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Public Expenditure on Education and Skill Formation: Is There a Simple Rule to Maximize Skills?

ROSSANA PATRON & MARCEL VAILLANT

ABSTRACT *The ratio of skilled-to-unskilled labour stocks in the economy is widely acknowledged to have an important role for development. Can education policy affect the evolution of this ratio? This paper shows that it can: it also shows that the effect of education policy, for a given budget size, depends on the allocation rule across educational levels, particularly in the presence of systemic inefficiency. Using a stylized hierarchical education model, the theoretical conditions under which the allocation rule would favour the accumulation of skills are determined. The analysis has implication for policymakers in developing countries, where skill formation is much needed, because it shows that their allocation rules usually violate the maximization condition by assigning higher-than-optimal resources to higher education.*

JEL Classification: I21, I22, I28

1. Introduction

Although the links between skills and growth, and hence education and growth, are well established theoretically, mainly in endogenous growth theory (see, e.g. Romer, 1986; Lucas, 1988), the empirical evidence is weak. To explain this, several authors (e.g. Gemmel, 1996; Birdsall *et al.*, 1998; Papageorgiou, 2003) have stressed the importance of the distinction between the different stages of human capital creation for development and, therefore, the relevance of considering the internal allocation rules of the education budget.

The skill level of the population is usually measured by the average number of years of schooling of the population; as skilled and unskilled workers are not perfect substitutes, the structure of the stock of human capital is a crucial aspect. The skilled-to-unskilled stock ratio in developing and developed countries shows significant differences. According to data from UNESCO (2009) and OECD (2009), the proportion of the adult population with less than upper secondary as their maximum educational attainment in OECD-member countries is, on average, 30% (data for 2007), whereas for developing countries the proportion is much higher; for instance, in Brazil the rate is 63%, and it is even higher in many African countries, with rates of over 90%.

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Explanations for this gap are fairly easy to find, as in developing countries the accumulation of skills is hindered by inefficient education systems, often aggravated by lack of resources. For this reason, the structure of the system matters because performance at earlier stages affects the output at higher levels; budget allocation rules should take this into account. To analyse this point, hierarchical education models have been used by Driskill & Horowitz (2002), Driskill *et al.* (2009), Su (2004, 2006), as well as Estevan & Verheyden (2010). However, this approach is not commonly used in discussing policy issues in education.

As noted by Su (2004), a hierarchical structure in educational systems implies that levels are not perfect substitutes, which means that different allocations of similar budget size have different effects on aggregate efficiency. Following this line of research, this article discusses the allocation rules of the educational budget in a two-level education system, using a stylized model characterized by internal inefficiency. In this model, contrary to that in Su (2004), an individual's decision to become educated or to enter the labour market is not modelled; instead, the simple model here assumes a particular dropout function that easily captures the elements in an individual's decision to study. Also, unlike the works of Su (2006) and Estevan & Verheyden (2010), which deal with the political economy of budget allocation, which is endogenously determined, the focus here is on the optimal conditions for budget allocation in a steady-state economy, so the growth approach of Driskill & Horowitz (2002) and Driskill *et al.* (2009) is not followed.

Although most of the results in the related literature converge, there is a trade-off between the model complexity in much of the previous literature and the simplicity of this model: while richer models allow the discussion of a wider set of related issues such as income distribution or political stability, in this model it is possible to determine the theoretical conditions for optimal budget allocation. The model allows us to determine the analytical conditions under which the allocation rule will favour a rise in the skilled-to-unskilled ratio; this approach also allows us to identify several clear-cut policy recommendations that may serve as easy-to-follow guides to policymakers in developing countries.

This paper is organized as follows. Section 2 presents an overview of educational budget allocation and the accumulation of skills in some developed and developing countries. Section 3 presents the model and its properties. In Section 4, the conditions necessary to maximize the stock of skills are discussed. The conclusion is presented in Section 5. In addition, an appendix with mathematical details is also presented.

2. Overview

The distribution of skills across countries varies considerably, especially between developed and developing countries (see Table 1). In developing countries, the majority of the population (more than a half) has primary education or less as their maximum educational attainment, whereas in developed countries this proportion is extremely low.

In many developing countries, education systems have several weaknesses, especially in quality and coverage. In most cases, the expenditure on public education per student is far below that in the developed world, but as argued by Birdsall *et al.* (1998), Gemmel (1996) and Papageorgiou (2003), both the size and the efficiency of allocation of public funds for education are relevant for the overall performance of the system.

Some idea of actual allocation rules can be obtained by examining Tables 2–4. For instance, Table 2 shows that, in general, there are significant differences in budget allocation

Table 1. Educational attainment of the adult population (selected countries); distribution of the population aged 25 and older, by highest level of education attained (in percentages)

Country	Year	Completed primary or less	Lower secondary	Upper secondary	Tertiary education
Argentina	2004	43.8	14.2	28.4	13.6
Bangladesh	2001	73.3	9.6	12.9	4.2
Botswana	2000	75.3	15.7	5.9	3.1
Brazil	2004	57.5	13.0	21.2	8.1
Chile	2004	24.0	26.0	36.9	13.2
Mauritius*	2000	60.5	18.6	17.6	2.6
Mozambique	2000	96.9	2.3	0.8	0.1
Uganda	2002	88.5	5.1	1.6	4.8
Uruguay	2006	52.8	22.4	15.1	9.6
Australia	2005	9.1	25.8	33.3	31.5
Finland*	2006	22.0	8.9	38.8	30.3
Ireland*	2006	23.7	16.3	31.2	26.4
New Zealand	2005		21.3	51.6	27.1
Republic of Korea	2005/06	11.9	12.6	43.9	31.6
UK	2004/05		14.4	55.9	29.6
USA	2005	6.3	8.5	49.0	36.2

Notes: Latest data available. *Total may differ from 100% because of missing information or rounding. Upper secondary includes post-secondary non-tertiary.
Source: Data from UNESCO/UIS WEI (www.uis.unesco.org/publications/wei2007); UNESCO (2009).

preferences across the levels, measured by public expenditure per pupil as a percentage of GDP per capita by education level. The table shows that some countries, such as the USA, have a perfectly flat allocation pattern (all levels "equally preferred"), while Korea, for example, allocates less than average to higher education. The situation among developing

Table 2. Public expenditure per pupil as a percentage of GDP per capita by education level (selected countries)

Country	Primary	Secondary	Tertiary	All levels
Argentina	12	19	13	14
Bangladesh	9	15	46	13
Botswana	16	41	450	34
Brazil	14	12	34	15
Chile	12	13	13	13
Mauritius	12	19	37	17
Mozambique	16	69	570	23
Uganda	11	32	179	14
Uruguay	8	10	18	11
Australia	17	16	24	18
Finland	18	32	35	28
Ireland	15	22	25	19
New Zealand	19	22	28	22
Republic of Korea	18	23	9	17
UK	20	25	30	24
USA	22	24	24	23

Source: Data from UNESCO database, http://stats.uis.unesco.org/unesco/ReportFolders/ReportFolders.aspx. Averages of available years 2004–06.

Table 3. Public expenditure per pupil (current dollars) (selected countries)

Country	Primary	Secondary	Tertiary	All levels
Argentina	536	816	572	631
Bangladesh	39	67	201	57
Botswana	822	2100	22 900	1725
Brazil	566	496	1360	603
Chile	706	783	764	746
Mauritius	622	999	1968	883
Mozambique	46	197	1633	67
Uganda	32	98	542	43
Uruguay	384	456	865	508
Australia	5255	4722	7206	5375
Finland	6864	12 122	13 063	10 583
Ireland	6018	8977	10 263	7823
New Zealand	4452	5176	6749	5254
Republic of Korea	2972	3806	1499	2828
UK	7473	9436	11 277	9044
USA	9573	10 680	10 664	10 267

Source: Data from UNESCO and World Bank, average of available years 2004–06,.http://stats.uis.unesco.org/unesco/ReportFolders/ReportFolders.aspx, http://data.worldbank.org/indicator/NY.GNP.PCAP.CD

countries is even more heterogeneous. For instance, while in countries such as Chile and Argentina the distribution is quite flat, there are many countries that display strong preferences for higher education, some of them extraordinarily high, such as Mozambique and Botswana. The variation in resource intensity per student across the country groups is even higher in absolute terms, as presented in Table 3.

A relevant point here is to distinguish between the shares allocated to different levels and the absolute expenditures devoted to them. For instance, if we take Argentina, while

Table 4. Public education budget allocation across levels (in percentages) (selected countries)

	Primary	Secondary	Tertiary	Total
Argentina	37	39	23	100
Bangladesh	63	5	32	100
Botswana	26	36	37	100
Brazil	36	41	23	100
Chile	35	36	29	100
Mauritius	40	32	28	100
Mozambique	65	20	15	100
Uganda	71	13	16	100
Uruguay	34	38	28	100
Australia	31	37	32	100
Finland	23	43	35	100
Ireland	36	38	26	100
New Zealand	27	41	32	100
Republic of Korea	45	37	18	100
UK	34	39	27	100

Source: Data from UNESCO, averages of available years 2004–06, http://stats.uis.unesco.org/unesco/ReportFolders/ReportFolders.aspx

the budget allocation rule might not be very different from that in Australia, as shown in Table 4, the size of the total budget introduces limitations to potential outcomes.

Taking the data in Tables 1–4, the observed differences in educational budget allocation rules and systemic performance, measured by the educational attainment of the population, lead us to consider the question of the role of budget allocation in skills formation. For instance, as noted by Gemmel (1996), there is a different skill level that contributes most to growth for each development stage: human capital effects on growth are most evident at the primary level in low-income countries; for higher-income developing countries, the key level is the secondary level, whereas in developed countries the tertiary level is the most relevant for economic growth. According to this approach, it seems apparent from the data that many African countries are using allocation rules that are contrary to their development needs. However, the evidence in less extreme cases is less easy to interpret. Analysis of the allocation rules for the more general case will be the focus of this paper. The role of budget allocation rules in the process of skills formation and the conditions necessary for efficient allocation will be discussed in the rest of the paper.

3. The Education Model

As the learning process is cumulative, the indicator f_m (human capital) is defined as $f_m = \sum_m q_j$, which is the knowledge accumulated per student who has completed up to level m, where q_j measures knowledge accumulation per level. A two-level education system is considered to consist of basic and higher education ($j = B, H$). The output per student q_j measures "school quality", where $q_j = q_j(k_j)$, k_j is the resource intensity per student, and $\partial q_j/\partial k_j > 0$, $\partial^2 q_j/\partial^2 k_j < 0$. Students leave the system early when the quality of education that they receive at the lower stage is poor; thus early exit rates θ can be expressed as $\theta = \theta(q_B)$, where $\partial\theta/\partial q_B < 0$ and $\partial^2\theta/\partial^2 q_B < 0$. So, q_j at different levels are not perfect substitutes, and hence the allocation of resources across them affects human capital accumulation.

Thus, the accumulation process is driven by

$$dL_U = \theta E_B f_B$$

$$dL_S = E_H f_H = (1 - \theta) E_B f_H,$$

where dL_U and dL_S are the inflow of units of unskilled and skilled labour, respectively. The marginal ratio of skilled-to-unskilled labour produced can be defined as

$$\xi(k_B, k_H) = \frac{dL_S}{dL_U} = \frac{1 - \theta}{\theta}\frac{f_H}{f_B}.$$

The ratio of skilled-to-unskilled labour in the economy is modified by ξ; thus, dealing with inefficiency optimally will allow the achievement of a maximum ratio of skilled-to-unskilled labour. As the marginal ratio ξ is dependent on the capital intensity of basic and higher education, total differentiation and some manipulation result in

$$\hat{\xi} = \left(\frac{\hat{f}_H}{f_B}\right) + \frac{\hat{S}}{\theta},$$

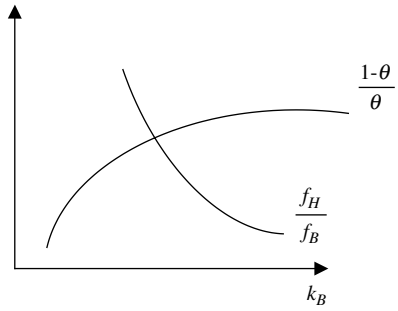

Figure 1. Variability of ξ components over k_B.

where a hat (^) placed over the variables denotes the rate of growth and S is the survival rate defined as $S = 1 - \theta$.

The evolution of ξ depends on the effects of allocation on the survival-to-exit rate and on the relative human capital accumulation across the levels. This is presented in Figure 1, considering

$$\partial\left(\frac{1-\theta}{\theta}\right)/\partial k_B > 0,\ \partial^2\left(\frac{1-\theta}{\theta}\right)/\partial^2 k_B < 0,\ \partial\left(f_H/f_B\right)/\partial k_B < 0$$

$$\text{and } \partial^2\left(f_H/f_B\right)/\partial^2 k_B > 0.$$

Specific conditions allow us to determine the sign of ξ (below). In general, $d\xi > 0$ when

$$\frac{d\left(f_H/f_B\right)}{f_H/f_B} > -\frac{d\left(1-\theta/\theta\right)}{(1-\theta)/\theta}.$$

4. Properties and Implications

This section analyses the properties of ξ. The underlying mechanism of the hierarchical model is simple: the allocation of more resources to higher education will increase the skilled-to-unskilled ratio (ξ), but if that budget increase implies lower quality in basic education, then the effect on ξ will be the opposite, because the number of potential entrants to higher education will necessarily shrink. The relative magnitude of both the effects will determine the net impact on ξ ; this section discusses the conditions for and properties of these effects.

The main properties and implications are analysed in the following.

Proposition 1. $\partial\xi/\partial k_H > 0$, *given* k_B.

The demonstration is straightforward as $\partial\xi/\partial k_H = [(1-\theta)/\theta](q_H/q_B)$, which is positive. The property applies to the effects of changes in the budget for higher education for a given budget for basic education.

Proposition 2. $\partial\xi/\partial k_B > 0$ *if* $\varepsilon_{\theta q_B} > s_H(1-\theta)$, *where* $s_H = q_H/f_H$ *and* $\varepsilon_{\theta q_B} = -\partial\theta/\partial q_B\, q_B/\theta$ (*see Appendix* 1 *for Demonstration* 1).

The property applies to changes in the budget for basic education for a given budget in higher education. From this property, it follows that ξ is a non-monotonic function of k_B. The capital intensity in basic education will have a positive effect on the marginal ratio of skilled-to-unskilled labour if the elasticity of the dropout variable to the quality of basic education ($\varepsilon_{\theta q_B}$) is high. Therefore, for a given s_H, if the survival parameter is too low, then the possibility that the increase in capital intensity in basic education will have a positive effect on the marginal ratio ξ is higher.

Proposition 3. *For a given budget,* $d\xi/dk_B > 0$, *if* $\varepsilon_{\theta q_B}/(\varepsilon_{q_B k_B} + \varepsilon_{q_H k_H}\varepsilon_{k_h k_B}) > s_H(1-\theta)$, *where* $\varepsilon_{\theta k_B} = -\partial\theta/\partial k_B\, k_B/\theta$, $\varepsilon_{k_H k_B} = -dk_H/dk_B\, k_B/k_H$ *and* $\varepsilon_{q_j k_j} = -\partial q_j/\partial k_j\, k_j/q_j$, $j = B, H$ (*see Appendix* 1 *for Demonstration* 2).

This property describes the effects of changes in the allocation rule for a given budget. The allocation of more resources to basic education is more likely to increase the marginal ratio of skilled-to-unskilled labour (ξ), if the survival parameter $(1-\theta)$ is low (as in Proposition 2).

Proposition 4. *Given* $B_B = K_B/K$, *when Proposition 3 holds,* $\partial\xi/\partial k > 0$ (*see Appendix* 1 *for Demonstration* 3).

This property highlights the fact that the effect of changing the resource intensity k, given a particular allocation rule, is positive on the skilled-to-unskilled ratio; it follows that for a given allocation, the marginal ratio is a positive function of the size of the budget.

5. Are There Clear-cut Policy Recommendations?

For a given budget, it is possible to find a rule to maximize the skilled-to-unskilled ratio of labour produced by maximizing $\xi(k_B, k_H)$ subject to $K = K_B + K_H$ and technology parameters.

From the first-order conditions, it follows that

$$\frac{d\xi}{dk_B} = 0 \Leftrightarrow \frac{\varepsilon_{\theta k_B}}{\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}} = S_H(1-\theta),$$

where $\varepsilon_{\theta k_B} = -\partial\theta/\partial k_B\, k_B/\theta$, $\varepsilon_{k_H k_B} = -dk_H/dk_B\, k_B/k_H$ and $\varepsilon_{q_j k_j} = -\partial q_j/\partial k_j\, k_j/q_j$, with $j = B, H$. The program has no closed solution, but some clear hints can be obtained. It can be shown that

$$\varepsilon_{\xi k_B} = \frac{\varepsilon_{\theta k_B}}{1-\theta} - s_H\left[\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}\right].$$

In addition, considering the "quasi-neutral" assumption on education technology that $\varepsilon_{q_B k_B} = \varepsilon_{q_H k_H} = \varepsilon_{q_j k_j}$, the above-mentioned expression can be written as

$$\varepsilon_{\xi k_B} = \frac{\theta}{1-\theta}\varepsilon_{\theta k_B}\left(\frac{1}{\theta} - s_H\varepsilon_{q_j k_j}\right) - \frac{s_H}{B_H}\varepsilon_{q_j k_j},$$

where $B_H = K_H/K$ is the participation of higher education in the total budget.

The determinants of the elasticity of ξ with respect to the resource intensity in basic education can be shown using the above expression. The elasticity of the marginal ratio of skilled-to-unskilled labour relative to the resource intensity in basic education is higher:

- the higher $\varepsilon_{\theta k_B}$, the responsiveness of the early exit rate to the resource intensity;
- the higher S_B ($s_B = 1 - s_H$), the contribution of basic education in total human capital accumulated;
- the lower B_B ($B_B = 1 - B_H$), the participation of basic education in the total budget.

The level of the early exit rate (θ) has an ambiguous role. The former aspect listed is a pure technology parameter, whereas the latter is a pure policy variable; the second value listed is a combination of technology and policy aspects. It must be noted that education technology plays a crucial role. For instance, in an extreme case $\varepsilon_{\theta k_B}$ could be zero, in which case the effect on ξ due to an increase in k_B would be negative.

These results imply that in many developing countries with bad systemic outcomes due to the poor performance of basic education, an increase in the share of resources to basic education ($B_B = K_B/K$) could be effective in increasing the amount of skilled labour in relation to unskilled labour. A simple example will provide more insight into this issue.

Table 5 presents the benchmark values of the variables of the model, for a reasonable set-up of parameters corresponding to a typical developing country, and the results of simulated changes in the size and allocation of the budget. The results displayed in this table correspond to the following exercises: (i) Benchmark: K low and $B_B = 0.74$ (typical developing country); (ii) Exercise 1: assumes K high (budget-rich country size) and $B_B = 0.74$; (iii) Exercise 2: assumes K high (budget-rich country size) and $B_B = 0.72$ (rich country allocation rule); (iv) Exercise 3: assumes K high (budget-rich country size) and $B_B = 0.80$, conveniently selected.

As the table shows, a higher budget would increase ξ (Exercise 1); but the rich country allocation rule is not the best option for the developing country (Exercise 2), as a higher allocation to basic education (Exercise 3) would be better.

Table 5. Simulation exercises

	Benchmark	Excercise 1	Exercise 2	Exercise 3
q_B	1.093	1.301	1.298	1.308
q_H	1.184	1.336	1.344	1.311
ξ	0.262	0.636	0.633	0.641
θ	88.816	76.130	76.281	75.738
k_H	1823	10233	11153	7788
k_B	585	7005	6802	7558
f_H/f_B	2.083	2.027	2.035	2.002
$(1 - \theta)/\theta$	0.126	0.314	0.311	0.320

Source: Authors' calculations. Simulation results using data from UNESCO database (average of selected countries with availability of all required data). Developed group: UK, Ireland, Finland and Korea; developing group: Argentina, Brazil, Chile and Uruguay.

6. Conclusions

The evolution of the ratio of skilled-to-unskilled labour stocks in the economy is relevant to development policy given the imperfect substitutability of both factors and the central role of human capital for development. Can education policy affect the evolution of this ratio? This paper has shown that it can, and has presented the general theoretical conditions necessary for maximizing it.

The paper follows an increasingly important line of research that focuses on the key role played by allocation of the education budget in generating skills. The contribution of the paper lies in the strategy used to analyse the optimal allocation, which has several advantages. In particular, in contrast to the previous literature, the analysis suggests theoretically optimal conditions for efficient budget allocation and offers easy-to-follow rules for policymakers. The stylized version of the hierarchical education model allows us to focus on the links between alternative budget allocation rules and the production of human capital. The analytical conditions under which the skilled-to-unskilled ratio will reach a maximum, given the total resources assigned to the education system, are discussed thoroughly. Drawing on the data for a set of real economies, it has been shown through a series of simulations that the optimal allocation rule for a developing country would require the allocation of relatively more resources (proportionally) to basic education in comparison with developed countries.

The analysis has implication for policymakers in developing countries, where skill formation is much needed, because it shows that the allocation rules observed in many developing countries usually violate the maximization condition by assigning higher-than-optimal resources to higher education. A further implication is that as long as the marginal skilled-to-unskilled ratio regulates the wage gap, a less-than-maximum value would worsen the wage distribution.

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Appendix 1

Preliminary Results

The educational budget constraint is

$$K = K_B + K_H.$$

By definition, $k = k_B + (1 - \theta)k_H$, where $k = K/E_B$. Thus,

$$dk_H = -\left[1 - \theta - (k - k_B)\theta'\right] dk_B/(1 - \theta)^2. \tag{A1}$$

Also, after some manipulations, this expression can be written as

$$\varepsilon_{k_H k_B} = \frac{K_B}{K_H} + \frac{\theta}{1 - \theta}\varepsilon_{\theta k_B}, \tag{A2}$$

where $\varepsilon_{\theta k_B} = -\partial\theta/\partial k_B\, k_B/\theta$ and $\varepsilon_{k_H k_B} = -dk_H/dk_B\, k_B/k_H$.

Demonstration 1

Total differentiation of ξ results in the following:

$$d\xi = \frac{\partial\xi}{\partial k_H} dk_H + \frac{\partial\xi}{\partial k_B} dk_B, \tag{A3}$$

where

$$\frac{\partial\xi}{\partial k_B} = -\frac{1}{\theta q_B}\left[(1 - \theta)\frac{q_H}{q_B} q_B^{'} + (q_B + q_H)\frac{\theta^{'}}{\theta}\right] \tag{A4}$$

and

$$\frac{\partial\xi}{\partial k_h} = \frac{1 - \theta}{\theta}\frac{q_H^{'}}{q_B}. \tag{A5}$$

While the sign of $\partial\xi/\partial k_H > 0$, that for $\partial\xi/\partial k_B$ is indeterminate. The conditions under which $\partial\xi/\partial k_B > 0$ are easy to find. Considering the following definitions, $S_H = q_H/f_H$ and $\varepsilon_{\theta q_B} = -\partial\theta/\partial q_B\, q_B/\theta = -\theta^{'} q_B/\theta$, and substituting in Equation (A3) after some manipulation, the following is obtained:

$$\frac{\partial\xi}{\partial k_B} > 0 \Leftrightarrow \quad \varepsilon_{\theta q_B} > S_H(1 - \theta).$$

Demonstration 2

Inserting Equations (A1), (A4) and (A5) in Equation (A3), as well as using the definition $\varepsilon_{\theta k_B} = -\partial\theta/\partial k_B\, k_B/\theta$, after some manipulation the following result is obtained:

$$\frac{d\xi}{dk_B} > 0 \Leftrightarrow \frac{\varepsilon_{\theta k_B}}{\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}} > S_H(1-\theta).$$

It can also be shown that

$$\varepsilon_{\xi k_B} = \frac{\varepsilon_{\theta k_B}}{1-\theta} - S_H\left[\varepsilon_{q_H k_H}\varepsilon_{k_H k_B} + \varepsilon_{q_B k_B}\right].$$

Inserting Equation (A2) in the above expression and defining $B_H = K_H/K$, and considering the "quasi-neutral" assumption that $\varepsilon_{q_B k_B} = \varepsilon_{q_H k_H} = \varepsilon_{q_j k_j}$, the following is obtained:

$$\varepsilon_{\xi k_B} = \frac{\varepsilon_{\theta k_B}}{1-\theta}\left[1 - S_H\varepsilon_{q_j k_j}\theta\right] - \frac{S_H}{B_H}\varepsilon_{q_j k_j}.$$

Demonstration 3

$\partial\xi/\partial k > 0$ as $\frac{\partial\xi}{\partial k} = \frac{\partial\xi}{\partial k_B}\frac{\partial k_B}{\partial k} + \frac{\partial\xi}{\partial k_H}\frac{\partial k_H}{\partial k}$, is positive, resulting from $\frac{\partial k_B}{\partial k}, \frac{\partial k_H}{\partial k}$ positive for given allocation structure, and $\frac{\partial\xi}{\partial k_B}, \frac{\partial\xi}{\partial k_H}$ positive for increasing budget allocated to basic and higher education (Proposition 2 holds).