# The Effect of Compressed Demographic Transition and Demographic Gift on Economic Growth

Shin, Inyong

Asia University

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# The Effect of Compressed Demographic Transition and Demographic Gift on Economic Growth

Inyong Shin*

**Abstract**

In this paper, we examine the demographic transition and its effect on economic growth using a cross-country data. We use a threshold regression model to verify the transition and to confirm whether the demographic transitions are compressed or not in developing countries. We found out that in general, the demographic transitions, including the decreasing birth and death rate, in developing countries start in an earlier development stage compared to the demographic transitions in developed countries. These results suggest that the aging population and the decreasing working-age fraction in developing countries can also start in an earlier development stage than the experiences of developed countries and that the demographic gift in developing countries can also be lost in an early stage.

JEL Classification Codes: J11, J13, O11

Keywords: economic growth, compressed demographic transition, latecomer's advantage, aging population, threshold model.

*Department of Economics, Asia University,
5-24-10 Sakai Musashino Tokyo 180-8629 Japan,
Tel.: +81-422-36-5259,
Fax: +81-422-36-4042,
e-mail addredss: shin@asia-u.ac.jp

# 1 Introduction

This paper analyzes the change of birth rate and death rate as the economy grows. The difference of birth rate and death rate is an important factor to verify the populations growth rate.[1] Both birth and death rate drop as a country develops from a pre-industrial to an industrialized economic system. Moreover, the death rate starts to decrease before the birth rate does. The transition from a pre-modern regime of high birth and death rates to a post-modern regime of low birth and death rates through the intermediate regime of high birth rate and low death rate is called the demographic transition. Weber (2010) and Galor (2011) are details for the survey on the demographic transition.

There are a lot of researches on the demographic transition and factors that explain the drop of death rate. Cutler, et al. (2006) gives nutrition, public health, urbanization, vaccination, medical treatments, education, etc. as the decrease factors in death rate. Tekce (1985) points out that a mother's academic level, equipment of the house (e.g. accessibility of safe water), householders' occupation and income, sanitary practices (e.g. washing hands), nutrient level, etc. exert a big influence on decreasing death rate. Weil (2013) illustrates the improvement of living standard, the improvement of the public health equipment and the improvement of medical treatment as the factors which decrease the death rate. In Omran (1971), the decline in death rate is called epidemiologic transition. Meanwhile, Momota et al. (2005), Pecchenino and Pollard (1997), Chakraborty (2004), Tabata (2005), Mizushima (2009), Chakraborty, et al. (2010) analyze death rate as endogenous variable with a two-period overlapping generations model.[2]

There are also many famous economic theoretical papers regarding the birth rate. For example, Easterlin (1966), Becker (1960) and Nerlove et al. (1978) are static studies, and Becker and Barro (1988), Barro and Becker (1989), Lapan and Enders (1990), Benhabib and Nishimura (1989), Becker et al. (1990), Kremer (1993), Galor and Weil (1996), Dahan and Tsiddon (1998) and Qi and Kanaya (2010) are dynamic studies. The determinants of birth rate have been sought in the decline of death rate, emphasizing the quality of children, the increase of the opportunity cost of the women, an increase in the status and education of women, urbanization (movement off the farms), social security systems, religious values, social values, etc.[3] Except for religious values and social values, the decrease factors in birth and death rate are deeply related to the economic development. By this, we consider both birth and death rate as functions of GDP per capita. We analyze the demographic transition with the stage of economic development, specifically with GDP per capita.

For further understading of the demographic transition, we use a conceptual graph in Figure 1. The demographic transition involves three regimes.[4] In the first regime, pre-industrial society, both birth rate and death rate are high and roughly in balance. The population grows slowly. In the second regime, the death rate declines rapidly while the birth rate remains high. The second regime sees a rise in population and this is called population explosion. In the third regime, both birth rate and death rate are low. Instead of the rapid growth of the second regime, population growth slows down.

Not only the demographic transition, but also birth rate and death rate involve three regimes and two

[1] To forecast the population of a particular country or region, we must also account for immigration and emigration. (Weil, 2013)

[2] Sen (1998) mentions that mortality is not in itself an economic phenomenon and that while birth rate is based on economic thoughts, death rate seems to be exogenous.

[3] Some researches (e.g., Doepke(2005), Murphy (2009), Fernandez-Villaverde (2001) etc.) report that an increase in the income makes fertility increase.

[4] On other researches, the number of regimes are devided into 4 or more.

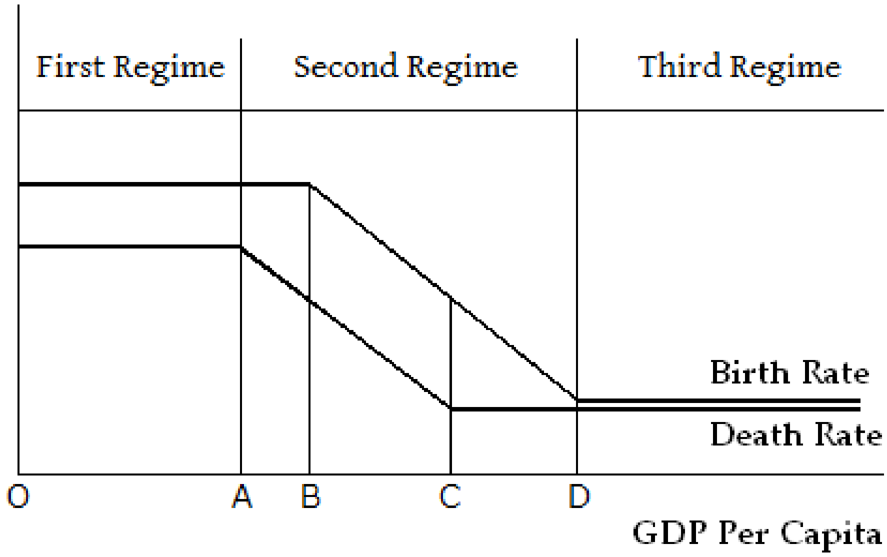


Figure 1: Conceptual graph

turning points. The first regime is the period that shows a gradual change before the demographic transition begins as the period OB in birth rate and the period OA in death rate in Figure 1. The second regime is the period that shows a rapid drop after the first regime as that of period BD and AC in birth rate and death rate, respectively. The third regime is the final period that shows a gradual change again as the over D period in birth rate and the over C period in death rate.

At the first analysis, we examine the trend of birth rate and death rate and confirm the demographic transition using a recent cross-country data from the World Bank. Almost all of the researches on the demographic transition deal with a specific country or region and use a fairly long-term data. For example, Doepke (2005) deals with the United Kingdom data for about one century from 1861 to 1951. Maddison (2001) and Murtin (2009) uses a panel data during 1870 to 2000, etc. We use the cross-country data for recent 50 years which is different from the previous researches. We estimate the GDP per capita level at each turning points and confirm that the turning points of death rate are earlier than those of birth rate, where in point A is earlier than point B and point C is also earlier than point D using the data.

At the second analysis, we verify a compressed development in the demographic transition using the cross-country data. The process of development followed by the latecomers has shifted to one that can be described as compressed development. According to Gerschenkron (1962), the latecomers can evade the bad points and can only imitate the good points which the advanced countries have experienced in their economic development.[5] The latecomers in economic development can skip several stages which the former had to go through by adopting their advanced technology, so the latecomers can grow more rapidly on borrowed technology from early starters. This is called latecomer's advantage. The development process of latecomers can be compressed into shorter periods comparing with that of the leaders. The latecomer's advantage and the compressed development are particularly common in structural changes in the process of industrialization.

The decline of death rates in the developed countries is an endogenous result of the their own efforts to research on the development on medical technology, diseases, new medicines and effective public health care,

[5]The hypothesis of the "latecomer's advantage" was advanced by Alexander Gerschenkron. See Gerschenkron (1962).

etc. Meanwhile, the decline of death rates in the developing countries is an exogenous result of the adoption of the experiences and technologies from the developed countries. It can be said that the developing countries enjoy the latecomer's advantage in the demographic transition as well as in the process of industrialization. If developing countries study on the developed countries, they can save their time and effort when they face a similar problem which developed countries have already faced. If the developing countries mimic the demographic transition of the developed countries, the demographic transition of the developing countries will occur at an earlier economic development stage compared with developed countries. We will describe this process as "the compressed demographic transition".

There are already some textbooks describing the compressed demographic transition, even though they did not name the "compressed" explicitly. We introduce two kinds of famous textbooks below. Weil (2013) gives an example that compares life expectancy and GDP per capita of France and India. The compressed demographic transition is referred in Weil (2013) as follows:

> To give an example, in India life expectancy at birth increased from 26.9 years in 1930 to 55.6 years in 1980. In France a roughly comparable change took more than three times as long: Life expectancy at birth was 27.9 years in 1755 and reached 56.7 years only in 1930.
>
> In addition to its speed, the crucial characteristic of the mortality transition in the developing world is its occurrence at a level of income per capita far below income in the rich countries when they went through a similar transition. For example, India achieved a life expectancy of 55.6 years in 1980 with income per capita of $1,239 (in 2000 dollars). By contrast, France achieved a life expectancy of 56.7 years in 1930 with income per capita of $4,998 (also in 2000 dollars).
>
> Weil (2013) Economic Growth, pp.119

While, Todaro and Smith (2009) refers on the compressed demographic transition as follows:

> Nevertheless, the average life span remains about 12 years greater in the developed countries. This gap has been sharply reduced in recent decades. For example, in 1950, life expectancy at birth for people in developing countries averaged 35 to 40 years, compared with 62 to 65 years in the developed world. By 1980, the difference had fallen to 16 years as life expectancy in the LDCs increased to 56 years (a gain of 42%) while in the industrial nations it had risen to 72 years (an increase of 13%).
>
> Todaro and Smith (2009) Economic Development, pp.280-281

Almost previous researches like Weil (2013) and Todaro and Smith (2009) remain on just giving an example to explain about the compressed demographic transition without analyzing it statistically. In this paper, we analyze it statistically. There is no research to verify the compressed demographic transition statistically by using the cross-country data like in our research. This is our new contribution in this research field.

Even though we did not find a new determinant of birth rate and death rate theoretically, our paper yields several important results statistically by an econometric analysis: (i) we show that the threshold levels of death rates appear in an earlier stage than those of the birth rates. (ii) we confirm that the birth and death rates in the developed counrties are located above than those of in the developing counrties. In other words, the birth and death rate of the developing countries are lower than that of the developed countries at the same income level. (iii) we found out that the turning points B and D in Figure 1 depend on the initial income level and the lower the initial income level is, the lower the turning point is. In other words, in case of birth rates, the developing countries turn their regimes at lower income level compared with the developed countries. (iv) we found out that the developing countries reach their turning points at higher level of birth and death rates compared with the developed countries. (v) the developing countries undergo a more intensive decrease in birth and death rate than the developed countries do. These results support the compressed demographic transition which we defined.

To sum up, the compressed demographic transition, including the decreasing birth and death rate, in developing countries start in the earlier deveolpment stage compared to the demographic transition in developed countries. The results suggest that the aging population and the decreasing working-age fraction in developing countries can start in an earlier development stage than the experiences of developed countries and that the demographic gift in developing countries can also be lost in an early stage.[6] The aging population and the decreasing fraction of working-age will not only affect the decrease of demographic gifts but also the decrease of pension revenue source. The latter makes it probable that the pension system cannot continue to be part of a stable system as it had in the past. The demographic bonus could be reversed to a demographic onus.

This paper is organized as follows: Section 2 describes the data we used and summarizes the basic statistics about the data. Section 3 analyzes the existence of the demographic transition. Section 4 analyzes whether the demographic transition is compressed or not in developing countries, and the effect of the compressed demographic transition on economic growth using a simulation method. We conclude in Section 5. And finally, we include an Appendix.

# 2 Data and Basic Statistics

## 2.1 Data

We use the GDP per capita, the birth rate and the death rate.[7] The data were drawn from the World Development Indicators (WDI) 2010 in the World Bank.

Table 1: Data source

| Indicators | URL |
|---|---|
| GDP per capita (current US$) | http://data.worldbank.org/indicator/NY.GDP.PCAP.CD |
| Birth rate, crude (per 1,000 people) | http://data.worldbank.org/indicator/SP.DYN.CBRT.IN |
| Death rate, crude (per 1,000 people) | http://data.worldbank.org/indicator/SP.DYN.CDRT.IN |

* accessed on 1st/Feb/2011.

Table 1 shows the detailed data source. According to the World Development Indicators, the definitions

[6] Demographic gift is a term in demographics used to describe the initially favorable effects of falling birth rates on the age-dependency ratio, the fraction of children and aged as compared to that of the working population. In general, the most developed countries, where fertility reduction took place earliest, have already been through a period in which the working-age fraction rose, and in the next several decades will be facing a decline in working-age fraction. In many developing countries, declining fertility over the last several decades is still producing a rise in the fraction of the population than that of the working age. In many cases, this "demographic gift" can have a large impact on economic growth. (Weil, 2013)

[7] Demographers measure the fertility rate in a variety of ways, like as crude birth rate, total fertilty rate, general fertility rate, age-specific fertility rate, etc. World Development Indicators (WDI) contains not only crude birth rate and crude death rate but also fertility rate and motality rate data. However, fertility rate and motality rate are collected every five years. The number of samples of the fertility and mortality rate are limited. The birth rate and death rate are annual data. So, we chose the crude birth rate and the crude death rate instead of the fertility and the mortality.

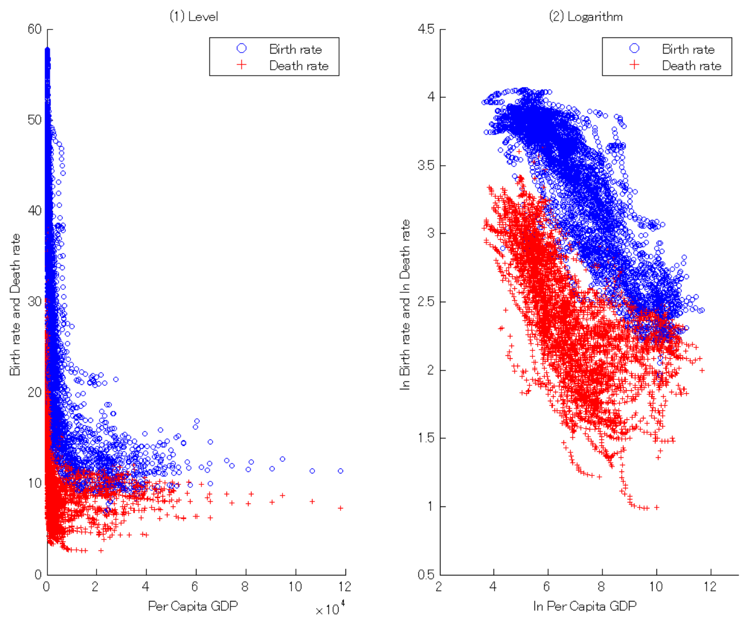


Figure 2: Per capita GDP and birth and death rates

are that the crude birth rate indicates the number of live births and the crude death indicates the number of deaths occurred each year, per 1,000 population estimated at midyear. The data are available for 49 years from 1960 to 2008. The WDI listed 213 countries. However, among the 213 countries, only 89 countries were fully collected for the three kinds of data and for 49 years. Therefore, we focus on these 89 countries. Table A1 in Appendix contains the basic information of the 89 countries.

## 2.2 Basic Statistics

Table 2 reports the information of 1) per capita GDP, 2) birth rate, 3) death rate, 4) the change of birth and death rate, 5) the change rate of birth and death rate, and 6) the income elasticity of birth and death rate. We denote per capita GDP, birth rate and death rate as $X$, $B$ and $D$, respectively. We take the logarithm to them and denote log per capita GDP, log birth rate and log death rate as $x$, $b$ and $d$, respectively, that is, $x = \ln X$, $b = \ln B$ and $d = \ln D$. Table 2 shows their min, max, mean, median and standard deviation. Table 2 (a) and (b) illustrate the basic statistics of level and logarithm value, respectively. The subscript $_1$ and $_T$ represent the first year 1960 and the last year 2008. In the last 50 years, the average decline in the birth rate ($\Delta B = -15.09$) is bigger than that in the death rate ($\Delta D = -7.61$). However, change rates are almost the same ($\frac{\Delta B}{B} = -0.39$ and $\frac{\Delta D}{D} = -0.4$). It suggests that the death rate has already started to decrease and has reached a stable phase, meanwhile, the birth rate is far from finished and is in the process of decreasing. This can be an evidence that the decreasing death rate is ahead of the decreasing birth rate.

Figure 2 plots the per capita GDP and the birth and death rates. In Figure 2 (1), the horizontal axis shows per capita GDP ($X$) and the vertical axis shows the birth rate ($B$) and the death rate ($D$). In Figure 2 (2), the horizontal axis shows log per capita GDP ($x$) and the vertical axis shows the log birth rate ($b$) and the log death rate ($d$). o's and +'s represent the birth rates and the death rates, respectively. It is difficult to find the demographic transition in Figure 2 (1) because the data are grossly left-sided, while in Figure 2 (2) we can visually find the demographic transition. Therefore, we used the logarithm for GDP per capita, birth rate and death rate in the following analysis, without considering the analysis with the level data.

Table 2: Basic statistics

(a) Level

| Variables | Min | Max | Mean | Median | Standard Deviation |
|---|---|---|---|---|---|
| 1) $X_1$ | 40.63 | 2881.10 | 485.35 | 247.06 | 607.78 |
| 1) $X_T$ | 144.77 | 117954.68 | 16026.63 | 4223.95 | 23247.80 |
| 1) $X$ | 37.77 | 117954.68 | 4975.01 | 901.09 | 10033.93 |
| 2) $B_1$ | 13.70 | 56.36 | 38.94 | 44.49 | 12.30 |
| 2) $B_T$ | 8.70 | 53.54 | 23.86 | 21.11 | 11.20 |
| 2) $B$ | 6.90 | 57.79 | 31.57 | 33.17 | 13.43 |
| 3) $D_1$ | 5.70 | 30.39 | 16.52 | 16.62 | 6.53 |
| 3) $D_T$ | 2.70 | 17.26 | 8.91 | 8.16 | 3.66 |
| 3) $D$ | 2.69 | 38.00 | 11.67 | 10.19 | 5.38 |
| 4) $\Delta B$ | -30.29 | 0.01 | -15.09 | -13.70 | 8.84 |
| 4) $\Delta D$ | -20.50 | 2.22 | -7.61 | -7.45 | 5.88 |
| 5) $\frac{\Delta B}{B}$ | -0.74 | 0.00 | -0.39 | -0.42 | 0.18 |
| 5) $\frac{\Delta D}{D}$ | -0.88 | 0.30 | -0.40 | -0.42 | 0.27 |
| 6) $e_B$ | -1.28 | 0.26 | -0.04 | -0.02 | 0.14 |
| 6) $e_D$ | -2.53 | 1.34 | -0.06 | -0.03 | 0.31 |

(b) Logarithm

| Variables | Min | Max | Mean | Median | Standard Deviation |
|---|---|---|---|---|---|
| 1) $x_1$ | 3.70 | 7.97 | 5.58 | 5.51 | 1.07 |
| 1) $x_T$ | 4.98 | 11.68 | 8.43 | 8.35 | 1.76 |
| 1) $x$ | 3.63 | 11.68 | 7.08 | 6.80 | 1.69 |
| 2) $b_1$ | 2.62 | 4.03 | 3.60 | 3.80 | 0.39 |
| 2) $b_T$ | 2.16 | 3.98 | 3.06 | 3.05 | 0.48 |
| 2) $b$ | 1.93 | 4.06 | 3.34 | 3.50 | 0.51 |
| 3) $d_1$ | 1.74 | 3.41 | 2.72 | 2.81 | 0.43 |
| 3) $d_T$ | 0.99 | 2.85 | 2.10 | 2.10 | 0.41 |
| 3) $d$ | 0.99 | 3.64 | 2.36 | 2.32 | 0.45 |
| 4) $\Delta b$ | -1.33 | 0.00 | -0.54 | -0.55 | 0.30 |
| 4) $\Delta d$ | -2.15 | 0.27 | -0.61 | -0.55 | 0.48 |
| 5) $\frac{\Delta b}{b}$ | -0.36 | 0.00 | -0.15 | -0.15 | 0.08 |
| 5) $\frac{\Delta d}{d}$ | -0.68 | 0.13 | -0.21 | -0.19 | 0.16 |
| 6) $e_b$ | -2.20 | 0.34 | -0.31 | -0.31 | 0.26 |
| 6) $e_d$ | -6.84 | 2.42 | -0.49 | -0.41 | 0.82 |

Note: $\Delta B = B_T - B_1,\ \Delta D = D_T - D_1,\ \frac{\Delta B}{B} = \frac{B_T - B_1}{B_1},\ \frac{\Delta D}{D} = \frac{D_T - D_1}{D_1},\ e_B = \frac{(B_T - B_1)/B_1}{(X_T - X_1)/X_1},\ e_D = \frac{(D_T - D_1)/D_1}{(X_T - X_1)/X_1},$

$\Delta b = b_T - b_1,\ \Delta d = d_T - d_1,\ \frac{\Delta b}{b} = \frac{b_T - b_1}{b_1},\ \frac{\Delta d}{d} = \frac{d_T - d_1}{d_1},\ e_b = \frac{(b_T - b_1)/b_0}{(x_T - x_1)/x_1},\ e_d = \frac{(d_T - d_1)/d_1}{(x_T - x_1)/x_1}$

Figure 3 shows the relationship between the initial income ($x_1$) and the three kinds of variations, which are 1) the changes – $\Delta b$ and $\Delta d$, 2) the change rates – $\frac{\Delta b}{b}$ and $\frac{\Delta d}{d}$, and 3) the elasticities in birth rate and death rate – $e_b$ and $e_d$.[8] Table 3 reports the correlation coefficients between the two variables and the estimated values by regression analysis with the initial income and the variations as dependent variables and independent varilables, respectively. Table 3 (a) and (b) are the results of birth rates and death rates, respectively. Based on the corr領ellation coefficients, we find the negative relationship between the initial income and the magnitude of the change in birth rates. The higher the initial income is, the larger the

[8] It would appear that there is one outlier in (3) and are two outliers in (6). The name of the country in (3) is Liberia, and the names of countries located above and below of (6) are Democratic Republic of the Congo and Liberia, respectively.

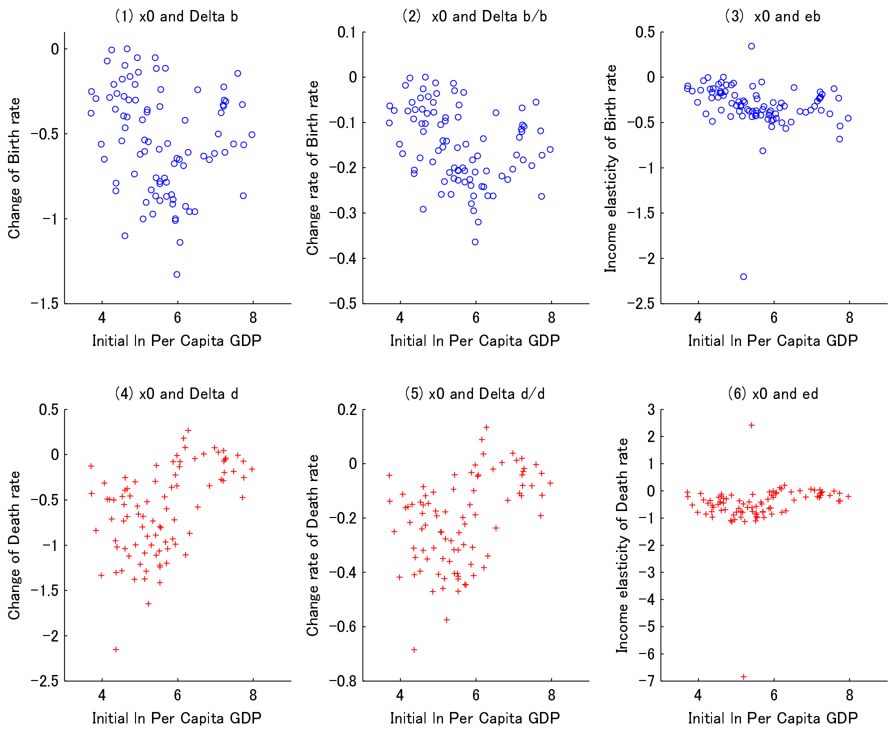


Figure 3: Initial income and the changes of birth and death rates

Table 3: Results of corellations and regressions

(a) Birth Rate

| | Variables | Correlation | Dependent Variables | Constant | Slope |
|---|---|---|---|---|---|
| (1) | $\Delta b, x_1$ | -0.180 | $\Delta b$ | -0.251 | -0.051 |
| | | | ($t$ value) | (-1.476) | (-1.711) |
| (2) | $\frac{\Delta b}{b}, x_1$ | -0.313 | $\frac{\Delta b}{b}$ | -0.014 | $-0.024^{**}$ |
| | | | ($t$ value) | (-0.322) | (-3.075) |
| (3) | $e_b, x_1$ | -0.211 | $e_b$ | -0.021 | $-0.052^{*}$ |
| | | | ($t$ value) | (-0.140) | (-2.016) |

(b) Death Rate

| | Variables | Correlation | Dependent Variables | Constant | Slope |
|---|---|---|---|---|---|
| (1) | $\Delta d, x_1$ | 0.494 | $\Delta d$ | $-1.859^{**}$ | $0.223^{**}$ |
| | | | ($t$ value) | (-7.769) | (5.301) |
| (2) | $\frac{\Delta d}{d}, x_1$ | 0.429 | $\frac{\Delta d}{d}$ | $-0.579^{**}$ | $0.066^{**}$ |
| | | | ($t$ value) | (-6.871) | (4.430) |
| (3) | $e_d, x_1$ | 0.198 | $e_d$ | $-1.336^{**}$ | 0.152 |
| | | | ($t$ value) | (-2.916) | (1.883) |

Notes: * and ** indicate statistical significance at the 5% and 1% levels, respectively.

change in birth rate is. Considering the compressed demographic transition, it may possibly seem to be counterintuitive, but when thinking about that the birth rate in the process of decreasing, it can make sense. We will explain them later in details with a conceptual graph.

On the other hand, we find the positive relationship between the initial income and the magnitude of the change in death rate. The regression results have the same features with the correlation coefficients. The

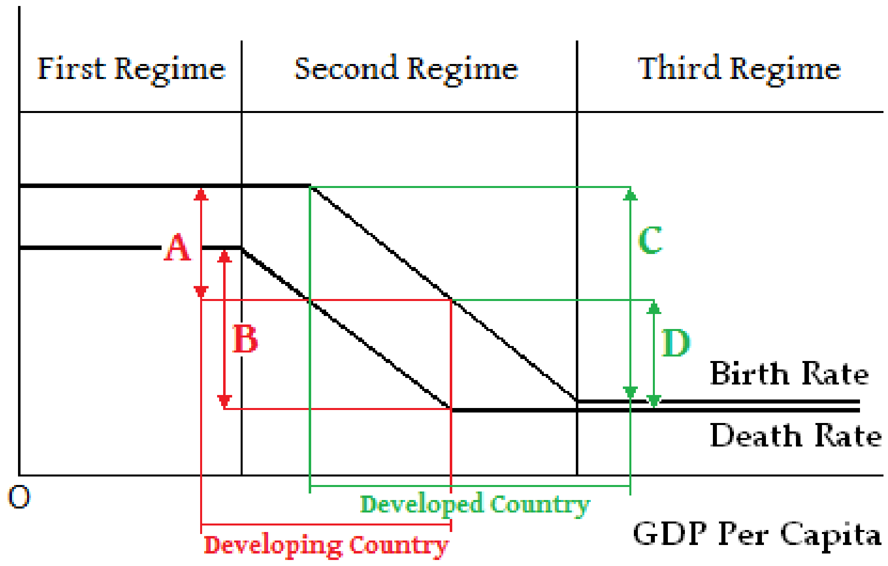


Figure 4: Conceptual graph

higher the initial income is, the smaller the change in the death rate is.

For further understanding of the results, we used a conceptual graph in Figure 4. At first, you can observe the change of death rate. In developed countries, the decline in death rate has already started at the initial point, the magnitude of the change of birth rate is small. To compare the magnitude of the change of death rate in the developed countries (the width D) and that of in the developing countries (the width B), the width B is longer than the width C (B>D). Otherwise, the birth rate in developing countries has not yet reached the third regime. To compare the magnitude of the change of birth rate in the developed countries (the width C) and that of in the developing countries (the width A), the width C is longer than the width A (C>A).

To confirm this interpretation from the data, we modify Figure 2 as Figure 5. We divide whole countries into two parts, in which the countries with initial income over the mean of initial income ($x_1 \geq 5.58$) and the countries with initial income below the mean of initial income ($x_1 \leq 5.58$). The numbers of countries over and below the initial income level are 39 and 50, repectively.

Figure 5 (1) and (2) show the birth and death rates in the countries with over the mean of initial income and those with below the mean of initial income, respectively. Figure 5 shows well the consistency with our interpretation about the relationship with Table 3 and Figure 4, that is, the higher the initial income is, the larger the change in birth rate is, however, the higher the initial income is, the smaller the change in the death rate is.

# 3 Demographic Transition

## 3.1 Model 1

### 3.1.1 Model

We used a threshold regression model to verify the demographic transition. We assume that the birth rate and the death rate involve three regimes, respectively. The first regime is a period which shows a gradual

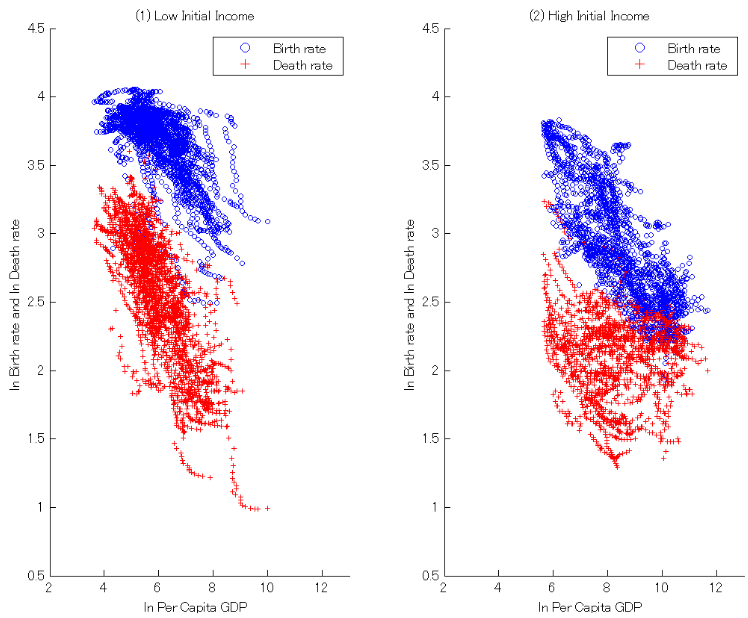


Figure 5: Birth and death rates in low initial income countries and high initial income countries

change with low income. The second regime is a period that shows a rapid drop with middle income. The third regime is a period that shows a gradual change again with high income. We estimated the change of both birth and death rates in each regime and the levels of per capita GDP at each turning points. We assume the econometric model as following:

$$y_{i,t} = \begin{cases} a_0 + a_1 x_{i,t} + \epsilon_{i,t} & \text{if } \tau^h \leq x_{i,t} \\ b_0 + b_1 x_{i,t} + \epsilon_{i,t} & \text{if } \tau^l \leq x_{i,t} < \tau^h \\ c_0 + c_1 x_{i,t} + \epsilon_{i,t} & \text{if } x_{i,t} < \tau^l \end{cases} \tag{1}$$

where subscript $_i$ and $_t$ represent countries and time, respectively. $i$=1,$\cdots$,89. $t$=1,$\cdots$,49. And, $y$, $x$ and $\epsilon$ are log birth rate (or log death rate), log per capita GDP and error term, respectively. To save the notation, we only use one of the regression equations like as Eq. (1) about the birth rate and the death rate, because the two kinds of regression equations have the same form. Let us assume that the errors $\epsilon_{i,t}$ are identically distributed, independent random variables with $\epsilon_{i,t} \sim N(0, \sigma^2)$.[9]

We assume that if per capital GDP is over $\tau^h$, the $y$ is in the third regime and if per capita GDP is between $\tau^l$ and $\tau^h$, the $y$ is in the second regime, and if per capita GDP is below $\tau^l$, the $y$ is the first regime. We estimate the nine variables in Eq. (1) and these are $a_0$, $a_1$, $b_0$, $b_1$, $c_0$, $c_1$, $\tau^h$, $\tau^l$ and $\sigma^2$.

### 3.1.2 Method

We estimated the variables by two kinds of methods which are the maximum likelihood estimation (MLE) and the Bayesian statistics. The maximum likelihood estimation is a method to maximize the likelihood function in estimating the variables. Bayesian statistics is a method to calculate the statistics using posterior which consists of likelihood function and prior. The mathematical representation of the model, with threshold variable, is given by:

[9]We leave the investigation of the results using other distributions and the use of different errors $\epsilon_{i,t}$ for each regime for further study.

$$\begin{aligned} y_{i,t} &= (a_0 + a_1 x_{i,t}) I(\tau^h \le x_{i,t}) + (b_0 + b_1 x_{i,t}) I(\tau^l \le x_{i,t} < \tau^h) + (c_0 + c_1 x_{i,t}) I(x_{i,t} < \tau^l) + \epsilon_{i,t} \\ &= h(a_0, a_1, b_0, b_1, c_0, c_1, \tau^h, \tau^l | x_{i,t}) + \epsilon_{i,t} \end{aligned} \tag{2}$$

where $h(\bullet|x_{i,t}) = (a_0 + a_1 x_{i,t}) I(\tau^h \le x_{i,t}) + (b_0 + b_1 x_{i,t}) I(\tau^l \le x_{i,t} < \tau^h) + (c_0 + c_1 x_{i,t}) I(x_{i,t} < \tau^l)$, $I(\bullet)$ is the indicator function. The likelihood function can be written as:

$$L(\mathbf{x}, \mathbf{y}|\theta) = \prod_{i=1}^{89} \prod_{t=1}^{49} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big\{ -\frac{1}{2\sigma^2} \big( y_{i,t} - h(\cdot|x_{i,t}) \big)^2 \Big\} \tag{3}$$

where $\theta = (a_0, a_1, b_0, b_1, c_0, c_1, \tau^h, \tau^l, \sigma^2)$, which is a variable vector. $\mathbf{x}$, $\mathbf{y}$ are data. The maximum likelihood estimation is a method to estimate the variables using likelihood function Eq. (3).

Bayes' Theorem for probability distribution is often stated as:

$$\underbrace{\pi(\theta|\mathbf{x}, \mathbf{y})}_{\text{Posterior}} \propto \underbrace{L(\mathbf{x}, \mathbf{y}|\theta)}_{\text{Likelihood}} \underbrace{p(\theta)}_{\text{Prior}} \tag{4}$$

where the symbol "$\propto$" means "is proportion to." Bayes estimation is a method to calculate the statistics using posterior Eq. (4). To calculate the posterior, we have to assume the distribution of prior, $a_0$, $a_1$, $b_0$, $b_1$, $c_0$, $c_1$, $\tau^h$, $\tau^l$ and $\sigma^2$. We assume the priors as follows: $a_0 \sim N(0, \sigma_{a_0})$, $a_1 \sim N(0, \sigma_{a_1})$, $b_0 \sim N(0, \sigma_{b_0})$, $b_1 \sim N(0, \sigma_{b_1})$, $c_0 \sim N(0, \sigma_{c_0})$, $c_1 \sim N(0, \sigma_{c_1})$, $\tau^h \sim U(h_a, h_b)$, $\tau^l \sim U(l_a, l_b)$ and $\sigma^{-2} \sim Ga(\alpha, \beta)$, where $N$, $U$ and $Ga$ are normal distribution, uniform distribution and gamma distribution, respectively. Moreover, we assume $\sigma_{a_0} = \sigma_{a_1} = \sigma_{b_0} = \sigma_{b_1} = \sigma_{c_0} = \sigma_{c_1} = \sigma_p$, then we can rewrite the prior like as Eq. (5).

$$\begin{aligned} p(\theta) &= p(a_0)p(a_1)p(b_0)p(b_1)p(c_0)p(c_1)p(\tau^h)p(\tau^l)p(\sigma^2) \\ &= \begin{cases} \left( \frac{1}{\sqrt{2\pi\sigma_p^2}} \right)^6 \exp\Big\{ -\frac{a_0^2 + a_1^2 + b_0^2 + b_1^2 + c_0^2 + c_1^2}{2\sigma_p^2} \Big\} \frac{\beta^\alpha}{\Gamma(\alpha)} \sigma^{-\alpha-1} e^{-\frac{\beta}{\sigma}} & \text{if } h_a \le \tau^h \le h_b,\ l_a \le \tau^l \le l_b, \sigma > 0 \\ 0 & \text{elsewhere} \end{cases} \end{aligned} \tag{5}$$

where $\Gamma(\bullet)$ is the gamma function. The posterior is expressed as the product of the proir Eq. (5) and the likelihood function Eq. (3).

#### 3.1.3 Estimation Results

We estimated the variables by two kinds of methods which are maximum likelihood estimation and Bayesian estimation. We used Newton method to maximize the likelihood function and Metropolis algorithm to calculate the statistics of the posterior.[10] We replaced the parameters as follows so that as much as possible the prior should not affect the posterior: $\sigma_p = 1,000$, $h_a$=6.0, $h_b$=12.0, $l_a$=2.0, $h_a$=8.0, $\alpha$=0.001 and $\beta$=0.001.

Table 4 (a) and (b) are the estimation results of birth rates and death rates, respectively. The left part and the right part of Table 4 are the results by the maximum likelihood estimation and Bayesian estimation, respectively. We report the estimated values and standard errors for the maximum likelihood estimation and simple summaries about the posterior mean, median, standard deviation, 95% posterior credible interval and Geweke's convergence diagnostic for the Bayesian estimation. The sampling was run with a burn-in of 1,000,000 iteration with 2,000,000. Based on the results of Geweke's convergence diagnostic,

[10]See Robert and Casella (2004) for details on Metropolis algorithm.

Table 4: Estimation result of Model 1

(a) Birth Rate

| | MLE | | Bayesian Method | | | | |
|---|---|---|---|---|---|---|---|
| Parameters | Estimated Value | Standard Error | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
| $a_0$ | 3.540 | 0.145 | 3.538 | 3.538 | 0.146 | **[3.253,3.823]** | -1.563 |
| $a_1$ | -0.101 | 0.015 | -0.101 | -0.101 | 0.015 | **[-0.130,-0.072]** | 1.572 |
| $b_0$ | 5.393 | 0.033 | 5.392 | 5.391 | 0.035 | **[5.325,5.461]** | -0.361 |
| $b_1$ | -0.285 | 0.005 | -0.285 | -0.285 | 0.005 | **[-0.295,-0.275]** | 0.315 |
| $c_0$ | 3.933 | 0.092 | 3.950 | 3.950 | 0.094 | **[3.767,4.134]** | 0.618 |
| $c_1$ | -0.024 | 0.018 | -0.027 | -0.027 | 0.019 | [-0.064,0.009] | -0.649 |
| $\tau^h$ | 8.878 | - | 8.878 | 8.877 | 0.008 | **[8.862,8.894]** | -0.653 |
| $\tau^l$ | 5.556 | - | 5.556 | 5.556 | 0.110 | **[5.347,5.766]** | 0.764 |
| $\sigma^2$ | 0.229 | 0.002 | 0.229 | 0.229 | 0.002 | **[0.225,0.234]** | 0.327 |

(b) Death Rate

| | MLE | | Bayesian Method | | | | |
|---|---|---|---|---|---|---|---|
| Parameters | Estimated Value | Standard Error | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
| $a_0$ | 1.622 | 0.061 | 1.629 | 1.631 | 0.070 | **[1.489,1.763]** | -0.305 |
| $a_1$ | 0.050 | 0.007 | 0.049 | 0.049 | 0.008 | **[0.034,0.065]** | 0.396 |
| $b_0$ | 5.152 | 0.081 | 5.020 | 5.018 | 0.124 | **[4.733,5.269]** | -0.291 |
| $b_1$ | -0.435 | 0.013 | -0.415 | -0.414 | 0.020 | **[-0.454,-0.377]** | 0.390 |
| $c_0$ | 3.857 | 0.136 | 3.772 | 3.819 | 0.221 | **[3.219,4.091]** | -1.450 |
| $c_1$ | -0.195 | 0.028 | -0.176 | -0.187 | 0.048 | **[-0.242,-0.054]** | 1.477 |
| $\tau^h$ | 7.298 | - | 7.299 | 7.306 | 0.132 | **[7.065,7.530]** | 0.783 |
| $\tau^l$ | 5.416 | - | 5.413 | 5.596 | 0.312 | **[4.800,5.750]** | -1.040 |
| $\sigma^2$ | 0.301 | 0.003 | 0.301 | 0.301 | 0.003 | **[0.295,0.308]** | -1.472 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic
The thick bold styles represent the cases that the 95% credible intervals do not include 0.

we can consider that this sampling has been converged. The credible interval in Bayesian statistics is similar to the confidential interval in classical statistics. For the readers who are not familar with Bayesian statistics, the credible interval will be interpreted in the same meaning of the confidential interval. We need to check if 95% credible interval includes 0 or not. If not, we use the term “significant” which is used in classical statistics.

In Table 4 (a) and (b), we know that all estimated results except $c_1$ in (a) birth rate are significant, in case of Bayesian method, the 95% credible intervals do not include 0, and $|b_1| > |a_1|$ and $|b_1| > |c_1|$. In other words, both birth rates and death rates drop more rapidly in the second regime than in both the first regime and third regime. And, we know that the first turning point of the birth rate $\tau^l$ (5.556, $e^{5.556} \doteq 259$) is bigger than that of the death rate $\tau^l$ (5.413, $e^{5.413} \doteq 224$), and the second turning point of the birth rate $\tau^h$ (8.878, $e^{8.878} \doteq 7,172$) is also bigger than that of the death rate $\tau^h$ (7.299, $e^{7.299} \doteq 1,479$). The decline of death rate starts earlier than the decline of birth rates. And the death rate reaches to third regime earlier than the birth rate does. This means the order, A<B and C<D in Figure 1. We can confirm the demographic transition in Model 1.

## 3.2 Model 2

### 3.2.1 Model

We modify the discontinuous at the turning points in Model 1. We add the continuous (no jump) constraint in Model 2. The constraints are as follows:

$$\begin{aligned} a_0 + a_1\tau^h &= b_0 + b_1\tau^h \ , \\ b_0 + b_1\tau^l &= c_0 + c_1\tau^l \ . \end{aligned} \tag{6}$$

We substitute the constraints in Eq. (6) to Eq. (1) and get Eq. (7).

$$\begin{aligned} b_0 &= \frac{\tau^l\tau^h(c_1 - a_1) + c_0\tau^h - a_0\tau^l}{\tau^h - \tau^l} \ , \\ b_1 &= \frac{a_0 + a_1\tau^h - c_1\tau^l - c_0}{\tau^h - \tau^l} \ . \end{aligned} \tag{7}$$

### 3.2.2 Estimation Results

Under the constraints in Eq. (6), we estimate the variables, $a_0$, $a_1$, $c_0$, $c_1$, $\tau^h$, $\tau^l$ and $\sigma$. And, we calculate $b_0$ and $b_1$ from the estimated values of $a_0$, $a_1$, $c_0$, $c_1$, $\tau^h$ and $\tau^l$ using Eq. (7). Bayesian statistics has a strong point that the distributions of $b_0$ and $b_1$ can be easily estimated from the samplings of $a_0$, $a_1$, $c_0$, $c_1$, $\tau^h$ and $\tau^l$.

Table 5 reports the estimation results. It contains the same reading in Table 4. Table 5 (a) and (b) are the estimation results of birth rates and death rates. The left part and the right part of Table 4 are the results of maximum likelihood estimation and Bayesian estimation. The sampling was run with a burn-in of 1,000,000 iteration with 2,000,000 like as what we did in Model 1. Based on the results of Geweke's convergence diagnostic, we can consider that this sampling has been converged.

Model 1 and Model 2 almost contain the same results. From Table 5 (a) and (b), we know that $|b_1| > |a_1|$ and $|b_1| > |c_1|$. And, the first turning point of the birth rate $\tau^l$ (5.746, $e^{5.746} \doteq 313$) is bigger than that of the death rate $\tau^l$ (5.205, $e^{5.205} \doteq 182$), and the second turning point of the birth rate $\tau^h$ (9.933, $e^{9.933} \doteq 20,599$) is also bigger than that of the death rate $\tau^h$ (7.292, $e^{7.292} \doteq 1,469$).

## 3.3 Fitness and Comparison

We draw the data and the theoretical values in Figure 6 to check the fitness of our models. Figure 6 (1) and (2) show the data and the theoretical values in Model 1 and Model 2, respectively. Both the theoretical values of Model 1 and Model 2 are in agreement with the data.

Figure 6 (3) and (4) show both the regression lines of Model 1 and Model 2 about birth rate and death rate, respectively. Based on both results in Model 1 and Model 2, we can say that there is no big difference, even though there is some difference when the value is mended from the logarithm. The second turning points of the birth rates in Model 1 and Model 2 are $\tau^h$ (8.878, $e^{8.878} \doteq 7,172$) and $\tau^h$ (9.933, $e^{9.933} \doteq 20,599$), respectively. The difference 1.055 (=9.933-8.878) in the logarithm is small, but the diffrence 13,427 (=20,599-7,172) in the level is not small.

Table 5: Estimation results of Model 2

(a) Birth rate

| | MLE | | Bayesian Method | | | | |
|---|---|---|---|---|---|---|---|
| Parameters | Estimated Value | Standard Error | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
| $a_0$ | 2.630 | 0.211 | 2.818 | 2.893 | 0.592 | **[1.547,3.748]** | -1.563 |
| $a_1$ | -0.014 | 0.021 | -0.032 | -0.039 | 0.056 | [-0.122,0.089] | 1.572 |
| $c_0$ | 3.983 | 0.062 | 4.002 | 4.002 | 0.102 | **[3.804,4.201]** | 0.618 |
| $c_1$ | -0.034 | 0.011 | -0.038 | -0.038 | 0.020 | [-0.078,0.002] | -0.649 |
| $\tau^h$ | 9.933 | - | 9.853 | 9.861 | 0.164 | **[9.565,10.140]** | -0.653 |
| $\tau^l$ | 5.715 | - | 5.746 | 5.736 | 0.092 | **[5.591,5.943]** | 0.764 |
| $\sigma^2$ | 0.231 | 0.002 | 0.231 | 0.231 | 0.002 | **[0.226,0.236]** | -1.036 |
| $b_0$ | - | - | 5.565 | 5.563 | 0.043 | **[5.489,5.657]** | -1.563 |
| $b_1$ | - | - | -0.310 | -0.310 | 0.006 | **[-0.322,-0.300]** | 1.572 |

(b) Death Rate

| | MLE | | Bayesian Method | | | | |
|---|---|---|---|---|---|---|---|
| Parameters | Estimated Value | Standard Error | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
| $a_0$ | 1.608 | 0.049 | 1.627 | 1.628 | 0.071 | **[1.488,1.762]** | -0.305 |
| $a_1$ | 0.052 | 0.006 | 0.050 | 0.049 | 0.008 | **[0.035,0.065]** | 0.396 |
| $c_0$ | 3.559 | 0.143 | 3.633 | 3.659 | 0.246 | **[3.114,4.038]** | -1.450 |
| $c_1$ | -0.128 | 0.029 | -0.144 | -0.151 | 0.054 | **[-0.231,-0.030]** | 1.477 |
| $\tau^h$ | 7.321 | - | 7.292 | 7.283 | 0.061 | **[7.180,7.406]** | 0.783 |
| $\tau^l$ | 5.120 | - | 5.205 | 5.195 | 0.172 | **[4.916,5.505]** | -1.040 |
| $\sigma^2$ | 0.301 | 0.003 | 0.301 | 0.301 | 0.003 | **[0.295,0.308]** | -0.455 |
| $b_0$ | - | | 5.091 | 5.082 | 0.107 | **[4.908,5.314]** | -0.305 |
| $b_1$ | - | - | -0.426 | -0.424 | 0.017 | **[-0.461,-0.396]** | 0.396 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic
The thick bold styles represent the cases that the 95% credible intervals do not include 0.

# 4 Compressed Demographic Transition

A latecomer's advantage is an idea that a developing country can potentially undergo a rapid economic development by using the experience in technology, knowledge and the development policy, etc. that the developed countries have made. If the developing country is enjoying the latecomer's advantage in demographic transition, it is possible that the demographic transition in developing countries occurs in earlier development stage and the demographic transition is compressed compared with that of the developed countries.

In this paper, the compressed demographic transition will be examined from three sides. First, as in the example of France and India in the quote from Weil (2013), the income level in developing countries are likely to be lower than that of the developed countries when they have gone or go through a similar transition. It means that the graph of the developing countries shows up on the leftside of that of the developed countries in the conceptual graph. We will call this as "the advancing of the transition".[11] The second and the third are about the turning points. The second is that in the case of developing countries, their turning points are likely to occur at the lower income level compared to the developed countries. The turning points in

[11]We compared the income level of developed and developing countries at the similar birth rate in this paper. However, we could also compare the birth rate at similar income level, but we did not. The range of income (8.05=11.68-3.63) is wider than those of the birth and death rates (2.13=4.06-1.93 and 2.65=3.64−0.99) as shown in Table 2. By doing the former, we could get more samples in the similar birth rate than in the similar income by doing the latter.

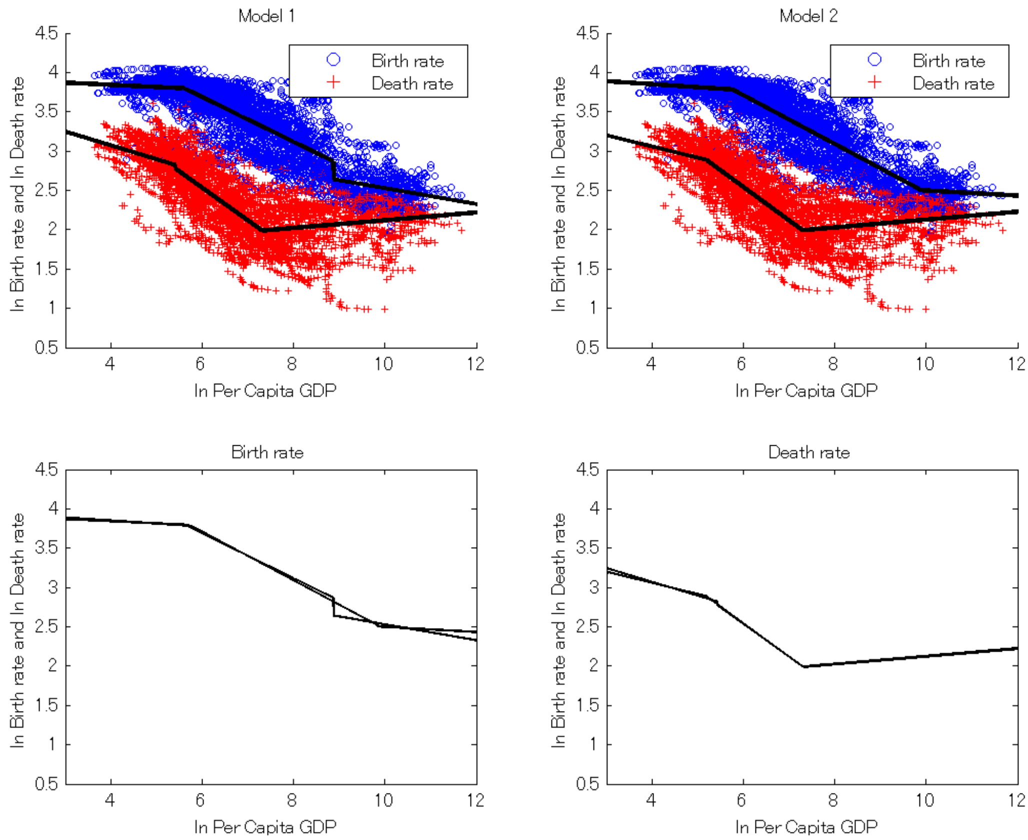


Figure 6: Fitness and Comparison

the developing countries show up on the leftside of the developed countries. We will call this as "the left movement of the turning point". The last is that in the case of developing countries, their turning points are likely to occur at the higher level of birth and death rates compared to the developed countries. The turning points in the developing countries show up on the upperside of that of the developed countries. We will call this as "the upper movement of the turning point".

## 4.1 Model 3

### 4.1.1 Model

We estimate the trend of birth and death rate in each countries and lead some common features in the whole cross country. To do so, we assume that the intercepts and slopes of the each countries are different and that the turning points of the each countries are also different. Moreover, we also assume that the intercepts, slopes, turning points are functions of initial income. We modify Eq. (1) as Eq. (8).

$$y_{i,t} = \begin{cases} a_{0,i} + a_{1,i}x_{i,t} + \epsilon_{i,t} & \text{if } \tau_i^h \leq x_{i,t} \\ b_{0,i} + b_{1,i}x_{i,t} + \epsilon_{i,t} & \text{if } \tau_i^l \leq x_{i,t} < \tau_i^h \\ c_{0,i} + c_{1,i}x_{i,t} + \epsilon_{i,t} & \text{if } x_{i,t} < \tau_i^l \ . \end{cases} \tag{8}$$

The diffrence with the Model 1 is that the variables, $a_0$, $a_1$, $b_0$, $b_1$, $c_0$, $c_1$, $\tau^h$ and $\tau^l$ are diffrent in each countries. In other words, the variables have the subscript $_i$ that represents a country like $a_{0,i}$, $a_{1,i}$, $b_{0,i}$, $b_{1,i}$, $c_{0,i}$, $c_{1,i}$, $\tau_i^h$ and $\tau_i^l$. The mathematical representation of the model, with threshold variable, is given by:

$$\begin{aligned} y_{i,t} &= (a_{0,i} + a_{1,i}x_{i,t})I(\tau_i^h \le x_{i,t}) + (b_{0,i} + b_{1,i}x_{i,t})I(\tau_i^l \le x_{i,t} < \tau_i^h) + (c_{0,i} + c_{1,i}x_{i,t})I(x_{i,t} < \tau_i^l) + \epsilon_{i,t} \\ &= h(a_{0,i}, a_{1,i}, b_{0,i}, b_{1,i}, c_{0,i}, c_{1,i}, \tau_i^h, \tau_i^l | x_{i,t}) + \epsilon_{i,t} \end{aligned} \tag{9}$$

where $h(\bullet|x_{i,t}) = (a_{0,i} + a_{1,i}x_{i,t})I(\tau_i^h \le x_{i,t}) + (b_{0,i} + b_{1,i}x_{i,t})I(\tau_i^l \le x_{i,t} < \tau_i^h) + (c_{0,i} + c_{1,i}x_{i,t})I(x_{i,t} < \tau_i^l)$, $I(\bullet)$ is the indicator function. Eq. (9) is very similar to Eq. (2). The likelihood function can be written as:

$$L(\mathbf{x}, \mathbf{y}|\theta) = \prod_{i=1}^{89}\prod_{t=1}^{49} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\Big\{-\frac{1}{2\sigma^2}(y_{i,t} - h(\cdot|x_{i,t}))^2\Big\} . \tag{10}$$

We assume that the variables, $a_{0,i}$, $a_{1,i}$, $b_{0,i}$, $b_{1,i}$, $c_{0,i}$, $c_{1,i}$, $\tau_i^h$ and $\tau_i^l$ are linear functions of the initial income ($x_{i,1}$) as follows:

$$\begin{aligned} a_{0,i} &= a_{00} + a_{01}x_{i,1} + u_{a_0,i} \ , \quad & a_{1,i} &= a_{10} + a_{11}x_{i,1} + u_{a_1,i} \ , \\ b_{0,i} &= b_{00} + b_{01}x_{i,1} + u_{b_0,i} \ , \quad & b_{1,i} &= b_{10} + b_{11}x_{i,1} + u_{b_1,i} \ , \\ c_{0,i} &= c_{00} + c_{01}x_{i,1} + u_{c_0,i} \ , \quad & c_{1,i} &= c_{10} + c_{11}x_{i,1} + u_{c_1,i} \ , \\ \tau_i^h &= h_0 + h_1 x_{i,1} + u_{h,i} \ , \quad & \tau_i^l &= l_0 + l_1 x_{i,1} + u_{l,i} \ , \end{aligned} \tag{11}$$

where $u_{a_0,i}$, $u_{a_1,i}$, $u_{b_0,i}$, $u_{b_1,i}$, $u_{c_0,i}$, $u_{c_1,i}$, $u_{h,i}$ and $u_{l,i}$ are error terms. We assume these as follows: $u_{a_0,i} \sim N(0, \sigma_{a_0}^2)$, $u_{a_1,i} \sim N(0, \sigma_{a_1}^2)$, $u_{b_0,i} \sim N(0, \sigma_{b_0}^2)$, $u_{b_1,i} \sim N(0, \sigma_{b_1}^2)$, $u_{c_0,i} \sim N(0, \sigma_{c_0}^2)$, $u_{c_1,i} \sim N(0, \sigma_{c_1}^2)$, $u_{h,i} \sim N(0, \sigma_{\tau^h}^2)$ and $u_{l,i} \sim N(0, \sigma_{\tau^l}^2)$.

We substitute the $a_{0,i}$, $a_{1,i}$, $b_{0,i}$, $b_{1,i}$, $c_{0,i}$, $c_{1,i}$ in Eq. (11) into Eq. (8) then we obtain:

$$y_{i,t} = \begin{cases} a_{00} + a_{01}x_{i,1} + a_{10}x_{i,t} + a_{11}x_{i,1}x_{i,t} + u_{a_0,i} + u_{a_1,i}x_{i,t} + \epsilon_{i,t} & \text{if } \tau_i^h \le x_{i,t} \\ b_{00} + b_{01}x_{i,1} + b_{10}x_{i,t} + b_{11}x_{i,1}x_{i,t} + u_{b_0,i} + u_{b_1,i}x_{i,t} + \epsilon_{i,t} & \text{if } \tau_i^l \le x_{i,t} < \tau_i^h \\ c_{00} + c_{01}x_{i,1} + c_{10}x_{i,t} + c_{11}x_{i,1}x_{i,t} + u_{c_0,i} + u_{c_1,i}x_{i,t} + \epsilon_{i,t} & \text{if } x_{i,t} < \tau_i^l \ . \end{cases} \tag{12}$$

In this representation, we have grand means ($a_{00}$, $b_{00}$, $c_{00}$) and individual adjustments to them ($u_{a_0,i}$, $u_{b_0,i}$, $u_{c_0,i}$), main effects of the initial income ($a_{01}$, $b_{01}$, $c_{01}$), main effects of income ($a_{10}$, $b_{10}$, $c_{10}$) and individual adjustments to them ($u_{a_1,i}$, $u_{b_1,i}$, $u_{c_1,i}$), interaction effects between income and initial income ($a_{11}$, $b_{11}$, $c_{11}$), and an error term ($\epsilon_{i,t}$). This equation shows that the composite error structures, ($u_{a_0,i}$, $u_{b_0,i}$, $u_{c_0,i}$, $u_{a_1,i}x_{i,t}$, $u_{b_1,i}x_{i,t}$, $u_{b_1,i}x_{i,t}$, $\epsilon_{i,t}$) have identification problems and heteroscedastics.

The variables, $a_{0,i}$, $a_{1,i}$, $b_{0,i}$, $b_{1,i}$, $c_{0,i}$, $c_{1,i}$, $\tau_i^h$ and $\tau_i^l$, have one hierarchical level (e.g., $a_0$ is a function of the variables, $a_{00}$, $a_{01}$ and $\sigma_{a_0^2}$.) and they can be expressed by a conditional probability. So, we used a hierarchical Bayesian model to estimiate the variables. When we calculate the hierarchical Bayesian model, we have to generate sequentially one conditional distribution after another.

The hierarchical model has the following structure:

$$\underbrace{\pi(\theta, \alpha|\mathbf{x}, \mathbf{y})}_{\text{Posterior}} \propto \underbrace{L(\mathbf{x}, \mathbf{y}|\theta)}_{\text{Likelihood}} \underbrace{p(\theta|\alpha)}_{\text{Prior}} \underbrace{p(\alpha)}_{\text{Hyperprior}} \tag{13}$$

where $\theta = (a_{0,1}, \cdots, a_{0,89}, a_{1,1}, \cdots, a_{1,89}, b_{0,1}, \cdots, b_{0,89}, b_{1,1}, \cdots, b_{1,89}, c_{0,1}, \cdots, c_{0,89}, c_{1,1}, \cdots, c_{1,89}, \tau_1^h, \cdots, \tau_{89}^h,$ $\tau_1^l, \cdots, \tau_{89}^l, \sigma^2)$, which is a parameter vector. $\alpha = (a_{00}, a_{01}, a_{10}, a_{11}, b_{00}, b_{01}, b_{10}, b_{11}, c_{00}, c_{01}, c_{10}, c_{11}, h_0, h_1, l_0, l_1,$ $\sigma_{a_0}^2, \sigma_{a_1}^2, \sigma_{b_0}^2, \sigma_{b_1}^2, \sigma_{b_0}^2, \sigma_{b_1}^2, \sigma_h^2, \sigma_l^2)$, which is a hyperparameter vector. To put the hierarchical model more precisely, the model is expressed as follow:

$$
\begin{aligned}
\pi(\theta,\alpha|\mathbf{x},\mathbf{y}) \propto & L(\mathbf{x},\mathbf{y}|\theta)p(\theta|\alpha)p(\alpha) \\
\propto L(\mathbf{x},\mathbf{y}|\theta) & \prod_{i=1}^{89} p(a_{0,i}|a_{00},a_{01},\sigma_{a_0}^2)\prod_{i=1}^{89} p(a_{1,i}|a_{10},a_{11},\sigma_{a_1}^2)\prod_{i=1}^{89} p(b_{0,i}|b_{00},b_{01},\sigma_{b_0}^2)\prod_{i=1}^{89} p(b_{1,i}|b_{10},b_{11},\sigma_{b_1}^2) \\
& \prod_{i=1}^{89} p(c_{0.i}|c_{00},c_{01},\sigma_{c_0}^2)\prod_{i=1}^{89} p(c_{1,i}|c_{10},c_{11},\sigma_{c_1}^2)\prod_{i=1}^{89} p(\tau_i^h|h_0,h_1,\sigma_{\tau^h}^2)\prod_{i=1}^{89} p(\tau_i^l|l_0,l_1,\sigma_{\tau^l}^2) \\
& p(a_{00})p(a_{01})p(a_{10})p(a_{11})p(b_{00})p(b_{01})p(b_{10})p(b_{11})p(c_{00})p(c_{01})p(c_{10})p(c_{11})p(h_0)p(h_1)p(l_0)p(l_1) \\
& p(\sigma_{a_0}^2)p(\sigma_{a_1}^2)p(\sigma_{b_0}^2)p(\sigma_{b_1}^2)p(\sigma_{c_0}^2)p(\sigma_{c_1}^2)p(\sigma_h^2)p(\sigma_l^2)p(\sigma^2)
\end{aligned} \tag{14}
$$

where $p(a_{0,i}|a_{00},a_{01},\sigma_{a_0}^2)$, $\cdots$, $p(\tau_i^l|l_0,l_1,\sigma_{\tau^l}^2)$ are

$$
\begin{aligned}
p(a_{0,i}|a_{00},a_{01},\sigma_{a_0}^2) &= \frac{1}{\sqrt{2\pi\sigma_{a_0}^2}}\exp\Big\{-\frac{(a_{0,i}-a_{00}-a_{01}x_{i,1})^2}{2\sigma_{a_0}^2}\Big\}\ , \\
p(a_{1,i}|a_{10},a_{11},\sigma_{a_1}^2) &= \frac{1}{\sqrt{2\pi\sigma_{a_1}^2}}\exp\Big\{-\frac{(a_{1,i}-a_{10}-a_{11}x_{i,1})^2}{2\sigma_{a_1}^2}\Big\}\ , \\
p(b_{0,i}|b_{00},b_{01},\sigma_{b_0}^2) &= \frac{1}{\sqrt{2\pi\sigma_{b_0}^2}}\exp\Big\{-\frac{(b_{0,i}-b_{00}-b_{01}x_{i,1})^2}{2\sigma_{b_0}^2}\Big\}\ , \\
p(b_{1,i}|b_{10},b_{11},\sigma_{b_1}^2) &= \frac{1}{\sqrt{2\pi\sigma_{b_1}^2}}\exp\Big\{-\frac{(b_{1,i}-b_{10}-b_{11}x_{i,1})^2}{2\sigma_{b_1}^2}\Big\}\ , \\
p(c_{0,i}|c_{00},c_{01},\sigma_{c_0}^2) &= \frac{1}{\sqrt{2\pi\sigma_{c_0}^2}}\exp\Big\{-\frac{(c_{0,i}-c_{00}-c_{01}x_{i,1})^2}{2\sigma_{c_0}^2}\Big\}\ , \\
p(c_{1,i}|c_{10},c_{11},\sigma_{c_1}^2) &= \frac{1}{\sqrt{2\pi\sigma_{c_1}^2}}\exp\Big\{-\frac{(c_{1,i}-c_{10}-c_{11}x_{i,1})^2}{2\sigma_{c_1}^2}\Big\}\ , \\
p(\tau_i^h|h_0,h_1,\sigma_{\tau^h}^2) &= \frac{1}{\sqrt{2\pi\sigma_{\tau^h}^2}}\exp\Big\{-\frac{(h_i-h_0-h_1x_{i,1})^2}{2\sigma_{\tau^h}^2}\Big\}I(\tau_a^h,\tau_b^h)\ , \\
p(\tau_i^l|l_0,l_1,\sigma_{\tau^l}^2) &= \frac{1}{\sqrt{2\pi\sigma_{\tau^l}^2}}\exp\Big\{-\frac{(l_i-l_0-l_1x_{i,1})^2}{2\sigma_{\tau^l}^2}\Big\}I(\tau_a^l,\tau_b^l).
\end{aligned} \tag{15}
$$

where $I(\bullet)$ is the indicator function. $\tau_i^h$ and $\tau_i^l$ are assumed as distributions truncated to the range $(\tau_a^h,\tau_b^h)$ and $(\tau_a^l,\tau_b^l)$, respectively.

To calculate the posterior, we have to assume the distribution of priors, $a_{00}$, $a_{01}$, $a_{10}$, $a_{11}$, $b_{00}$, $b_{01}$, $b_{10}$, $b_{11}$, $c_{00}$, $c_{01}$, $c_{10}$, $c_{11}$, $h_0$, $h_1$, $l_0$, $l_1$, $\sigma_{a_0}^2$, $\sigma_{a_1}^2$, $\sigma_{b_0}^2$, $\sigma_{b_1}^2$, $\sigma_{c_0}^2$, $\sigma_{c_1}^2$, $\sigma_h^2$, $\sigma_l^2$ and $\sigma^2$. We assume the priors as follows: $a_{00}\sim N(0,\sigma_{a_{00}})$, $a_{01}\sim N(0,\sigma_{a_{01}})$, $a_{10}\sim N(0,\sigma_{a_{10}})$, $a_{11}\sim N(0,\sigma_{a_{11}})$, $b_{00}\sim N(0,\sigma_{b_{00}})$, $b_{01}\sim N(0,\sigma_{b_{01}})$, $b_{10}\sim N(0,\sigma_{b_{10}})$, $b_{11}\sim N(0,\sigma_{b_{11}})$, $c_{00}\sim N(0,\sigma_{c_{00}})$, $c_{01}\sim N(0,\sigma_{c_{01}})$, $c_{10}\sim N(0,\sigma_{a_{10}})$, $c_{11}\sim N(0,\sigma_{c_{11}})$, $h_0\sim N(0,\sigma_{h_0})$, $h_1\sim N(0,\sigma_{h_1})$, $l_0\sim N(0,\sigma_{l_0})$, $l_1\sim N(0,\sigma_{l_1})$, $\sigma_{a_0}^{-2}\sim Ga(\alpha,\beta)$, $\sigma_{a_1}^{-2}\sim Ga(\alpha,\beta)$, $\sigma_{b_0}^{-2}\sim Ga(\alpha,\beta)$, $\sigma_{b_1}^{-2}\sim Ga(\alpha,\beta)$, $\sigma_{c_0}^{-2}\sim Ga(\alpha,\beta)$, $\sigma_{c_1}^{-2}\sim Ga(\alpha,\beta)$, $\sigma_h^{-2}\sim Ga(\alpha,\beta)$, $\sigma_l^{-2}\sim Ga(\alpha,\beta)$ and $\sigma^{-2}\sim Ga(\alpha,\beta)$. Moreover, we assume $\sigma_{a_{00}}=\sigma_{a_{01}}=\sigma_{a_{10}}=\sigma_{a_{11}}=\sigma_{b_{00}}=\sigma_{b_{01}}=\sigma_{b_{10}}=\sigma_{b_{11}}=\sigma_{c_{00}}=\sigma_{c_{01}}=\sigma_{c_{10}}=\sigma_{c_{11}}=\sigma_{h_0}=\sigma_{h_1}=\sigma_{l_0}=\sigma_{l_1}=\sigma_p$, then we can rewrite the hyperprior in Eq. (14) as follows:

$$
\begin{aligned}
& p(a_{00})p(a_{01})p(a_{10})p(a_{11})p(b_{00})p(b_{01})p(b_{10})p(b_{11})p(c_{00})p(c_{01})p(c_{10})p(c_{11})p(h_0)p(h_1)p(l_0)p(l_1) \\
=& \Big(\frac{1}{\sqrt{2\pi\sigma_p^2}}\Big)^{16}\exp\Big\{-\frac{a_{00}^2+a_{01}^2+a_{10}^2+a_{11}^2+b_{00}^2+b_{01}^2+b_{10}^2+b_{11}^2+c_{00}^2+c_{01}^2+c_{10}^2+c_{11}^2+h_0^2+h_1^2+l_0^2+l_1^2}{2\sigma_p^2}\Big\}\ .
\end{aligned} \tag{16}
$$

$$p(\sigma_{a_0}^2)p(\sigma_{a_1}^2)p(\sigma_{b_0}^2)p(\sigma_{b_1}^2)p(\sigma_{c_0}^2)p(\sigma_{c_1}^2)p(\sigma_h^2)p(\sigma_l^2)p(\sigma^2)$$
$$=\left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^9(\sigma_{a_0}\sigma_{a_1}\sigma_{b_0}\sigma_{b_1}\sigma_{c_0}\sigma_{c_1}\sigma_{\tau^h}\sigma_{\tau^l}\sigma)^{-\alpha-1}e^{-\beta\left(\frac{1}{\sigma_{a_0}}+\frac{1}{\sigma_{a_1}}+\frac{1}{\sigma_{b_0}}+\frac{1}{\sigma_{b_1}}+\frac{1}{\sigma_{c_0}}+\frac{1}{\sigma_{c_1}}+\frac{1}{\sigma_{\tau^h}}+\frac{1}{\sigma_{\tau^l}}+\frac{1}{\sigma}\right)} \,. \tag{17}$$

The posterior is expressed as the product of likelihood function Eq. (10), prior Eq. (15) and hyperprior Eq. (16) and (17). Because the posterior is very complex, we use Gibbs sampling to calculate the posterior.[12]

### 4.1.2 Estimation Results

As we did in Model 1 and Model 2, we also replaced the same values in Model 3 by using the following parameters: $\sigma_p$=1,000, $\alpha$=0.001, $\beta$=0.001, $\tau_a^h$=6, $\tau_b^h$=10, $\tau_a^l$=4 and $\tau_b^l$=8. Table 6 reports the estimation results.[13] The readings in Table 6 is the same with the readings in Table 4. Table 6 (a) and (b) are the estimation results of birth rates and death rates, respectively.[14] The sampling was run with a burn-in of 500,000 iteration with 1,000,000. Based on the results of Geweke's convergence diagnostic, we can consider that this sampling has been converged.

**(a) Birth rate**

Let us examine the results of the birth rate in Table 6 (a).

**(a-1) The left movement of the turning point**

The results of the turning points are,

$$\tau_i^h = \underbrace{5.275}_{h_0^*} + \underbrace{0.478}_{h_1^*} x_{i,1} + u_{\tau^h,i} \qquad \tau_i^l = \underbrace{3.919}_{l_0^*} + \underbrace{0.374}_{l_1^*} x_{i,1} + u_{\tau^l,i} \ ,$$

where $*$'s mean that the 95% interval does not include zero. As we mentioned in Section 3, we interpret the credible interval in the same meaning of the confidential interval in the classical statistics. All $h_0$, $h_1$, $l_0$ and $l_1$ are significant. And $h_1$ and $l_1$ are positive. This means that the smaller the initial income is, the earlier the first and the second turning points are. In other words, the turning points of a country with low initial income are located more on the left side. On the contrary, the turning points of the country with high initial income are located more on the right side. Based on the results that $h_1$ and $l_1$ are positive and significant, we can find the left movement of the turning points in the birth rates.

**(a-2) The upper movement of the turning point**

The estimation results of each regimes are,

$$a_{0,i} = \underbrace{9.836}_{a_{00}^*} - \underbrace{0.867}_{a_{01}^*} x_{i,1} + u_{a_0,i} \qquad a_{1,i} = -\underbrace{0.728}_{a_{10}^*} + \underbrace{0.087}_{a_{11}^*} x_{i,1} + u_{a_1,i} \ ,$$

[12] There is another method which maximizes the posterior to estimate variables. This method is called as MAP (maximization a posterior). In this case, it is difficult to use MAP, because this model contains more than 730 dimensions to be maximized. So, we used Gibbs sampling to calculate the posterior.

[13] We used WinBUGS Vesion 1.4 for calculation of Table 6, 7, 9, 11, 12 and 13. BUGS is a generic tool which can be used in a wide variety of situations, because BUGS makes the complex calculation easy and is very simple to code. See Ntzoufras (2009) for details about BUGS.

[14] The results in Table 6 (a) and Table 7 are calculated together. For convenience of explanation, we divided them into two tables. The same applies to Table 6 (b) and Table 9.

Table 6: Estimation results of Model 3

(a) Birth rate

| Parameters | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| $a_{00}$ | 9.836 | 9.839 | 0.549 | [ **8.757 , 10.900** ] | 0.179 |
| $a_{01}$ | -0.867 | -0.867 | 0.085 | [ **-1.035 , -0.701** ] | -0.207 |
| $a_{10}$ | -0.728 | -0.728 | 0.058 | [ **-0.840 , -0.616** ] | -0.079 |
| $a_{11}$ | 0.087 | 0.087 | 0.009 | [ **0.070 , 0.104** ] | 0.110 |
| $b_{00}$ | 3.943 | 3.944 | 0.256 | [ **3.449 , 4.461** ] | -0.215 |
| $b_{01}$ | 0.030 | 0.029 | 0.044 | [ -0.057 , 0.115 ] | 0.525 |
| $b_{10}$ | 0.081 | 0.082 | 0.042 | [ -0.005 , 0.161 ] | -0.031 |
| $b_{11}$ | -0.033 | -0.033 | 0.007 | [ **-0.046 , -0.019** ] | -0.257 |
| $c_{00}$ | 2.714 | 2.723 | 0.287 | [ **2.135 , 3.263** ] | 0.734 |
| $c_{01}$ | 0.396 | 0.395 | 0.060 | [ **0.282 , 0.515** ] | -0.858 |
| $c_{10}$ | 0.154 | 0.153 | 0.054 | [ **0.051 , 0.267** ] | -0.559 |
| $c_{11}$ | -0.065 | -0.065 | 0.011 | [ **-0.087 , -0.045** ] | 0.733 |
| $h_0$ | 5.275 | 5.256 | 0.668 | [ **4.019 , 6.640** ] | 0.993 |
| $h_1$ | 0.478 | 0.481 | 0.112 | [ **0.250 , 0.690** ] | -0.688 |
| $l_0$ | 3.919 | 3.901 | 0.680 | [ **2.636 , 5.303** ] | -0.039 |
| $l_1$ | 0.374 | 0.378 | 0.125 | [ **0.120 , 0.608** ] | 0.065 |
| $\sigma^2$ | 0.080 | 0.080 | 0.001 | [ **0.078 , 0.082** ] | -1.020 |
| $\sigma^2_{a_0}$ | 0.300 | 0.289 | 0.063 | [ **0.206 , 0.453** ] | 0.844 |
| $\sigma^2_{a_1}$ | 0.024 | 0.023 | 0.006 | [ **0.015 , 0.039** ] | 1.243 |
| $\sigma^2_{b_0}$ | 0.139 | 0.140 | 0.042 | [ **0.052 , 0.221** ] | 0.925 |
| $\sigma^2_{b_1}$ | 0.035 | 0.034 | 0.004 | [ **0.026 , 0.044** ] | 0.567 |
| $\sigma^2_{c_0}$ | 0.074 | 0.074 | 0.022 | [ **0.031 , 0.117** ] | 1.091 |
| $\sigma^2_{c_1}$ | 0.020 | 0.020 | 0.003 | [ **0.015 , 0.027** ] | -1.013 |
| $\sigma^2_{\tau^h}$ | 0.785 | 0.779 | 0.082 | [ **0.639 , 0.960** ] | 0.377 |
| $\sigma^2_{\tau^l}$ | 0.864 | 0.860 | 0.105 | [ **0.668 , 1.080** ] | -0.812 |

(b) Death rate

| Parameters | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| $a_{00}$ | 2.300 | 2.123 | 1.085 | [ **0.616 , 4.982** ] | 0.757 |
| $a_{01}$ | 0.053 | 0.072 | 0.145 | [ -0.290 , 0.291 ] | -1.029 |
| $a_{10}$ | -0.130 | -0.105 | 0.137 | [ -0.479 , 0.076 ] | -0.330 |
| $a_{11}$ | 0.010 | 0.007 | 0.018 | [ -0.019 , 0.055 ] | 0.527 |
| $b_{00}$ | 6.133 | 6.192 | 0.727 | [ **4.503 , 7.447** ] | 0.508 |
| $b_{01}$ | -0.611 | -0.618 | 0.126 | [ **-0.845 , -0.340** ] | -0.471 |
| $b_{10}$ | -0.477 | -0.483 | 0.104 | [ **-0.671 , -0.253** ] | -0.279 |
| $b_{11}$ | 0.071 | 0.072 | 0.018 | [ **0.035 , 0.105** ] | 0.172 |
| $c_{00}$ | 3.241 | 3.241 | 0.497 | [ **2.264 , 4.219** ] | 0.612 |
| $c_{01}$ | 0.277 | 0.276 | 0.101 | [ **0.080 , 0.476** ] | -0.315 |
| $c_{10}$ | -0.100 | -0.100 | 0.092 | [ -0.282 , 0.080 ] | -0.842 |
| $c_{11}$ | -0.050 | -0.050 | 0.018 | [ **-0.086 , -0.014** ] | 0.538 |
| $h_0$ | 8.395 | 8.545 | 1.235 | [ **5.530 , 10.440** ] | 1.706 |
| $h_1$ | -0.083 | -0.117 | 0.237 | [ -0.461 , 0.475 ] | -1.590 |
| $l_0$ | 5.930 | 5.914 | 0.728 | [ **4.552 , 7.397** ] | 0.663 |
| $l_1$ | 0.053 | 0.056 | 0.133 | [ -0.216 , 0.303 ] | -0.589 |
| $\sigma^2$ | 0.108 | 0.108 | 0.001 | [ **0.105 , 0.110** ] | 1.561 |
| $\sigma^2_{a_0}$ | 0.381 | 0.378 | 0.074 | [ **0.245 , 0.536** ] | -0.351 |
| $\sigma^2_{a_1}$ | 0.044 | 0.043 | 0.009 | [ **0.027 , 0.063** ] | -0.723 |
| $\sigma^2_{b_0}$ | 0.206 | 0.213 | 0.078 | [ **0.043 , 0.339** ] | 0.361 |
| $\sigma^2_{b_1}$ | 0.044 | 0.045 | 0.008 | [ **0.028 , 0.060** ] | 0.850 |
| $\sigma^2_{c_0}$ | 0.353 | 0.350 | 0.059 | [ **0.247 , 0.479** ] | 1.199 |
| $\sigma^2_{c_1}$ | 0.067 | 0.067 | 0.010 | [ **0.049 , 0.090** ] | 1.691 |
| $\sigma^2_{\tau^h}$ | 0.885 | 0.883 | 0.108 | [ **0.678 , 1.104** ] | -0.696 |
| $\sigma^2_{\tau^l}$ | 0.853 | 0.849 | 0.086 | [ **0.696 , 1.035** ] | 0.722 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic

The thick bold styles represent the cases that the 95% credible intervals do not include 0.

$$b_{0,i} = \underbrace{3.943}_{b^*_{00}} + \underbrace{0.030}_{b_{01}} x_{i,1} + u_{b_0,i} \qquad b_{1,i} = \underbrace{0.081}_{b_{10}} - \underbrace{0.033}_{b^*_{11}} x_{i,1} + u_{b_1,i} \ ,$$

$$c_{0,i} = \underbrace{2.714}_{c^*_{00}} + \underbrace{0.396}_{c^*_{01}} x_{i,1} + u_{c_0,i} \qquad c_{1,i} = \underbrace{0.154}_{c^*_{10}} - \underbrace{0.065}_{c^*_{11}} x_{i,1} + u_{c_1,i} \ .$$

$b_{01}$ and $c_{01}$ are positive, but $b_{01}$ is not significant. $b_{11}$ and $c_{11}$ are negative and significant. The bigger the initial income is, the bigger $c_0$ is. And the bigger the initial income is, the smaller $b_1$ and $c_1$ are. We think that the jumps at the turning points are one of the reasons why $b_{01}$ is not significant.

In the first regime, the graph of a country with high initial income has an upper intercept and a steeper slope. On the contrary, the graph of a country with low initial income has a lower intercept and a glacis slope. The result that $a_{11}$ is positive and significant is in accordance with the recent rising trend in the birth rates in some developed countries, e.g., Sweden, United Kingdom, Spain, Italy, Finland, etc.

Let us analyze the relationship between initial income and birth rates at the turning points. We evaluate the effects of initial income on birth rates at the turning points. We substitute the $\tau_i^l$ and $\tau_i^h$ in Eq. (11) into $x_{i,t}$'s in Eq. (12) and diffrentiate it with respect to initial income $x_{i,1}$. The error terms are deemed to 0. Because of the jumps, there are two values at each turning point as follows:

$$\frac{\partial y_{i,t}}{\partial x_{i,1}}\Big|_{x_{i,t}=\tau_i^l} = \begin{cases} c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x_{i,1} & \text{at the end of the first regime} \\ b_{01} + b_{10}l_1 + b_{11}l_0 + 2b_{11}l_1x_{i,1} & \text{at the beginning of the second regime} \end{cases} \tag{18}$$

$$\frac{\partial y_{i,t}}{\partial x_{i,1}}\Big|_{x_{i,t}=\tau_i^h} = \begin{cases} b_{01} + b_{10}h_1 + b_{11}h_0 + 2b_{11}h_1x_{i,1} & \text{at the end of the second regime} \\ a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x_{i,1} & \text{at the beginning of the third regime} \end{cases} \tag{19}$$

Eq. (18) and Eq. (19) are evaluated at the first and the second turning point, respectively. We use three kinds of initial income for $x_{i,1}$. $x^h_{i,1} = 7.97$, $x^m_{i,1} = 5.58$ and $x^l_{i,1} = 3.70$. The values are maximum, mean and minimum of the log income in 1960 as seen on Table 2. Table 7 shows the estimation results of the effect of the initial income on birth rate. (11) and (12) – the effects of initial income evaluated with $x^m_{i,1}$ and $x^l_{i,1}$ on birth rate at the end of the first regime – , and (1) and (2) – the effects of initial income evaluated with $x^h_{i,1}$ and $x^m_{i,1}$ on birth rate at the beginning of the third regime – are not significant. But the other effects are negative and significant. We can consider that the birth rates in developed and developing countries converge at almost the same level in the third regime. That is why the effects of the initial income evaluated with $x^h_{i,1}$ and $x^m_{i,1}$ on birth rate at the beginning of the third regime is not significant. The 4 cases of 12 cases are not significant, but the other 8 cases from (3) to (10) are negative and significant. From these results, it is considered generally applicable that the higher the level of initial income is, the lower the birth rates at the first and the second turning points are. As compared to developed countries, their turning points occur at the higher level of birth rate.

### (a-3) The advancing of the transition

It is difficult to investigate the advancing of the transition with the three regimes, because, the regimes might be different according to the countries even if they are in the same income level. For example, when the income level is 10,000, some countries can be in the first regime, and some countries can be in the second regime, and some countries can be in the third regime. Then, we have to compare it using a different

Table 7: Estimation results using birth rate

| Effects | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| (1) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^h_{i,1}$ | -0.093 | -0.094 | 0.060 | [ -0.209 , 0.029 ] | -0.245 |
| (2) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^m_{i,1}$ | -0.270 | -0.271 | 0.058 | [ -0.382 , 0.153 ] | 0.177 |
| (3) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^l_{i,1}$ | -0.448 | -0.449 | 0.084 | **[ -0.609 , -0.297 ]** | 0.284 |
| (4) $b_{01} + b_{10}h_1 + b_{11}h_0 + 2b_{11}h_1x^h_{i,1}$ | -0.358 | -0.357 | 0.055 | **[ -0.469 , -0.253 ]** | 0.245 |
| (5) $b_{01} + b_{10}h_1 + b_{11}h_0 + 2b_{11}h_1x^m_{i,1}$ | -0.290 | -0.290 | 0.037 | **[ -0.363 , -0.217 ]** | 0.368 |
| (6) $b_{01} + b_{10}h_1 + b_{11}h_0 + 2b_{11}h_1x^l_{i,1}$ | -0.222 | -0.222 | 0.028 | **[ -0.278 , -0.167 ]** | 0.230 |
| (7) $b_{01} + b_{10}l_1 + b_{11}l_0 + 2b_{11}l_1x^h_{i,1}$ | -0.267 | -0.266 | 0.047 | **[ -0.361 , -0.176 ]** | 0.436 |
| (8) $b_{01} + b_{10}l_1 + b_{11}l_0 + 2b_{11}l_1x^m_{i,1}$ | -0.214 | -0.214 | 0.032 | **[ -0.277 , -0.150 ]** | 0.956 |
| (9) $b_{01} + b_{10}l_1 + b_{11}l_0 + 2b_{11}l_1x^l_{i,1}$ | -0.161 | -0.161 | 0.027 | **[ -0.214 , -0.107 ]** | 0.675 |
| (10) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^h_{i,1}$ | -0.190 | -0.188 | 0.065 | **[ -0.363 , -0.029 ]** | 0.274 |
| (11) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^m_{i,1}$ | -0.086 | -0.085 | 0.049 | [ -0.183 , 0.010 ] | -0.059 |
| (12) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^l_{i,1}$ | 0.019 | 0.019 | 0.028 | [ -0.038 , 0.073 ] | -0.930 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic
The thick bold styles represent the cases that the 95% credible intervals do not include 0.
$x^h_{i,1} = 7.97$, $x^m_{i,1} = 5.58$ and $x^l_{i,1} = 3.70$. The values are maximum, mean and minimum log income in 1960, respectively.

estimated values even though they have the same income level. So, when we analyze the advancing of the transition, we decide not to consider the regimes.

To analyze the relationship between initial income and birth rate, we calculate the correlation coefficients between the initial income and the income of the countries when they went or go through a similar birth rate. It is considered that developing countries achieve a similar birth rate in the earlier development stage compared with the developed countries. Under a similar birth rate, the developed countries which have a higher level of initial income have also a higher level of income. We calculate the correlation coefficients of the initial income and the income when the birth rate is controlled at the similar level. Table 8 shows the ranges of the log birth rate, the number of countries in the range and the correlation coefficients. For example, the number of the countries which are or were in the range from 2.0 to 2.5 is 20, and the correlation coefficients between their initial income (per capita GDP in 1960) and the income when they are or were in the range from 2.0 to 2.5 is 0.229. The correlation coefficients of all of the ranges are positive. The results mean that the developing countries pass through the same birth rate at a lower income level than the developed countries have done. Therefore, it becomes “the advancing of the transition”.

Table 8: Correlation coefficient

| Range of log birth rate | Number of countries | Correlation coefficient* | Correlation coefficient** |
|---|---|---|---|
| 1.0 - 1.5 | 0 | - | - |
| 1.5 - 2.0 | 1 | - | - |
| 2.0 - 2.5 | 20 | 0.229 | 0.428 |
| 2.5 - 3.0 | 36 | 0.077 | 0.274 |
| 3.0 - 3.5 | 55 | 0.278 | 0.350 |
| 3.5 - 4.0 | 65 | 0.957 | 0.987 |
| 4.0 - 4.5 | 4 | 0.196 | 0.310 |
| 4.5 - 5.0 | 0 | - | - |

* and ** are calculated using the level and the logarithm value of GDP per capita, respectively.

**(a-4) The findings from birth rate in Model 3**

To summarize it, our results can be interpreted as follows: in case of the countries with the smaller initial income compared to the countries with the higher initial income, 1) their income levels are lower at the similar birth rate level, 2) their turning points occur at lower income levels, and 3) their turning points occur at higher birth rate levels. These results satisfy the three features which may be shown in the compressed demographic transition, that is, the advancing of the transition, the left movement of the turning point and the upper movement of the turning point.

**(b) Death rate**

Let us examine the results of the death rate in Table 6 (b).

**(b-1) The left movement of the turning point**

The results of the turning points are,

$$\tau_i^h = \underbrace{8.395}_{h_0^*} - \underbrace{0.083}_{h_1} x_{i,1} + u_{\tau^h,i} \qquad \tau_i^l = \underbrace{5.930}_{l_0^*} + \underbrace{0.053}_{l_1} x_{i,1} + u_{\tau^l,i} \ .$$

$h_0$ and $l_0$ are significant, while $h_1$ and $l_1$ are not significant. We cannot find the left movement of the turning points in the death rate.

**(b-2) The upper movement of the turning point**

The estimation results of each regimes are,

$$a_{0,i} = \underbrace{2.300}_{a_{00}^*} - \underbrace{0.053}_{a_{01}} x_{i,1} + u_{a_0,i} \qquad a_{1,i} = -\underbrace{0.130}_{a_{10}} + \underbrace{0.010}_{a_{11}} x_{i,1} + u_{a_1,i} \ .$$

$$b_{0,i} = \underbrace{6.133}_{b_{00}^*} - \underbrace{0.611}_{b_{01}^*} x_{i,1} + u_{b_0,i} \qquad b_{1,i} = -\underbrace{0.477}_{b_{10}^*} + \underbrace{0.071}_{b_{11}^*} x_{i,1} + u_{b_1,i} \ .$$

$$c_{0,i} = \underbrace{3.241}_{c_{00}^*} + \underbrace{0.277}_{c_{01}^*} x_{i,1} + u_{c_0,i} \qquad c_{1,i} = -\underbrace{0.100}_{c_{10}} - \underbrace{0.055}_{c_{11}^*} x_{i,1} + u_{c_1,i} \ .$$

$c_{01}$ is positive and $c_{11}$ is negative. The bigger the initial income is, the bigger the $c_0$ is and the smaller the $c_1$ is. In the first regime, the graph of a country with high initial income has an upper intercept and a steeper slope. On the contrary, the graph of a country with low initial income has a lower intercept and a glacis slope.

Table 9 shows the estimation results of the effect of the initial income on death rate. Only (6), (7), (8) and (9) are significant and negative. The effects of the initial income just work at the beginning of the second regime.

**(b-3) The advancing of the transition**

In Table 10, all of the correlation coefficients are positive. As with the birth rate, the result means that the developing countries pass through the same death rate at lower income level than the developed countries have done. Therefore, it also becomes “the downward at the same income level”.

Table 9: Estimation results using death rate

| Effects | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| (1) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^h_{i,1}$ | 0.137 | 0.138 | 0.076 | [ -0.015 , 0.287 ] | -0.245 |
| (2) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^m_{i,1}$ | 0.135 | 0.135 | 0.078 | [ -0.017 , 0.290 ] | -0.245 |
| (3) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^l_{i,1}$ | 0.134 | 0.133 | 0.085 | [ -0.032 , 0.035 ] | 0.284 |
| (4) $b_{01} + b_{10}h_1 + b_{11}h_0 + 2b_{11}h_1x^h_{i,1}$ | -0.074 | -0.075 | 0.098 | [ -0.261 , 0.115 ] | 0.115 |
| (5) $b_{01} + b_{10}h_1 + b_{11}h_0 + 2b_{11}h_1x^m_{i,1}$ | -0.041 | -0.039 | 0.065 | [ -0.177 , 0.082 ] | 0.245 |
| (6) $b_{01} + b_{10}h_1 + b_{11}h_0 + 2b_{11}h_1x^l_{i,1}$ | -0.009 | -0.013 | 0.091 | **[ -0.174 , -0.181 ]** | 0.230 |
| (7) $b_{01} + b_{10}l_1 + b_{11}l_0 + 2b_{11}l_1x^h_{i,1}$ | -0.152 | -0.150 | 0.070 | **[ -0.296 , -0.019 ]** | 0.436 |
| (8) $b_{01} + b_{10}l_1 + b_{11}l_0 + 2b_{11}l_1x^m_{i,1}$ | -0.169 | -0.167 | 0.060 | **[ -0.292 , -0.058 ]** | 0.436 |
| (9) $b_{01} + b_{10}l_1 + b_{11}l_0 + 2b_{11}l_1x^l_{i,1}$ | -0.186 | -0.185 | 0.075 | **[ -0.335 , -0.042 ]** | 0.675 |
| (10) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^h_{i,1}$ | -0.064 | -0.063 | 0.141 | [ -0.344 , 0.209 ] | 0.274 |
| (11) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^m_{i,1}$ | -0.053 | -0.055 | 0.124 | [ -0.295 , 0.193 ] | 0.274 |
| (12) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^l_{i,1}$ | -0.043 | -0.044 | 0.113 | [ -0.260 , 0.185 ] | -0.930 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic
The thick bold styles represent the cases that the 95% credible intervals do not include 0.
$x^h_{i,1} = 7.97$, $x^m_{i,1} = 5.58$ and $x^l_{i,1} = 3.70$. The values are maximum, mean and minimum log income in 1960, respectively.

Table 10: Correlation coefficient

| Range of log birth rate | Number of countries | Correlation coefficient* | Correlation coefficient** |
|---|---|---|---|
| 0.0 - 0.5 | 0 | - | - |
| 0.5 - 1.0 | 1 | - | - |
| 1.0 - 1.5 | 6 | 0.383 | 0.118 |
| 1.5 - 2.0 | 42 | 0.532 | 0.526 |
| 2.0 - 2.5 | 72 | 0.647 | 0.646 |
| 2.5 - 3.0 | 61 | 0.966 | 0.867 |
| 3.0 - 3.5 | 32 | 0.968 | 0.992 |
| 3.5 - 4.0 | 1 | - | - |
| 4.0 - 4.5 | 0 | - | - |

* and ** are calculated using the level and the logarithm value of GDP per capita, respectively.

**(b-4) The findings from death rate in Model 3**

To summarize it, our results can be interpreted as follows: in case of the countries with the smaller initial income compared to the countries with the higher initial income, 1) their death rates are lower at the same income level, 2) there is no leftward in the turning points, and 3) the first turning point occurs at the higher death rate levels but the second turning point does not.[15]

## 4.2 Model 4

### 4.2.1 Model

As we did in Model 2, we modify the discontinuous at the turning points in Model 3. We add the continuous (no jump) constraint in Model 4. The constraints are as follows:

[15] At the beginning of the second regime, the effects – (7), (8) and (9) – are significant, but at the end of the first regime, the effects – (10), (11) and (12) – are not significant. As seen later, in Model 4 without jumps, we get the significant results. We judge this results in concert with the results of Model 4.

$$
\begin{aligned}
a_{0,i} + a_{1,i}\tau_i^h &= b_{0,i} + b_{1,i}\tau_i^h \ , \\
b_{0,i} + b_{1,i}\tau_i^l &= c_{0,i} + c_{1,i}\tau_i^l \ .
\end{aligned}
\tag{20}
$$

We substitute the constraints in Eq. (20) to Eq. (8) and get the Eq. (21).

$$
\begin{aligned}
b_{0,i} &= \frac{\tau_i^l \tau_i^h (c_{1,i} - a_{1,i}) + c_{0,i}\tau_i^h - a_{0,i}\tau_i^l}{\tau_i^h - \tau_i^l} \ , \\
b_{1,i} &= \frac{a_{0,i} + a_{1,i}\tau_i^h - c_{1,i}\tau_i^l - c_{0,i}}{\tau_i^h - \tau_i^l} \ .
\end{aligned}
\tag{21}
$$

### 4.2.2 Estimation Results

Under the constraints in Eq. (20), we estimate the variables, $a_{00}$, $a_{01}$, $a_{10}$, $a_{11}$, $a_{00}$, $c_{00}$, $c_{01}$, $c_{10}$, $c_{11}$, $h_0$, $h_1$, $l_0$, $l_1$, $\sigma^2_{a_0}$, $\sigma^2_{a_1}$, $\sigma^2_{c_0}$, $\sigma^2_{c_1}$, $\sigma^2_{\tau^h}$, $\sigma^2_{\tau^l 1}$, etc. Table 11 reports the estimation results.[16] The sampling was run with a burn-in of 500,000 iteration with 1,000,000 as what we did before. Based on the results of Geweke's convergence diagnostic, we can consider that this sampling has been converged.

#### (a) Birth rate

Let us examine the results of the birth rate in Table 11 (a).

#### (a-1) The left movement of the turning point

The results of the turning points are,

$$
\tau_i^h = \underbrace{7.469}_{h_0^*} + \underbrace{0.171}_{h_1} x_{i,1} + u_{\tau^h,i} \qquad \tau_i^l = \underbrace{4.388}_{l_0^*} + \underbrace{0.326}_{l_1^*} x_{i,1} + u_{\tau^l,i} \ .
$$

$h_0$, $l_0$ and $l_1$ are significant, while $h_1$ is not significant. The smaller the initial income is, the earlier the first turning point is. Even though $h_1$ is positive, $h_1$ is not significant. We can find the leftward in the turning point only at the first turning point, but we cannot find it in the second turning point.

#### (a-2) The upper movement of the turning point

The estimation results of the third and first regimes are,

$$
a_{0,i} = \underbrace{7.898}_{a_{00}^*} - \underbrace{0.626}_{a_{01}^*} x_{i,1} + u_{a_0,i} \qquad a_{1,i} = -\underbrace{0.496}_{a_{10}^*} + \underbrace{0.058}_{a_{11}^*} x_{i,1} + u_{a_1,i} \ .
$$

$$
c_{0,i} = \underbrace{3.434}_{c_{00}^*} + \underbrace{0.136}_{c_{01}^*} x_{i,1} + u_{c_0,i} \qquad c_{1,i} = \underbrace{0.187}_{c_{10}^*} - \underbrace{0.050}_{c_{11}^*} x_{i,1} + u_{c_1,i} \ .
$$

$c_{01}$ is positive and $c_{11}$ is negative. The bigger the initial income is, the bigger the $c_0$ is and the smaller the $c_1$ is. In the first regime, the graph of a country with high initial income has an upper intercept and a steeper slope. On the contrary, the graph of a country with low initial income has a lower intercept and a glacis slope. As mentioned before, the result that $a_{11}$ is positive and significant represents the recent rising trend in the birth rates in some developed countries.

---

[16] The results in Table 11 (a) and Table 12 are calculated together. For convenience of explanation, we divided them into two tables. The same applies to Table 11 (b) and Table 13.

Table 11: Estimation results of Model 4

(a) Birth rate

| Parameters | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| $a_{00}$ | 7.898 | 7.910 | 0.936 | [ **6.091 , 9.696** ] | -0.202 |
| $a_{01}$ | -0.626 | -0.628 | 0.135 | [ **-0.886 , -0.364** ] | 0.249 |
| $a_{10}$ | -0.496 | -0.497 | 0.111 | [ **-0.709 , -0.282** ] | -0.128 |
| $a_{11}$ | 0.058 | 0.058 | 0.016 | [ **0.027 , 0.088** ] | 0.089 |
| $c_{00}$ | 3.434 | 3.433 | 0.344 | [ **2.761 , 4.104** ] | -0.410 |
| $c_{01}$ | 0.136 | 0.136 | 0.069 | [ **0.001 , 0.273** ] | 0.439 |
| $c_{10}$ | 0.187 | 0.187 | 0.058 | [ **0.073 , 0.300** ] | 0.310 |
| $c_{11}$ | -0.050 | -0.050 | 0.011 | [ **-0.073 , -0.028** ] | -0.356 |
| $h_0$ | 7.469 | 7.472 | 0.780 | [ **5.934 , 8.990** ] | -0.171 |
| $h_1$ | 0.171 | 0.170 | 0.126 | [ -0.075 , 0.420 ] | 0.214 |
| $l_0$ | 4.388 | 4.371 | 0.752 | [ **2.965 , 5.913** ] | 0.205 |
| $l_1$ | 0.326 | 0.330 | 0.137 | [ **0.048 , 0.583** ] | -0.236 |
| $\sigma^2$ | 0.084 | 0.084 | 0.001 | [ **0.082 , 0.086** ] | -0.255 |
| $\sigma^2_{a0}$ | 0.526 | 0.522 | 0.067 | [ **0.407 , 0.670** ] | 1.325 |
| $\sigma^2_{a1}$ | 0.053 | 0.052 | 0.007 | [ **0.040 , 0.069** ] | 1.329 |
| $\sigma^2_{c0}$ | 0.147 | 0.149 | 0.031 | [ **0.081 , 0.205** ] | 0.039 |
| $\sigma^2_{a1}$ | 0.028 | 0.028 | 0.004 | [ **0.021 , 0.037** ] | -0.350 |
| $\sigma^2_{\tau h}$ | 0.808 | 0.803 | 0.086 | [ **0.653 , 0.989** ] | -0.868 |
| $\sigma^2_{\tau l}$ | 0.933 | 0.927 | 0.093 | [ **0.766 , 1.129** ] | -0.722 |

(b) Death rate

| Parameters | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| $a_{00}$ | 0.912 | 0.896 | 0.687 | [ -0.392 , 2.311 ] | -1.110 |
| $a_{01}$ | 0.222 | 0.224 | 0.104 | [ **0.011 , 0.421** ] | 1.078 |
| $a_{10}$ | 0.148 | 0.150 | 0.085 | [ -0.024 , 0.307 ] | 1.138 |
| $a_{11}$ | -0.027 | -0.028 | 0.013 | [ **-0.052 , -0.002** ] | -1.113 |
| $c_{00}$ | 3.104 | 3.101 | 0.527 | [ **2.084 , 4.134** ] | 0.259 |
| $c_{01}$ | 0.276 | 0.276 | 0.109 | [ **0.062 , 0.487** ] | -0.165 |
| $c_{10}$ | 0.049 | 0.047 | 0.100 | [ -0.142 , 0.249 ] | -0.314 |
| $c_{11}$ | -0.075 | -0.075 | 0.020 | [ **-0.116 , -0.036** ] | 0.212 |
| $h_0$ | 7.138 | 7.147 | 0.887 | [ **5.375 , 8.850** ] | 0.356 |
| $h_1$ | 0.088 | 0.084 | 0.164 | [ -0.223 , 0.419 ] | -0.260 |
| $l_0$ | 5.586 | 5.583 | 0.674 | [ **4.275 , 6.921** ] | 0.544 |
| $l_1$ | 0.029 | 0.030 | 0.125 | [ -0.219 , 0.272 ] | -0.567 |
| $\sigma^2$ | 0.113 | 0.113 | 0.001 | [ **0.110 , 0.115** ] | -0.568 |
| $\sigma^2_{a0}$ | 0.491 | 0.487 | 0.059 | [ **0.385 , 0.617** ] | 0.120 |
| $\sigma^2_{a1}$ | 0.050 | 0.050 | 0.007 | [ **0.038 , 0.065** ] | 1.185 |
| $\sigma^2_{c0}$ | 0.206 | 0.205 | 0.041 | [ **0.128 , 0.291** ] | 0.196 |
| $\sigma^2_{c1}$ | 0.051 | 0.050 | 0.007 | [ **0.038 , 0.065** ] | 1.710 |
| $\sigma^2_{\tau h}$ | 0.819 | 0.814 | 0.091 | [ **0.657 , 1.011** ] | -0.539 |
| $\sigma^2_{\tau l}$ | 0.889 | 0.884 | 0.079 | [ **0.746 , 1.058** ] | 0.335 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic
The thick bold styles represent the cases that the 95% credible intervals do not include 0.

Table 12 shows the effects of initial income on birth rate at the turning points. In model 4, the values – birth rate and death rate – at the end of the first regime and at the beginning of the second regime are the same. And the values at the end of the second regime and at the beginning of the third regime are also the same. Because there is no jump under the constraints in Eq. (20). (2) to (6) are significant and negative. Only (1) is not significant. As with the birth rate in Model 3, in the case of the countries with smaller initial income, the birth rates at both turning points are high.

Table 12: Estimation results using birth rate

| Effects | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| (1) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^h_{i,1}$ | -0.121 | -0.121 | 0.079 | [ -0.275 , 0.036 ] | 0.040 |
| (2) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^m_{i,1}$ | -0.163 | -0.164 | 0.079 | [ **-0.316 , -0.007** ] | 0.100 |
| (3) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^l_{i,1}$ | -0.206 | -0.207 | 0.092 | [ **-0.384 , -0.022** ] | 0.064 |
| (4) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^h_{i,1}$ | -287 | -0.286 | 0.065 | [ **-0.417 , -0.162** ] | -0.341 |
| (5) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^m_{i,1}$ | -0.215 | -0.216 | 0.035 | [ **-0.282 , -0.144** ] | 0.337 |
| (6) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^l_{i,1}$ | -0.143 | 0.144 | 0.029 | [ **-0.199 , -0.086** ] | 1.274 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic
The thick bold styles represent the cases that the 95% credible intervals do not include 0.
$x^h_{i,1} = 7.97$, $x^m_{i,1} = 5.58$ and $x^l_{i,1} = 3.70$. The values are maximum, mean and minimum log income in 1960, respectively.

### (a-3) The findings from birth rate in Model 4

To summarize it, our results can be interpreted as follows: in case of the countries with the smaller initial income compared to the countries with the higher initial income, 1) the first turning point of each country occurs at the lower income levels, but the second turning point of each country does not, 2) the first and second turning points occur at the higher birth rate levels.

### (b) Death rate

Let us examine the results of the death rate in Table 11 (b).

### (b-1) The left movement of the turning point

The results of the turning points are,

$$\tau^h_i = \underbrace{7.138}_{h^*_0} + \underbrace{0.088}_{h_1} x_{i,1} + u_{\tau^h,i} \qquad \tau^l_i = \underbrace{5.586}_{l_0} + \underbrace{0.029}_{l_1} x_{i,1} + u_{\tau^l,i} \ .$$

$h_0$ and $l_0$ are significant, while $h_1$ and $l_1$ are not significant. We cannot find the leftward in the turning point in the death rate.

### (b-2) The upper movement of the turning point

The estimation results of the third and first regimes are,

$$a_{0,i} = \underbrace{0.912}_{a_{00}} + \underbrace{0.222}_{a^*_{01}} x_{i,1} + u_{a_0,i} \qquad a_{1,i} = -\underbrace{0.148}_{a_{10}} - \underbrace{0.027}_{a^*_{11}} x_{i,1} + u_{a_1,i} \ .$$

$$c_{0,i} = \underbrace{3.104}_{c^*_{00}} + \underbrace{0.276}_{c^*_{01}} x_{i,1} + u_{c_0,i} \qquad c_{1,i} = \underbrace{0.049}_{c^*_{10}} - \underbrace{0.075}_{c^*_{11}} x_{i,1} + u_{c_1,i} \ .$$

$a_{01}$ and $c_{01}$ are positive and $a_{11}$ and $c_{11}$ are negative. The bigger the initial income is, the bigger the $a_0$ and $c_0$ are and the smaller the $a_1$ and $c_1$ are. In the first and third regimes, the graph of a country with high initial income has an upper intetcept and a steeper slope. On the contrary, the graph of a country with low initial income has a lower intercept and a glacis slope.

In Table 13, only (5) to (6) are significant and negative. The others, (1) to (4), are not significant. As with death rate in Model 3, in case of developing countries, the first turning point occurs at the higher death rate levels but the second turning point does not.

Table 13: Estimation results using death rate

| Effects | Mean | Median | Standard Deviation | 95% HPDI | Geweke's CD |
|---|---|---|---|---|---|
| (1) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^h_{i,1}$ | -0.001 | 0.000 | 0.074 | [ -0.149 , 0.142 ] | -0.342 |
| (2) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^m_{i,1}$ | 0.011 | 0.010 | 0.067 | [ -0.121 , 0.143 ] | 0.186 |
| (3) $a_{01} + a_{10}h_1 + a_{11}h_0 + 2a_{11}h_1x^l_{i,1}$ | 0.022 | 0.022 | 0.068 | [ -0.111 , 0.156 ] | 0.583 |
| (4) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^h_{i,1}$ | -0.177 | -0.175 | 0.096 | [ -0.372 , 0.008 ] | 0.869 |
| (5) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^m_{i,1}$ | -0.169 | -0.168 | 0.066 | **[ -0.300 , -0.038 ]** | 0.758 |
| (6) $c_{01} + c_{10}l_1 + c_{11}l_0 + 2c_{11}l_1x^l_{i,1}$ | -0.160 | -0.159 | 0.055 | **[ -0.270 , -0.054 ]** | 0.971 |

HPDI: Highest Posterior Density Interval, CD: Convergence Diagnostic
The thick bold styles represent the cases that the 95% credible intervals do not include 0.
$x^h_{i,1} = 7.97$, $x^m_{i,1} = 5.58$ and $x^l_{i,1} = 3.70$. The values are maximum, mean and minimum log income in 1960, respectively.

**(b-3) The findings from death rate in Model 4**

To summarize it, our results can be interpreted as follows: in case of the countries with the smaller initial income compared to the countries with the higher initial income, 1) there is no leftward in the turning points, and 2) the first turning point occurs at the higher death rate levels but the second turning point does not.

**Summary**

We summarize the results of Model 3 and Model 4 on the compressed demographic transition in Table 14. Table 14 indicates whether or not there is a compressed demographic transition. At first, we found the downward in the same income level in both birth and death rate. The other results vary with the regimes. The thick bold styles represent the cases that the results in Model 3 and Model 4 are the same. We found out the same results in Model 3 and 4 that 1) there is a leftward in the turning point of the birth rate at the first turning point, and that 2) both Model 3 and 4 have no leftward in the turning point of the death rate and that 3) in the case of the countries with smaller initial income, the birth rates at both the first and the second turning points are high, however, the death rate at the first turning point is high, but the death rate at the second turning point is not high. Both the birth and death rate of the developing countries drop more steeply than those of the developed countries.

## 4.3 Compressed Demographic Transition and Demographic Gift

Demographic gift (or demographic bonus) which was used by Bloom and Williamson (1998) means the economic benefits of a high ratio of working-age to dependent population during the demographic transition.[17]

[17] Bloom and Williamson (1998) shows that East Asia's demographic transition resulted in its working-age population growing at a much faster pace than its dependent population during the period of 1965-1990, thereby expanding the per capita productive capacity of East Asian economies.

Table 14: Summary

| | The advancing of the transition |
|---|---|
| Birth Rate | Yes |
| Death Rate | Yes |

Model 3

| | The leftward in the turning point | | The vlaue at the turning point | | | |
|---|---|---|---|---|---|---|
| | The 1st turning point | The 2nd turning point | The 1st turning point | | The 2nd turning point | |
| Birth Rate | **Yes** | Yes | **high** | $\frac{4}{6}$ | **high** | $\frac{4}{6}$ |
| | | | not high | $\frac{2}{6}$ | not high | $\frac{2}{6}$ |
| Death Rate | **No** | **No** | **high** | $\frac{3}{6}$ | high | $\frac{1}{6}$ |
| | | | not high | $\frac{3}{6}$ | **not high** | $\frac{5}{6}$ |

Model 4

| | The leftward in the turning point | | The vlaue at the turning point | | | |
|---|---|---|---|---|---|---|
| | The 1st turning point | The 2nd turning point | The 1st turning point | | The 2nd turning point | |
| Birth Rate | **Yes** | No | **high** | $\frac{3}{3}$ | **high** | $\frac{2}{3}$ |
| | | | not high | $\frac{0}{3}$ | not high | $\frac{1}{3}$ |
| Death Rate | **No** | **No** | **high** | $\frac{2}{3}$ | high | $\frac{0}{3}$ |
| | | | not high | $\frac{1}{3}$ | **not high** | $\frac{3}{3}$ |

*) The thick bold styles represent the cases that the results in Model 3 and Model 4 are the same.
The figures (e.g., $\frac{4}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{3}{3}$, etc.) show the ratio of the number of cases which are significant to the number of all cases of each turning points in Table 7, 9, 12 and 13.

The growth rate of GDP per capita is calculated as follows:[18]

$$\begin{aligned} \text{growth rate of GDP per capita} &= \text{growth rate of GDP per worker} \\ &\quad + \text{growth rate of working age fraction of population} \end{aligned} \tag{22}$$

Even though there is no growth of GDP per worker, depending on the fraction of working age, the growth rate of GDP per capita can be changed. An important implication of the compressed demographic transition is that future demographic change will tend to depress growth rates in latecomers' economies.

To show the effect of the compressed demographic transition on the change of working-age fraction of the population, we simulate the Model 4 using time instead of income. And to represent the compressed demographic transition, we set the suitable parameter values as Table 15. The number of population at the initial period is 200.

Even though the values of these parameters are arbitrary, they satisfy the characteristics in our models: 1) the threshold levels of death rates appear in an earlier stage than those of the birth rates. 2) the birth and death rates in the developed counrties are located above than those of in the developing counrties. 3) the demographic transition in developing countries starts at a higher level of birth and death rate. 4) the turning point of birth rate in developing countries starts at a lower level of income. Working-age fraction is defined as population aged 15 to 64 over total population. Normally, an age-specific survivorship function

[18] From the definition, GDP per worker = $\frac{\text{GDP}}{\text{number of workers}}$, and GDP per capita = $\frac{\text{GDP}}{\text{total population}}$, then GDP per capita = GDP per worker × $\frac{\text{number of workers}}{\text{total population}}$. Taking the logarithm to both sides and differentiating both sides with respect to time, we can get the Eq. (22). See Weil (2013) for details.

Table 15: The values of parameters

| | Developed country | | Developing country | |
|---|---|---|---|---|
| | Birth rate | Death rate | Birth rate | Death rate |
| at the initial time | 60 | 51 | 57 | 50 |
| at the 1st turining point | 45 | 38 | 50 | 42 |
| at the 2nd turining point and beyond | 10 | 10 | 10 | 10 |
| the time of the 1st turining point | 100 | 45 | 55 | 45 |
| the time of the 2nd turining point | 200 | 100 | 125 | 100 |

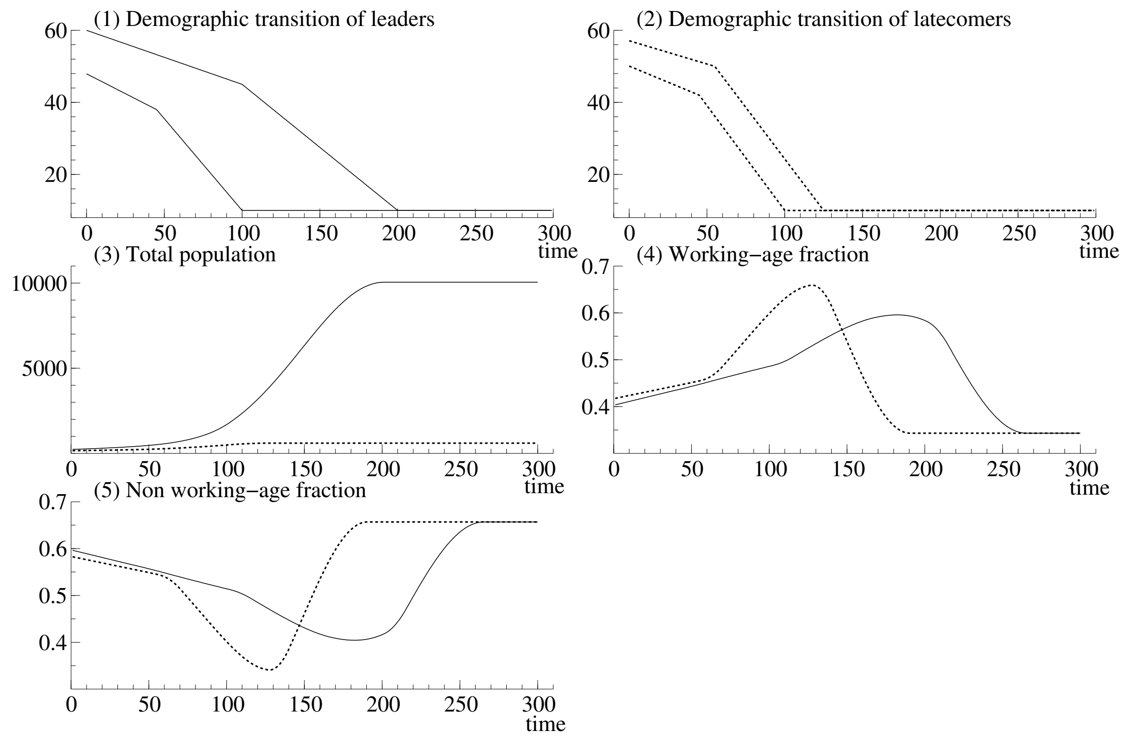


Figure 7: The results of simulation

which shows the probability that will be alive at different ages is used for population forcasting. However, for simplication, we apply the same percentage of survivorship to everyone regardless of their age.

Figure 7 shows the simulation results. The vertical axises in (1) to (5) show the demographic transition of leaders, the demographic transition of latecomers, total population, working-age fraction and non working-age fraction, respectively. The horizontal axis show time not income. The thick bold lines and dashed lines show the leaders and the latercomers, respectively. In Figure 7 (4), we can find that the curved line of leaders is a longer mountain-like shape than that of the latecomers, even though the height is lower. This means that in terms of the area under the line, the demographic gifts of leaders are bigger than that of latercomers. In Figure 7 (5), the increasing speed of non working-age fraction in the latecomers is higher than that of the leaders.

The aging population and the decrease in working-age fraction will not only affect the decrease of demographic gift but also the decrease of pension revenue source. The latter makes it probable that the pension system cannot continue to be part of a stable system as it had in the past. The demographic bonus could be opposed to a demographic onus.

# 5 Concluding Remarks

In this paper, we examined the relationship between economic development and the birth and death rate which are important factors to identify the population growth rate. Demographic transition is well known that both variables decrease with economic growth, and the decrease of birth rate follows that of the death rate. We confirmed the demographic transition using the cross-country data and the threshold econometric model. We estimated and compared the turning points which show that both birth rate and death rate start to change their movements. The turning points of death rate appears in an earlier stage than that of the birth rate. This result shows that our threshold model explains the demographic transition very well. We also examined the compresssed demographic transition. Even though the compresssed demographic transition depends on the development regimes, we found that the demographic transition in developing countries starts at a lower level of income and at higher levels of birth and death rates. And we also found that the developing countries undergo a more intensive decrease in birth and death rate than the developed countries do.

Therefore, we conclude that the compressed demographic transition, including the birth and death rate, in developing countries start at an earlier stage compared to that in the developed countries. This result suggests that the aging population and the decrease in working-age fraction in developing countries can start in an earlier development stage than the experiences of developed countries and the demographic gifts in developing countries can also be lost at an early stage.

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# Appendix

Table A1: Per capita GDP, birth rate and death rate

| Country Name | Level | | | | | | Logarithm | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Per Capita GDP | | Birth rate | | Death rate | | Per Capita GDP | | Birth rate | | Death rate | |
| | 1960 | 2008 | 1960 | 2008 | 1960 | 2008 | 1960 | 2008 | 1960 | 2008 | 1960 | 2008 |
| Algeria | 252.19 | 4974.46 | 50.70 | 20.76 | 20.24 | 4.92 | 5.530 | 8.512 | 3.926 | 3.033 | 3.008 | 1.594 |
| Austria | 935.40 | 49739.05 | 17.90 | 9.33 | 12.70 | 9.01 | 6.841 | 10.815 | 2.885 | 2.233 | 2.542 | 2.198 |
| Bangladesh | 78.96 | 497.21 | 47.26 | 21.43 | 24.22 | 6.59 | 4.369 | 6.209 | 3.856 | 3.065 | 3.187 | 1.885 |
| Barbados | 378.84 | 14380.71 | 30.78 | 11.21 | 9.57 | 7.65 | 5.937 | 9.574 | 3.427 | 2.417 | 2.259 | 2.035 |
| Belgium | 1278.51 | 47193.99 | 17.00 | 11.67 | 12.50 | 9.49 | 7.153 | 10.762 | 2.833 | 2.457 | 2.526 | 2.250 |
| Belize | 308.33 | 4218.26 | 43.15 | 24.70 | 10.30 | 3.64 | 5.731 | 8.347 | 3.765 | 3.207 | 2.332 | 1.292 |
| Benin | 99.68 | 771.49 | 43.41 | 39.40 | 25.92 | 9.17 | 4.602 | 6.648 | 3.771 | 3.674 | 3.255 | 2.216 |
| Bolivia | 168.01 | 1720.04 | 46.36 | 27.10 | 22.32 | 7.54 | 5.124 | 7.450 | 3.836 | 3.300 | 3.106 | 2.020 |
| Botswana | 57.97 | 7050.38 | 47.00 | 24.54 | 16.51 | 12.06 | 4.060 | 8.861 | 3.850 | 3.200 | 2.804 | 2.490 |
| Brazil | 208.48 | 8532.12 | 42.87 | 16.19 | 13.26 | 6.38 | 5.340 | 9.052 | 3.758 | 2.785 | 2.585 | 1.852 |
| Burkina Faso | 70.00 | 528.15 | 47.53 | 47.21 | 26.39 | 12.98 | 4.248 | 6.269 | 3.861 | 3.855 | 3.273 | 2.563 |
| Burundi | 66.66 | 144.77 | 45.87 | 34.47 | 22.81 | 13.89 | 4.200 | 4.975 | 3.826 | 3.540 | 3.127 | 2.631 |
| Cameroon | 114.41 | 1243.45 | 43.37 | 36.86 | 22.44 | 14.22 | 4.740 | 7.126 | 3.770 | 3.607 | 3.111 | 2.654 |
| Canada | 2294.57 | 45002.85 | 26.70 | 11.25 | 7.80 | 7.25 | 7.738 | 10.714 | 3.285 | 2.420 | 2.054 | 1.981 |
| Central African Republic | 74.61 | 458.17 | 43.68 | 35.42 | 27.97 | 16.96 | 4.312 | 6.127 | 3.777 | 3.567 | 3.331 | 2.831 |
| Chad | 105.72 | 765.75 | 45.68 | 45.69 | 24.35 | 16.71 | 4.661 | 6.641 | 3.822 | 3.822 | 3.193 | 2.816 |
| Chile | 550.78 | 10167.27 | 38.96 | 14.94 | 12.87 | 5.40 | 6.311 | 9.227 | 3.662 | 2.704 | 2.555 | 1.686 |
| China | 92.01 | 3413.59 | 20.86 | 12.14 | 25.43 | 7.06 | 4.522 | 8.136 | 3.038 | 2.497 | 3.236 | 1.954 |
| Colombia | 252.46 | 5389.19 | 44.59 | 20.40 | 12.18 | 5.51 | 5.531 | 8.592 | 3.797 | 3.016 | 2.500 | 1.706 |
| Congo, Dem. Rep. | 222.77 | 180.33 | 47.24 | 44.87 | 22.79 | 16.96 | 5.406 | 5.195 | 3.855 | 3.804 | 3.126 | 2.831 |
| Congo, Rep. | 130.27 | 3261.07 | 42.55 | 34.51 | 17.36 | 12.86 | 4.870 | 8.090 | 3.751 | 3.541 | 2.854 | 2.554 |
| Costa Rica | 380.43 | 6564.02 | 45.37 | 16.68 | 11.03 | 4.11 | 5.941 | 8.789 | 3.815 | 2.814 | 2.401 | 1.412 |
| Cote d'Ivoire | 158.56 | 1137.08 | 53.05 | 34.95 | 24.06 | 10.84 | 5.066 | 7.036 | 3.971 | 3.554 | 3.181 | 2.383 |
| Denmark | 1364.10 | 62035.78 | 16.60 | 11.84 | 9.50 | 9.94 | 7.218 | 11.035 | 2.809 | 2.471 | 2.251 | 2.296 |
| Dominican Republic | 200.76 | 4602.30 | 51.69 | 22.53 | 15.96 | 5.89 | 5.302 | 8.434 | 3.945 | 3.115 | 2.770 | 1.773 |
| Ecuador | 227.57 | 4056.39 | 44.43 | 20.80 | 15.68 | 5.16 | 5.427 | 8.308 | 3.794 | 3.035 | 2.752 | 1.640 |
| Egypt, Arab Rep. | 149.08 | 1997.33 | 45.98 | 24.70 | 19.66 | 5.85 | 5.005 | 7.600 | 3.828 | 3.207 | 2.978 | 1.766 |
| El Salvador | 225.54 | 3604.03 | 48.09 | 20.24 | 16.62 | 6.83 | 5.419 | 8.190 | 3.873 | 3.007 | 2.811 | 1.921 |
| Fiji | 285.12 | 4223.95 | 44.81 | 20.95 | 10.55 | 6.62 | 5.653 | 8.349 | 3.802 | 3.042 | 2.357 | 1.890 |
| Finland | 1179.26 | 50905.01 | 18.50 | 11.20 | 9.00 | 9.24 | 7.073 | 10.838 | 2.918 | 2.416 | 2.197 | 2.224 |
| France | 1344.21 | 44471.50 | 17.90 | 12.86 | 11.40 | 8.56 | 7.204 | 10.703 | 2.885 | 2.554 | 2.434 | 2.147 |
| Gabon | 291.28 | 10036.65 | 30.60 | 27.27 | 25.46 | 9.71 | 5.674 | 9.214 | 3.421 | 3.306 | 3.237 | 2.273 |
| Ghana | 179.29 | 1221.66 | 46.93 | 32.36 | 18.66 | 11.09 | 5.189 | 7.108 | 3.849 | 3.477 | 2.927 | 2.406 |
| Greece | 533.99 | 31173.57 | 18.90 | 10.28 | 7.30 | 9.52 | 6.280 | 10.347 | 2.939 | 2.330 | 1.988 | 2.254 |
| Guatemala | 252.04 | 2860.26 | 46.36 | 33.01 | 19.07 | 5.63 | 5.530 | 7.959 | 3.836 | 3.497 | 2.948 | 1.727 |
| Guyana | 299.41 | 1518.44 | 42.62 | 17.87 | 14.82 | 8.15 | 5.702 | 7.325 | 3.752 | 2.883 | 2.696 | 2.098 |
| Honduras | 167.58 | 1908.69 | 50.28 | 27.48 | 19.87 | 5.04 | 5.121 | 7.554 | 3.918 | 3.314 | 2.989 | 1.618 |
| Hong Kong SAR, China | 429.52 | 30863.26 | 35.33 | 11.30 | 6.38 | 5.90 | 6.063 | 10.337 | 3.565 | 2.425 | 1.852 | 1.775 |
| Iceland | 1411.57 | 52932.10 | 28.00 | 15.23 | 6.60 | 6.26 | 7.252 | 10.877 | 3.332 | 2.723 | 1.887 | 1.834 |
| Ireland | 684.31 | 60178.22 | 21.50 | 16.91 | 11.50 | 6.45 | 6.528 | 11.005 | 3.068 | 2.828 | 2.442 | 1.864 |
| Israel | 1365.68 | 27651.80 | 26.90 | 21.50 | 5.70 | 5.30 | 7.219 | 10.227 | 3.292 | 3.068 | 1.740 | 1.668 |
| Italy | 804.49 | 38384.51 | 18.10 | 9.62 | 9.60 | 9.69 | 6.690 | 10.555 | 2.896 | 2.264 | 2.262 | 2.271 |
| Japan | 470.87 | 38267.92 | 17.30 | 8.70 | 7.60 | 9.10 | 6.155 | 10.552 | 2.851 | 2.163 | 2.028 | 2.208 |
| Kenya | 97.64 | 774.70 | 51.26 | 38.77 | 20.21 | 11.64 | 4.581 | 6.652 | 3.937 | 3.658 | 3.006 | 2.455 |
| Lesotho | 40.63 | 777.69 | 42.26 | 28.94 | 19.20 | 16.92 | 3.704 | 6.656 | 3.744 | 3.365 | 2.955 | 2.828 |
| Liberia | 179.96 | 222.10 | 54.80 | 38.33 | 25.74 | 10.46 | 5.193 | 5.403 | 4.004 | 3.646 | 3.248 | 2.348 |
| Luxembourg | 2235.39 | 117954.68 | 15.90 | 11.45 | 11.80 | 7.36 | 7.712 | 11.678 | 2.766 | 2.438 | 2.468 | 1.996 |
| Madagascar | 131.90 | 495.14 | 48.55 | 35.90 | 24.89 | 9.19 | 4.882 | 6.205 | 3.883 | 3.581 | 3.214 | 2.218 |
| Malawi | 46.18 | 287.79 | 53.92 | 40.22 | 28.28 | 12.26 | 3.833 | 5.662 | 3.988 | 3.694 | 3.342 | 2.506 |
| Malaysia | 299.87 | 8211.51 | 44.69 | 20.38 | 14.87 | 4.48 | 5.703 | 9.013 | 3.800 | 3.014 | 2.699 | 1.499 |
| Mauritania | 105.46 | 1101.19 | 50.15 | 33.59 | 20.50 | 10.35 | 4.658 | 7.004 | 3.915 | 3.514 | 3.020 | 2.337 |
| Mexico | 353.44 | 10247.99 | 45.74 | 18.33 | 12.30 | 4.85 | 5.868 | 9.235 | 3.823 | 2.908 | 2.510 | 1.580 |
| Morocco | 174.73 | 2768.74 | 50.40 | 20.42 | 21.09 | 5.82 | 5.163 | 7.926 | 3.920 | 3.017 | 3.049 | 1.761 |
| Nepal | 53.28 | 437.87 | 44.49 | 25.39 | 24.35 | 6.41 | 3.976 | 6.082 | 3.795 | 3.234 | 3.192 | 1.858 |
| Netherlands | 1068.79 | 53075.91 | 20.80 | 11.23 | 7.60 | 8.21 | 6.974 | 10.879 | 3.035 | 2.418 | 2.028 | 2.105 |
| New Zealand | 2312.76 | 27598.80 | 26.50 | 15.06 | 8.80 | 6.83 | 7.746 | 10.226 | 3.277 | 2.712 | 2.175 | 1.921 |
| Nicaragua | 128.04 | 1035.39 | 51.47 | 24.62 | 18.67 | 4.70 | 4.852 | 6.943 | 3.941 | 3.204 | 2.927 | 1.548 |
| Niger | 138.70 | 364.13 | 56.36 | 53.54 | 26.39 | 14.92 | 4.932 | 5.898 | 4.032 | 3.980 | 3.273 | 2.703 |
| Nigeria | 92.94 | 1369.72 | 47.58 | 39.83 | 25.98 | 16.37 | 4.532 | 7.222 | 3.862 | 3.685 | 3.257 | 2.796 |
| Norway | 1441.85 | 94567.91 | 17.30 | 12.69 | 9.10 | 8.75 | 7.274 | 11.457 | 2.851 | 2.541 | 2.208 | 2.169 |
| Oman | 78.25 | 21648.57 | 50.65 | 21.96 | 23.19 | 2.70 | 4.360 | 9.983 | 3.925 | 3.089 | 3.144 | 0.992 |
| Pakistan | 80.85 | 986.64 | 39.12 | 30.09 | 19.19 | 6.92 | 4.393 | 6.894 | 3.667 | 3.404 | 2.954 | 1.934 |
| Panama | 369.08 | 6821.19 | 40.58 | 20.64 | 10.36 | 5.04 | 5.911 | 8.828 | 3.703 | 3.027 | 2.338 | 1.618 |
| Papua New Guinea | 110.79 | 1217.97 | 42.45 | 31.43 | 24.31 | 7.93 | 4.708 | 7.105 | 3.748 | 3.448 | 3.191 | 2.071 |
| Peru | 252.10 | 4477.25 | 46.72 | 21.11 | 18.64 | 5.39 | 5.530 | 8.407 | 3.844 | 3.050 | 2.925 | 1.684 |
| Philippines | 247.06 | 1843.95 | 44.65 | 24.73 | 13.89 | 4.80 | 5.510 | 7.520 | 3.799 | 3.208 | 2.631 | 1.568 |
| Portugal | 357.06 | 23707.70 | 23.90 | 9.85 | 10.60 | 9.82 | 5.878 | 10.074 | 3.174 | 2.287 | 2.361 | 2.284 |
| Rwanda | 41.21 | 458.49 | 52.90 | 41.13 | 22.23 | 14.46 | 3.719 | 6.128 | 3.968 | 3.717 | 3.101 | 2.671 |
| Senegal | 257.30 | 1078.91 | 48.84 | 38.44 | 24.29 | 10.82 | 5.550 | 6.984 | 3.889 | 3.649 | 3.190 | 2.381 |
| Sierra Leone | 142.89 | 351.60 | 46.29 | 40.31 | 30.39 | 15.77 | 4.962 | 5.862 | 3.835 | 3.696 | 3.414 | 2.758 |
| Singapore | 394.65 | 39949.51 | 38.50 | 10.20 | 6.20 | 4.40 | 5.978 | 10.595 | 3.651 | 2.322 | 1.825 | 1.482 |
| South Africa | 422.06 | 5665.79 | 42.27 | 22.04 | 17.36 | 15.20 | 6.045 | 8.642 | 3.744 | 3.093 | 2.854 | 2.721 |
| Spain | 396.39 | 35000.35 | 21.70 | 11.39 | 8.60 | 8.51 | 5.982 | 10.463 | 3.077 | 2.433 | 2.152 | 2.142 |
| St. Vincent and the Grenadines | 161.42 | 5331.18 | 47.88 | 17.58 | 14.78 | 7.48 | 5.084 | 8.581 | 3.869 | 2.867 | 2.693 | 2.013 |
| Sudan | 96.39 | 1403.52 | 46.40 | 31.29 | 21.28 | 10.22 | 4.568 | 7.247 | 3.837 | 3.443 | 3.058 | 2.324 |
| Suriname | 343.21 | 5888.09 | 44.79 | 18.98 | 10.87 | 7.58 | 5.838 | 8.681 | 3.802 | 2.944 | 2.386 | 2.025 |
| Swaziland | 100.47 | 2431.89 | 47.59 | 29.90 | 20.18 | 15.64 | 4.610 | 7.796 | 3.863 | 3.398 | 3.004 | 2.750 |
| Sweden | 1984.34 | 52884.46 | 13.70 | 11.86 | 10.00 | 9.92 | 7.593 | 10.876 | 2.617 | 2.473 | 2.303 | 2.294 |
| Switzerland | 1775.97 | 65699.35 | 17.60 | 10.06 | 9.70 | 8.07 | 7.482 | 11.093 | 2.868 | 2.308 | 2.272 | 2.088 |
| Syrian Arab Republic | 185.62 | 2648.82 | 48.35 | 27.98 | 17.54 | 3.38 | 5.224 | 7.882 | 3.879 | 3.331 | 2.865 | 1.219 |
| Thailand | 99.88 | 4042.78 | 43.64 | 14.52 | 13.26 | 8.95 | 4.604 | 8.305 | 3.776 | 2.676 | 2.584 | 2.192 |
| Togo | 77.34 | 448.79 | 46.93 | 32.88 | 21.07 | 8.16 | 4.348 | 6.107 | 3.849 | 3.493 | 3.048 | 2.099 |
| Trinidad and Tobago | 635.43 | 19442.64 | 38.68 | 14.83 | 8.40 | 8.05 | 6.454 | 9.875 | 3.655 | 2.697 | 2.128 | 2.085 |
| Turkey | 495.70 | 9880.87 | 46.10 | 18.23 | 17.99 | 5.95 | 6.206 | 9.198 | 3.831 | 2.903 | 2.890 | 1.784 |
| Uganda | 62.34 | 456.17 | 49.53 | 46.15 | 20.71 | 12.67 | 4.133 | 6.123 | 3.903 | 3.832 | 3.030 | 2.539 |
| United Kingdom | 1381.02 | 43360.77 | 17.50 | 12.94 | 11.50 | 9.44 | 7.231 | 10.677 | 2.862 | 2.560 | 2.442 | 2.245 |
| United States | 2881.10 | 47208.54 | 23.70 | 14.30 | 9.50 | 8.09 | 7.966 | 10.762 | 3.165 | 2.660 | 2.251 | 2.090 |
| Uruguay | 490.18 | 9351.27 | 22.20 | 14.58 | 8.70 | 9.40 | 6.195 | 9.143 | 3.100 | 2.680 | 2.163 | 2.241 |
| Zambia | 229.53 | 1165.17 | 48.11 | 42.88 | 19.46 | 17.26 | 5.436 | 7.061 | 3.874 | 3.758 | 2.968 | 2.848 |