Settling the theory of saving

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19 February 2013

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Abstract

There is no way around it: each theory rests on a tiny set of foundational propositions. Standard economics rests on behavioral axioms. After a long intellectual detour it should be clear by now that behavioral axioms are the wrong formal departure point. Being beyond repair, they have to be replaced by objective structural axioms. This paper deals with saving and its relation to investment and profit. It starts from the fact that there is no such thing as a real economy. Hence economic phenomena are only explicable as the outcome of the interaction of real and nominal variables.

JEL B41, C65, E20

Keywords new framework of concepts; structure-centric; axiom set; triangle theorem; income; profit; investment; productivity; time shift

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> To be absolutely clear, what Sonnenschein, Mantel and Debreu showed is that there is no hope of a general result for stability nor indeed of uniqueness of equilibria, if we wish to build a model based only on individuals who satisfy the standard axioms of rationality. (Kirman, 2010, p. 508)

There is no way around it: each theory rests on a tiny set of foundational propositions. Standard economics rests on a set of behavioral axioms. After a long intellectual detour it should be clear by now that behavioral axioms are the wrong formal departure point. Being beyond repair, they have to be replaced by a set of objective structural axioms.

This paper deals with saving and its interaction with real and nominal key variables, in particular with the relation between saving, investment and profit. It starts from the fact that there is no such thing as a real economy. Hence economic phenomena are only explicable as the outcome of the interaction of real and nominal variables.

Section 1 provides the 'reasonably small number' of equations and variables that is required for a start. From these the properties of the elementary consumption economy and the triangle theorem are derived in Section 2. In Section 3 the interaction of monetary and real time shifts and the emergence of interest is considered in great detail. The analysis of the investment economy in Section 4 delivers the correct relation between saving, investment, and profit. Section 5 concludes.

1 The reasonably small number

> The highest ambition an economist can entertain who believes in the scientific character of economics would be fulfilled as soon as he succeeded in constructing a simple model displaying all the essential features of the economic process by means of a reasonably small number of equations connecting a reasonably small number of variables. (Schumpeter, 1946, p. 3)

1.1 Axioms

The formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of real and nominal variables that constitutes the monetary economy.

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises. Three suffice for the beginning.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |t \tag{1}$$

If DN is set to zero then total income consists only of wage income.

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no government.

The economic meaning is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. Profit and distributed profit are different things.

1.2 Definitions

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms (Boylan and O'Gorman, 2007, p. 431). With (4) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{5}$$

A sales ratio $\rho_X = 1$ indicates that the quantity sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \tag{6}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditure C are equal to total income Y, in other words, that the household sector's budget is balanced.

We define the factor cost ratio as:

$$\rho_F \equiv \frac{W}{PR} \quad |t. \tag{7}$$

The factor cost ratio ρ_F summarizes the internal conditions of the firm. A value of $\rho_F < 1$ signifies that the real wage is lower than the productivity or, in other words, that unit wage costs are lower than the price, or in still other words, that the value of output exceeds the value of input.

We finally define the distributed profit ratio as:

$$\rho_D \equiv \frac{Y_D}{Y_W} \quad |t. \tag{8}$$

1.3 The – economic – triangle theorem

Axioms and definitions put together give a single equation that formally integrates the three constituents of the pure consumption economy: the firm ρ_F, the commodity market ω, and the income distribution ρ_D.

$$\rho_F \, \omega \, (1 + \rho_D) = 1 \quad \text{with} \quad \omega \equiv \frac{\rho_E}{\rho_X} \quad |t \tag{9}$$

The triangle theorem asserts that the product of the three key ratios which characterize the firm, the market outcome, and the distribution is always equal to unity. The respective ratios are unit-free.[1] In analogy to the geometric triangle, the third ratio/angle can be calculated exactly when two ratios/angles are known. It is important to note that all axiomatic variables, and thus the ratios, are measurable in principle. The triangle theorem is testable and involves no behavioral assumptions. The subjectivity of homo oeconomicus is no part of the objective triangle. The structural axiomatic approach claims that, when the ratios for the pure consumption economy are measured, eq. (9) will turn out to be true.

[1] "This procedure is in accordance with the principle of objectivity requiring that the whole theory and its interpretations have to be independent of the choice of the units of measurement. And this requirement is met, if the theory is unit-free, the necessary condition stated in Buckingham's Π-theorem." (Schmiechen, 2009, p. 176).

The form of (9) precludes any notion of causality. The equation simply states the interdependence of the key ratios. However, the relation as a whole is absolutely deterministic. If, for example, the distributed profit ratio changes then either the factor cost ratio or the market outcome ratio or both *must* change, but we cannot say in what proportion. Only if we fix one ratio the dependency between the other two ratios is absolutely deterministic. But, since each ratio itself consists of multiple variables, there is more than one configurations of axiomatic variables that is compatible with the new deterministic ratio. Although (9) is deterministic, a simple causal relation between any two axiomatic variables does not exist. However, when measured ex post the triangle theorem will always be satisfied. This in turn means that, if we can control two key ratios, the third can be determined with certainty. This is essential knowledge for economic policy consulting.

2 Properties of the pure consumption economy

From (9) follows the price as dependent variable:

$$P = \frac{\rho_E}{\rho_X}\frac{W}{R} \quad \text{if} \quad \rho_D = 0 \quad |t. \tag{10}$$

From this the market clearing price follows:

$$P = \rho_E \frac{W}{R} \quad \text{if} \quad \rho_X = 1 \quad |t. \tag{11}$$

The market clearing price is equal to unit wage costs if the expenditure ratio is unity. In the case of budget balancing the profit per unit is therefore zero. In the general case, the market clearing price is above or below unit wage costs. All changes of the expenditure ratio, the wage rate and the productivity affect the market clearing price in the period under consideration. This is what we understand here under perfect price flexibility. Note that this concept has no behavioral connotation. If price flexibility is less than perfect the business sector's stock of products changes (see Section 3.5).

The business sector's financial profit in period t is defined with (12) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:[2]

$$Q_{fi} \equiv C - Y_W \quad |t. \tag{12}$$

Because of (3) and (4) this is identical with:

[2] Nonfinancial profit is treated at length in (2012b).

$$Q_{fi} \equiv PX - WL \quad |t. \tag{13}$$

From this in combination with (10) and (5) follows:

$$Q_{fi} = WL(\rho_E - 1) \quad \text{if} \quad \rho_D = 0 \quad |t. \tag{14}$$

Financial profit is zero if the expenditure ratio is unity, i.e. if the household sector's budget is balanced. The business sector makes a profit if the expenditure ratio is above unity and makes a loss if it is below unity. Total profit does not depend on productivity, that is, an economy with a low efficiency may, all other things equal, show the same total profit as one with a high efficiency. Total profit depends alone on the expenditure ratio if distributed profit is zero.

What finally has to be determined is the concrete initial conditions. From (11) follows that we need the wage rate and the productivity in order to determine the market clearing price P_0 under the balanced budget condition $\rho_{E0} = 1$, if distributed profit is set to zero in the first round. The productivity R_0 is objectively given by the actual production conditions. The wage rate W_0 is set arbitrarily. Under these conditions total profit Q_{fi0} is zero. With regard to the initial employment L_0 it is assumed that the pure consumption economy operates at full employment. This initial configuration is reproducible for an indefinite time span. All changes of productivity or wage rate affect the price but profit is zero because the expenditure ratio is unity and the distributed profit ratio is zero.

At first there is no change; all subsequent periods are perfectly identical with the initial period. Consumption expenditures are equal in all periods. The same holds for the quantity produced, sold and consumed. Now the question arises: how can households ever shift consumption between periods?

3 Monetary and real time shifts

> The possibility that saving could disrupt the circulation of commodities through a lack of demand was recognized at least as early as the Physiocrats. (Bleaney, 1987, p. 1)

3.1 Saving and dissaving

It is assumed that output consists of nondurables. In this case production and consumption happen in the same period and no physical transfer of output between periods is possible. However, what is impossible for the household sector as a whole is apparently feasible for a single household or a subset of households.

Financial saving is defined as difference between total income (1) and consumption expenditures (3):[3]

$$S_{fi} \equiv Y - C \equiv (1 - \rho_E) Y \quad |t. \tag{15}$$

Compared to the initial period with an expenditure ratio of $\rho_{E0} = 1$ we have in period$_1$ $\rho_{E1} < 1$. Saving is > 0. From (11) follows that the market clearing price falls, i.e. $P_1 < P_0$. From (14) follows that the zero profit of the initial period turns into a loss, i.e. $Q_{fi1} < 0 = Q_{fi0}$. The business sector sells the whole output to the household sector, the product market is cleared. Saving causes no glut of unsold output. Worse, saving produces a loss in period$_1$.

In period$_2$ the households reverse their saving completely and dissave the same amount. We now have $\rho_{E2} > 1$ with $\rho_{E2} = 2 - \rho_{E1}$. From (11) follows that the market clearing price increases, i.e. $P_2 > P_0 \gg P_1$. From (14) follows that the business sector posts a profit, i.e. $Q_{fi2} > 0 \gg Q_{fi1}$. Over both periods profit and loss cancel out and cumulated profit is zero.

Savers abstain in period$_1$ from buying the quantity $X_1^S = \frac{S_1}{P_0}$. The lower price in period$_1$ enables the nonsavers to take over this quantity with their unchanged consumption expenditures. The market is cleared because of $\rho_X = 1$, that is, the whole output is sold at a lower price. Compared to the initial period the consumption of savers decreases and that of nonsavers increases. The redistribution is effected via the market clearing price. The nonsavers are passive players.

With their dissaving in period$_2$ the former savers now absorb the quantity $X_2^S = \frac{S_1}{P_2}$. The higher market clearing price reduces the quantity the passive nonsavers can buy with their unchanged consumption expenditures. The savers/dissavers successfully put the intended time transfer of consumption goods into practice. However, the quantities in the successive periods are not equal. It holds $X_2^S < X_1^S$. This translates into a negative real rate of interest:

$$\mathsf{I}^{real} = \frac{X_2^S - X_1^S}{X_1^S} < 0 \quad |t. \tag{16}$$

The calculated real interest rate depends on the magnitude of the price effects which in turn depend on the amount of saving/dissaving. If this amount is small the price effect is small or even negligible. This is, roughly, what the single saver/dissaver expects: that he can buy the same product at the same price in a future period. In the limiting case the calculated real interest rate approaches zero but it cannot become positive. This makes discounting a pointless exercise. In real terms the single household seems to shift a certain quantity of the consumption good from period$_1$ to period$_2$. As the subset of savers/dissavers grows the price falls in period$_1$ and increases in period$_2$. This, however, is an unintended effect. Nobody knows in

[3] Nonfinancial saving is discussed at length in (2012b, Sec. 4.2)

period_1 how dissaving in subsequent periods will affect the market clearing price. This produces an obvious paradox: if many agents act on the premise that the price will remain constant, it certainly will not. What is true for an individual becomes false when generalized.

While the individual saver/dissaver may have the subjective impression that he can shift consumption goods from one period to the next, nothing of the sort happens in reality. For the household sector as a whole there is no time shift at all. It is the complementary group that unintentionally helps to create the illusion of a choice between present and future goods. What really happens is a redistribution of the unchanged output in each period between different households. This redistribution neither increases nor diminishes the available quantities for the household sector as a whole. It is the sequence of in-period redistributions that creates the impression of a real time shift of consumption.

3.2 Transactions and the store of value

Money follows consistently from the given axiom set (for details see 2011a). If income is higher than consumption expenditures the household sector's stock of money increases. It decreases when the expenditure ratio ρ_E is greater than unity. The change of the household sector's stock of money in period t is defined as:

$$\Delta \mathsf{M}_{\mathbf{H}} \equiv^{m} Y - C \equiv^{m} Y\,(1-\rho_E) \quad |t. \tag{17}$$

The identity sign's superscript m indicates that the definition refers to the monetary sphere.

The stock of money at the end of an arbitrary number of periods is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\mathsf{M}_{\mathbf{H}} \equiv \sum_{t=1}^{t} \Delta \mathsf{M}_{\mathbf{H}t} + \mathsf{M}_{\mathbf{H}0}. \tag{18}$$

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta \mathsf{M}_{\mathbf{B}} \equiv^{m} C - Y \equiv^{m} Y\,(\rho_E - 1) \quad |t. \tag{19}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\mathsf{M}_{\mathbf{B}} \equiv \sum_{t=1}^{t} \Delta \mathsf{M}_{\mathbf{B}t} + \mathsf{M}_{\mathbf{B}0}. \tag{20}$$

In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions are carried out without costs by the central bank. The stock of money then takes the form of current deposits or current overdrafts. Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (18) the current overdrafts of the business sector are of equal amount according to (20) and vice versa if the business sector owns current deposits. Money and credit are symmetrical; the stock of money of each sector can be either positive or negative. The current assets and liabilities of the central bank are equal by construction. From its perspective the quantity of money at the end of an arbitrary number of periods is given by the absolute value either from (18) or (20):

$$\mathsf{M}_t \equiv \left| \sum_{t=1}^{t} \Delta \mathsf{M}_{\mathbf{H},\mathbf{B}t} \right| \quad \text{with} \quad \mathsf{M}_{\mathbf{H},\mathbf{B}0} = 0. \tag{21}$$

The quantity of money is always ≥ 0. Equation (21) implies for a start that the central bank plays an accommodative role and provides elastic currency.

The quantity of money at the end of period$_1$ follows from (17) as:

$$\mathsf{M}_1 = |Y_1 (1 - \rho_{E1})| \quad \text{with} \quad \rho_{E1} < 1. \tag{22}$$

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 1 results. In the initial period income and consumption expenditures are equal, i.e. $\rho_{E0} = 1$. The monthly income $\mathrm{Y}/12$ is paid out at mid-month, expenditures are evenly distributed over the month.

At the beginning of period$_1$ some households start to save and thereby their current deposits increase until period end. Business, taken as a whole, cannot recoup total wage income and by consequence its current overdrafts increase as an exact mirror image. It is just the other way round when the household sector dissaves.

While real saving and dissaving is impossible with a nondurable output of consumption goods, monetary saving and dissaving is possible at any time. How can the real and monetary side get out of phase? In the final analysis the reason lies in the nature of money. When wages are paid the concrete conditions of production are despecified in all dimensions. The amount of money in the hands of an agent does not tell anything about how, where or when it has been earned (remember Vespasian's axiom). Vice versa, in the act of buying a certain consumption good money is respecified. As a result, part of the income from producing cheese is spent on a TV-set and, in an indirect barter, part of the income from producing TV-sets is spent on cheese. Let us call this despecification and respecification the switch-yard function of money. Because the nondurable output is dated and cannot be shifted to the next period while money bears no time stamp and can be spent in any future period, the real and monetary side can get out of phase. The temporal despecification is the precondition that money assumes the function of a store of

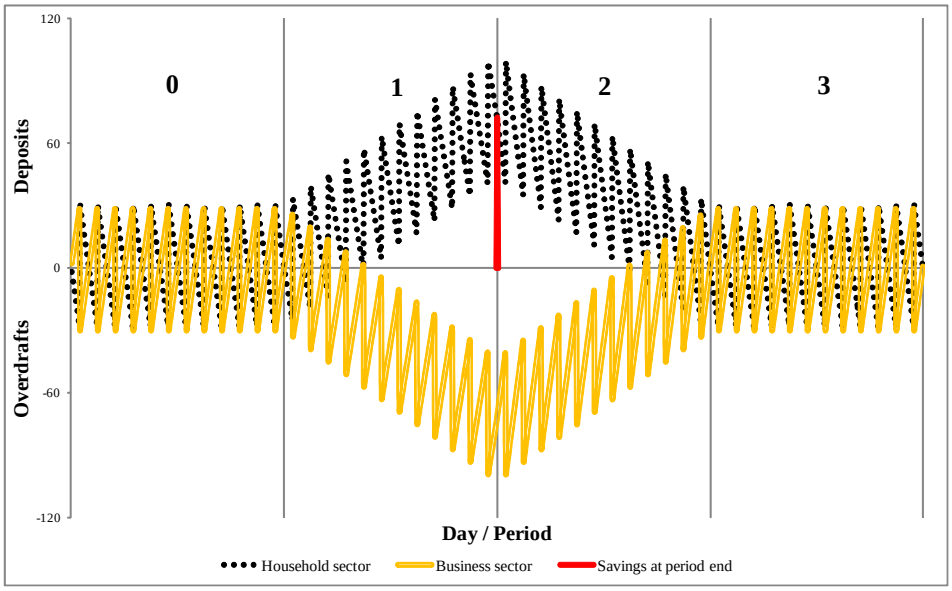

Figure 1: Money transactions between the household and the business sector: the quantity of money at the end of period$_1$ results from household sector saving

value. As Figure 1 makes clear this value is not "covered" by something tangible. The quantity of output is zero at the end of period$_1$. The quantity of money in the hand of savers is at any point of time exactly "covered" by the liabilities of the business sector. The store of value consists of unspecified claims on consumption goods which, however, are at the moment nonexistent.

3.3 Concurrent change of economic conditions

The dissaving in period$_2$ effects a price increase that in turn enables the anonymous redistribution of the period output from the nonsavers to the savers/dissavers. This price effect has been derived under the ceteris paribus condition. In the general case, though, productivity and wage rate may change from one period to the next. This modifies the outcome of the saving/dissaving process.

Let us assume first that the wage rate increases in period$_2$. From (11) follows the new market clearing price as:

$$P_2^{\bullet} = \rho_{E2}\frac{W_2}{R_0} \quad \text{if} \quad \rho_{X2} = 1. \tag{23}$$

The effect of $\rho_{E2} > 1$ on the price is amplified by the higher wage rate $W_2 > W_0$. The higher price in turn reduces the quantity the dissavers can buy, i.e. $X_2^{S\bullet} = \frac{S_1}{P_2^{\bullet}} < X_2^S$. The calculated negative real rate of interest becomes even more negative.

Let us assume next that the productivity increases in period$_2$. From (11) follows the new market clearing price as:

$$P_2^\circ = \rho_{E2}\frac{W_0}{R_2} \quad \text{if} \quad \rho_{X2} = 1. \tag{24}$$

The higher productivity dampens the price increase that stems from $\rho_{E2} > 1$ or may even effect a decrease. The dissavers participate in the output growth and can buy more, i.e. $X_2^{S\circ} = \frac{S_1}{P_2^\circ} > X_2^S$. Now the calculated real rate of interest turns positive. It is obvious, however, that the productivity effect is unforeseeable and that the savers/dissavers cannot take a positive real interest rate for granted. There is no causal link between saving in period$_1$ and a productivity increase in period$_2$.

As a limiting case it can be imagined that the productivity effect exactly counteracts the increase of the expenditure ratio, thus keeping the price constant. This enables a time shift with the quantity foregone by the savers in period$_1$ exactly equal to the quantity in period$_2$, i.e. $X_2^S = X_1^S$. In this case the calculated real rate of interest is zero.

Changes of wage rate and productivity in period$_2$ lead to a redistribution of output *within* the household sector. The profit of the business sector sums up to zero over all periods, independently of wage rate or productivity changes. The price acts as anonymous redistributor and brings the temporal unspecified nominal claims in line with the actual real outputs. Ultimately, the savers/dissavers force this redistribution upon the rest of the economy. What is optimal for the savers/dissavers may be detrimental to the complementary households or the business sector.

It is important to keep the questions of distribution and real time shifts of consumption apart. Imagine for a moment the alternative case of a durable consumption good. If a saver buys a certain quantity in period$_1$ and stores it for consumption in period$_2$ the time shift does in the ideal case neither increase nor diminish the quantity. If this physical fact is taken as a benchmark then follows as a rule of distribution that all changes in (11) should compensate such that the market clearing price remains constant. Thus the savers/dissavers neither gain nor loose in real terms by a time shift of consumption. This entails that the real rate of interest is zero; discounting future real consumption is therefore impossible.

3.4 The emergence of interest

Unwittingly, the savers play a dangerous game. In their attempt to shift consumption from the present to the future they impose a loss on the business sector. This is not a comfortable situation in the longer run. In the pure consumption economy saving must be followed by dissaving within a relatively short time span. To be more precise, in the case under consideration the time span depends on the willingness of the banking industry to finance a loss making firm.

The banking industry consists, for a straightforward beginning, of the central bank that handles the autonomous money transactions between the household and the business sector and thereby creates money and credit. Accordingly, the business sector consists of a consumption goods producing firm A and the central bank as the second firm B. The explicit inclusion of the central bank entails that the given resources of the business sector L_0 have first to be reallocated:

$$L_A + L_B = L_0 \quad \text{with} \quad L_A \downarrow, L_B \uparrow \quad |t. \tag{25}$$

As a consequence total income changes from (1) to:

$$Y = \underbrace{W_A}_{W} L_A + \underbrace{W_B}_{W} L_B + \underbrace{(D_A N_A + D_B N_B)}_{Y_D=0} \quad |t. \tag{26}$$

Since the wage rates are assumed to be identical, total income does not change with a reallocation of labor input between firms. If L_A decreases the output of consumption goods decreases. The concomitant increase of L_B increases the service output of the central bank. Full employment L_0 is maintained by assumption. Only the composition of the business sector's output changes, total wage income remains unaffected. In real terms more banking services are bought with less consumption output.

While the saving households accumulate demand deposits in period$_1$ as shown in Figure 1, the business sector accumulates overdrafts. Overdrafts are the most elementary form of credit. In the usual course of events they are replaced by longer term loans or other forms of financing. In our simple scheme of things it is the central bank that produces all forms of credit. The central bank can formally be treated like the consumption good producer with the specific feature that the output consists of credit services.

Overall financial profit (13) is differentiated for the two firms:

$$\begin{aligned} Q_{fiA} &\equiv P_A X_A - W L_A - \mathsf{I}_B \bar{\mathsf{A}}_B \\ Q_{fiB} &\equiv \mathsf{I}_B \bar{\mathsf{A}}_B - W L_B \quad |t. \end{aligned} \tag{27}$$

Firm A includes the interest payments $\mathsf{I}_B \bar{\mathsf{A}}_B$ to the central bank under the cost items. At the central bank the interest payment appear correspondingly as revenues. When the business sector is consolidated the interest payments drop out, they play only a role for the distribution of profits between firm A and B. They play no role for the profit of the business sector as a whole. Interest is, at this stage, neither part of profit nor of income.

As a first approximation it is assumed that profit in each firm is zero. Under this simplifying condition the market clearing price for the consumption good P_A and the rate of interest I_B for a given stock of loans $\bar{\mathsf{A}}_\mathsf{B}$ can be readily determined as:

$$P_A = \frac{W}{R_A}\left(1+\frac{L_B}{L_A}\right) \quad \text{if} \quad \rho_{XA} = 1$$
$$\mathsf{l}_B = \frac{W}{R_B} \quad \text{if} \quad \rho_{XB} = 1, \quad \text{with} \quad R_B \equiv \frac{\bar{\mathsf{A}}_B}{L_B} \quad |t. \tag{28}$$

The zero profit condition defines the relation of the rate of interest and the commodity price. For the general case with individual profits different from zero the relation is a bit more sophisticated (for details see 2011b, Sec. 7). The major determinant of the interest–price relation, which is sometimes referred to as real rate of interest, is the production conditions in both firms that find their expression in the respective productivities. Saving provokes, first of all, the emergence of interest on loans to the business sector. The loan interest rate l_B is, in the simplest case, objectively determined by the production conditions of the central bank.

Eq. (28) shows that the introduction of the banking industry has an effect on the market clearing price of the consumption good that depends on the relative weight of the banking industry measured in labor inputs. While total income and consumption expenditures remain unchanged the price of the consumptions good goes up and the quantity goes down. The new composition of output, less consumption goods and more banking services, is reflected in the higher price of the consumption good.

The central bank pays no interest on demand deposits. In our simple scheme demand deposits are money and money bears no interest. It is further assumed that interest-bearing alternatives are not available to the households in the elementary consumption economy. Hence there is no direct relation between saving and interest. The households have only the alternative to keep their money in the most liquid form or to spend it on consumption goods. Not before the central bank offers a kind of longer term savings account that bears interest does rentier income emerge. This interest is a compensation for parting with liquidity, not a compensation for waiting or abstinence or other psychological idiosyncrasies. Saving as such increases demand deposits and does not yield interest. It is the parting with accumulated liquidity that yields interest. Hence interest can motivate saving only indirectly. Saving first of all forces the business sector to pay interest on overdrafts or longer term loans. As long as households cannot put their money into saving accounts or something similar interest cannot become a separate form of income. Somewhat paradoxically, the savers unintentionally bring at first the *loan* interest rate into being. Even more paradoxical, saving always creates a loan demand of exactly equal amount. As a matter of principle, the household sector can always finance the deficit of the business sector, no matter how big it is.

3.5 Changes of the stock of products

Saving in period$_1$, i.e. $\rho_{E1} < 1$, may either result in a reduction of the market clearing price with the quantity sold constant and equal to output or of a reduction

of the quantity sold with the price constant or something in between. For analytical purposes solely the limiting cases are of interest. We now consider the case with a time transfer via the inventory of the business sector. This presupposes that output consists of a durable consumption good.

With a constant price P_0 the firm cannot sell the unchanged output in period$_1$. The increase of the stock of products is given by:

$$\Delta O_1 \equiv O_0 - X_1 \quad |t. \tag{29}$$

The quantity sold X_1 follows from (3) and (6) and this makes the change of the stock of products directly dependent on the expenditure ratio:

$$\Delta O_1 = O_0 - \frac{\rho_{E1} Y_0}{P_0} \quad \Rightarrow \quad \Delta O_1 = O_0 (1 - \rho_{E1}) \quad \text{if} \quad \rho_{X0} = 1. \tag{30}$$

Business does not interpret the drop of the quantity sold as lack of demand but keeps production and output on the same level. And rightly so, because dissaving in period$_2$, i.e. $\rho_{E2} > 1$, reduces the stock of products again to zero. The quantity sold is greater than the unchanged output. This time shift conforms to the benchmark rule of Section 3.3. The quantity the saving households give up in period$_1$, i.e. ΔO_1, is exactly equal to ΔO_2 because of $\rho_{E2} = 2 - \rho_{E1}$.

The transaction pattern and the resulting financing requirement is the same as shown in Figure 1. As a mirror image of saving the business sector posts a financial loss in period$_1$. However, the change of the valued stock of products amounts to a nonfinancial profit. The sum of financial loss and nonfinancial profit is zero if the stock is valued with the price P_0. As a mirror image of dissaving the business sector posts a financial profit in period$_2$. The nonfinancial loss from the depletion of the stock of products is of equal amount. The sum of financial profit and nonfinancial loss is zero. Over all periods financial and nonfinancial profit and loss add up to zero. The monetary side is the same as in Section 3.2, the real side, however, is different from Section 3.1. The quantity of money is now "covered" by the stock of products.

The business sector cannot accumulate stocks and loans indefinitely. This calls for a quick reversal of saving in the pure consumption economy. In the final analysis it is a matter of indifference whether the price or the quantity sold is reduced as a reaction to households' saving, what the business sector really needs is a timely dissaving.

4 The investment economy

> For the best part of the last two centuries, this equality [of saving and investment] ... has been at the heart of long and protracted debates.

> Indeed, two of the most fruitful periods in the history of economic thought were entirely devoted to this riddle which, still today, is far from being satisfactorily solved. (Bridel, 1987, p. 1)

4.1 The differentiated triangle

When the axioms (1) to (3) are differentiated for the consumption goods industry and the investment goods industry we have in strict formal analogy for period t:

$$Y = W_{\mathbf{C}} L_{\mathbf{C}} + W_{\mathbf{I}} L_{\mathbf{I}} + D_{\mathbf{C}} N_{\mathbf{C}} + D_{\mathbf{I}} N_{\mathbf{I}} \tag{31}$$

$$\begin{aligned} O_{\mathbf{C}} &= R_{\mathbf{C}} L_{\mathbf{C}} \\ O_{\mathbf{I}} &= R_{\mathbf{I}} L_{\mathbf{I}} \end{aligned} \tag{32}$$

$$\begin{aligned} C &= P_{\mathbf{C}} X_{\mathbf{C}} \\ I &= P_{\mathbf{I}} X_{\mathbf{I}} \end{aligned} \tag{33}$$

With the appropriate definitions this boils down to the triangle theorem which looks a bit more sophisticated now but is composed of the same three basic constituents as (9):

$$\left(\rho_{F\mathbf{C}} \frac{\rho_{E\mathbf{C}}}{\rho_{X\mathbf{C}}} + \rho_{F\mathbf{I}} \frac{\rho_{E\mathbf{I}}}{\rho_{X\mathbf{I}}} \right) (1 + \rho_D) = 1 \quad |t. \tag{34}$$

From this a host of limiting cases follows, e.g. market clearing $\rho_{X\mathbf{C}} = 1$, $\rho_{X\mathbf{I}} = 1$, e.g. equal factor cost ratios $\rho_{F\mathbf{C}} = \rho_{F\mathbf{I}}$, e.g. overall budget balancing $\rho_{E\mathbf{C}} + \rho_{E\mathbf{I}} = 1$, e.g. zero profit distribution $\rho_D = 0$. All we have to say in the following about saving and investment is succinctly summarized in the differentiated triangle theorem.

4.2 Consumer sovereignty

Saving in period$_1$ may either result in a reduction of the market clearing price with the quantity sold constant or in a reduction of the quantity sold with the price constant or a mixture of both. The third alternative is that the business sector keeps the price of the consumption good constant and reduces output. This in turn reduces employment in the consumption goods industry. Total employment is given by $L_{\mathbf{C}} + L_{\mathbf{I}} = L$ and if we want to maintain full employment L_0 then labor input in the investment goods industry must increase by the same amount. What is required, then, is a shift of labor between industries and a change of the composition of output. Under the condition of

$$Y = \underbrace{W_{\mathbf{C}}}_{W} L_{\mathbf{C}} + \underbrace{W_{\mathbf{I}}}_{W} L_{\mathbf{I}} + \underbrace{D_{\mathbf{C}} N_{\mathbf{C}} + D_{\mathbf{I}} N_{\mathbf{I}}}_{Y_D=0} \quad |t \tag{35}$$

the compensatory reallocation of labor input leaves total income unchanged.

The respective expenditure ratios are defined as:

$$\rho_{E\mathbf{C}} \equiv \frac{C}{Y} \qquad \rho_{E\mathbf{I}} \equiv \frac{I}{Y} \quad |t. \tag{36}$$

In order to make the periods comparable it must hold $\rho_{E\mathbf{C}1} + \rho_{E\mathbf{I}1} = \rho_{E0}$, that is, the overall expenditure ratio remains unaltered. From this the investment expenditures that conform to all conditions can be calculated. Note that we have not lost a word about how the coordination of labor input and expenditures comes about. What we have derived is the investment expenditures that are compatible with full employment and the amount that households actually save. The household sector determines the composition of output, the business sector adapts employment accordingly.

As a result of the reallocation of labor, that is set in motion by the savers, consumption output is reduced in period$_1$ by:

$$\Delta O_{\mathbf{C}1} = O_{C0} - O_{C1}. \tag{37}$$

At the same time the output of the investment good is increased by:

$$\Delta O_{\mathbf{I}1} = O_{\mathbf{I}1} - O_{\mathbf{I}0}. \tag{38}$$

From the equality of $L_{\mathbf{C}0} - L_{\mathbf{C}1} = L_{\mathbf{I}1} - L_{\mathbf{I}0}$ follows the equivalence of $\Delta O_{\mathbf{C}1}$ and $\Delta O_{\mathbf{I}1}$. It can be said, then, that the quantity of unproduced consumption goods is equivalent to the quantity of produced investment goods. What cannot be said is that real saving is equal to real investment. This would be semantic nonsense. Saving is defined with (15) and investment is defined with (33). Whether these two *nominal* magnitudes are equal is quite another matter.

The investment goods output becomes the vehicle for the savers' time shift of consumption goods. Business reduces the production of present goods and replaces it by intermediate investment goods. In the last instance the household sector determines the composition of output under the condition of full employment.

In period$_2$ the investment goods are combined with the full employment labor input in the production of the consumption good. The investment goods industry vanishes again. The households dissave exactly what they have saved in period$_1$, that is, the expenditure ratio is $\rho_{E2} > 1$. The combination of labor and machines boosts productivity. The market clearing price (11) is now given by:

$$P_2 = \frac{\rho_{E2}}{R_2} W_0 \quad \text{if} \quad \rho_{X2} = 1. \tag{39}$$

If the productivity effect is strong the market clearing price falls despite the increase of the expenditure ratio. In this case the calculated real interest rate (16) is positive for the savers/dissavers; they participate in the output growth. However, if the productivity effect is relatively weak the market clearing price increases. In order to exclude all intertemporal redistribution the price must be kept constant. This implies that the wage rate increases if the productivity effect is strong, and decreases if it is weak according to:

$$W_2 = \frac{R_2}{\rho_{E2}} P_0 \quad \text{if} \quad \rho_{X2} = 1. \tag{40}$$

This equation ensures that the quantity foregone in the saving phase is equal to that of the dissaving phase. The time shift is quantitatively neutral for the savers/dissavers. The disproportionate output growth that is due to the input of investment goods is absorbed by the employees that produce the current output in period$_2$. This distributional neutrality is in accordance with the benchmark rule of Section 3.3. A wage increase in (40) signals that the time shift of the savers/dissavers has been beneficial for the household sector as a whole. The growth effect, that is ultimately due to the input of investment goods, is entirely decoupled from the time shift of consumption goods.

Note that we do not employ the concept of a production function. Production functions are needed for the application of the subjective optimization calculus but have no empirical rationale (Shaik, 1980).

The money transactions look like Figure 1. With regard to the monetary side it makes not much difference whether the business sector reduces the price, accumulates stocks or produces investment goods. However, the claim that is implicit in the quantity of money is now "covered" with the stock of investment goods.

What we have done is to replace a certain part of former consumption expenditures by investment expenditures. From this the all-important relation between saving, investment and profit can be derived.

4.3 Saving, investment, and profit

From (13) follows for the financial profit of the consumption and investment goods industry, respectively:

$$
\begin{aligned}
Q_{fi\mathbf{C}} &\equiv C - W_{\mathbf{C}} L_{\mathbf{C}} \\
\underline{Q_{fi\mathbf{I}} &\equiv I - W_{\mathbf{I}} L_{\mathbf{I}}} \\
Q_{fi} &\equiv C + I - WL \\
\text{with} \quad WL &= W_{\mathbf{C}} L_{\mathbf{C}} + W_{\mathbf{I}} L_{\mathbf{I}} \quad |t.
\end{aligned}
\tag{41}
$$

Financial profit of the business sector as a whole is given as difference of total expenditures and total wage income.

In combination with the 1st axiom and the definition of saving (15) this yields the general relation:

$$
\begin{aligned}
Q_{fi} &\equiv C + I - (Y - Y_D) \\
Q_{fi} &\equiv I - (Y - C) + Y_D \\
Q_{fi} - Y_D &\equiv I - S \quad |t.
\end{aligned}
\tag{42}
$$

The difference between profit and distributed profit is by definition equal to the difference between investment expenditures and household sector saving. If profit and distributed profit happen to be equal, then, as a corollary, investment expenditure and household saving must be equal too. Vice versa, if it happens that household saving is equal to investment expenditures then, as a corollary, profit and distributed profit must be equal too. In reality, though, profit and distributed profit are *never* equal and correspondingly household saving and investment are not equal either. This is the general case.

We have simplified the analysis from the very beginning by setting distributed profit to zero. If, in addition, investment expenditures are zero in (42) then saving produces a financial loss of equal magnitude:

$$
\begin{aligned}
Q_{fi} &= -S \\
\text{if} \quad Y_D = 0, I = 0 \quad |t.
\end{aligned}
\tag{43}
$$

In period$_1$ total income remains unchanged but consumption expenditures decline. This would produce a loss according to (43) were it not for the fact that the consumption goods industry buys investment goods from the investment goods industry. For convenience it is assumed that the investment expenditures exactly compensate the decline in consumption expenditures. Hence financial profit for the business sector as a whole is zero in (42) under the condition of $Y_D = 0$. The investment expenditures are financed by the central bank. The household sector's deposits

in Figure 1 are exactly equal to the business sector's overdrafts. These are in the normal course of events replaced by longer term loans. The diverse variants of financing are passed over here.

In period_2 total income remains again unchanged but consumption expenditures now exceed income. The amount that has been saved in period_1 is now dissaved. According to (43) the business sector makes a profit that is equal to the amount dissaved, i.e. to the saving in period_1. The business sector is therefore in the position to redeem the loan. However, we have still to take depreciation into account. It is assumed for convenience that the investment goods are fully depreciated in period_2, i.e. depreciation is exactly equal to the investment expenditures of period_1.

Total profit is given with the 5th axiom as sum of financial and nonfinancial profit (for details see 2012b, Sec. 6):

$$Q = Q_{fi} + Q_{nf} \quad |t. \tag{44}$$

Since financial profit is equal to dissaving and nonfinancial profit is equal to depreciation and both are in turn equal to the investment expenditures of period_1 total profit is zero. In our simple scheme of things total profit is zero in both periods. The productivity effect of the input of investment goods does not translate into higher profits for the business sector as a whole. According to (39) and (40) a higher productivity translates either into a lower market clearing price or a higher wage rate. In either case the productivity effect benefits the household sector. The productivity effect is irrelevant for the profit of the business sector as a whole. This does not preclude that it may be of great importance to the investing firm. The individual experience of a single firm, though, cannot be generalized for the whole economy.

We have fully reversed investment in period_2. This, clearly, is an analytical limiting case. If the operating life of the investment goods in longer than one period and if the production of investment goods is repeated over a longer time span then the stock of investment goods on the one hand and the stock of loans on the other accumulates. In the growing economy investment and disinvestment continuously overlap.

4.4 The business sector takes the lead

We now consider the case that business determines the quantity of investment goods that is produced in period_1. Under the condition that full employment L_0 is maintained an increase of labor input in the investment goods industry translates into a cutback of consumption goods production. Total income is, according to (35), not affected by the reallocation of labor input. There is no need to speculate about why the business sector switches from consumption goods to investment goods production. The known answer is, because somebody expects that he can make more profit in this way. This, of course, is vacuous psychologism.

> Everything can be 'explained' if we place no restrictions on what we mean by 'explanation'. (Blaug, 1994, p. 123)

In any case there is no necessity whatsoever for the business sector to wait with investment until the households make up their minds and start to save. Schumpeter's entrepreneur simply overrules the households' preferences. The optimal distribution of real consumption over time is no longer an issue.

It is assumed that the households keep their consumption expenditures unchanged. The market clearing price therefore increases because $L_{\mathbf{C}1} < L_0$:

$$P_{\mathbf{C}1} = \frac{Y_0}{R_{\mathbf{C}0} L_{\mathbf{C}1}} \quad \text{if} \quad \rho_{XC} = 1. \tag{45}$$

Since the households do not save in period_1 the transaction pattern in Figure 1 is the same as in the initial period. Neither deposits nor overdrafts accumulate.

Financial profit of the business sector as a whole is, according to (42), equal to investment expenditures because saving is zero. The business sector finances investment expenditures out of current profit.

In period_2 the market clearing price drops because of the productivity effect of the investment goods input and of the reallocation of labor from the investment goods industry back to the consumption goods production, i.e. $L_0 > L_{\mathbf{C}1}$:

$$P_{\mathbf{C}2} = \frac{Y_0}{R_{\mathbf{C}1} L_0} \quad \text{if} \quad \rho_{XC} = 1. \tag{46}$$

According to (42) financial profit is now zero. Because of the depreciation total profit according to (44) is negative. The nonfinancial loss is, under the given conditions, equal to the investment expenditures of period_1. The profit of period_1 and the loss of period_2 add up to zero. There is no difference to the outcome with the household sector in the driver seat. Over the full investment cycle total profit sums up to zero. The productivity effect of the investment goods input is fully absorbed by the wage earners in period_2 and depends in no way on a time shift of consumption via saving/dissaving. It is important to keep the effects of time shift and of growth apart. The productivity effect is due to the use of machines and not to the abstinence of savers.

4.5 Unacceptable

The fundamental flaw of value theory lies in the *premise* that the value of output is equal to factor incomes. Profit, though, is not a factor income and is entirely different from distributed profit (for details see 2013). From the wrong profit theory follows, first, the perennial commonplace that total income is the sum of wages

and profits, and, second, the I=S/I≡S tenet. The latter gives rise to the question of how saving and investment are equalized. The classics identified the rate of interest as coordinating mechanism, Keynes the income multiplier. Both conceptions are misleading because saving and investment are never equal – neither ex ante nor ex post. Their difference is equal to the difference of profit and distributed profit. A glance at aggregated annual statements confirms that profit and distributed profit are never equal and this amounts to an indisputable refutation of the the I=S/I≡S tenet.

Indeed, I and S are independent and there is no mechanism and no need to make them equal. The difference is always made good by retained profit. This result holds independently of whether the economy is at full employment or not (for the structural-axiomatic employment multiplier see 2012a, Sec. 6). It is a remarkable fact, that the untenable IS-schedule of the Hicksean IS/LM construct has never been challenged and that the model was discarded, after a long and happy life in the textbooks, for some other reasons.

The alternatives to the Keynesian I=S/I≡S confusion are even worse because they ignore the interaction of nominal and real variables which is the defining characteristic of the economy we happen to live in. DSGE starts from the equivocation that total output is also total income and is built upon the equality of real saving and real investment $i_t = s_t$ (Wickens, 2008, p. 13). In this hapless construct the existence and role of profit gets completely out of sight. Accordingly, in (Ljungqvist and Sargent, 2004) profit and loss do not appear any longer in the index. In a real economy profit cannot exist (for other indefensible properties see Quiggin, 2010, pp. 80-134). An economy without profits, though, is like a car without wheels.

5 Conclusion

> As is well known, and taking over that argument from Smith, both Ricardo and Malthus in England and Say and Sismondi on the Continent always identified decisions to save with decisions to invest. Since they all considered as a fact that 'what is annually saved is as regularly consumed as what is annually spent, and nearly in the same time too' (Smith, 1776, vol. I, pp. 337–8), the question of a possible divergence between saving and investment was never asked. (Bridel, 1987, p. 1)
>
> That is, economists have persisted with a model that is theoretically flawed and systematically contradicted by the empirical evidence. (Kirman, 2010, p. 512)

From the set of structural axioms follows: The difference between investment expenditures and household sector saving is by definition equal to the difference between profit and distributed profit. The equality of saving and investment (real or nominal) does not take place and never has been nor ever will be observed.

References

Blaug, M. (1994). Why I am Not a Constructivist. Confessions of an Unrepetant Popperian. In R. E. Backhouse (Ed.), *New Directions in Economic Methodology*, pages 109–136. London, New York, NY: Routledge.

Bleaney, M. (1987). Over-Saving. In J. Eatwell, M. Milgate, and P. Newman (Eds.), *The New Palgrave Dictionary of Economics Online*, pages 1–4. Palgrave Macmillan, 1st edition. URL http://www.dictionaryofeconomics.com/article?id=pde1987_X001637.

Boylan, T. A., and O'Gorman, P. F. (2007). Axiomatization and Formalism in Economics. *Journal of Economic Surveys*, 21(2): 426–446.

Bridel, P. (1987). Saving equals Investment. In J. Eatwell, M. Milgate, and P. Newman (Eds.), *The New Palgrave Dictionary of Economics Online*, pages 1–5. Palgrave Macmillan, 1st edition. URL http://www.dictionaryofeconomics.com/article?id=pde1987_X001941.

Kakarot-Handtke, E. (2011a). Reconstructing the Quantity Theory (I). *SSRN Working Paper Series*, 1895268: 1–26. URL http://ssrn.com/abstract=1895268.

Kakarot-Handtke, E. (2011b). The Wine Maker's Business and the Origin of Interest in a Monetary Economy. *SSRN Working Paper Series*, 1939915: 1–22. URL http://ssrn.com/abstract=1939915.

Kakarot-Handtke, E. (2012a). Keynes's Employment Function and the Gratuitous Phillips Curve Desaster. *SSRN Working Paper Series*, 2130421: 1–19. URL http://ssrn.com/abstract=2130421.

Kakarot-Handtke, E. (2012b). Primary and Secondary Markets. *Levy Economics Institute Working Papers*, 741: 1–27. URL http://www.levyinstitute.org/publications/?docid=1654.

Kakarot-Handtke, E. (2013). Confused Confusers: How to Stop Thinking Like an Economist and Start Thinking Like a Scientist. *SSRN Working Paper Series*, 2207598: 1–15. URL http://ssrn.com/abstract=2207598.

Kirman, A. (2010). The Economic Crisis is a Crisis for Economic Theory. *CESifo Economic Studies*, 56(4): 498–535. DOI 10.1093/cesifo/ifq017.

Ljungqvist, L., and Sargent, T. J. (2004). *Recursive Macroeconomic Theory*. Cambridge, MA, London: MIT Press, 2nd edition.

Quiggin, J. (2010). *Zombie Economics. How Dead Ideas Still Walk Among Us*. Princeton, NJ, Oxford: Princeton University Press.

Schmiechen, M. (2009). *Newton's Principia and Related 'Principles' Revisited*, volume 1. Norderstedt: Books on Demand, 2nd edition.

Schumpeter, J. A. (1946). The Decade of the Twenties. *American Economic Review*, 36(2): 1–10. URL http://www.jstor.org/stable/1818192.

Shaik, A. (1980). Laws of Production and Laws of Algebra: Humbug II. In E. J. Nell (Ed.), *Growth, Profits, and Property*, pages 80–95. Cambridge, New York, NY, Melbourne: Cambridge University Press.

Wickens, M. (2008). *Macroeconomic Theory. A Dynamic General Equilibrium Approach*. Princeton, NJ, Oxford: Princeton University Press.