue for future research.

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## A Appendix

### A.1 Proof of Proposition 1

The rate of return from cooperation,  $r_{asym}^*$ , is given in (13). The components of  $r_{asym}^*$  are  $\pi^{COLL}$ ,  $\pi_H^{CH,I}$ , and  $\pi^{NC}$ . Below we first calculate  $\pi^{NC}$ . Then we find  $\pi^{COLL} - \pi^{NC}$  and  $\pi_H^{CH,I} - a_H^2/4n$ . Throughout let

$$A_\phi = \phi a_H + (1 - \phi)a_L = \phi(a_H - a_L) + a_L.^9 \quad (\text{A1})$$

From equation (7),  $\pi^{NC}$  depends on  $\pi^{NC,U}$  and  $\pi^{NC,I}$ . To find  $\pi^{NC,U}$ , substitute (2), (3) and (4) into (5) to get

$$\begin{aligned} \pi^{NC,U} = \frac{\phi A_\phi}{n+1} \left( \frac{(n+1)a_H - (n-1)A_\phi}{2(n+1)} \right) \\ + \frac{(1-\phi)A_\phi}{n+1} \left( \frac{(n+1)a_L - (n-1)A_\phi}{2(n+1)} \right). \end{aligned} \quad (\text{A2})$$

After considerable simplification this becomes

$$\pi^{NC,U} = \frac{A_\phi^2}{(n+1)^2}. \quad (\text{A3})$$

Next, to get  $\pi^{NC,I}$ , substitute equations (3) and (4) into equation (6), obtaining

$$\begin{aligned} \pi^{NC,I} = \phi \left( \frac{(n+1)a_H - (n-1)A_\phi}{2(n+1)} \right)^2 \\ + (1-\phi) \left( \frac{(n+1)a_L - (n-1)A_\phi}{2(n+1)} \right)^2. \end{aligned} \quad (\text{A4})$$

After simplification this becomes

$$\pi^{NC,I} = \phi \frac{a_H^2}{4} + (1-\phi) \frac{a_L^2}{4} - \frac{(n-1)(n+3)A_\phi^2}{4(n+1)^2}. \quad (\text{A5})$$

Combining (A3) and (A5) in (7) we get

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<sup>9</sup>Notice this is merely the intercept of the expected demand curve.

$$\pi^{NC} = \phi \frac{a_H^2}{4n} + (1 - \phi) \frac{a_L^2}{4n} - \frac{(n-1)^2 A_\phi^2}{4n(n+1)^2}. \quad (\text{A6})$$

Combining (1) and (A6) we get

$$\pi^{COLL} - \pi^{NC} = \frac{(n-1)^2 A_\phi^2}{4n(n+1)^2}. \quad (\text{A7})$$

This gives the numerator of equation (13).

Now we turn to the denominator of equation (13), where we subtract  $a_H^2/4n$  from the cheating profits in (9). This gives

$$\pi_H^{CH,I} - \frac{a_H^2}{4n} = \left( \frac{2na_H - (n-1)a_L}{4n} \right)^2 - \frac{a_H^2}{4n} = \frac{n-1}{16n^2} ((2a_H - a_L)^2 n - a_L^2). \quad (\text{A8})$$

Finally, substituting (A7) and (A8) into (13) gives

$$r_{asym}^* = \frac{\text{Equation (A7)}}{\text{Equation (A8)}} = \frac{4n(n-1)A_\phi^2}{(n+1)^2((2a_H - a_L)^2 n - a_L^2)}. \quad (\text{A9})$$

## A.2 Proof of Proposition 2.

To prove proposition 2 we need to first find  $r_{sym}^*$  and  $r_0^*$ . To find  $r_{sym}^*$  we need to compare the expected payoff from cheating with the expected payoff from colluding under symmetric information.

When information about demand is symmetric the cheating profit in the high demand state is

$$\pi_{sym}^{CH} = \left( \frac{(n+1)a_H}{4n} \right)^2. \quad (\text{A10})$$

There is no difference between monopoly profits for the symmetric information and the asymmetric information cases. However, the (noncooperative) profit in

the punishment phase when demand is high is

$$\pi_{H,sym}^{NC} = \left( \frac{a_H}{n+1} \right)^2. \quad (\text{A11})$$

When demand is low it is

$$\pi_{L,sym}^{NC} = \left( \frac{a_L}{n+1} \right)^2. \quad (\text{A12})$$

Thus, using (A11) and (A12), expected punishment profits for each firm when information is symmetric is

$$\pi_{sym}^{NC} = \phi \left( \frac{a_H}{n+1} \right)^2 + (1-\phi) \left( \frac{a_L}{n+1} \right)^2. \quad (\text{A13})$$

Using these results, the return to collusion is

$$r_{asym}^* = \frac{\pi^{COLL} - \pi_{sym}^{NC}}{\pi^{CH} - \frac{a_H^2}{4n}}. \quad (\text{A14})$$

Substituting (1), (A10), and (A13) into (A14) yields

$$r_{sym}^* = \frac{4n}{(n+1)^2} \left( \phi + (1-\phi) \frac{a_L^2}{a_H^2} \right) \quad (\text{A15})$$

Let  $\varepsilon = \frac{a_H - a_L}{a_L}$ . Then, plugging  $a_H = (1 + \varepsilon)a_L$  into (A15) and taking the first order linear approximation of  $r_{sym}^*$  around  $\varepsilon = 0$  gives

$$r_{sym}^{*LE} = \frac{4n}{(n+1)^2} - \frac{8n(1-\phi)}{(n+1)^2} \varepsilon. \quad (\text{A16})$$

Similarly, plugging  $a_H = (1 + \varepsilon)a_L$  into (14) and taking the first order linear approximation of  $r_{asym}^*$  around  $\varepsilon = 0$  gives

$$r_{asym}^{*LE} = \frac{4n}{(n+1)^2} - \frac{4n}{(n+1)^2} \left( \frac{4n - 2\phi(n-1)}{n-1} \right) \varepsilon. \quad (\text{A17})$$

Also when there is no demand fluctuation, and therefore no scope for informational asymmetry in our model, the return to cooperation becomes (setting  $a_H = a_L$  in (A15))

$$r_0^* = \frac{4n}{(n+1)^2}. \quad (\text{A18})$$

Now the approximate proportion of the fall in the return to cooperation due to asymmetric information is

$$\frac{r_{sym}^{*LE} - r_{asym}^{*LE}}{r_0^* - r_{asym}^{*LE}} \quad (\text{A19})$$

Substituting (A16), (A17) and (A18) into (A19), gives  $\frac{n+1}{2n-(n-1)\phi}$ . This proves Proposition 2.

### A.3 Supporting Material for the Proof of Proposition 3.

We have derived  $r_H^I(q^U)$  in equation (26). We can similarly derive  $r_L^I(q^U)$  and  $r^U(q^U)$ . In what follows we first show that  $r_H^I(q^U)$  and  $r_L^I(q^U)$  are equal and then show that their common value,  $r^I(q^U)$ , is decreasing in  $q^U$ . Then we show

that  $r^U(q^U)$  is increasing in  $q^U$ . This allows us to prove Proposition 3 in the main text.

**A.3.1 Proof that  $r_H^I(q^U)$  and  $r_L^I(q^U)$  are equal and decreasing in  $q^U$ .**

Equation (26) gives us  $r_H^I(q^U)$ . Replacing  $\pi_{H,NT}^{COLL,I}$  and  $\pi_{H,NT}^{CH,I}$  in equation (26) with  $\pi_{L,NT}^{COLL,I}$  and  $\pi_{L,NT}^{CH,I}$  from equation (17) and (22) respectively gives us  $r_L^I(q^U)$ . The informed firm will therefore honor the collusive agreement when current demand is low iff

$$r \leq r_L^I(q^U) = \frac{\pi_{NT}^{COLL} - \pi^{NC}}{\pi_{L,NT}^{CH,I} - \pi_{L,NT}^{COLL,I}}. \quad (\text{A20})$$

Notice that equation (26) for  $r_H^I(q^U)$  and (A20) for  $r_L^I(q^U)$  differ only in denominator. In what follows we first calculate the denominators for (26) and (A20) separately and show that they are equal. We then derive the common numerator.

When demand is high current collusive profit to the informed firm is given by equation (16) and cheating profit to the informed firm is given by equation (21). Substituting these values into the denominator in (26) gives us

$$\begin{aligned} \pi_{H,NT}^{CH,I} - \pi_{H,NT}^{COLL,I} &= \left(\frac{a_H - (n-1)q^U}{2}\right)^2 - \left(\frac{a_H}{2} - (n-1)q^U\right)\frac{a_H}{2} \\ &= \frac{1}{4}(q^U)^2(n-1)^2. \end{aligned} \quad (\text{A21})$$

Similarly, when demand is low then the current collusive profit to the informed firm is given by equation (17) and cheating profit to the informed firm is given by equation (22). Substituting these values into the denominator of (A20) also gives us (A21). Thus, both the numerator and the denominator of (26) are identical to those of (A20). In other words,  $r_L^I(q^U) = r_H^I(q^U)$ . Thus,

we represent both  $r_H^I(q^U)$  and  $r_L^I(q^U)$  by  $r^I(q^U)$  and equations (26) and (A20) can be replaced by

$$r^I(q^U) = \frac{\pi_{NT}^{COLL} - \pi^{NC}}{\frac{1}{4}(q^U)^2(n-1)^2}. \quad (\text{A22})$$

We now derive the numerator of (A22). First we substitute equations (16) through (19) into equation (20) to give us expected collusive profits in the future in terms of  $\phi$ ,  $a_H$ ,  $a_L$ , and  $n$ . On the other hand,  $\pi^{NC}$  is defined in (A6). Upon simplification we get

$$\begin{aligned} \pi_{NT}^{COLL} - \pi^{NC} &= \frac{1}{4n} (a_i^2 + \phi(a_H^2 - a_L^2)) - \phi \frac{a_H^2}{4n} - (1 - \phi) \frac{a_L^2}{4n} + \frac{(n-1)A_\phi^2}{4n(n+1)^2} \\ &= \frac{(n-1)^2 A_\phi^2}{4n(n+1)^2} \end{aligned} \quad (\text{A23})$$

Thus, after substituting (A23) in (A22) we get

$$\begin{aligned} r^I(q^U) &= \frac{\frac{(n-1)^2 A_\phi^2}{4n(n+1)^2}}{\frac{1}{4}(q^U)^2(n-1)^2} \\ &= \frac{A_\phi^2}{n(q^U)^2(n+1)^2} \end{aligned} \quad (\text{A24})$$

Notice that  $q^U$  only appears in the denominator of (A24).<sup>10</sup> The numerator of (A24) is positive and the denominator rises as  $q^U$  rises. Therefore  $r^I(q^U)$

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<sup>10</sup>IS THIS FOOTNOTE EVEN NECESSARY NOW???????????

decreases as  $q^U$  rises.

### A.3.2 Proof that $r^U(q^U)$ is increasing in $q^U$ .

Equation (19) gives the current expected collusive profit to an uninformed firm and equation (23) gives the expected cheating profit to an uninformed firm. Further, recall that (A20) and (A6) give the expected future collusive profits and the expected future punishment profits for all firms. Thus, substituting these values into

$$r^U(q^U) = \frac{\pi_{NT}^{COLL} - \pi^{NC}}{\pi_{NT}^{CH,U} - \pi_{NT}^{COLL,U}} \quad (\text{A25})$$

gives us  $r^U(q^U)$ .

Notice that once again the numerator of (A25) is the same as that for (A21) and is independent of  $q^U$ . The denominator however, upon substituting (19) and (23), simplifies to

$$\frac{1}{16} (A_\phi - 2q^U)^2 \quad (\text{A26})$$

Now, upon taking the first derivative of (A26) w.r.t.  $q^U$  we find

$$\frac{d}{dq^U} \left( \frac{1}{16} (A_\phi - 2q^U)^2 \right) = -\frac{1}{2} \left( \frac{A_\phi}{2} - q^U \right).$$

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Recall that  $q^U$  is the quantity produced by uninformed firms when they believe they are colluding. Thus,  $\pi^{NC}$  is independent of  $q^U$  since  $q^U$  is not relevant in the punishment phase.

Expected collusive profits are also independent of  $q^U$ . As  $q^U$  rises the expected collusive profits for the informed firm falls at a constant rate while the expected collusive profits for the uninformed firm rises at a constant rate. In fact, the fall in expected collusive profits for the uninformed firm is always  $(n-1)$  times the rise in expected collusive profits for the uninformed firm. But any firm has an  $(n-1)/n$  chance of being uninformed and a  $1/n$  chance of being informed. Thus, as  $q^U$  rises, the fall in *expected future* profits given the likelihood that a firm is informed, is exactly matched by a rise in the *expected future* profits given the likelihood that the firm is uninformed. Thus a rise in  $q^U$  has no impact on future collusive profits. Similarly a fall in  $q^U$  will have no impact on future collusive profits either. This happens because under collusion, the informed firm adjusts its output each period to maximize industry profits for that period given demand. This profit is independent of firms' market shares because marginal cost is constant in our model.



Thus, the first derivative of (A26) w.r.t.  $q^U$  is negative for all values of  $q^U < \frac{A_\phi}{2}$ . Also,  $q^U$  will never be  $\geq \frac{A_\phi}{2}$ . Industry profit maximizing cartels will *never* choose  $q^U \geq \frac{A_\phi}{2}$  because then the quantity produced by the informed firm would exceed the industry profit maximizing amount when demand is low, since  $A_\phi > a_L$ . Thus for all relevant values of  $q^U$  the expression  $\frac{1}{16} (A_\phi - 2q^U)^2$  decreases as  $q^U$  increases. That is, the denominator (A25) falls as  $q^U$  rises for all relevant values of  $q^U$ . Note, incidentally, that  $q^U = \frac{A_\phi}{2n}$  lies in this range. This particular value,  $q^U = \frac{A_\phi}{2n}$ , is important because at this value of  $q^U$ ,  $r^U(q^U) = r^I(q^U)$ , as proven in the next section of this appendix.

The numerator of (A25) is identical to the numerator for (A22) which, recall, is independent of  $q^U$ . Thus, the numerator of (A25) is also independent of  $q^U$ . Therefore, since its denominator is decreasing in  $q^U$ ,  $r^U(q^U)$ , is increasing in  $q^U$ .

**A.3.3 Proof that  $r^I(q^U) = r^U(q^U) = r_0^*$  when  $q^U = \frac{A_\phi}{2n}$ .**

We derived  $r^I(q^U)$  in section A.3.1.<sup>11</sup> We also derived  $r^U(q^U)$  in section A.3.2. We also noted, in section A.3.2 that the expressions for  $r^I(q^U)$  and  $r^U(q^U)$  as shown in (A22) and (A25) respectively differed only in the denominator and that the numerator was independent of  $q^U$ . Thus, setting the denominators of (A22) and (A25) equal to each other will give us the values for which  $r^I(q^U) = r^U(q^U)$ .

We therefore solve for  $q^U$  by setting

$$\frac{1}{4} (q^U)^2 (n-1)^2 = \frac{1}{16} (A_\phi - 2q^U)^2. \quad (\text{A27})$$

After taking the square root on both sides of (A26) we get

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<sup>11</sup>Recall, that in Section A.3.1. we showed that  $r^I(q^U) = r_H^I(q^U) = r_L^I(q^U)$ .

$$\frac{1}{2}q^U (n - 1) = \pm \frac{1}{4} (A_\phi - 2q^U). \quad (\text{A28})$$

Now solving for  $q^U$  gives us

$$q^U = \frac{A_\phi}{2n}. \quad (\text{A29})$$

Thus,  $r^I(q^U) = r^U(q^U)$  when  $q^U = \frac{A_\phi}{2n}$ .<sup>12</sup> We substitute  $q^U = \frac{A_\phi}{2n}$ , (A23), and (A6) into (A22) and (A25) and simplify. This gives us

$$r^I(q^U) = r^U(q^U) = \frac{4n}{(n+1)^2} = r_0^*. \quad (\text{A30})$$

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<sup>12</sup>The negative root gives us  $q^U < 0$  which is meaningless. Therefore we do not use the negative root.

I think the above is a more obvious and neater reasoning than recalling  $q^U < A_\phi/2$ , then showing that  $A_\phi - 2q^U > 0$  and so on.