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June 2005

Online at https://mpra.ub.uni-muenchen.de/462/ MPRA Paper No. 462, posted 16 Nov 2006 UTC

Taxes and labour supply under interdependent preferences

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Warsaw, June 2005

Abstract

In this paper we identify how changes in the income tax rate affect the labour supply under interdependent utility functions. To reach that aim we create a model of the economy in which households choosing their optimal labour supply take into account not only their income, tax rate and individual consumption but also so called relative consumption level (see Garbicz 1997). Taking into account the last issue we significantly modify the well known Becker model (1965).

We conduct a comparative statics exercise using na lattice and supermodular game theory. Thanks to which we show sufficient and necessary conditions for a labour supply to be monotonic function of the income tax rate. We analyze the economic behaviour under static and dynamic setup.

Under quite general assumptions concerning the household utility function we show that the higher the tax rate the lower the macroeconomic labour supply. Additionally we show the possibility of multiple equilibria in the economy that offers the explanation of differences in the working time between e.g. European countries and the US as well as discrepancies between micro and macroeconomic elasticity of labour supply (see Alesina, Glaeser, and Sacerdote 2005).

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1 Introduction

Certain behavioural assumptions are necessary to model households' labour supply decisions. From Becker (1965) onwards it has been assumed as a rule, that households maximize utility over joint consumptions of goods, services and leisure, subject to budget and time constraints. This model includes however one dubious assumption suggesting that consumers care exclusively for the absolute consumption level. Empirical studies (Zizzo 2003) indicates that social environment exerts enormous influence on attitude and behaviour of an average household. Economic, social and political preferences are not after all accidental but in most cases socially predefined. The consumers buy many goods because they want to behave like those who are considered authority in the field of fashion, spending free time or other consumer customs. They strive to equal other agents, to find their respect, recognition and acceptance but above all, they want to avoid the threat of social exclusion. Therefore, reducing human consumption motives only to maximization of absolute level of consumption does not seem to be justified. Consumer reactions, basing on relative level of wealth or poverty, could be also very important (Sobel 2005). That is why both motives (relative as well as absolute consumption) are taken into account, even though their weight can be arguable. The paper attempts to modify the traditional model of labour supply with respect to social aspects of human behaviour.

We present the modified labour supply model using relative consumption as an additional argument of the household's utility function. In comparison with Garbicz (1997), who presented the discussed idea, this paper includes (i) endogenization of the relative consumption, (ii) transformation of the whole analysis to a dynamic context, (iii) generalizing analyzed utility functions to a class of functions. Two cases are examined: (a) strategic complementarity, and (b) strategic substitutability, in which the households react to the increase of consumption of the reference group with increase or decrease of its own propensity to consumption, respectively. In this way this paper delivers quite robust conclusions concerning the influence of relative consumption on households labour supply.

The traditional model of labour supply does not imply any interdependence between consumers. If individual households behaviour is a consequence only of the aspiration to maximize their own, absolute consumption, the social environment does not exert any influence on the consumer's optimum. Under these conditions analysis of individual households' behaviours can be conducted independently and in the spirit of the representative agent. When we add the relative consumption to the household's utility function (with an appropriate weight), we change not only the set of arguments of the function but above all it requires the entirely different approach to the modeling of household's behaviour. It is no more plausible to determine the market demand by simple adding up of the individual optima. The idea of the representative agent has to be rejected. In the new situation households are involved in a strategic game with other subjects, which leads to a Nash equilibrium. In such a market structure agents' behaviours can be complementary. It means that households will increase their own consumption as a reaction to the increase of the consumption of other households (and expand working time – reducing leisure). The shape of the reaction curve can however lead to multiple Nash equilibria on the labour market. This situation is intuitively easy to understand. For if individual household assigns high rank to the relative level of consumption (main desire to be like others), and low to the absolute level of consumptions, all consumers (households) can choose together either high level of consumption and short leisure time or low level of the absolute consumption (and long leisure time). Historical experience, customs, cultural norms and the exogenous shock could determine whether the system achieves high or low level of consumption.

We use this pattern of households' behaviour to examine the influence of income tax changes on the amount of labour supply. If the household seeks its optimum – taking into account only the absolute level of consumption – the increase of taxation levied on work results in most cases in a decrease of the labour supply, since this situation is equivalent to the reduction of real wages. The only exception to this rule is the backwards-bending labour supply curve, but this case is unlikely for the wide class of utility functions. In case of relative consumption the situation looks different. Changes in the tax rates have the nature of exogenous shocks. This kind of shock starts the process of interdependent adjustments of households to the new state of equilibrium. At the same time, shifts of behaviour of some households modify conditions of the optimization for other subjects. It is an intriguing question whether in the described situation households will behave in the same way as subjects in the model with the absolute level consumption.

We established in this paper two essential facts concerning this issue. First of all, quite general assumptions concerning the household's utility function allow conclusion that the increase of fiscal charges results in decrease of labour supply on the macroeconomic level. In principle it confirms current conclusions of economic theory. On the other hand, the potential existence of multiple equilibria on labour market gives us the possible explanation of the differences in the length of working time in European countries and the USA. The reasons of this phenomenon are currently a subject of intensive studies and disputes. Empirical examinations (e.g. Alesina, Glaeser, and Sacerdote 2005) are suggesting that almost whole difference in the level of GDP per capita between Western Europe and the US can be assigned to the duration of working time¹. The differences in labour productivity per hour are minimal between countries mentioned above. Prescott (2004)

¹Europe in comparison with the US is characterized by shorter working week, longer leave and higher unemployment.

assigns shorter working time in Europe to higher taxation. Other authors point out that the sources of this phenomena are institutional factors in the labour market (see Alesina, Glaeser, and Sacerdote 2005). Finally, we can find "cultural" explanation - suggesting that Europeans work less, because it matches better their aspiration and norms of behaviour. This paper is referring to the current of thought, represented by the conception of the social multiplier (Glaeser, Sacerdote, and Scheinkman 2002). This idea is based on the observation (e.g. Sobel 2005) that in the world of social interactions the individual tendency to behave in certain way is strengthened, when others behave alike. The authors call this phenomenon – social multiplier. There is a close analogy between the social multiplier and the model of the behaviour of agents involved in the strategic game presented in our article. Glaeser's paper however does not concentrate on the behaviours of households which influence the labour supply; it is more general in the character. The present paper is more specific in this particular area (independently of the different language and approach to modeling). Allowing the existence of possible multiple equillibria of market system means, that people (even equipped with the same utility functions) can choose both long working time (i.e. high consumption of goods and the short relaxation) if this is the way the others behave, as well as short working time (i.e. the low consumption of goods and the long leisure time) if such social pattern is typical. Under such condition it is plausible to talk about the importance of inertia of the system to tax changes. We can believe that culture norms are responsible for such community conduct. It means that the tax rates leveling will not necessarily be enough to get the analogous result in the US and Western Europe. The text shows that in order to break the existing inertia the system can require more fundamental tax changes, i.e. such changes, which would raze the multiple equilibria and substitute them with one state being characterized by a high level of the working time. Finally the discussed inertia can also explain discrepancies in labour supply elasticities between countries as well as between the micro and macro level (Alesina, Glaeser, and Sacerdote 2005).

The rest of the paper is organized in the following way: chapter 2 introduces the basic model of single household's choice of the labour supply (Becker 1965); it points out necessary and sufficient conditions for the increase (decrease) of labour supply as the consequence of the reduction (rise) of a tax rate. In chapter 3 we extend the preferences of households including the relative consumption in order to examine how the analyzed situation changes after taking the relative consumption into consideration; we show appropriate conditions, for the increase (drop) of labour supply with the cuts (increases) in tax rates. Chapter 3.1 presents conclusions concerning comparative statics. In chapter 3.2 we examined the dynamics of the labour supply of given group of households resulting from the changes in the tax rate. The paper ends with conclusions (chapter 4) and proofs of the theorems (chapter 5).

2 One household economy

In this section we examine the household's reaction on changes in the income tax rate. To reach that aim we present the necessary and sufficient conditions for the optimal choice of labour supply to be a monotonic function of the income tax rate.

Consider a single household with its preferences represented by a realvalued utility function: U(c, r) where $c \in \mathbb{R}_+ \cup \{0\}$ represents consumption level and $r \in [0, 1]$ time spend on leisure². The rest of the time l = 1 - r is spend on work. Household faces the budget constraint: $w(1-r)(1-\theta)+t = c$, where $w \in \mathbb{R}_+$ represents the wage level, $t \in \mathbb{R}_+ \cup \{0\}$ other income while parameter $\theta \in [0, 1) = \Theta$ stands for the income tax³. We assume that the preferences do not change during the analysis period and w, t, θ are exogenous parameters. We assume also that U is a increasing function of r and c.

Households solves the following problem:

$$\max_{c,r} U(c,r), u.c. w(1-r)(1-\theta) + t = c.$$
(2.1)

Using the budget constraint we get:

$$U(c,r) = U(w(1-r)(1-\theta) + t, r) \equiv F(r,\theta),$$

where for simplicity we have reformulated the utility function to: $F(r, \theta) \equiv U(w(1-r)(1-\theta)+t, r)$. Now the optimization problem (see (2.1)) takes the form of:

$$\max_{r \in [0,1]} F(r,\theta), \tag{2.2}$$

where θ is a parameter. The fall of θ (decrease in the tax rate) may result in increase as well as decrease in the optimal leisure choice (decrease as well as increase in labour supply). These two cases are presented in graph 1. Below we show the necessary and sufficient conditions for the optimal labour supply to be a monotonic function of the tax rate.

Let the R^*_{θ} (with elements $r^*(\theta)$) denote the set of solutions of the maximization problem (2.2) for the parameter θ . Put differently: $R^*_{\theta} = \{r : r = \arg \max_r F(r,\theta), \theta \in \Theta^*\}$, where $\Theta^* = \{\theta \in \Theta : R^*_{\theta} \neq \emptyset\}$. The following theorem gives the necessary and sufficient conditions for the set R^*_{θ} to be ascending⁴ on Θ^* .

²For simplicity we normalize the time available to 1, where r denotes the proportion of time spend on leisure.

³We skip the case of $\theta = 1$ as trivial.

 $^{^{4}}$ Following e.g. Topkis (1998) we use terms: ascending, descending, increasing and decreasing in their weak sense.

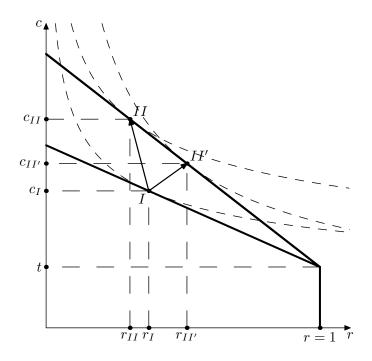


Figure 1: On the fall of θ household may react with increase $(I \to II)$ or decrease $(I \to II')$ labour supply.

Theorem 2.1 Function $F(r, \theta)$ satisfy the single crossing property (SCP) *i.e.* $\forall \theta < \theta', r < r'$:

$$F(r,\theta) \le F(r',\theta) \Rightarrow F(r,\theta') \le F(r',\theta'),$$
(2.3)

$$F(r,\theta) < F(r',\theta) \Rightarrow F(r,\theta') < F(r',\theta'), \tag{2.4}$$

iff. the set of R_{θ}^* is ascending on Θ^* .

Single crossing property means that: if for some θ it is (weakly) profitable to increase leisure from r to r' then it should as well hold for the higher value of the parameter: θ' . From theorem 2.1 we know that SCP is sufficient and necessary condition for the set of R_{θ}^* to be ascending on Θ^* .

Sufficient and necessary condition for the set of maximal solutions (R^*_{θ}) to be descending shows the following theorem:

Theorem 2.2 Function $F(r, \theta)$ satisfies the reverse single crossing property (RSCP) i.e. $\forall \theta < \theta', r < r'$:

$$F(r',\theta) \le F(r,\theta) \Rightarrow F(r',\theta') \le F(r,\theta'), \tag{2.5}$$

$$F(r',\theta) < F(r,\theta) \Rightarrow F(r',\theta') < F(r,\theta'),$$
(2.6)

iff. the set R^*_{θ} is descending on Θ^* .

Reverse single crossing property means that if for some θ it is (weakly) profitable to decrease the leisure choice from r' to r then it should as well hold for the higher value of the parameter θ' . From the theorem 2.2 we get that it is necessary and sufficient condition for set R^*_{θ} to be descending on Θ^* .

Let's stress that theorems 2.1 and 2.2 do not require the function F to be concave neither the maximal solution to be non corner one.

In the case of multiple solutions of the maximization problem (2.2) theorems 2.1 and 2.2 say only that single crossing property (RSCP respectively) is necessary and sufficient condition for the set of maximizers R_{θ}^* to be ascending (descending), i.e. the maximal and minimal selections $r^*(\theta)$ to be increasing (decreasing) with θ . These theorems do not say, however, anything on the monotonocity of the "middle" solutions. There are two ways to reach stronger results: to constraint the set of solutions by adding the convexity assumption of F (see theorem 2.3) or to add stronger assumptions on the reaction of F on changes in θ and r (see theorem 2.4).

Theorem 2.3 If the function $F(r, \theta)$ is strictly concave with respect to r on [0, 1] then the problem (2.2) has only one global maximum.

The concavity assumption of $F(r, \theta)$ within the \mathscr{C}^2 class is satisfied if $U_{cc} < 0, U_{rr} < 0$ and $U_{cr} > 0$. However outside this class these are not the necessary conditions.

Joining together theorems 2.1 (or 2.2) and 2.3 we get conditions for the only solution of (2.2) to be increasing (or decreasing) with θ .

Theorem 2.4 gives conditions for the every selection of the optimal labour supply to be an increasing function of the tax rate without the concavity assumptions of F i.e. allowing for multiple solutions of 2.2.

Theorem 2.4 Consider the utility function U of class \mathscr{C}^2 . If the following condition holds:

$$\frac{\partial U(c,r)}{\partial c} + (1-r)\left(w(1-\theta)\frac{\partial^2 U}{\partial c^2} - \frac{\partial^2 U}{\partial c\partial r}\right) > 0$$
(2.7)

then every $r^*(\theta)$ is strictly increasing on Θ^* . If we substitute > with < in inequality (2.7) then every $r^*(\theta)$ will be strictly decreasing on Θ^* .

Theorem 2.4 says that if the inequality (2.7) is satisfied growth of θ would result in the increase of optimal choice(-es) of r i.e. the decrease of labour supply l (if the solutions of (2.2) exist).

If the inequality (2.7) is satisfied and only one solution of (2.2) exists theorem 2.4 implies, that after the growth in the tax rate labour supply will decrease. In the case of multiple solutions theorem says only that all of the optimal r^* will move monotonically with the parameter.

To sum up: for the concave F (e.g. for $U_{cc} < 0, U_{rr} < 0$ and $U_{rc} > 0$ where U is \mathscr{C}^2) there is only one solution of the optimization problem, which increases with the increase in the tax rate when the condition (2.7) is satisfied. When we substitute > with < in the inequality (2.7) labour supply will be an increasing function of the tax rate. And finally when the inequality (2.7) changes signs for $\hat{r}(\theta) \in [0, 1]$ further change in the tax rate would result with the opposite move of labour supply.

We will finish this section with the following example using the above theorems:

Example 2.1

Consider a real Cobb-Douglas utility function: $U(c,r) = c^{\alpha}r^{\beta}$, where $\alpha, \beta > 0$. The corresponding F function takes the form of: $F(r,\theta) = r^{\beta}(t+(1-r)(1-\theta)w)^{\alpha}$. Using the Spence-Mirrlees condition (see Milgrom and Shannon 1994) for analyzing single crossing property and theorem 2.1 we have that the set of the optimal solutions R^*_{θ} is ascending with θ . We may check this result by calculating the labour supply function: $l^*(\theta) \equiv 1 - r^*(\theta) = 1 - \frac{\alpha}{\alpha+\beta} \left(\frac{t}{w(1-\theta)} + 1\right)$ and noticing that $l^*(\theta)$ strictly decreases with θ .

We shall add the ordinal conditions given in theorem 2.4 are very general although often not easily implementable. \diamond

The next section generalizes this results for the *I*-household economy with interdependent preferences.

3 Multi household economy

In this section we analyze how the above results change when a given household cares about so called relative consumption level, i.e. we assume that each household cares not only about its own consumption level but also consumption of all the other households.

Consider $I \in \mathbb{N}$ single households, any (denoted *i*, where $i \in \{1, 2, ..., I\}$) represented by the utility function: $U_i(c_i, r_i, c_{-i})$, where $c_i \in \mathbb{R}_+ \cup \{0\}, r_i \in [0, 1]$ denote consumption level and leisure time of household *i* while $c_{-i} = [c_1, c_2, ..., c_{i-1}, c_{i+1}, ..., c_I]$ vector of the consumption levels of all the other households. Each household faces the budget constraint given by: $w_i(1 - r_i)(1 - \theta) + t_i = c_i$. The income tax rate is the same for all agents: $\forall_i \theta_i = \theta \in [0, 1) = \Theta$. We assume also that: $l_i = 1 - r_i$ and preferences do not change during the period of our analysis. We treat w_i, t_i, θ as exogenous parameters. Each household *i* treats c_{-i} as given and solves:

$$\max_{c_i, r_i} U(c_i, r_i, c_{-i}),
u.c. w_i (1 - r_i) (1 - \theta) + t_i = c_i.$$
(3.8)

Using the budget constraint we reformulate the utility function:

$$F_i(r_i, r_{-i}, \theta) \equiv U_i((1-\theta)(1-r_i)w_i + t_i, r_i, \forall j \neq i (1-\theta)(1-r_j)w_j + t_j),$$

where $r_{-i} = [r_1, r_2, \ldots, r_{i-1}, r_{i+1}, \ldots, r_I]$ denotes the vector of time devoted to relax by households other then *i*.

Reformulating the analyzed model under game theoretic approach we get a game Γ between I players with strategies $(r_i, r_{-i}) = [0, 1]^I$ and the payoff function F_i for each i. The solution of (3.8) for any agent corresponds to Nash equilibria of Γ .

We will analyze this game in a static and dynamic framework to find out how changes in the tax rate influence the behaviour of the economy under study.

3.1 Static setup

Similarly to one household economy we will find the conditions under which the set of optimal solutions (Nash equilibria of Γ) is monotonic with respect to θ .

Theorem 3.1 (Equilibria under complementarities) If for each *i* the following holds:

- (a) for any r_{-i} and θ function F_i is upper semicontinuous⁵ with respect to r_i on [0, 1],
- (b) for any θ function F_i has increasing differences⁶ on r_i and r_{-i} ,

then:

- (i) the set of Nash equilibria of Γ in pure strategies is a nonempty complete lattice⁷ and has its greatest and least element reached by iterated deletion of strictly dominated strategies,
- (ii) if in addition for any i and given r_{-i} function F has increasing (decreasing) differences⁸ on r_i and θ then the greatest and least Nash equilibrium in pure strategies is increasing (decreasing) with θ ,
- (iii) if in addition in (b) we assume strictly increasing differences, $\forall i t_i = t, w_i = w$ and that the payoff functions are the same for each player (game is symmetric), then the game do not have any asymmetric Nash equilibrium.

Condition (a) allows for upward jumps in F only. Any continues function satisfies this condition. Point (b) says that the payoff function should have

⁵I.e. for any sequence r_i^n : $\limsup_{r_i^n \downarrow \inf(r_i^n)} F(r_i^n, r_{-i}, \theta) \leq F(\inf(r_i^n), r_{-i}, \theta)$ and $\limsup_{r_i^n \uparrow \sup(r_i^n)} F(r_i^n, r_{-i}, \theta) \leq F(\sup(r_i^n), r_{-i}, \theta).$

⁶I.e. $\forall r_i < r'_i \ i \ \forall r_{-i} < r'_{-i} \ F(r_i, r_{-i}, \theta) - F(r'_i, r_{-i}, \theta) \le F(r_i, r'_{-i}, \theta) - F(r'_i, r'_{-i}, \theta).$

⁷Partially ordered set X is a lattice iff. $\forall x, y \in X \quad x \lor y \in X \ni x \land y$. Lattice X is complete if for each nonempty $T \subset X$: $\inf_X(T) \in X$ and $\sup_X(T) \in X$.

⁸I.e. $\forall r_i < r'_i \text{ and } \forall \theta < \theta' F(r_i, r_{-i}, \theta) - F(r'_i, r_{-i}, \theta) \le F(r_i, r_{-i}, \theta') - F(r'_i, r_{-i}, \theta').$

increasing differences on r_i and r_{-i} , i.e. strategies r_i and r_{-i} are complementary. (Strict) complementarity means that the marginal utility of i must be increasing (strictly) with respect to the other players' strategies.

(Strict) increasing differences in (b) under the differentiable U takes the form of:

$$\forall i \neq j \quad (1-\theta)w_i \frac{\partial^2 U_i}{\partial c_i \partial c_j} - \frac{\partial^2 U_i}{\partial r_i \partial c_j} (>) \ge 0, \tag{3.9}$$

while (strictly) increasing differences condition in (ii) is equivalent to⁹:

$$\forall i \forall r_{-i} \quad \frac{\partial^2 F}{\partial r_i \partial \theta} \ge (\le)0. \tag{3.10}$$

Let's begin from interpretation of the condition set on differences of F_i with respect to r_i and r_{-i} , i.e. if U_i is twice differentiable:

$$\forall i \forall r_{-i} \forall j \neq i \quad (1-\theta)^2 w_j w_i \frac{\partial^2 U_i}{\partial c_i \partial c_j} - (1-\theta) w_j \frac{\partial^2 U_i}{\partial r_i \partial c_j}.$$
 (3.11)

This term describes the change in household's marginal utility $\left(\frac{\partial U_i}{\partial r_i} - (1 - \frac{\partial U_i}{\partial r_i})\right)$ $\theta w_i \frac{\partial U_i}{\partial c_i}$) resulting from the increase of leisure time choices of all the other households. Put differently increasing differences of F_i between (r_i, r_{-i}) (compare with (3.11)) says how the net increase in the utility function from reallocating the unit of labour to leisure time (and loosing this way some consumption) changes with resect to the choices of leisure time of the other households. Under twice differential U_i the direction of *i*-household reaction to changes in the decision of the others (i.e. expression (3.11)) is determined by the sign of the expression:

$$(1-\theta)w_i\frac{\partial^2 U_i}{\partial c_i\partial c_j} - \frac{\partial^2 U_i}{\partial r_i\partial c_j}.$$
(3.12)

When the above term is positive *i*-household is more willing to devote more time for leisure (consume less) when the others increase their leisure choices (decrease consumption levels). We say that strategies of i and -i are complementary. And the opposite under decreasing returns (when expression (3.12)) is negative) we say about substitutive strategies of i and -i.

The assumption of complementarities (in theorem 3.1) can be explained the following way. Taking into account so called relative consumption level

⁹This is rather strong condition because θ is not only present in c_i but also in c_{-i} . This assumption can be, however, reformulated using the corresponding game Γ' with payoffs: $F'_i(c_i, c_{-i}, \theta) = U_i(c_i, 1 - \frac{c_i - t_i}{w_i(1-\theta)}, c_{-i})$. Theorem 3.1 holds then for (a) upper semicontinuous U_i with (b) increasing differences on c_i and c_{-i} (all the same in Γ and Γ'). Condition (*ii*) takes a simpler form of: $\forall i \forall r_{-i} \quad \frac{\partial^2 U_i}{\partial c_i \partial \theta} \leq (\geq)0$ (compare with inequality (2.7)). Simpler conditions on U to reach the results in theorem 3.1 result from the fact that

theorem gives sufficient but not necessary conditions for the comparative statics results.

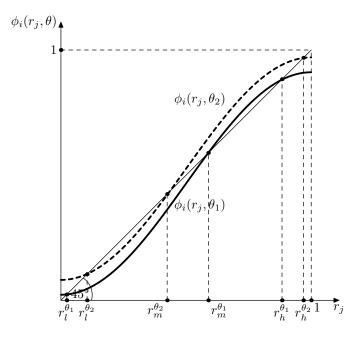


Figure 2: The set of Nash equilibria of Γ is ascending under complementarity.

the increase of -i households consumption choices will not improve the *i* relative situation. This way such a change would increase *i* inclination to increase its consumption as well.

Point (i) in theorem 3.1 guarantees existence of at least one Nash equilibrium of Γ (i.e. equilibrium of the analyzed economy) and describes the structure of the equilibrium set. Additionally when the assumption in (ii) is satisfied (in differential case inequality (3.10 holds) with¹⁰ >) theorem says how the economy reacts to changes in the tax rate θ : each household's inclination to enlarge leisure time will raise and this will incline (through complementarity) the others to raise their leisure choices as well. The set of Nash equilibria of Γ will ascend, i.e. the greatest and least Nash equilibrium will growth with θ . It cannot be implied however that the middle equilibria will increase as well.

Figure 2 shows household *i* best response correspondence to r_j for a symmetric game Γ . The fact that the correspondence in increasing shows that complementarity assumption is satisfied. And the fact that this correspondence moved up after the change from θ_1 to θ_2 shows that *F* has increasing differences on (r_i, θ) . The analyzed economy has 3 symmetric equilibria (and has no asymmetric ones). Growth in parameter θ results in the increase of the greatest and least equilibrium. However the middle equilibrium declines.

¹⁰Compare with (2.7).

Note that at this stage we cannot surely say that after the growth of the tax rate (under complementarity and increasing differences on (r, θ) the labour supply will decline because middle equilibria of unknown monotonicity may exist. In addition when a few equilibria exist economy may jump from the chosen equilibrium to some other after the change in θ ; as a result households may change the equilibrium from one (e.g. with higher leisure and lower labour supply) to another (e.g with lower leisure and higher labour supply) even if all equilibria increase after the change in θ . Conditions excluding such cases will be presented in section 3.2.

Before that we will analyze the case of substitutive strategies.

Theorem 3.2 (Equilibria under substitutive strategies) If for each i the following holds:

- (a) for any r_{-i} and θ function F_i is continuous with respect to r_i in [0, 1]and
- (b) for any θ function F_i has decreasing differences on r_i and r_{-i} ,

then:

- (i) the game has at least one Nash equilibrium,
- (ii) if additionally the game is symmetric, for each i and any given r_{-i} function F_i has increasing (decreasing) differences on r_i and θ and we assume strictly decreasing differences in (b) then Γ has only one symmetric Nash equilibrium. This equilibrium is increasing (decreasing) with θ .

Under twice differentiable U_i decreasing differences assumption can be checked using the following inequality:

$$(1-\theta)w\frac{\partial^2 U}{\partial c_i \partial c_j} - \frac{\partial^2 U}{\partial r_i \partial c_j} \le 0.$$
(3.13)

Substitutive strategies mean that the increase in leisure time (decrease in consumption level) by all -i households results in household *i* higher inclination to decrease its leisure time choice (higher consumption). This assumption seems rather awkward because household tend to imitate the others behaviour (Zizzo 2003). Nevertheless it can be justified by so called self awarding character of the alternative when households gain satisfaction behaving differently to the others.

Point (i) in theorem 3.2 guarantees existence of at least one Nash equilibrium. Point (ii) says that even under substitutive strategies symmetric game will have one symmetric equilibrium, which for increasing (decreasing) differences with respect to r and θ will be increasing (decreasing) with θ . Under

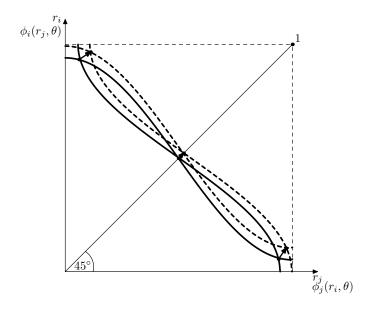


Figure 3: Under substitutive strategies asymmetric equilibria may be present in a symmetric game. In such a game only one symmetric equilibrium exists, monotonic with θ .

substitutive strategies there is a possibility of multiple asymmetric equilibria to occur even in symmetric games (which was impossible under strict complementarities). This result should not be surprising, however, because of substitutive strategies themselves. More interestingly despite substitutive strategies monotonic symmetric equilibrium exists. That is: under increasing differences with respect to r and θ increase in the tax rate may result in a decrease in labour supply of all the households; substitution effect of a change in labour supply will be outweighed by the complementary effect between change in the tax rate and leisure choice). Figure 3 exemplifies this results. In addition, it shows that condition in theorem 3.1 are not necessary for a growth of r^* with θ .

Figure 4 shows that in a game with substitutive strategies growth in θ may result in a increase of labour supply $(I \rightarrow III)$ of all the households. It is possible, however, that some part of households react with increase and some with decrease (when substitution effect outweigh the complementary effect) in labour supply $(I \rightarrow II)$. Sum of this reactions both may result in a growth or decline in the aggregated labour supply.

Comparing one with multi household economy we may draw some interesting conclusions. Condition (2.7) satisfied for each player is no longer sufficient for a growth in labour supply after a decline in the tax rate. Even complementarities condition does not guarantee this result. The choice of the greatest or least equilibrium and guarantee that the economy will not

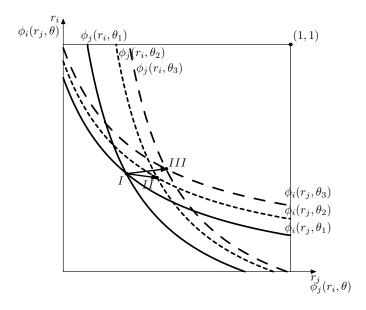


Figure 4: Under substitutive strategies households may all decrease their labour supply $(I \rightarrow III)$ after growth of θ as well as some of them may increase while some may decrease their labour supply $(I \rightarrow II)$.

jump from higher to the lower equilibrium are necessary for the result to hold. Finally we may say that growth in labour supply after increase in the tax rate is more likely in a multi household economy.

Asymmetric equilibria in symmetric games are possible only under substitutive strategies. Asymmetric equilibria for a homogenous players mean that each of identical players taking into account the others behaviour may react differently to the change in θ . The aggregated effect of this changes is ambiguous.

For a heterogenous agents the fall of labour supply after growth in the tax rate is certain only for greatest or least equilibria under complementarities, increasing differences between r and θ and relevant dynamic of the economy.

We shall end this section with one more theorem concerning the set of equilibria of Γ when the number of players changes.

Theorem 3.3 Consider game Γ with N players with strategies $r_n \in [0,1]$ and payoffs $F_n(r_n, r_{-n}, \theta)$ defined earlier. If for each $n \in N$, any θ and r_{-n} the following holds:

- F_n is upper semicontinuous,
- F_n has increasing differences on r_n and r_{-n} ,

then for any $i \in I$ where $\emptyset \neq I \subset N$ its strategies in greatest and least equilibrium of Γ increase with I.

Increase in the number of players under complementary strategies will result in the growth of the greatest and least equilibrium. Example 3.2

Consider economy composed of I households, each represented by utility function: $U_i(c_i, r_i, c_{-i}) = c_i^{\alpha} r_i^{\beta} \left(\frac{\sum_{j \neq i} c_j}{l-1}\right)^{\gamma}$, where $\alpha, \beta > 0$. Because monotonic transformation of U will not changer qualitative results we will create a new function: $\tilde{U}_i(c_i, r_i, c_{-i}) = \ln U_i(c_i, r_i, c_{-i})$. Using remarks from the footnote 9 increasing differences of \tilde{U}_i (on c_i, c_{-i}) condition is always satisfied. In addition \tilde{U}_i has strictly increasing differences on $(c_i, -\theta)$. As a result the set of Nash equilibria of the economy is nonempty complete lattice with greatest and least equilibrium, that (r_i) increase with θ . For identical players our economy does not have asymmetric equilibria. Notice that the results are independent of the sign of γ , i.e. positive of negative influence of the relative consumption on a players i utility.

To specify: the economy has only one Nash equilibrium because best response correspondences are constant. \diamond

3.2 Dynamic setup

The results (reached in the previous section) concerning comparative statics exercise under complementary and substitutive strategies does not rule out the possibility of multiple equilibria to occur. Under multiplicity we have shown the possibility of equilibria moving in the opposite direction to the changes in parameter's value. In addition we have not ruled out the possibility that the economy jumps out from the chosen equilibrium after the change in parameter.

This section defines the dynamics of the economy in order to analyze the stability of equilibria in Γ . It will allow us to eliminate the cases described in the previous paragraph.

We shall start from defining (see Echenique 2002) dynamics of the economy and as well its stability:

Definition 3.1 Let $\phi : X \to X$ be a correspondence, where X is a lattice. A sequence $\{x_k\}$ in X is called generalized adaptive dynamics from ϕ if there is some $\gamma \in \mathbb{N}$ such that $x_k \in [\phi(\inf H_k^{\gamma}), \bar{\phi}(\sup H_k^{\gamma})]$ for each $k \geq 1$, where H_k^{γ} denotes the history of length γ at time k: $H = \{x_{k-\gamma}, \ldots, x_{k-1}\}$ and $\phi = \inf(\phi(x)), \ \bar{\phi} = \sup(\phi(x))$ for any x. Let $\mathcal{D}(x_0, \phi)$ be the set of all sequences that are generalized adaptive dynamics from ϕ starting at x_0 .

Generalized adaptive dynamics from ϕ rule out moves, that cannot be "justified" be a response to some history. It does not have to be, however, the best response to a previous move neither the moves should equally exactly the best response for a given history. An important case of generalized adaptive dynamics is the set of simple adaptive dynamics:

$$\mathcal{A}(x_0,\phi) = \{\{x_k\}_{k=0}^{\infty} : x_k \in \phi(x_{k-1}), k \ge 1\},\$$

a la tâtonnement.

Definition 3.2 Let $\phi: X \to X$ be a correspondence. A point $\hat{x} \in X$ is best case stable if there is a neighborhood O of \hat{x} in X such that for all $x \in O$ there is a sequence $\{x_k\} \in \mathcal{D}(x_0, \phi)$ with $x_k \to \hat{x}$. A point $\hat{x} \in X$ is worst case stable if there is a neighborhood O of \hat{x} in X, such that for all $x \in O$ and all sequences $\{x_k\} \in \mathcal{D}(x_0, \phi)$, $x_k \to \hat{x}$.

Using the above definitions and denoting $\varepsilon(\theta)$ the set of equilibria of Γ for θ we get:

Theorem 3.4 (On stability of equilibria) Consider game Γ such that for each player its payoff function F_i is upper semicontinuous with respect to r_i for a given θ and r_{-i} , has increasing differences on (r_i, r_{-i}) for each θ and strictly increasing differences on (r_i, θ) for any r_{-i} . If

- (a) ϵ is a continuous correspondence Θ on $\{\varepsilon(\theta) : \theta \in \Theta\}$ and
- (b) ϵ is nowhere increasing $[\underline{\theta}, \overline{\theta}] \subset \Theta$

then for any $\theta \in [\underline{\theta}, \overline{\theta}] \epsilon(\theta)$ is not best case stable. If we substitute point (b) with:

(b') ϵ is strictly increasing on $[\underline{\theta}, \overline{\theta}] \subset \Theta$ and $\epsilon(\theta)$ is isolated¹¹ in θ

then $\epsilon(\theta)$ is worst case stable.

Theorem says that for a game with complementary strategies and strictly increasing differences on r_i and θ and continuous moves of equilibrium: if after the increase in parameter equilibrium increased (decreased) this equilibrium is stable (unstable). Because of the fact that for continuous strategy spaces probability that the economy will end up in a unstable equilibrium is zero, growth of labour supply after growth in the tax rate (moving down with the middle equilibrium) is no longer possible. Therefore any unstable equilibrium is not a point of rest but rather the point that divides basins of attraction between two stable equilibria.

Theorem 3.5 (see Echenique 2002) describes how the economy will behave when shifted "upwards" (e.g. with the growth of θ):

Theorem 3.5 Let $\phi : X \to X$, where $X = [0,1]^I$ be a best response correspondence under complementarity. If $X \ni x \preceq \inf \phi(x)$ then $F(x,\phi) = \{z \in X : \exists \{x_k\} \in \mathcal{D}(x,\phi), z = \lim_k x_k\}$ is nonempty with the least element: $\inf F(x,\phi) = \inf \{z \in \varepsilon : x \preceq z\}$ oraz $\inf F(x,\phi) \in \varepsilon$.

¹¹I.e. there is a neighborhood O of $\epsilon(\theta)$ that $O \cap \epsilon(\theta) = {\epsilon(\theta)}$, i.e. it is the only one equilibrium in this neighborhood.

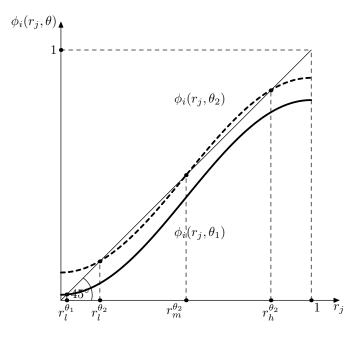


Figure 5: With the growth in θ two new equilibria emerge. Their stability cannot be implied from the changes in the tax rate between θ_1 and θ_2 . We may imply only that the equilibrium r_l is stable.

Put differently when the economy is in some point x but lower than the best response correspondence value at this point it will converge to an equilibrium that lays higher than x. Therefore when the economy is in equilibrium growth in θ cannot result in the fall of r. To interpret: under complementarities and increasing differences on (r_i, θ) growth in the tax rate cannot result in the growth in labour supply¹². It is because of the fact that basin of attraction of the greatest equilibrium is enlarging (unstable equilibrium is moving down). It means that whatever equilibrium (stable or unstable) the economy is in before the change in the tax rate the labour supply will not growth when the tax rate increases.

In figure 2 the middle equilibrium is unstable. It divides the basins of attraction between greatest and least equilibrium. If the economy will fall into one of these basins it will converge to its equilibrium.

In theorem 3.4 the condition that $\epsilon(\theta)$ is isolated in some neighborhood is to exclude the case of *continuum* of equilibria. It this case we cannot imply how the economy will behave. The condition that equilibrium selection is continuous with θ is designed to exclude equilibria that emerge or disappear after the change in θ (compare with the case on figure 5). Note that this

¹²We have excluded this way the possibility of downward jumps between equilibria when the parameter growths.

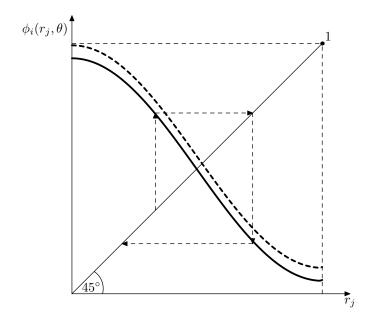


Figure 6: Under symmetric substitutive strategies despite increase of a symmetric equilibrium with θ it is unstable for *tâtonnement* dynamics.

theorem is no longer necessary for the results under substitutive strategies. Counterexample is shown in the figure 6.

The last example shows the possible consequences of multiplicity of equilibria and their monotonicity with the tax rate. Example 3.3

Consider the utility function: $U(c_i, r_i, c_{-i}) = \left(\lambda c_i - (1 - \lambda) \left(\frac{\sum_{j \neq i} c_j}{I - 1}\right)^2\right)^{\alpha} r_i^{\beta}$. Suppose that: $\alpha = 1/3$, $\beta = 2/3$, $\lambda = 9/10$, $w_i = 10$ i $t_i = 1$. For simplicity and to enable graphical representation we assume that I = 2. For this given values of parameters we face complementary strategies. In addition U has increasing differences on $(c_i, -\theta)$ (see footnote 9).

Suppose that we start under the tax rate of 13%. Than our economy has 3 symmetric equilibria with the following consumption levels 5.4, 8.1 i 9.7 (and corresponding labour supply l_i : 0.5, 0.8, 1). After the growth of the tax rate up to 17% all equilibria move down to: 4.8, 8.7 and 9.3 (corresponding labour supplies move down to l_i : 0.46, 0.9, 1). It means that that the set of Nash equilibria has descended while the set of equilibrium r_i ascended (see figure 7). The middle equilibrium (II and II') is unstable for any generalized adaptive dynamics.

For tax rates of 13%, 17% and all in between the state of our economy is undetermined: we cannot *a priori* say which equilibrium the economy is in.

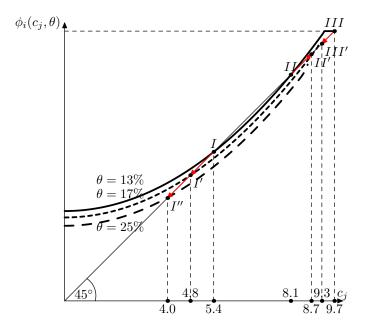


Figure 7: Best response function (of consumption i) for a given consumption level of -i. For $\theta = 13\%$ (and $\theta = 17\%$) the game has three symmetric Nash equilibria: I,II i III (I',II' i III'). For $\theta = 25\%$ there is only one equilibrium: I''.

For the tax rate of 25% there is only one symmetric equilibrium with consumption level of 4 (corresponding l_i : 0.4).

The results reached in the example 3.2 may be interpreted the following way: suppose that (when $\theta = 13\%$) our economy is in equilibrium *III*. After the growth in the tax rate (up to $\theta = 17\%$) households will reduce (not increase) their labour supply (see theorem 3.5) either by moving with equilibrium (from *III* to *III'*) or jumping to equilibrium *I'*. Assuming however that the dynamics of our system is a la tâtonnement, our economy will rest in equilibrium *III'*. After the next growth in the tax rate (to 25%) the economy will end up in *I''* whatever the earlier equilibria. Consequently labour supply will fall down from 1 to 0.4.

Suppose now that we would like to come back the high labour supply state: lowering the tax rate to 17% (or even 13%) may not be sufficient. The economy may got stuck in equilibrium I'(I) and for changes in the tax rate react with only small changes in labour supply (from 0.4 through 0.46 to 0.5). Only lowering of the tax rate down to 8% enforces (there is only one equilibrium) come back to the high labour supply state.

Let us stress that this scenario is specific: after the decrease in the tax rate (e.g. from 25% to 17%) our economy may jump to equilibrium III' at once. In addition the chosen utility function and parameters value to reach multiple equilibria seems to be rather rare e.g. change in λ for some lower value result in only one equilibrium.

4 Conclusion

The analysis of the reaction of the labour supply changes to fiscal charges used to be dominated by the belief that each household establishes its strategy both separately and independently of other agent choices. This assumption permitted simple adding up of individual households' reactions but it was not realistic for obvious reasons. Considering these facts, we examined what kind of new results could be obtained assuming that households establish their behaviour strategies with respect to behaviours of other households belonging to the same reference group. We call this situation interdependence of preferences. We model it by including relative consumption into household's preferences (e.g. relation of own consumption to the consumption in the reference group). Using lattice theory and the idea of supermodular games we obtained results based on very general assumptions concerning the shape of households utility functions. The achieved results can be encapsulated in the following four points.

First, including household's relative consumption in the utility function does not fundamentally change previous general conclusions concerning the negative impact of the increase of income tax rate on the size of the labour supply on the macroeconomic level. Second, the interdependence of preferences affects strategies of households in the way that they become either substitutive or complementary in relation to other agents' behaviour. We believe that the case of strategic complementarity seems to be typical in real world; at the same time it has potentially very interesting consequences – as far as it leads to multiple equilibria. The existence of multiple equilibria implies that the economy can be characterized either by high labour supply (households choose short leisure), or with low labour supply (households prefer long leisure). Both these situations are possible; the empirical state is a consequence of some exogenous factors or is due to certain path of historical development.

Third, the existence of multiple equilibria may imply that the reaction of the economy to tax rate changes could be relatively weak. Speaking precisely: in spite of mechanism strengthening the subjects reaction to external shocks (characteristic to the strategic complementarity), changes in labour supply, which follow the changes in the tax rate, can be limited to little shifts in local equilibria. It can be conditioned by the mechanism of "anchoring" decisions of one household in the scope for activity of other subjects. The strong inertia of the system in response to external shocks can certainly be interpreted as a result of cultural conditioning. We do not feel competent to discuss the legitimacy of this point of view in a detailed way. In our opinion, the inertia of the economy is a result of certain historical factors. These factors created expectations concerning behaviour of the majority and in this way, they determine strategies of individual households, to a large extent. The mechanism of hysteresis, well known in a different context of dynamic analysis of the labour market, can be easily recognized here.

Fourth, from practical point of view this reasoning can deliver important explanations of existing huge differences between countries in the size of the labour supply as well as in the elasticity of labour supply with respect to taxes (or to wages). The model, which includes the complementary character of households' decisions, shows that two identical economies remaining in different equilibrium states and characterized by different tax rate will not behave identically after leveling the tax rate. Due to mechanism of hysteresis the economy needs a very strong exogenous shock (in this case change in taxes) to jump to the new equilibrium state. In order to eliminate the multiple equilibria and obtain one attracting point the tax impulse must exceed a critical level of changes.

5 Proofs

Proof (Theorem 2.1)

This theorem follows directly from theorem 4 in (Milgrom and Shannon 1994). $\hfill \square$

Proof (Theorem 2.2)

RSCP for function F is equivalent to the SCP property for $G(l, \theta) \equiv F(-r, \theta)$. The rest follows directly from theorem 4 in (Milgrom and Shannon 1994). **Proof** (Theorem 2.3)

Compare with the conditions necessary for minimization of the concave function.

Proof (Theorem 2.4)

$$\frac{\partial F(r,\theta)}{\partial r} = \frac{\partial U(w(1-r)(1-\theta) + t, r)}{\partial r} = -w(1-\theta)\frac{\partial U(c,r)}{\partial c} + \frac{\partial U(c,r)}{\partial r},$$

and:

$$\frac{\partial^2 F(r,\theta)}{\partial r \partial \theta} = w \frac{\partial U(c,r)}{\partial c} + w^2 (1-\theta)(1-r) \frac{\partial^2 U}{\partial c^2} - w(1-r) \frac{\partial U}{\partial c \partial r}.$$

Knowing additionally that $r \in [0, 1]$ and that any function with arguments in \mathbb{R} is supermodular we may use the theorems concerning comparative statics showed in (Amir 1996) and (Edlin and Shannon 1998).

Proof (Theorem 3.1, equilibria under complementarities) We will show that this game is supermodular:

$$\frac{\partial F_i(r_1, r_2, \dots, r_I, \theta)}{\partial r_i} = \frac{\partial U_i(w_i(1-r_i)(1-\theta)+t_i, r_i, w_{-i}(1-r_{-i})(1-\theta)+t_{-i})}{\partial r_i} = -w_i(1-\theta)\frac{\partial U_i(c_i, r_i, c_{-i})}{\partial c_i} + \frac{\partial U_i(c_i, r_i, c_{-i})}{\partial r_i},$$

and for $j \neq i$

$$\frac{\partial^2 F_i(r_1, r_2, \dots, r_I, \theta)}{\partial r_i \partial r_j} = (1 - \theta) w_j \left((1 - \theta) w_i \frac{\partial^2 U_i(c_i, r_i, c_{-i})}{\partial c_i \partial c_j} - \frac{\partial U_i(c_i, r_i, c_{-i})}{\partial r_i \partial c_j} \right).$$

For $(1-\theta)w_i\frac{\partial^2 U}{\partial c_i\partial c_j} - \frac{\partial^2 U}{\partial r_i\partial c_j} > 0$ Γ is a supermodular game. Additionally:

$$\frac{\frac{\partial^2 F_i(r_1, r_2, \dots, r_I, \theta)}{\partial r_i \partial \theta}}{\frac{\partial U_i}{\partial c_i} = w_i \frac{\partial U_i}{\partial c_i} + (1 - r_j) w_j \left((1 - \theta) w_i \frac{\frac{\partial^2 U_i(c_i, r_i, c_{-i})}{\partial c_i \partial c_j}}{\frac{\partial U_i(c_i, r_i, c_{-i})}{\partial r_i \partial c_j}} \right) + (1 - r_i) w_i \left((1 - \theta) w_i \frac{\frac{\partial^2 U_i(c_i, r_i, c_{-i})}{\partial c_i^2}}{\frac{\partial C_i^2}{\partial c_i^2}} - \frac{\frac{\partial U_i(c_i, r_i, c_{-i})}{\partial r_i \partial c_i}}{\frac{\partial r_i \partial c_i}{\partial c_i}} \right),$$

i.e. for condition (2.7) the payoff function exhibits increasing differences on (r_i, θ) and given r_{-i} . Then using the theorems 4.2.1 i 4.2.2 from (Topkis 1998) we get (i) and (ii).

We will show point (*iii*) (see Amir, Jakubczyk, and Knauff 2006) by contradiction. For simplicity we will skip the parameter θ in the notation. Assume that for a symmetric supermodular game the asymmetric equilibrium exists: (a_i, b_{-i}) . Knowing that the game is symmetric we get that (b_i, a_{-i}) is also the Nash equilibrium and additionally:

$$F((a_i, b_{-i})) \ge F((b_i, b_{-i})), \quad F((b_i, a_{-i})) \ge F((a_i, a_{-i})).$$

Summing these two inequalities we get:

$$F((a_i, b_{-i})) + F((b_i, a_{-i})) \ge F((a_i, a_{-i})) + F((b_i, b_{-i}))$$
(5.14)

From (i) we know that the set of the equilibria is a complete lattice. Additionally knowing that the payoff function is strictly supermodular we get:

$$F((a_i, b_{-i}) \lor (b_i, a_{-i})) + F((a_i, b_{-i}) \land (b_i, a_{-i})) =$$

= $F((a_i, a_{-i})) + F((b_i, b_{-i})) > F((b_i, a_{-i})) + F((a_i, b_{-i})).$

Comparing this inequality with (5.14) we get a contradiction.

Proof (Theorem 3.2, equilibria under substitutive strategies) We will show point (*ii*) by contradiction. For simplicity we will skip the parameter θ in the notation. Assume that there exist two symmetric Nash equilibria $(a_i, a_{-i}) < (b_i, b_{-i})$. We know that:

$$F((a_i, a_{-i})) \ge F((b_i, a_{-i})), \quad F((b_i, b_{-i})) \ge F((a_i, b_{-i})).$$

Strictly decreasing differences imply the function F to be strictly submodular. As a result we get:

$$F((a_i, b_{-i}) \lor (b_i, a_{-i})) + F((a_i, b_{-i}) \land (b_i, a_{-i})) =$$

= $F((a_i, a_{-i})) + F((b_i, b_{-i})) < F((b_i, a_{-i})) + F((a_i, b_{-i})).$

Summing this inequalities we get a contradiction.

The rest of point (ii) follows directly from the fact that the best response function decreases (increases) with θ .

Proof (Theorem 3.4, stability of equilibria)

It follows directly from theorem 4 and 6 in (Echenique 2002).

References

- ALESINA, A., E. GLAESER, AND B. SACERDOTE (2005): "Work and leisure in the U.S. and Europe: why so different?," *NBER Working Paper*, No. 11278.
- AMIR, R. (1996): "Sensitivity analysis of multisector optimal economic dynamics," Journal of Mathematical Economics, 25, 123–141.
- AMIR, R., M. JAKUBCZYK, AND M. KNAUFF (2006): "Symmetric Versus Asymmetric Equilibria In Symmetric N-Player Supermodular Games," Ph.D. thesis of M.Knauff, CORE, UCL.
- BECKER, G. (1965): "A theory of the allocation of time," *Economic Journal*, 75, 493–517.
- ECHENIQUE, F. (2002): "Comparative statics by adaptive dynamics and the correspondence principle," *Econometrica*, 70(2), 833–844.
- EDLIN, A. S., AND C. SHANNON (1998): "Strict Monotonicity in Comparative Static," *Journal of Economic Theory*, 81(1), 201–219.
- GARBICZ, M. (1997): "Model reakcji podaży pracy na zmiany płac realnych," *Ekonomista*, 2.
- GLAESER, E., B. SACERDOTE, AND J. SCHEINKMAN (2002): "The Social Multiplier," *NBER Working Paper*, No. 9153.
- MILGROM, P., AND C. SHANNON (1994): "Monotone comparative statics," *Econometrica*, 62(1), 157–180.
- PRESCOTT, E. (2004): "Why Do Americans Work So Much More Than Europeans?," *NBER Working Paper*, No. 10316.
- SOBEL, J. (2005): "Interdependent Preferences and Reciprocity," Journal of Economic Literature, 43, 392–436.
- TOPKIS, D. M. (1998): Supermodularity and complementarity, Frontiers of economic research. Princeton University Press.
- ZIZZO, D. J. (2003): "Empirical evidence on interdependent preferences: nature or nurture?," *Cambridge Journal of Economics*, 27(6), 867–880.