The Calculating Auctioneer, Enlightened Wage Setters, and the Fingers of the Invisible Hand

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Abstract

The formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of real and nominal variables that constitutes the monetary economy. This is a cogent conclusion from the persistent collapse of behavioral and real models. Conceptual rigor demands, first, to take objective-structural axioms as a formal point of departure and, secondly, to clarify the interrelations of the fundamental concepts income and profit. The present paper reconstructs the characteristic properties of a Walrasian economy in structural axiomatic terms, generalizes them and explores the consequences for our understanding of the working of the economy we happen to live in.

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The realization of the Walrasian program has, apart from mathematical feats, not been a success story (Ackerman and Nadal, 2004). The program cannot be abandoned, though, as long as no promising alternative is on the horizon. It is a bit like Waiting for Godot.

> There is another alternative: to formulate a completely new research program and conceptual approach. As we have seen, this is often spoken of, but there is still no indication of what it might mean. (Ingrao and Israel, 1990, p. 362)

Formal rigor is certainly a strong point of the Walrasian program. Heterodox critics either overlook or ignore that to start from axioms is a logical imperative for any theoretical approach. Thus far Debreu was right. But to start from behavioral axioms means to start with the wrong foot.

> The great contradiction revealed is as follows: one of the theories greatest strength – its claim to deduce significant results from very general hypotheses about the behavior of economic agents – turns out to be its greatest weakness. (Ingrao and Israel, 1990, p. 364)

It has been argued elsewhere that subjective-behavioral thinking has to be replaced by something fundamentally different (2013a). Conceptual rigor demands, first, to take objective-structural axioms as a formal point of departure and, secondly, to clarify the interrelations of the fundamental concepts income and profit. Conceptual consistency deserves as much attention as formal consistency. An axiomatic approach that has no real world interpretation is of no use in theoretical economics.

The present paper reconstructs the characteristic properties of a Walrasian economy in structural axiomatic terms, generalizes them and explores the consequences for our understanding of the working of the economy we happen to live in.

We start from the objective givens of the monetary economy. Section 1 provides the formal point of departure with the set of three structural axioms. In Section 2 the competitive structure is derived from the conditions of market clearing, budget balancing, zero profit and the proportionality of expenditures and labor input. It turns out as the most elementary law of value that the exchange ratio is inverse to the transformation ratio. In Section 3 preferences, productivity, and employment are varied and the effects on the economy as a whole analyzed. Special attention is given to the effects on profits because the neo-Walrasian profit theory is known to be deficient. In Section 4 the Walrasian regime is restated in structural axiomatic terms and generalized. It is demonstrated, inter alia, how the pervasive deflationary bias can be corrected. Section 5 concludes.

1 Conceptual uniqueness

> The effort toward rigor substitutes correct reasonings and results for incorrect ones, but it offers other rewards too. It usually leads to a deeper understanding of the problems to which it is applied, and this has not failed to happen in the present case. (Debreu, 1959, p. x)

1.1 Axioms

The formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of real and nominal variables that constitutes the monetary economy. This is a cogent conclusion from the persistent collapse of behavioral and real models.

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises. Three suffice for the beginning.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |t \tag{1}$$

If DN is set to zero then total income consists but of wage income.

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment, no foreign trade, and no government.

The economic content of the structural axioms is plain. The sole point to mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. It is an imperative of rigorous analysis to keep profit and distributed profit apart.

1.2 Definitions

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (4) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{5}$$

A sales ratio $\rho_X = 1$ indicates that the quantity sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \tag{6}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to total income Y, in other words, that the household sector's budget is balanced.

2 Market clearing, budget balancing and other desiderata

> Surely the greatest oddity in contemporary economics, granted that 'Smith's Problem' is the central issue of economics, is the absence, more than 200 years after the publication of Smith's *Wealth of Nations*, of an intellectually satisfying account of the *modus operandi* of the 'invisible hand'; so the question whether actual monetary exchange economics are in some relevant sense self adjusting ... not only remains to be resolved, *it has yet to be seriously addressed.* (Clower and Howitt, 1997, p. 24), original emphasis

2.1 One firm

From (3), (5), and (6) follows the price as dependent variable:

$$P = \frac{\rho_E}{\rho_X}\frac{W}{R} \quad \text{if} \quad Y_D = 0 \quad |t. \tag{7}$$

Under the condition of market clearing and budget balancing follows:

$$\begin{gathered} P = \frac{W}{R} \\ \text{if} \quad Y_D = 0,\, \rho_X = 1,\, \rho_E = 1 \quad |t. \end{gathered} \tag{8}$$

The market clearing price is equal to unit wage costs if the expenditure ratio is unity. In the case of budget balancing the profit per unit is therefore zero. All changes of the wage rate and the productivity affect the market clearing price in the period under consideration. Let us refer to this property as conditional price flexibility because (8) involves no assumption about human behavior.

From (8) follows:

$$\begin{gathered} \frac{W}{P} = R \\ \text{if} \quad Y_D = 0,\, \rho_X = 1,\, \rho_E = 1 \quad |t. \end{gathered} \tag{9}$$

The real wage is equal to the productivity. This implies that the real wage is not separately determined in the labor market. It has been derived in direct lineage from the axiom set and the conditions of budget balancing and product market clearing. This in turn implies that a variation of the wage rate can never affect the real wage but only the market clearing price. In the simplest case the real wage is exactly equal to the hourly product of a worker which is objectively determined by the production process. Labor gets the whole product. It is immaterial whether the labor union pushes the wage rate up or the firm pushes it down. The wage rate assumes the role of the numéraire that co-determines all other nominal variables (Minsky, 2008, p. 58).

Since the real wage is given in any period independently of employment it cannot possibly determine the level of employment. It is assumed for a start that the economy operates at full employment L_0. Total income is then given by:

$$Y = WL_0 \quad \text{if} \quad Y_D = 0 \quad |t. \tag{10}$$

With this our elementary consumption economy is completely specified. It displays a host of desirable properties (market clearing, budget balancing, conditional product price flexibility, full employment) and is reproducible for an indefinite time span, provided no external hindrances occur.

One desirable property of the elementary consumption economy is easy expandability. If the available labor supply grows the firm hires additional workers at the going wage rate. With more labor input output increases according to (2). On the other hand income and consumption expenditures rise in step according to (10) and

$\rho_E = 1$. Under the conditions of budget balancing and market clearing the price (8) remains unchanged. Supply creates its own demand; Say's Law holds because the expenditure ratio and the sales ratio is unity.

Not to forget, any expansion requires a larger average stock of transaction money and must be supported with 'elastic currency' by the central bank (for details see 2011b, Sec. 3). The expansion happens at constant price if wage rate and productivity either remain unaltered or change with the same rate.

In sum: The structural axioms and the conditions of market clearing and budget balancing render the most elementary formal description of a reproducible consumption economy that operates at any level of employment. Full employment growth conjoined with price stability is possible under additional conditions.

2.2 Two firms

The axioms and definitions have first to be differentiated. Period income changes from (1) to:

$$Y = \underbrace{W_A}_{W} L_A + \underbrace{W_B}_{W} L_B + \underbrace{D_A N_A + D_B N_B}_{Y_D=0} \quad |t. \tag{11}$$

The full employment labor input is now allocated between two firms:

$$L_0 \equiv L_A + L_B \quad |t. \tag{12}$$

Since distributed profits are set to zero in order to keep things simple for the beginning and the wage rates of the two firms are assumed to be identical, total income does not change with a reallocation of labor input between firms. Full employment L_0 is maintained by assumption. Only the composition of the business sector's output changes with a reallocation of labor input.

The partitioning of the consumption expenditures in period t is given by:

$$\begin{aligned} C_A &= P_A X_A \\ C_B &= P_B X_B. \end{aligned} \tag{13}$$

For the relative prices of two products then follows directly from (13) in combination with the differentiated sales ratio (5):

$$\frac{P_A}{P_B} = \frac{R_B}{R_A}\frac{L_B}{L_A}\frac{C_A}{C_B} \quad \text{if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \tag{14}$$

If the markets for both products are cleared the relation of prices is inversely proportional to the relation of productivities and the relation of labor inputs and directly proportional to the relation of consumption expenditures for the two products. A straightforward result materializes if the labor inputs of the two firms stand in the same proportion as the expenditures for both products:

$$\frac{P_A}{P_B} = \frac{R_B}{R_A}$$
$$\text{if} \quad \frac{L_A}{L_B} = \frac{C_A}{C_B} = \frac{\rho_{EA}}{\rho_{EB}} \quad \text{and} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \tag{15}$$

If labor input is allocated according to the consumers' preferences, which are revealed by their expenditure ratios, then relative prices are inversely proportional to the productivities in the two lines of production. The productivities are measurable in principle. Hence relative prices depend in the simplest case with equal wage rates on the objective ratio of productivities. The subjective partitioning of consumption expenditures has no effect on relative prices if it corresponds to the allocation of labor input. We refer to this unique configuration of labor inputs and expenditure ratios as the *competitive structure*.

We define the exchange ratio as quotient of market clearing prices and the transformation ratio as quotient of productivities:

$$\rho_P \equiv \frac{P_A}{P_B} \quad \text{and} \quad \rho_R \equiv \frac{R_A}{R_B}$$
$$\frac{\text{units of}\,B}{\text{unit of}\,A} \leftarrow \frac{\frac{€}{\text{unit of}\,A}}{\frac{€}{\text{unit of}\,B}} \quad \text{and} \quad \frac{\text{units of}\,A}{\text{unit of}\,B} \leftarrow \frac{\frac{\text{units of}\,A}{h}}{\frac{\text{units of}\,B}{h}} \tag{16}$$

The exchange ratio is different from the price relation with regard to the dimension but has the same numerical value. The exchange ratio is the real counterpart of relative prices. In a strictly real analysis only exchange ratios are admissible. Likewise for the transformation ratio. In real terms (15) boils down to:

$$\rho_P = \frac{1}{\rho_R} \qquad |t. \tag{17}$$

In the competitive structure the real exchange ratio is inverse to the transformation ratio. This is the most elementary form of the objective relation between exchange and production. This real law of value is entirely free of subjective connotations.

From (15) in combination with (12) follows under the condition of budget balancing:

$$\frac{L_A}{L_0 - L_A} = \frac{\rho_{EA}}{1 - \rho_{EA}} \quad \Rightarrow \quad L_A = \rho_{EA}\, L_0$$
$$\text{if} \quad \rho_{EA} + \rho_{EB} = 1 \quad |t. \tag{18}$$

The employment of firm A is determined by that part of total income that the households spend on product A. Under the condition of full employment the labor input of firm B is then also known.

The argument of the present section is not affected if L_A (or L_B) is split between an arbitrary number of firms of different size, i.e. $L_A = L_{A1} + L_{A2} + \ldots + L_{An}$. The only condition that must hold is that all firms have the same productivity. The general case with different productivities has been dealt with elsewhere (2011c). The arbitrary number of firms with equal productivity then constitutes the supply side of market A.

The business sector's financial profit in period t is defined with (19) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:[1]

$$Q_{fi} \equiv C - Y_W \quad |t. \tag{19}$$

Because of (3) and (4) this is identical with:

$$Q_{fi} \equiv PX - WL \quad |t. \tag{20}$$

This form is well-known from the theory of the firm.

From (19) and (1) finally follows:

$$Q_{fi} \equiv C - Y + Y_D \quad |t. \tag{21}$$

The three equations are formally equivalent and show profit under different perspectives. Eq. (21) tells us that overall profit is zero if $\rho_E = 1$ and $Y_D = 0$. It is important to recall that we discuss until Section 4.3 the simplified case with zero distributed profit. Hence profit for the business sector as a whole depends at the moment solely on the relation of consumption expenditures and income, i.e. on the expenditure ratio. Then, with an expenditure ratio of unity profit of the business sector as a whole is zero.

For firm A eq. (20) reads in the case of market clearing:

$$Q_{fiA} \equiv P_A R_A L_A \left(1 - \frac{W_A}{P_A R_A}\right) \quad \text{if} \quad \rho_{XA} = 1 \quad |t. \tag{22}$$

Financial profit of firm A is zero under the condition that the quotient of wage rate, price, and productivity is unity. This holds independently of the level of employment or the size of the firm. From the zero profit condition follows:

[1] Nonfinancial profit is treated at length in (2012).

$$P_A = \frac{W_A}{R_A} \quad |t. \tag{23}$$

The price is equal to unit wage costs. In the same way we get the price P_B. Taken together, the zero profit condition – Walras's 'ni bénéfice ni perte' – gives for relative prices again (15) under the condition of equal wage rates:

$$\frac{P_A}{P_B} = \frac{\frac{W_A}{R_A}}{\frac{W_B}{R_B}} = \frac{\frac{W}{R_A}}{\frac{W}{R_B}} = \frac{R_B}{R_A} \quad \Rightarrow \quad \rho_P = \frac{1}{\rho_R} \tag{24}$$
$$\text{if} \quad Q_{fiA} = 0,\, Q_{fiB} = 0,\, \rho_{EA} + \rho_{EB} = 1,\, W_A = W_B = W$$

In other words, in the competitive structure with employment in strict proportion to the expenditure ratios profits of both firms are zero if the wage rates are equal. The relation of prices is unequivocally determined by the inverse productivities independently of the partitioning of the consumption expenditures. No matter how the household sector distributes total income between the two products, the markets are cleared, wage rates are equal and profits are zero.

Since we have from the standard theory of consumer demand the marginalistic behavioral condition that the marginal rate of substitution MRS be equal to the price ratio we are in the position to synthesize the structural formalism and the marginalistic behavioral assumption. From the definition of the expenditure ratio (6) in combination with (3) follows:

$$\frac{\rho_{EA}}{\rho_{EB}} = \frac{\frac{C_A}{Y}}{\frac{C_B}{Y}} = \frac{P_A X_A}{P_B X_B} \quad |t. \tag{25}$$

When, by applying the rule $MRS = \frac{P_A}{P_B}$, the optimal quantities X_A, X_B are determined in the usual way as coordinates of the tangential point of budget constraint and indifference map, then the optimal partitioning of consumption expenditures ρ_{EA}, ρ_{EB} is also determined. This implies that *any* configuration of expenditure ratios can be interpreted as a consumer optimum. In the competitive structure subjective factors like the marginal rate of substitution or the underlying preferences play no longer any role for relative prices because these are determined by the objective productivities.

The task of the auctioneer in the elementary consumption economy is rather straightforward. At the beginning of period t he looks at the productivities and the wage rate, calculates the prices according to (23) and makes them publicly known. The households partition the total consumption expenditures, which are equal to the full employment income, and the business sector allocates the total labor input in identical proportion. Since the marginal rate of substitution is a nonentity it can always be said, but neither be proved nor disproved, that the competitive structure is Pareto optimal. There is no point, therefore, in devoting much words to rationality and optimality. The key question is how the partitioning of full employment labor input can be brought in harmony with the arbitrary partitioning of consumption expenditures between both lines of production. The auctioneer knows the solution because he has the structural axiom set at the back of his mind, but he cannot communicate it to the agents by telling them only the market clearing prices. In order to determine the allocation of labor input the firms need, according to (18), the respective expenditure ratios and the full employment input L_0.

Since changes of the uniform wage rate make no difference in our simple consumption economy they cannot be used as signal. With a flexible market clearing price profit is zero with any wage rate at any level of employment. Thus the firm can be perfectly indifferent between all employment levels. To break the impasse it is necessary to introduce a behavioral assumption. Full employment can only be achieved if the firm hires workers at the going wage rate as long as there is supply available. In the case of a positive profit per unit this rule guarantees the biggest absolute profit at the capacity limit. In the case of zero profit per unit, however, this biggest absolute profit is zero.

3 Signals, interpretations, and the fingers of the invisible hand

> It is correct, of course, to assert that coordination is performed by an "invisible hand" or the "price system", correct, to be sure, but just as surely insane, because such a response is no better than an appeal to Jupiter or Providence. An intellectually respectable answer should consist of something more than tired clichés; observable economic events derive ultimately not from unspecified coordinating mechanisms, whether invisible hands, price systems, or neo-Walrasian "auctioneers", but … from definable actions of real people. What we economists have yet to explain is the working of the fingers of the "invisible hand". (Clower, 1998, p. 410)

3.1 Change of preferences

Our point of departure is the initial period as described in Section 2.2 with the competitive structure at full employment. It is assumed now that well before the beginning of period$_1$ the firms poll the households and learn that that the expenditure ratio for the product of firm A, i.e. ρ_{EA1}, will be up and correspondingly ρ_{EB1} will be down, such that the overall expenditure ratio is still unity.

The firms decide to adapt output accordingly and to maintain the competitive structure. By consequence L_A goes up and L_B is reduced in strict proportion to the expenditure ratios. There is no further change. Relative prices remain unchanged since productivities stay where they are. Total income is not affected by the reallocation of labor input and with an overall expenditure ratio of unity total consumption expenditures are equal to those in the initial period. A change of preferences does not affect the product prices, only the allocation of labor input. Relative prices play no role for the reallocation. What the firms really need is accurate prior information about shifts of the expenditure ratios. Then the respective labor inputs can be calculated with (18) under the premise that total full employment input is known. The adaptation of the labor inputs implies that the workers are, with equal wage rates, indifferent between the firms and move freely between them. This, of course, is an idealization that helps to focus here on the question of price–quantity adaptations of output. The separate effects of wage rate changes are discussed in Section 3.4. Since the wage rates are here uniform they cannot assume the role of a signal. The workers must take their clue from another signal.

As an alternative scenario imagine now that the firms learn about the demand shift as before but cannot change the allocation of labor input immediately. The structure in period_1 is no longer competitive. In order to clear the product market the firms adapt the prices. From (3) in combination with (5), (6), and (11) follows for the general case:

$$P_A = \rho_{EA} \frac{W_A}{R_A} \left(1 + \frac{W_B}{W_A} \left(\frac{L_0}{L_A} - 1 \right) \right) \qquad \text{if} \quad \rho_{XA} = 1, Y_D = 0 \quad |t. \tag{26}$$

The price of product A depends under the condition of market clearing on demand, expressed by the expenditure ratio, unit wage costs and the structure of wage costs in both firms. Mutatis mutandis for firm B. If wage rates are equal in both firms (26) reduces to:

$$P_A = \rho_{EA} \frac{W}{R_A} \frac{L_0}{L_A} \qquad \text{if} \quad \rho_{XA} = 1, Y_D = 0, W_A = W_B = W \quad |t. \tag{27}$$

It holds that $P_{A1} > P_{A0}$ because $\rho_{EA1} > \rho_{EA0}$ with unchanged production conditions. Correspondingly, the market clearing price of firm B falls. Thus, relative prices are no longer in inverse proportion with the unaltered productivities as in (15).

The profit of firm A is now greater than zero because the price is higher while wage costs do not change. As a mirror image firm B makes a loss of equal magnitude. Total profit of the business sector is zero as it was in the initial period. This is a situation than cannot last for long. With continuing losses firm B drops eventually out and the structure of the business sector changes. Product B vanishes and employment may fall below full employment. The adaptation of prices clears the markets but disturbs the initial equality of zero profits. Logically this implies that one firm may get lost in the process. This aspect of price adaptation is usually overlooked.

The only action that leads to a stable outcome is known. It consists of the reallocation of labor input. An appropriate increase of L_A in (27) counteracts the increase of the expenditure ratio and leaves the price at the former level. The move of labor input from firm B to A is in accordance with the profit situation, however, firm A is not forced in any way to increase employment. In stark contrast, firm B is under pressure to reduce employment. It is assumed here that labor takes vacancies as signal and moves smoothly at equal and constant wage rates from firm B to A. There is no point, obviously, in watching price signals. The final result is the same as it was without the detour of price changes except for the redistribution of money between the firms that takes the form of profit and loss (for details about the monetary side see 2011b).

In sum: If the households' preferences between the products A and B change then a purely quantitative adaptation of labor input is sufficient. All prices and the wage rate can be held constant. An adaption of product prices to demand shifts destabilizes the economy and has strong distributive side effects. To answer a shift of preferences the quantity mechanism is needed instead of the price mechanism.

3.2 Change of productivity

Our point of departure is again the initial period with the competitive structure at full employment. It is assumed now that firm A knows at the beginning of period$_1$ that the productivity increases, i.e. that $R_{A1} > R_{A0}$. In order to maintain the competitive structure relative prices should therefore change according to (15). In the simplest case firm A reduces the price and everything else is left unchanged. The lower market clearing price follows from (23). At this price the additional output is, as a first step, fully absorbed by the household sector with unchanged consumption expenditures. The lower price is not only a signal but de facto enables the households to buy the increased quantity. In other words, with unchanged total nominal income (11) real income increases. For deflationary effects on the price level and advisable counter measures see Section 4.2.

The simple price/quantity adaptation, which fits the accustomed idea of supply up – price down, changes the relation of consumption goods and may give rise to a second round adaptation. The new relation of quantities bought, which indicates a quantitative improvement, is given by:

$$\frac{X_{A1}}{X_{B0}} > \frac{X_{A0}}{X_{B0}}. \tag{28}$$

The households, though, may wish to return to the previous relation:

$$\frac{X_{A2}}{X_{B2}} = \frac{X_{A0}}{X_{B0}}. \tag{29}$$

In order to restore the initial relation in period$_2$ the expenditure ratio for product A has to be lowered and that for product B has to be increased. This demand shift leaves relative prices (15) unchanged.

From (25) follows in combination with (15) under the condition that the relation of quantities X_A and X_B remains constant:

$$\frac{\rho_{EA}}{\rho_{EB}} = \rho_{\kappa}\frac{R_B}{R_A} \quad |t. \tag{30}$$

With a constant ratio ρ_{κ} of the quantities bought and consumed the expenditure ratio ρ_{EA} decreases if the productivity R_A increases. As a consequence labor input is reallocated from firm A to B and the households buy in the second round more of product B and less of A. This restoration of the initial relation of the quantities bought happens at constant prices. In the final analysis the productivity push in firm A increases the quantities bought and consumed of both products.

The properties of the competitive structure suggest that a price reduction is the correct first round answer to a productivity increase. The firm, however, has no idea of the systemic interrelations and may decide to sell the same quantity at the same price as before and to reduce labor input thus causing a small version of technological unemployment. If the firm sees the market price as fixed this reaction is not improbable. After all, the idea to lower employment in order to increase profit is rather commonplace. If the firm succeeds total income diminishes and with constant expenditure ratios consumption expenditures in both lines of production diminish thus causing a loss in firm B that is exactly equal to the profit in firm A. This is not stable situation in the longer run.

In sum: Since no firm, however small, is a price taker but fixes its selling price autonomously (with an eye to overall market conditions) the enlightened price setter takes a productivity increase as a signal to reduce the price. The myopic price setter tries to increase profit by reducing labor input thus causing trouble in the rest of the economy. It is quite improbable that the myopic price setter interprets this trouble as a signal to change his behavior. He interprets, rather, his rising profit as a signal to continue with the reduction of labor input, thus aggravating the situation.

3.3 Change of the employment level

Our point of departure is again the initial period with the competitive structure at full employment. Now the overall supply of labor increases and the full employment level reaches a new height, i.e. $L_1 > L_0$.

It is assumed first that both firms act in accordance with (18), that is, both increase labor input at the going wage rate in proportion to the given expenditure ratios. This presupposes that the firms know at the beginning of period$_1$ the exact amount of the new full employment labor input L_1 and the respective expenditure ratios. It would be the wrong course of action to reduce the uniform wage rate because this would, as we know from Section 2.1, only translate into a fall of the market clearing prices. For the labor market as a whole the accustomed idea of supply up – wage rate down is inapplicable, because of the interdependence with the product market as a whole. The interdependence is established by the balanced budget condition which is expressed by an overall expenditure ratio of unity.

Total income increases as both firms move in step by applying (18) and increase labor input to L_{A1}, respectively L_{B1}. Output grows in both firms according to (2). The relation of expenditures, expressed by the respective expenditure ratios, remains the same. Absolute prices stay where they are and the price relation (15) remains unchanged. With a constant wage rate a proportional increase of supply creates proportional demand in both lines of production. Under the proportionality condition Say's Law holds in an economy with an arbitrary number of firms and markets. It nearly goes without saying that the employment expansion requires a larger average stock of transaction money (for details see 2011b, Sec. 3). Otherwise, the monetary side becomes a hindrance to growth.

If the newly hired workers display a different expenditure pattern the expenditure ratios change and by consequence the allocation of labor input between the firm must change in order to maintain the competitive structure. This case can be analytically decomposed into a proportional increase of labor input and a change of preferences as discussed in Section 3.1.

If the total increase of labor supply, i.e. $\Delta L_1 = L_1 - L_0$, is fully allocated to firm B then costs increase by $W\Delta L_1$. However, consumption expenditures increase only by the fraction ρ_{EB} of this amount. Starting from the zero profit situation in the initial period this reduces profit by:

$$\Delta Q_{fiB1} = W\Delta L_1 \left(\rho_{EB} - 1\right) \tag{31}$$

and leaves firm B with a loss. Firm A's labor input and costs remain unchanged but because of the increased total income the market clearing price rises with consumption expenditures C_A. The resulting profit is equal to firm B's loss. This is an untenable situation in the longer run. The solution is known: employment has to move from B to A until the proportionality of the competitive structure is again established.

A Schumpeterian entrepreneur disturbs the competitive structure if the expenditure ratio for his product is disproportionate compared to his labor input. His profit, and the losses in the rest of the economy, vanishes with an increase of labor input.

3.4 Differentiation of the wage rate

Our point of departure is again the initial period with the competitive structure at full employment and an equal wage rate W in both firms. It is assumed now that firm A increases the wage rate at the beginning of period_1 and firm B reduces it, such that total income (11) remains unchanged.

The consequences are obvious. Firm A makes a loss and B a profit of equal magnitude. All other things remain unchanged. Again the situation is untenable. The remedy consists in a reduction of labor input in firm A and an expansion in B. This brings us to the general conditions of the competitive structure with different wage rates.

4 Generalizations

> Wherever entrepreneurs make profits (beyond the market return on their own land, labor, and capital) they expand production; wherever they incur losses, production is contracted. In equilibrium therefore, there are neither profits nor losses. Walras thus created the abstraction of the zero-profit entrepreneur under perfect competition. (Niehans, 1994, p. 214)

4.1 The Walrasian regime in structural axiomatic terms

Equal wage rates is the most convenient assumption to start with. Reality strongly suggests to abandon this simplification first. Hence the 1st axiom reads now:

$$Y = W_A L_A + W_B L_B + \underbrace{D_A N_A + D_B N_B}_{Y_D = 0} \quad |t. \tag{32}$$

We do not stop for an explanation but take it as simple fact that the wage rate is higher in one line of production than in the other. The wage rate is, more precisely, an average that is compatible with extremely different distributions of wage rates among individual workers (the generic term workers includes all levels of management). For the world outside the firm this makes no difference if the expenditure ratios of all workers are equal. This is assumed for the moment.

The wage differentiation splits the labor market. At least one characteristic must be different between the workers in firm A and B. Since the working hours L_A and L_B are now qualitatively different they cannot, in the strict sense, be added up to overall employment. The perfectly free movement of labor between the two lines of production is no longer possible. Hence full employment has now to be defined separately for each line of production. The more we differentiate the occupations and levels the more occasions for mismatches arise. With progressive differentiation the notion of overall full employment gets progressively out of sight.

It is assumed that the expenditure ratios are given and independent of the level of total income. Whether each agent realizes his utility maximum or not is an empty question because utility is a nonentity. From (3) and (6) then follows the market clearing price of product A:

$$P_A = \frac{\rho_{EA}}{R_A}\left(W_A + W_B\frac{L_B}{L_A}\right) \quad \text{if} \quad \rho_{XA} = 1 \quad |t. \tag{33}$$

In can be said that product A's price depends on demand expressed by the expenditure ratio and supply expressed by the production- and cost conditions in firms A and B. It has to be emphasized that (33) gives a numerical answer to the question of the market clearing price while crossing demand and supply schedules never did and never will. The price of product A depends also on the cost situation in firm B. It would be illegitimate to assume away this interdependence with the Marshallian *ceteris paribus*.

The market clearing price is inserted into the profit equation (20):

$$Q_{fiA} \equiv W_A L_A (1 - \rho_{EA}) + \rho_{EA} W_B L_B \quad \text{if} \quad \rho_{XA} = 1 \quad |t. \tag{34}$$

Under the condition of market clearing profit depends on total wage income and the expenditure ratio for product A. Profit does not depend, for example, on productivity or monopoly power or exploitation. Under the additional conditions of zero profit and budget balancing (34) yields:

$$\rho_{EB} W_A L_A = \rho_{EA} W_B L_B \quad \text{if} \quad \rho_{XA} = 1,\ Q_{fiA} = 0,\ \rho_{EA} + \rho_{EB} = 1 \quad |t. \tag{35}$$

From the market clearing price of firm B, i.e. from

$$P_B = \frac{\rho_{EB}}{R_B}\left(W_B + W_A\frac{L_A}{L_B}\right) \quad \text{if} \quad \rho_{XB} = 1, \tag{36}$$

follows under the condition of zero profit and budget balancing again (35).

From (35) in turn the general rule of allocation can be derived as:

$$\frac{\rho_{EA}}{\rho_{EB}} = \frac{W_A L_A}{W_B L_B}$$
$$\text{if } \rho_{XA} = 1,\ Q_{fiA} = 0,\ \rho_{XB} = 1,\ Q_{fiB} = 0,\ \rho_{EA} + \rho_{EB} = 1 \quad |t. \tag{37}$$

The competitive structure is established if weighted labor input is allocated in the same proportion as the expenditure ratios. Hence, given the expenditure ratio ρ_{EA}, a higher wage rate corresponds to a lower labor input in the respective line of production. For equal wage rates (37) reduces to the already known rule of straightforward proportionality of expenditure ratios and labor input as stated with (15). With regard to firm A (37) says explicitly:

$$L_A = \frac{1}{\dfrac{1}{\rho_{EA}} - 1} \frac{W_B}{W_A} L_B$$
$$\text{if } \rho_{XA} = 1,\ Q_{fiA} = 0,\ \rho_{XB} = 1,\ Q_{fiB} = 0,\ \rho_{EA} + \rho_{EB} = 1 \quad |t. \tag{38}$$

Given the expenditure ratio ρ_{EA} and employment in firm B, employment in firm A varies with relative wage rates. An increase of the relative wage rate of firm A corresponds to a lower labor input. In order to determine the labor input in firm A it is assumed that L_B and W_B is known. The wage rate of firm B assumes in this case the role of the nominal numéraire. Then it is sufficient to determine the wage rate W_A such that full employment in this line of production is established.

Mutatis mutandis for firm B:

$$L_B = \frac{1}{\dfrac{1}{\rho_{EB}} - 1} \frac{W_A}{W_B} L_A$$
$$\text{if } \rho_{XA} = 1,\ Q_{fiA} = 0,\ \rho_{XB} = 1,\ Q_{fiB} = 0,\ \rho_{EA} + \rho_{EB} = 1 \quad |t. \tag{39}$$

Note that labor input L_B does not change at all if a lower wage rate W_A is exactly compensated by a higher employment L_A, such that the product of both variables remains constant. We refer to this special case as hyperbolic variation. An alternative characterization would be to say that the elasticity of employment with regard to the wage rate is unity.

Equations (38) and (39) are mutually compatible under the condition of budget balancing.

Relative prices follow from (33) and (36):

$$\frac{P_A}{P_B} = \frac{\rho_{EA}}{\rho_{EB}} \frac{R_B}{R_A} \frac{L_B}{L_A} \tag{40}$$

$$\text{if } \rho_{XA} = 1,\, Q_{fiA} = 0,\, \rho_{XB} = 1,\, Q_{fiB} = 0,\, \rho_{EA} + \rho_{EB} = 1 \quad |t.$$

Now we substitute the general condition for the competitive structure (37) and this finally gives:

$$\frac{P_A}{P_B} = \frac{\frac{W_A}{R_A}}{\frac{W_B}{R_B}} \tag{41}$$

$$\text{if } \rho_{XA} = 1,\, Q_{fiA} = 0,\, \rho_{XB} = 1,\, Q_{fiB} = 0,\, \rho_{EA} + \rho_{EB} = 1 \quad |t.$$

In the competitive structure the relation of market clearing prices is equal to the relation of unit wage costs. With equal wage rates this reduces to (15) and ultimately to (17).

To put the formalism to work imagine a scenario with initial full employment in both lines of production followed by an increase of labor supply L_B. The expenditure ratios remain fix. According to the accustomed idea of the price mechanism the wage rate is bid down. The fall of W_B and the increase of employment L_B in (38) is here hyperbolic, that is, their product remains constant. By consequence L_A remains unchanged. Hence, total income remains unchanged too. However, with a falling wage rate and increasing employment L_B and output O_B on the one hand, the market clearing price P_B on the other hand must fall. Since the price P_A remains unaffected the price level, measured as a weighted average of both prices, declines. With increasing employment in firm B the composition of output changes. In the first round of adaptation the households buy more of product B. The previous composition can be reestablished in a separate step with a change of expenditure ratios as described in Section 3.1. Note that our scenario follows the logic of the competitive structure. Behavioral assumptions like ‘maximization of an utility indicator’ (Ingrao and Israel, 1990, p. 297) are not applied. In particular the assumption of profit maximization is not applied. These conceptions cannot be justified (for an alternative formalization of human behavior see 2011a).

The generality of the structural axioms enables a reconstruction of the Walrasian regime as special case. The well-known characteristics are present: market clearing, budget balancing, zero profit, full employment in both lines of production, and optimal partitioning of the expenditures between product A and B. It is important to emphasize, though, that an analog to unit wage costs cannot be formulated in real terms, that is, eq. (41) cannot be expressed in a Walrasian context. Or, put otherwise, a consistent Walrasian regime works on the implicit assumption of equal wage rates because only in this case (41) reduces to (17) which expresses a relation of real magnitudes. The Walrasian approach cannot, according to its own rules, admit nominal or monetary magnitudes.

> ... the prerequisite for the elaboration of the theory of value is the exclusion of the nominal form in which economic magnitudes occur, and beyond that, the elimination of all monetary magnitudes. (Benetti and Cartelier, 1997, p. 207)

This is the main difference in comparison to the structural axiomatic approach which has built the interaction of real and nominal magnitudes into the formal foundations. The quantity of money follows directly from the axiom set (for details see 2011b). In marked contrast there is no proper place for money in the Walrasian regime. The auxiliary role of money is restricted to the determination of the price level. In the structural axiomatic approach the quantity of money is not among the determinants of the market clearing price.

4.2 Price level stability

Price level stability is an additional desideratum. In the Walrasian regime this is achieved by holding the quantity of money, respectively its growth rate, stable. The mechanism in the structural axiomatic regime is quite different.

From the simplest possible case of Section 2.1 we know that the market clearing price is determined by the wage rate and the productivity. In Section 4.1 we have found that an employment expansion in one line of production is accompanied by a fall of the wage rate and of the market clearing price. Since the price level for our simple economy is defined as weighted average of the market clearing prices in the two lines of production, i.e. as

$$\begin{aligned} &P \equiv P_A\Theta_A + P_B\Theta_B \\ &\text{with} \quad \Theta_A + \Theta_B = 1 \end{aligned} \tag{42}$$

the price level declines with a fall of P_B. This is not a desirable outcome if one holds that price stability is preferable to both inflation and deflation.

In eq. (39) labor input L_B depends on the relation of wage rates in the two lines of production. The commonplace idea is that the wage rate W_B must fall in order to increase employment L_B. This move is correct as a first step. With correct is meant that the variation of relative wage rates is in accordance with the competitive structure. This, though, is not the end of the story if we also want price stability.

In order to keep the price level constant a compensating wage increase that leaves relative wage rates undisturbed is required. That is, in a second round the wage rates in both lines of production have to be increased. According to (41) relative prices are not affected by a proportional wage rate increase. The net result of the two steps is that the wage rate W_A rises bit and W_B falls a bit, but less so than at the first step. Needless to emphasize that the myopic decision makers in firm A do not see any necessity for wage rate increases in order to counteract the deflationary bias that is inherent in the employment adaptation of firm B. As we have seen in Section 3.2, this bias is amplified by productivity increases. What is required in a Walrasian regime, then, is an enlightened wage setter. The quantity of money has no direct impact on the price level (for details see 2011b), but the wage rate has. All in all, as a guide to economic policy the Walrasian regime and the attendant quantity theory can neither be taken seriously as a simplified theoretical description of the elementary interdependencies nor as an ideal. Price signals and price taking agents are not sufficient to keep the economy in a reproducible state. Moreover, the price taker is a nonentity.

4.3 Profit and profit distribution

We finally have to allow for positive profit and profit distribution. The 1st axiom now takes the general form:

$$Y = W_A L_A + W_B L_B + \underbrace{D_A N_A + D_B N_B}_{Y_D > 0} \quad |t. \tag{43}$$

From the profit definition (19) in combination with (6) follows:

$$Q_{fiA} \equiv \rho_{EA} \left(W_A L_A + W_B L_B + Y_D \right) - W_A L_{A.} \quad |t. \tag{44}$$

Profit in firm A depends on the expenditure ratio, the wage costs in both firms and the total amount of distributed profits. The formula for firm B is analogous.

For a comparison between firms of different size the profit ratio is needed. It is defined as:

$$\rho_{QA} \equiv \frac{Q_{fiA}}{W_A L_A} \quad |t. \tag{45}$$

The profit ratio, which is different from the profit rate, is the quotient of absolute profit and costs. It carries no dimension.

We now introduce the additional condition that the profit ratios are equal in all lines of production. Therefrom follows:

$$\rho_{QA} = \rho_{QB} \quad \Rightarrow \quad \frac{\rho_{EA}}{\rho_{EB}} = \frac{W_A L_A}{W_B L_B} \tag{46}$$

Again we get the condition of the competitive structure (37), i.e. weighted labor input in the two lines of production stands in the same relation as the respective expenditure ratios. That is to say, that the zero profit condition is only a limiting case of equal profit ratios. In the general case budget balancing, i.e. $\rho_{EA} + \rho_{EB} = 1$, is not required. In the competitive structure holds for the price relation (41), that is, the market clearing prices are proportional to unit wage costs. Absolute prices are, of course, higher with distributed profits than without them.

With distributed profits greater than zero disturbances of the competitive structure do no longer result in a profit in one line of production and a loss in the other but in a higher or lower profit ratio. Profit in both lines are greater that zero. This, of course, stabilizes the economy. A firm can survive for an indefinite time with a small profit but not with a loss. Without profit buffers, which are created by profit distribution, the zero profit economy displays the highest possible degree of instability.

What is needed in a general Walrasian regime is not only a mechanism for overall market clearing but also a mechanism for overall profit ratio equalization. This is a theoretical requirement. Under the zero profit condition this point is implicitly answered: if all profits are zero, all profit ratios are equal. We do not know whether profit ratios in fact tend to equalize in the real world but we can be absolutely sure that the zero profit configuration has no counterpart wherever. Apart from the question of profit ratio equalization one crucial point is that the neo-Walrasian profit theory is indefensible (for details see 2013a). This, however, is not to say that Post-Keynesianism is any better in this regard (for details see 2013b).

The original Walrasian attempt consisted in a sheer multiplication of the one-fits-all basic idea of demand function–supply function–equilibrium. All three elements are nonentities. How this hapless construction could ever find support among economists is as great a puzzle as how epicycles could find support among astronomers for more than thousand years. Whether economists can get out faster of the protoscientific stage remains to be seen.

5 Conclusion

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. In the present paper assumptions like maximization of an utility indicator are not applied. In particular the assumption of profit maximization is not applied. These conceptions cannot be justified. Therefore, three non-behavioral axioms constitute the formal groundwork for the reconstruction and generalization of a Walrasian economy. The main results of the analysis are:

- The structural axioms and the conditions of market clearing and budget balancing render the most elementary formal description of a reproducible consumption economy that operates at any level of employment.

- In the competitive structure subjective factors like the marginal rate of substitution or the underlying preferences play no longer any role for relative prices because these are determined by the objective productivities.
- In the competitive structure the real exchange ratio is inverse to the transformation ratio.
- If the households' preferences between the products A and B change then a purely quantitative adaptation of labor input is sufficient. All prices and the wage rate can be held constant. An adaption of product prices to demand shifts destabilizes the economy and has strong distributive side effects. To answer a shift of preferences the quantity mechanism is needed instead of the price mechanism.
- Since no firm is a price taker but fixes its selling price autonomously (with an eye to overall market conditions) the enlightened price setter takes a productivity increase as a signal to reduce the price. The myopic price setter tries to increase profit by reducing labor input thus causing trouble in the rest of the economy.
- What is required in a Walrasian regime is an enlightened wage setter to counteract the pervasive deflationary bias.
- The generality of the structural axioms enables a reconstruction of the Walrasian regime as special case. The well-known characteristics are present: market clearing, budget balancing, zero profit, full employment in both lines of production, and optimal partitioning of the expenditures between products.

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