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Abstract

This study analyzes the growth and welfare effects of monetary policy in a two-country Schumpeterian growth model with cash-in-advance constraints on consumption and R&D investment. We find that an increase in the domestic nominal interest rate decreases domestic R&D investment and the growth rate of domestic technology. Given that economic growth in a country depends on both domestic and foreign technologies, an increase in the foreign nominal interest rate also decreases economic growth in the domestic economy. When each government conducts its monetary policy unilaterally to maximize the welfare of only domestic households, the Nash-equilibrium nominal interest rates are generally higher than the optimal nominal interest rates chosen by cooperative governments who maximize the welfare of both domestic and foreign households. This difference is caused by a cross-country spillover effect of monetary policy arising from trade in intermediate goods. Under the CIA constraint on consumption (R&D investment), a larger market power of firms decreases (increases) the wedge between the Nash-equilibrium and optimal nominal interest rates. We also calibrate the two-country model to data in the Euro Area and the UK and find that the cross-country welfare effects of monetary policy are quantitatively significant.

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1 Introduction

In this study, we analyze the effects of monetary policy on economic growth and social welfare in an open economy. Specifically, we develop a two-country version of the Schumpeterian growth model and introduce money demand into the model via a cash-in-advance (CIA) constraint on R&D investment in each country.[1] Empirical evidence supports the view that R&D investment is severely affected by liquidity requirements. For example, Hall (1992), Himmelberg and Petersen (1994), Opler *et al.* (1999) and Brown and Petersen (2009) find a positive and significant relationship between R&D and cash flow in US firms. Recent studies by Brown and Petersen (2011) and Brown *et al.* (2012) provide evidence that firms tend to smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves.[2] Given these liquidity requirements on R&D investment, the nominal interest rate that determines the opportunity cost of cash holdings affects R&D investment, economic growth and social welfare. In an open economy, monetary policy may have spillover effects across countries through international trade.

The results from our growth-theoretic analysis confirm the above intuition and can be summarized as follows. An increase in the domestic nominal interest rate decreases domestic R&D investment and the growth rate of domestic technology. Given that economic growth in a country depends on the growth rate of domestic technology as well as the growth rate of foreign technology, an increase in the foreign nominal interest rate also decreases economic growth in the domestic economy. When each government conducts its monetary policy unilaterally to maximize the welfare of only domestic households, the Nash-equilibrium nominal interest rates are generally different from the optimal nominal interest rates chosen by cooperative governments who maximize the aggregate welfare of domestic and foreign households. Specifically, we find that under the special case of inelastic labor supply, the Nash-equilibrium nominal interest rates coincide with the optimal nominal interest rates. However, under the more general case of elastic labor supply, the Nash-equilibrium nominal interest rates are higher than the optimal nominal interest rates because there exists a cross-country spillover effect of monetary policy. The intuition of this result can be explained as follows. When the government in a country reduces the nominal interest rate, the welfare gain from higher economic growth is shared by the other country through trade in intermediate goods, whereas the welfare cost of increasing labor supply falls entirely on domestic households. As a result, the governments do not lower the nominal interest rates sufficiently in the Nash equilibrium.

The Nash-equilibrium nominal interest rates depend on the market power of firms. Under the CIA constraint on consumption, a larger markup reduces the wedge between the Nash-equilibrium and optimal nominal interest rates. This finding is consistent with the insight of Arseneau (2007), who shows that the market power of firms has a dampening effect on the inflationary bias from monetary policy competition in Cooley and Quadrini (2003). However, under the CIA constraint on R&D investment, we have the opposite result that a larger markup magnifies the inflationary bias from monetary policy competition. These different implications highlight the importance of the differences between the two CIA constraints.

[1] See also Chu and Cozzi (2013), who introduce a CIA constraint on R&D investment into a closed-economy version of the Schumpeterian growth model and analyze the effects of monetary policy.

[2] See also Berentsen *et al.* (2012) for a discussion of other empirical studies on the importance of cash holdings on R&D investment.

The difference between the CIA constraint on consumption and the CIA constraint on R&D is that under the latter, an increase in the nominal interest rate leads to a reallocation of labor from R&D to production. As a result, higher nominal interest rates would be chosen by the governments in the Nash equilibrium to depress R&D as the negative R&D externality in the form of a business-stealing effect determined by the markup becomes stronger. In contrast, under the CIA constraint on consumption, this reallocation effect is absent because an increase in the nominal interest rate decreases both R&D and production labors. Given that the economy features an undersupply of labor due to the distortion of monopolistic competition, the governments would choose lower nominal interest rates in the Nash equilibrium as this monopolistic distortion determined by the markup becomes stronger.

We also calibrate the two-country model to aggregate data in the Euro Area and the United Kingdom (UK). We find that the cross-country welfare effects of monetary policy are quantitatively significant. For example, achieving price stability in the Euro Area would lead to a welfare gain of 0.287% of consumption per year in the UK in addition to a welfare gain of 0.979% of consumption per year in the Euro Area. If we consider a larger decrease in the nominal interest rate in the Euro Area to achieve the Friedman rule (i.e., a zero nominal interest rate), the welfare gains would be even more significant at 0.706% of consumption per year in the UK and 2.447% of consumption per year in the Euro Area. Furthermore, given a rising volume of trade between the two economies, the cross-country welfare effects of monetary policy are likely to become even stronger in the future.

This study relates to the literature of inflation and economic growth; see Stockman (1981) and Abel (1985) for seminal studies of the CIA constraint on capital investment in the Neoclassical growth model. Instead of analyzing the effects of monetary policy in the Neoclassical growth model, we consider an R&D-based growth model in which economic growth is driven by innovation and endogenous technological progress. The seminal study in this literature of inflation and innovation-driven growth is Marquis and Reffett (1994), who analyze a CIA constraint on consumption in a Romer variety-expanding model.[3] In contrast, we consider a Schumpeterian quality-ladder model and analyze the effects of monetary policy via a CIA constraint on R&D investment as in Chu and Cozzi (2013).[4] Huang *et al.* (2013) also analyze the effects of monetary policy via CIA constraints on R&D investment in a Schumpeterian model with endogenous market structure. The present study differs from the closed-economy analyses in Chu and Cozzi (2013) and Huang *et al.* (2013) by considering a two-country setting with trade in intermediate goods across countries. To our knowledge, this is the first study that analyzes the effects of monetary policy in a growth-theoretic framework that features R&D-driven innovation in an open economy.

The rest of this study is organized as follows. In Section 2, we present the open-economy monetary Schumpeterian growth model. Section 3 analyzes the effects of monetary policy on economic growth. Section 4 analyzes the effects of monetary policy on social welfare. Section 5 concludes.

[3] Chu *et al.* (2013) provide an analysis of the CIA constraint on consumption in a hybrid growth model in which economic growth in the long run is driven by both variety expansion and capital accumulation.

[4] See Chu and Lai (2013) for an analysis of the money-in-utility approach to model money demand in the quality-ladder growth model.

2 The model

In this section, we develop an open-economy version of a monetary Schumpeterian growth model. The underlying quality-ladder model is based on the seminal work of Grossman and Helpman (1991).[5] In summary, we modify the Grossman-Helpman model by introducing money demand via CIA constraints on consumption and R&D investment as in Chu and Cozzi (2013) and extending the closed-economy model into a two-country setting with trade in intermediate goods. The home country is denoted with a superscript h, whereas the foreign country is denoted with a superscript f. Both countries invest in R&D, but we allow for asymmetry across the two countries in a number of structural parameters. Following a common treatment in this type of two-country models, we assume balanced trade. Given that the quality-ladder model has been well-studied, we will describe the familiar components briefly but discuss the new features in details. Furthermore, to conserve space, we will only present the equations for the home country h, but the readers are advised to keep in mind that for each equation that we present, there is an analogous equation for the foreign country f.

2.1 Households

In each country, there is a representative household. The lifetime utility function of the household in country h is given by

$$U^h = \int_0^\infty e^{-\rho t} \left[\ln C_t^h + \theta^h \ln(1 - L_t^h) \right] dt, \tag{1}$$

where C_t^h denotes consumption goods in country h at time t. L_t^h denotes the supply of labor in country h, and labor is assumed to be immobile across countries. The parameters $\rho > 0$ and $\theta^h \geq 0$ determine respectively subjective discounting and leisure preference. We allow for asymmetry in θ^h across the two countries, so that the countries may have different sizes of the labor force.

The household maximizes (1) subject to the following asset-accumulation equation:

$$\dot{V}_t^h + \dot{M}_t^h = R_t^h V_t^h + W_t^h L_t^h + T_t^h - P_t^h C_t^h + R_t^h B_t^h. \tag{2}$$

V_t^h is the nominal value of financial assets (in the form of equity shares in monopolistic firms in country h) owned by the household.[6] R_t^h is the nominal interest rate in country h. W_t^h is the nominal wage rate and T_t^h is the nominal value of a lump-sum transfer (or tax if $T_t^h < 0$) from the government to the household. P_t^h is the price of consumption goods in country h. M_t^h is the nominal value of domestic currency held by the household partly to facilitate purchases of consumption goods in country h. The CIA constraint on consumption goods is given by $\xi^h P_t^h C_t^h \leq M_t^h - B_t^h$, where $\xi^h \geq 0$ parameterizes the strength of the

[5] See also Aghion and Howitt (1992) and Segerstrom *et al.* (1990) for the other seminal studies of the quality-ladder model.

[6] Here we assume home bias in asset holding (i.e., domestic monopolistic firms are owned by domestic households) in order to allow the rates of return on assets to differ across countries.

CIA constraint on consumption in country h. Finally, B_t^h is the nominal value of domestic currency borrowed by R&D entrepreneurs to finance their R&D investment, and the rate of return on B_t^h is the nominal interest rate R_t^h.[7]

From standard dynamic optimization, the optimality condition for consumption in country h is

$$P_t^h C_t^h = \frac{1}{\eta_t^h(1+\xi^h R_t^h)}, \tag{3}$$

where η_t^h is the Hamiltonian co-state variable on (2). The optimality condition for labor supply is

$$L_t^h = 1 - \frac{\theta^h P_t^h C_t^h (1+\xi^h R_t^h)}{W_t^h}. \tag{4}$$

Finally, the intertemporal optimality condition is

$$-\frac{\dot{\eta}_t^h}{\eta_t^h} = R_t^h - \rho. \tag{5}$$

In the case of a constant nominal interest rate R^h, (3) and (5) simplify to the familiar Euler equation $\dot{C}_t^h / C_t^h = r_t^h - \rho$, where $r_t^h = R^h - \dot{P}_t^h / P_t^h$ is the real interest rate in country h.

2.2 Consumption goods

Consumption goods in country h are produced by competitive firms that aggregate two types of final goods using a Cobb-Douglas aggregator[8] given by

$$C_t^h = \frac{(Y_t^{h,h})^{1-\alpha}(Y_t^{h,f})^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}, \tag{6}$$

where $Y_t^{h,f}$ denotes country h's final goods that are produced with intermediate goods imported from country f, and $Y_t^{h,h}$ denotes country h's final goods that are produced with domestic intermediate goods. The parameter $\alpha \in [0,1]$ determines the importance of foreign goods in domestic consumption. From profit maximization, the conditional demand functions for $Y_t^{h,h}$ and $Y_t^{h,f}$ are respectively

$$Y_t^{h,h} = (1-\alpha) P_t^h C_t^h / P_t^{h,h}, \tag{7}$$

$$Y_t^{h,f} = \alpha P_t^h C_t^h / P_t^{h,f}, \tag{8}$$

where $P_t^{h,h}$ is the price of $Y_t^{h,h}$, and $P_t^{h,f}$ is the price of $Y_t^{h,f}$. The familiar price index of consumption goods in country h is $P_t^h = (P_t^{h,h})^{1-\alpha}(P_t^{h,f})^{\alpha}$.

[7] It can be easily shown as a no-arbitrage condition that the rate of return on B_t^h must be equal to the nominal interest rate.

[8] We consider a Cobb-Douglas aggregator instead of a more general CES aggregator in order to allow $Y_t^{h,h}$ and $Y_t^{h,f}$ to grow at different rates on the balanced growth path.

2.3 Final goods

Final goods $Y_t^{h,h}$ and $Y_t^{h,f}$ are also produced by competitive firms. Competitive firms in country h produce $Y_t^{h,h}$ by aggregating a unit continuum of domestic intermediate goods $X_t^{h,h}(i)$ for $i \in [0,1]$. The standard Cobb-Douglas aggregator is given by

$$Y_t^{h,h} = \exp\left(\int_0^1 \ln X_t^{h,h}(i)di\right). \tag{9}$$

Similarly, competitive firms in country h produce $Y_t^{h,f}$ by aggregating a unit continuum of foreign intermediate goods $X_t^{h,f}(j)$ for $j \in [0,1]$. The Cobb-Douglas aggregator is given by

$$Y_t^{h,f} = \exp\left(\int_0^1 \ln X_t^{h,f}(j)dj\right). \tag{10}$$

From profit maximization, the conditional demand functions for $X_t^{h,h}(i)$ and $X_t^{h,f}(j)$ are respectively

$$X_t^{h,h}(i) = P_t^{h,h} Y_t^{h,h} / P_t^{h,h}(i), \tag{11}$$

$$X_t^{h,f}(j) = P_t^{h,f} Y_t^{h,f} / P_t^{h,f}(j), \tag{12}$$

where $P_t^{h,h}(i)$ is the price of $X_t^{h,h}(i)$, and $P_t^{h,f}(j)$ is the price of $X_t^{h,f}(j)$. Finally, the price index of final goods $Y_t^{h,h}$ is $P_t^{h,h} \equiv \exp\left(\int_0^1 \ln P_t^{h,h}(i)di\right)$, and the price index of final goods $Y_t^{h,f}$ is $P_t^{h,f} \equiv \exp\left(\int_0^1 \ln P_t^{h,f}(j)dj\right)$. All these prices are denominated in the domestic currency.

2.4 Intermediate goods

In country h, there is a unit continuum of differentiated intermediate goods indexed by $i \in [0,1]$. In each industry $i \in [0,1]$, there is an industry leader who dominates the market temporarily until the arrival of the next innovation.[9] The industry leader employs workers in country h to produce $X_t^{h,h}(i)$ for sales in country h and $X_t^{f,h}(i)$ for sales in country f.[10] The industry leader's production of $X_t^{h,h}(i)$ and $X_t^{f,h}(i)$ uses the same technology except for the presence of an iceberg transportation cost $c \in (0,1)$ for $X_t^{f,h}(i)$. Specifically, the production functions are given by

$$X_t^{h,h}(i) = (z^h)^{q_t^h(i)} L_{x,t}^{h,h}(i), \tag{13}$$

$$X_t^{f,h}(i) = (1-c)(z^h)^{q_t^h(i)} L_{x,t}^{f,h}(i). \tag{14}$$

$z^h > 1$ is the step size of innovation in country h, and we allow this parameter to differ across countries. $q_t^h(i)$ is the number of quality improvements that have occurred in industry i as of time t.[11] The total number of production workers employed in industry i of country h is $L_{x,t}^h(i) = L_{x,t}^{h,h}(i) + L_{x,t}^{f,h}(i)$.

[9] This is known as the Arrow replacement effect in the literature. See Cozzi (2007) for a discussion of the Arrow effect.

[10] In order to keep the analysis tractable, we do not consider production offshoring in this study.

[11] It is useful to note that we here adopt a cost-reducing view of quality improvement as in Peretto (1998).

Given $(z^h)^{q_t^h(i)}$ in industry i, the leader's marginal cost functions for $X_t^{h,h}(i)$ and $X_t^{f,h}(i)$ are respectively

$$MC_t^{h,h}(i) = \frac{W_t^h}{(z^h)^{q_t^h(i)}}, \tag{15}$$

$$MC_t^{f,h}(i) = \frac{W_t^h}{(1-c)(z^h)^{q_t^h(i)}}. \tag{16}$$

Standard Bertrand price competition leads to markup pricing. The markup ratio is assumed to equal the step size z^h of innovation in the original Grossman-Helpman model. Here we allow for variable patent breadth similar to Li (2001) and Iwaisako and Futagami (2013) by assuming that the markup $\mu^h > 1$ is a policy instrument determined by the patent authority.[12] For simplicity, we focus on the case in which $\mu^h = \mu^f = \mu$, and this assumption can be partly justified by the harmonization of patent protection across countries as a result of the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) effective since 1996.[13] Therefore, the price of $X_t^{h,h}(i)$ is

$$P_t^{h,h}(i) = \mu MC_t^{h,h}(i) = \mu \frac{W_t^h}{(z^h)^{q_t^h(i)}}. \tag{17}$$

Similarly, the price of $X_t^{f,h}(i)$ denominated in country h's currency is

$$\mathcal{E}_t P_t^{f,h}(i) = \mu MC_t^{f,h}(i) = \mu \frac{W_t^h}{(1-c)(z^h)^{q_t^h(i)}}, \tag{18}$$

where $P_t^{f,h}(i)$ is the price of $X_t^{f,h}(i)$ denominated in country f's currency, and $\mathcal{E}_t$ denotes the nominal exchange rate.

Given (17), the amount of monopolistic profit from selling $X_t^{h,h}(i)$ in country h is

$$\omega_t^{h,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{h,h}(i) X_t^{h,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{h,h} Y_t^{h,h}, \tag{19}$$

where the second equality follows from (11). Similarly, the amount of monopolistic profit (denominated in country f's currency) from selling $X_t^{f,h}(i)$ in country f is

$$\omega_t^{f,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{f,h}(i) X_t^{f,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{f,h} Y_t^{f,h}, \tag{20}$$

[12] To model patent breadth, we first make a standard assumption in the literature, see for example Howitt (1999) and Segerstrom (2000), that once the incumbent leaves the market, she cannot threaten to reenter the market due to a reentry cost. As a result of the incumbent stopping production, the entrant is able to charge the unconstrained monopolistic markup, which is infinity due to the Cobb-Douglas specification in (9) and (10), under the case of complete patent breadth. However, with incomplete patent breadth, potential imitation limits the markup. Specifically, the presence of monopolistic profits attracts imitation; therefore, stronger patent protection allows monopolistic producers to charge a higher markup without the threat of imitation. This formulation of patent breadth captures Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

[13] See Grossman and Lai (2004) for an analysis of the harmonization of patent protection under TRIPS.

where the second equality follows from country f's analogous condition of (12). Therefore, the total amount of monopolistic profits (denominated in country h's currency) earned by the leader in industry i is

$$\omega_t^h(i) = \omega_t^{h,h}(i) + \mathcal{E}_t \omega_t^{f,h}(i) = \frac{\mu - 1}{\mu} \left(P_t^{h,h} Y_t^{h,h} + \mathcal{E}_t P_t^{f,h} Y_t^{f,h} \right). \tag{21}$$

Finally, wage income paid to industry i's workers in country h is

$$W_t^h L_{x,t}^h(i) = W_t^h L_{x,t}^{h,h}(i) + W_t^h L_{x,t}^{f,h}(i) = \frac{1}{\mu} \left(P_t^{h,h} Y_t^{h,h} + \mathcal{E}_t P_t^{f,h} Y_t^{f,h} \right). \tag{22}$$

2.5 R&D

Denote $V_t^h(i)$ as the nominal value of the monopolistic firm in industry $i \in [0, 1]$ of country h. Because $\omega_t^h(i) = \omega_t^h$ for $i \in [0, 1]$ from (21), $V_t^h(i) = V_t^h$ in a symmetric equilibrium that features an equal arrival rate of innovation across industries within a country.[14] In this case, the familiar no-arbitrage condition for V_t^h is

$$R_t^h = \frac{\omega_t^h + \dot{V}_t^h - \lambda_t^h V_t^h}{V_t^h}. \tag{23}$$

This condition equates the nominal interest rate R_t^h in country h to the rate of return per unit of domestic asset. The asset return is the sum of (a) monopolistic profit ω_t^h, (b) potential capital gain $\dot{V}_t^h$, and (c) expected capital loss $\lambda_t^h V_t^h$ due to creative destruction, where λ_t^h is the arrival rate of the next innovation in country h.

There is a unit continuum of R&D entrepreneurs indexed by $k \in [0, 1]$ in each country, and they hire R&D labor for innovation. In country h, entrepreneur k's wage payment to R&D labor is $W_t^h L_{r,t}^h(k)$. However, to facilitate this wage payment, the entrepreneur needs to borrow $B_t^h(k) = W_t^h L_{r,t}^h(k)$ units of domestic currency from the domestic household. Following Chu and Cozzi (2013), we impose a CIA constraint on R&D investment, and the cost of borrowing is $R_t^h B_t^h(k)$. Therefore, the total cost of R&D is $(1 + R_t^h) W_t^h L_{r,t}^h(k)$. Free entry implies zero expected profit such that

$$V_t^h \lambda_t^h(k) = (1 + R_t^h) W_t^h L_{r,t}^h(k), \tag{24}$$

where the *firm-level* arrival rate of innovation is $\lambda_t^h(k) = \varphi^h L_{r,t}^h(k)$, and we allow the R&D productivity parameter φ^h to differ across countries. Finally, the *aggregate* arrival rate of innovation in country h is

$$\lambda_t^h = \int_0^1 \lambda_t^h(k) dk = \varphi^h L_{r,t}^h. \tag{25}$$

[14] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian growth model.

2.6 Monetary authority

The growth rate of the nominal money supply in country h is $\dot{M}_t^h/M_t^h$. By definition, the real money balance in country h is $m_t^h = M_t^h/P_t^h$, where P_t^h is the price of consumption goods in country h. Therefore, the growth rate of m_t^h is $\dot{m}_t^h/m_t^h = \dot{M}_t^h/M_t^h - \pi_t^h$, where $\pi_t^h \equiv \dot{P}_t^h/P_t^h$ is the inflation rate of the price of consumption goods in country h. The policy instrument that we consider is the nominal interest rate R_t^h, which is exogenously chosen by the monetary authority in country h. Given R_t^h, the inflation rate in country h is endogenously determined according to the Fisher identity $\pi_t^h = R_t^h - r_t^h$ where r_t^h is the real interest rate in country h, and then the growth rate of the nominal money supply M_t^h in country h is endogenously determined according to $\dot{M}_t^h/M_t^h = \dot{m}_t^h/m_t^h + \pi_t^h$. Finally, the monetary authority in country h returns the seigniorage revenue as a lump-sum transfer $T_t^h = \dot{M}_t^h$ to the domestic household.

2.7 Decentralized equilibrium

The equilibrium is a time path of allocations $\{L_t^h, L_t^f, C_t^h, C_t^f, Y_t^{h,h}, Y_t^{h,f}, Y_t^{f,f}, Y_t^{f,h}, X_t^{h,h}(i), X_t^{h,f}(j), X_t^{f,f}(j), X_t^{f,h}(i), L_{x,t}^h(i), L_{x,t}^f(j), L_{r,t}^h(k), L_{r,t}^f(k)\}_{t=0}^{\infty}$, a time path of prices $\{W_t^h, W_t^f, P_t^h, P_t^f, P_t^{h,h}, P_t^{h,f}, P_t^{f,f}, P_t^{f,h}, P_t^{h,h}(i), P_t^{h,f}(j), P_t^{f,f}(j), P_t^{f,h}(i), V_t^h, V_t^f, \mathcal{E}_t\}_{t=0}^{\infty}$ and a time path of monetary policies $\{R_t^h, R_t^f\}_{t=0}^{\infty}$. Also, at each instance of time,

- the representative household in country h maximizes lifetime utility taking $\{R_t^h, W_t^h, P_t^h\}$ as given;
- the representative household in country f maximizes lifetime utility taking $\{R_t^f, W_t^f, P_t^f\}$ as given;
- competitive consumption-good firms in country h produce $\{C_t^h\}$ to maximize profit taking $\{P_t^h, P_t^{h,h}, P_t^{h,f}\}$ as given;
- competitive consumption-good firms in country f produce $\{C_t^f\}$ to maximize profit taking $\{P_t^f, P_t^{f,f}, P_t^{f,h}\}$ as given;
- competitive final-good firms in country h produce $\{Y_t^{h,h}, Y_t^{h,f}\}$ to maximize profit taking $\{P_t^{h,h}, P_t^{h,f}, P_t^{h,h}(i), P_t^{h,f}(j)\}$ as given;
- competitive final-good firms in country f produce $\{Y_t^{f,f}, Y_t^{f,h}\}$ to maximize profit taking $\{P_t^{f,f}, P_t^{f,h}, P_t^{f,f}(j), P_t^{f,h}(i)\}$ as given;
- monopolistic intermediate-good firm $i \in [0,1]$ in country h produces $\{X_t^{h,h}(i), X_t^{f,h}(i)\}$ and chooses $\{P_t^{h,h}(i), P_t^{f,h}(i)\}$ to maximize profit taking $\{W_t^h\}$ as given;
- monopolistic intermediate-good firm $j \in [0,1]$ in country f produces $\{X_t^{f,f}(j), X_t^{h,f}(j)\}$ and chooses $\{P_t^{f,f}(j), P_t^{h,f}(j)\}$ to maximize profit taking $\{W_t^f\}$ as given;
- competitive R&D entrepreneurs $k \in [0,1]$ in country h employ $\{L_{r,t}^h(k)\}$ to maximize expected profit taking $\{R_t^h, W_t^h, V_t^h\}$ as given;

- competitive R&D entrepreneurs $k \in [0,1]$ in country f employ $\{L^f_{r,t}(k)\}$ to maximize expected profit taking $\{R^f_t, W^f_t, V^f_t\}$ as given;
- the market-clearing condition for labor holds in both countries such that $L^h_{x,t}+L^h_{r,t} = L^h_t$ and $L^f_{x,t} + L^f_{r,t} = L^f_t$; and
- the value of trade in intermediate goods is balanced such that $\int_0^1 P^{h,f}_t(j)X^{h,f}_t(j)dj = \mathcal{E}_t\int_0^1 P^{f,h}_t(i)X^{f,h}_t(i)di$.

2.8 Aggregate economy

Substituting (13) into (9) yields the aggregate production function for final goods $Y^{h,h}_t$ given by

$$Y^{h,h}_t = Z^h_t L^{h,h}_{x,t}, \tag{26}$$

where aggregate technology Z^h_t in country h is defined as

$$Z^h_t \equiv \exp\left(\int_0^1 q^h_t(i)di \ln z^h\right) = \exp\left(\int_0^t \lambda^h_v dv \ln z^h\right). \tag{27}$$

The second equality of (27) applies the law of large numbers. Differentiating the log of (27) with respect to t yields the growth rate of aggregate technology in country h given by

$$\frac{\dot{Z}^h_t}{Z^h_t} = \lambda^h_t \ln z^h = (\varphi^h \ln z^h)L^h_{r,t}. \tag{28}$$

Similarly, substituting country f's analogous condition of (14) into (10) yields the aggregate production function for final goods $Y^{h,f}_t$ given by

$$Y^{h,f}_t = (1-c)Z^f_t L^{h,f}_{x,t}, \tag{29}$$

where aggregate technology Z^f_t in country f is defined as

$$Z^f_t \equiv \exp\left(\int_0^1 q^f_t(j)dj \ln z^f\right) = \exp\left(\int_0^t \lambda^f_v dv \ln z^f\right). \tag{30}$$

Differentiating the log of (30) with respect to t yields the growth rate of aggregate technology in country f given by

$$\frac{\dot{Z}^f_t}{Z^f_t} = \lambda^f_t \ln z^f = (\varphi^f \ln z^f)L^f_{r,t}. \tag{31}$$

As for the dynamics of the model, Proposition 1 shows that the economy jumps to a unique and saddle-point stable balanced growth path (BGP). On the BGP, the allocations of labor in both countries are stationary. Therefore, taking the log of (6) and differentiating it with respect to t yields the balanced growth rate of consumption goods C^h_t in country h given by

$$\frac{\dot{C}^h_t}{C^h_t} = (1-\alpha)\frac{\dot{Y}^{h,h}_t}{Y^{h,h}_t} + \alpha\frac{\dot{Y}^{h,f}_t}{Y^{h,f}_t} = (1-\alpha)\frac{\dot{Z}^h_t}{Z^h_t} + \alpha\frac{\dot{Z}^f_t}{Z^f_t} = (1-\alpha)(\varphi^h \ln z^h)L^h_r + \alpha(\varphi^f \ln z^f)L^f_r. \tag{32}$$

Proposition 1 *Given constant nominal interest rates $\{R^h, R^f\}$ in the two countries, the economy immediately jumps to a unique and saddle-point stable balanced growth path along which each variable grows at a constant (possibly zero) rate.*

Proof. See Appendix A. ■

2.9 Equilibrium labor allocations

In this subsection, we sketch out the derivations of the equilibrium labor allocations in country h and relegate the detailed proof to Appendix A. From (24), the free-entry condition in the R&D sector becomes $\varphi^h V_t^h = (1 + R^h) W_t^h$, where the value of an innovation is $V_t^h = \omega_t^h / (\rho + \lambda^h) = \omega_t^h / (\rho + \varphi^h L_r^h)$ on the BGP as implied by (23) and (25). Then, substituting ω_t^h from (21) and W_t^h from (22) into the R&D free-entry condition, we derive the following equilibrium relationship between L_r^h and L_x^h:

$$L_r^h = \frac{\mu - 1}{1 + R^h} L_x^h - \frac{\rho}{\varphi^h}. \tag{33}$$

The second condition is the household's labor supply function in (4), which can be reexpressed as

$$L^h = 1 - \mu\theta^h (1 + \xi^h R^h) L_x^h. \tag{34}$$

The third condition for solving the equilibrium labor allocations is the resource constraint on labor given by

$$L_x^h + L_r^h = L^h, \tag{35}$$

where $L_x^h = L_x^{h,h} + L_x^{f,h}$. Finally, we combine the conditional demand functions in (7) and (8) with (22) and the balanced-trade condition that can be reexpressed as $P_t^{h,f} Y_t^{h,f} = \mathcal{E}_t P_t^{f,h} Y_t^{f,h}$ to solve for the following equilibrium relationship between $L_x^{h,h}$ and $L_x^{f,h}$:

$$L_x^{f,h} = \frac{\alpha}{1 - \alpha} L_x^{h,h}. \tag{36}$$

Solving (33)-(36), we derive the equilibrium labor allocations in country h.

Proposition 2 *The equilibrium labor allocations in country h are given by*

$$L_r^h = \frac{\mu - 1}{\mu + R^h + \mu\theta^h (1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \tag{37}$$

$$L_x^{h,h} = (1 - \alpha) L_x^h = \frac{(1 - \alpha)(1 + R^h)}{\mu + R^h + \mu\theta^h (1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \tag{38}$$

$$L_x^{f,h} = \alpha L_x^h = \frac{\alpha(1 + R^h)}{\mu + R^h + \mu\theta^h (1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \tag{39}$$

$$L^h = 1 - \frac{\mu\theta^h (1 + \xi^h R^h)(1 + R^h)}{\mu + R^h + \mu\theta^h (1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right). \tag{40}$$

Proof. See Appendix A. ■

3 Effects of monetary policy on economic growth

In this section, we analyze the effects of the domestic and foreign nominal interest rates on economic growth. Equation (37) shows that R&D labor L_r^h is decreasing in the domestic nominal interest rate R^h through the CIA constraints on R&D investment and consumption. Given the CIA constraint on R&D investment, an increase in the domestic nominal interest rate R^h increases the financing cost of R&D, and hence, it has a direct negative effect on R&D labor L_r^h. This financing-cost channel via the CIA constraint on R&D investment is present regardless of whether labor supply is elastic (i.e., $\theta^h > 0$) or inelastic (i.e., $\theta^h = 0$). Under the CIA constraint on consumption, the negative effect of the domestic nominal interest rate R^h on R&D operates through elastic labor supply. Equation (40) shows that labor supply L^h is decreasing in the nominal interest rate R^h partly due to the CIA constraint on consumption (i.e., $\xi^h > 0$). In other words, an increase in the domestic nominal interest rate raises the cost of consumption and causes the household to increase leisure. The resulting decrease in labor supply L^h in turn reduces R&D labor L_r^h. This labor-supply channel via the CIA constraint on consumption is present only if labor supply is elastic (i.e., $\theta^h > 0$). Given that R&D labor L_r^h is decreasing in the domestic nominal interest rate, the growth rate of technology in country h is also decreasing in the domestic nominal interest rate R^h; in other words,

$$\frac{\partial \dot{Z}_t^h / Z_t^h}{\partial R^h} = (\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^h} < 0. \tag{41}$$

Equation (37) also shows that R&D labor is independent of the foreign nominal interest rate R^f. Therefore, the growth rate of technology in country h must be also independent of R^f; in other words,

$$\frac{\partial \dot{Z}_t^h / Z_t^h}{\partial R^f} = (\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^f} = 0. \tag{42}$$

Nevertheless, economic growth in country h (i.e., the growth rate of consumption goods) is decreasing in the foreign nominal interest rate R^f. To see this result, we first write down country f's analogous expression of (37) for the equilibrium allocation of R&D labor L_r^f given by

$$L_r^f = \frac{\mu - 1}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \tag{43}$$

which is decreasing in the foreign nominal interest rate R^f. Therefore, the growth rate of consumption goods in country h is decreasing in the foreign nominal interest rate R^f through the growth rate of foreign technology; in other words,

$$\frac{\partial g^h}{\partial R^f} = (1 - \alpha)(\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^f} + \alpha(\varphi^f \ln z^f) \frac{\partial L_r^f}{\partial R^f} = \alpha(\varphi^f \ln z^f) \frac{\partial L_r^f}{\partial R^f} < 0, \tag{44}$$

where $g^h \equiv \dot{C}_t^h / C_t^h$. The growth rate of consumption goods in country h is also decreasing in the domestic nominal interest rate R^h through the growth rate of domestic technology; in other words,

$$\frac{\partial g^h}{\partial R^h} = (1 - \alpha)(\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^h} + \alpha(\varphi^f \ln z^f) \frac{\partial L_r^f}{\partial R^h} = (1 - \alpha)(\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^h} < 0. \tag{45}$$

As foreign goods become more important in domestic consumption (i.e., a larger α), the negative growth effect of the foreign nominal interest rate becomes stronger, whereas the negative growth effect of the domestic nominal interest rate becomes weaker. Proposition 3 summarizes the above results.

Proposition 3 *The growth rate of domestic technology is decreasing in the domestic nominal interest rate but independent of the foreign nominal interest rate. The growth rate of foreign technology is decreasing in the foreign nominal interest rate but independent of the domestic nominal interest rate. The growth rate of domestic consumption is decreasing in both the domestic and foreign nominal interest rates. The relative magnitude of these negative growth effects of the domestic and foreign nominal interest rates depends on α (i.e., the importance of foreign goods in domestic consumption).*

Proof. Proven in text. ■

Using the Fisher identity $\pi^h = R^h - r^h$ and the Euler equation $r^h = g^h + \rho$, we can write down an expression for the equilibrium inflation rate given by $\pi^h = R^h - g^h(R^h) - \rho$, where g^h is decreasing in the nominal interest rate R^h. Therefore, differentiating π^h with respect to R^h yields

$$\frac{\partial \pi^h}{\partial R^h} = 1 - \frac{\partial g^h}{\partial R^h} > 0.$$

Together with Proposition 3, we have the following empirical implications. First, an increase in the nominal interest rate leads to an increase in the inflation rate and a decrease in R&D investment. This finding is consistent with the empirical evidence in Chu and Lai (2013), who document a negative relationship between inflation and R&D using cross-country data. Second, an increase in the nominal interest rate leads to an increase in the inflation rate and a decrease in the growth rate of technology. This finding is consistent with the empirical results in Bruno and Easterly (1998) and Evers *et al.* (2007), who provide evidence for a negative relationship between inflation and the growth rate of total factor productivity. Finally, an increase in the nominal interest rate leads to an increase in the inflation rate and a decrease in the growth rate of output. This negative relationship between inflation and economic growth is supported by the empirical results in Vaona (2012).

4 Effects of monetary policy on social welfare

In this section, we analyze the effects of the domestic and foreign nominal interest rates on social welfare. Given the balanced-growth behavior of the economy, the lifetime utility of the representative household in country h simplifies to

$$U^h = \frac{1}{\rho}\left[\ln C_0^h + \frac{g^h}{\rho} + \theta^h \ln(1 - L^h)\right]. \tag{46}$$

Substituting (6), (26), (29) and (32) into (46) and then dropping the exogenous terms (including Z_0^h and Z_0^f) yield

$$\rho U^h = (1-\alpha)\ln L_x^{h,h} + \alpha \ln L_x^{h,f} + \frac{(1-\alpha)(\varphi^h \ln z^h)}{\rho} L_r^h + \frac{\alpha(\varphi^f \ln z^f)}{\rho} L_r^f + \theta^h \ln(1-L^h). \quad (47)$$

The analogous condition of (39) in country f implies that $L_x^{h,f}$ is given by

$$L_x^{h,f} = \frac{\alpha(1+R^f)}{\mu + R^f + \mu\theta^f(1+\xi^f R^f)(1+R^f)} \left(1 + \frac{\rho}{\varphi^f}\right), \quad (48)$$

which depends on the foreign nominal interest rate R^f. In summary, the set of variables $\{L_x^{h,h}, L_r^h, L^h\}$ in (47) depends on the domestic nominal interest rate R^h, whereas the set of variables $\{L_x^{h,f}, L_r^f\}$ depends on the foreign nominal interest rate R^f.

In the following subsections, we will derive (a) the nominal interest rate that is unilaterally chosen by each government to maximize domestic welfare and (b) the nominal interest rates that are chosen by cooperative governments who maximize the aggregate welfare of the two countries. Given that the results differ under the following three scenarios, we analyze them separately. In Section 4.1, we consider the case of inelastic labor supply. In Section 4.2, we consider elastic labor supply with only the CIA constraint on R&D investment. In Section 4.3, we consider elastic labor supply with only the CIA constraint on consumption.

4.1 Inelastic labor supply

In this subsection, we consider the case of inelastic labor supply (i.e., $\theta^h = \theta^f = 0$). In this case, (37), (38), (43) and (48) simplify to

$$L_r^h = \frac{\mu - 1}{\mu + R^h}\left(1 + \frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \quad (49)$$

$$L_x^{h,h} = \frac{(1-\alpha)(1+R^h)}{\mu + R^h}\left(1 + \frac{\rho}{\varphi^h}\right), \quad (50)$$

$$L_r^f = \frac{\mu - 1}{\mu + R^f}\left(1 + \frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \quad (51)$$

$$L_x^{h,f} = \frac{\alpha(1+R^f)}{\mu + R^f}\left(1 + \frac{\rho}{\varphi^f}\right), \quad (52)$$

and $L^h = 1$. Due to inelastic labor supply, the effect of the nominal interest rate operates solely through the CIA constraint on R&D investment. Substituting (49)-(52) into (47) and then differentiating U^h with respect to R^h, we obtain the following domestic nominal interest rate that is unilaterally chosen by the government in country h to maximize the domestic household's welfare:

$$R_{ne}^h = \max\left\{\frac{\mu - \Omega^h}{\Omega^h - 1}, 0\right\}, \quad (53)$$

where $\Omega^h \equiv (1 + \varphi^h/\rho) \ln z^h > 1$.[15] Equation (53) also takes into account the zero lower bound on the nominal interest rate, which would be binding if $\Omega^h > \mu$. The analogous foreign nominal interest rate that is unilaterally chosen by country f's government to maximize the welfare of the household in country f is given by

$$R^f_{ne} = \max\left\{\frac{\mu - \Omega^f}{\Omega^f - 1}, 0\right\}, \tag{54}$$

where $\Omega^f \equiv (1 + \varphi^f/\rho) \ln z^f > 1$.[16] The comparative statics are quite intuitive. Recall that domestic R&D investment is decreasing in the domestic nominal interest rate. Therefore, a larger innovation step size z^h (z^f) would decrease the nominal interest rate R^h_{ne} (R^f_{ne}) because R&D has a larger social benefit in this case. Similarly, a higher R&D productivity φ^h (φ^f) would also decrease R^h_{ne} (R^f_{ne}) for the same reason. In contrast, a higher discount rate ρ would increase R^h_{ne} and R^f_{ne} because R&D that leads to economic growth has a smaller social benefit when households discount future consumption more heavily.

We refer to the pair $\{R^h_{ne}, R^f_{ne}\}$ as the Nash-equilibrium nominal interest rates because each government pursues its own objective taking the other government's action as given. An interesting observation is that R^f_{ne} is also the foreign nominal interest rate that would be preferred by the government in country h. To see this result, we differentiate U^h with respect to R^f and find that the optimal foreign nominal interest rate for country h is also R^f_{ne}. Finally, we consider cooperative governments who choose $\{R^h, R^f\}$ to maximize aggregate welfare defined as $U^h + U^f$, and we refer to these nominal interest rates as the optimal nominal interest rates denoted as $\{R^h_*, R^f_*\}$. We find that $\{R^h_*, R^f_*\} = \{R^h_{ne}, R^f_{ne}\}$; in other words, the unilateral action of each government gives rise to an internationally optimal outcome; however, in the next subsection, we will show that this special result is due to the restriction of inelastic labor supply. We summarize the above results in Proposition 4.

Proposition 4 *Under inelastic labor supply, the Nash-equilibrium nominal interest rate unilaterally chosen by each government coincides with the optimal nominal interest rate chosen by cooperative governments who maximize aggregate welfare of the two countries. The optimal nominal interest rate in each country is decreasing in the domestic step size of innovation and domestic R&D productivity but increasing in the discount rate.*

Proof. See Appendix A. ■

4.2 Elastic labor supply with CIA on R&D only

In this subsection, we consider the case of elastic labor supply with the CIA constraint on R&D. However, we remove the CIA constraint on consumption by setting $\xi^h = \xi^f = 0$. In

[15] In an unpublished appendix (see Appendix B), we derive the first-best allocation of R&D labor in country h and show that the parameter restriction on this optimal R&D labor to be positive also implies $\Omega^h > 1$.

[16] We show in an unpublished appendix (see Appendix B) that this parameter restriction holds for a similar reason as $\Omega^h > 1$.

this case, (37), (38), (40), (43) and (48) simplify to

$$L_r^h = \frac{\mu - 1}{\mu + R^h + \mu\theta^h(1 + R^h)}\left(1 + \frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \tag{55}$$

$$L_x^{h,h} = \frac{(1-\alpha)(1 + R^h)}{\mu + R^h + \mu\theta^h(1 + R^h)}\left(1 + \frac{\rho}{\varphi^h}\right), \tag{56}$$

$$L^h = 1 - \frac{\mu\theta^h(1 + R^h)}{\mu + R^h + \mu\theta^h(1 + R^h)}\left(1 + \frac{\rho}{\varphi^h}\right), \tag{57}$$

$$L_r^f = \frac{\mu - 1}{\mu + R^f + \mu\theta^f(1 + R^f)}\left(1 + \frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \tag{58}$$

$$L_x^{h,f} = \frac{\alpha(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + R^f)}\left(1 + \frac{\rho}{\varphi^f}\right). \tag{59}$$

Substituting these conditions into (47) and then differentiating U^h with respect to R^h, we obtain the following domestic nominal interest rate that is unilaterally chosen by the government in country h to maximize the domestic household's welfare:

$$R_{ne}^h = \max\left\{\frac{\mu - \Phi^h}{\Phi^h - 1}, 0\right\}, \tag{60}$$

where[17]

$$\Phi^h \equiv \frac{1-\alpha}{1-\alpha+\theta^h}\left(1 + \frac{\varphi^h}{\rho}\right)(1 + \mu\theta^h)\ln z^h - \mu\theta^h > 1.$$

The analogous foreign nominal interest rate that is unilaterally chosen by country f's government to maximize the welfare of the household in country f is given by

$$R_{ne}^f = \max\left\{\frac{\mu - \Phi^f}{\Phi^f - 1}, 0\right\}, \tag{61}$$

where[18]

$$\Phi^f \equiv \frac{1-\alpha}{1-\alpha+\theta^f}\left(1 + \frac{\varphi^f}{\rho}\right)(1 + \mu\theta^f)\ln z^f - \mu\theta^f > 1.$$

We also consider cooperative governments who choose $\{R^h, R^f\}$ to maximize aggregate welfare $U^h + U^f$, and these optimal nominal interest rates are given by

$$R_*^h = \max\left\{\frac{\mu - \Psi^h}{\Psi^h - 1}, 0\right\}, \tag{62}$$

$$R_*^f = \max\left\{\frac{\mu - \Psi^f}{\Psi^f - 1}, 0\right\}, \tag{63}$$

[17] In an unpublished appendix (see Appendix B), we derive the condition under which this parameter restriction holds.

[18] In an unpublished appendix (see Appendix B), we derive the condition under which this parameter restriction holds.

where[19]

$$\Psi^h \equiv \frac{1}{1+\theta^h}\left(1+\frac{\varphi^h}{\rho}\right)(1+\mu\theta^h)\ln z^h - \mu\theta^h > 1,$$

$$\Psi^f \equiv \frac{1}{1+\theta^f}\left(1+\frac{\varphi^f}{\rho}\right)(1+\mu\theta^f)\ln z^f - \mu\theta^f > 1.$$

We see that $\Psi^h > \Phi^h$ and $\Psi^f > \Phi^f$ because $\alpha > 0$; therefore, $R_*^h < R_{ne}^h$ and $R_*^f < R_{ne}^f$. In other words, the unilateral action of each government leads to excessively high nominal interest rates in the Nash equilibrium due to a cross-country spillover effect of monetary policy under elastic labor supply, and the degree of this cross-country spillover effect is measured by α (i.e., the importance of foreign goods in domestic consumption). Intuitively, when a country lowers its nominal interest rate, the welfare gain from higher economic growth is shared by the other country, whereas the welfare cost of increasing labor supply (L_h in (57) is decreasing in R^h) falls entirely on the domestic household. As a result, the government does not lower the domestic nominal interest rate sufficiently in the Nash equilibrium; in contrast, cooperative governments would internalize the welfare gain from a higher growth rate in the other country. We summarize these results in Proposition 5.

Proposition 5 *Under elastic labor supply with only a CIA constraint on R&D, the Nash-equilibrium nominal interest rate unilaterally chosen by each government is higher than the optimal nominal interest rate chosen by cooperative governments who maximize aggregate welfare of the two countries. The Nash-equilibrium nominal interest rates are increasing in α, whereas the optimal nominal interest rates are independent of α.*

Proof. See Appendix A. ■

Substituting Φ^h into R_{ne}^h in (60), we find that R_{ne}^h is increasing in the markup μ. Similarly, substituting Ψ^h into R_*^h in (62), we find that R_*^h is also increasing in the markup μ. Intuitively, a larger markup strengthens the negative R&D externality, commonly known as the business-stealing effect, in the Schumpeterian growth model. As a result of this stronger negative R&D externality, the government would raise the nominal interest rate to decrease the equilibrium allocation of R&D labor. Taking the difference between R_{ne}^h and R_*^h and differentiating it with respect to μ, we find that

$$\frac{\partial(R_{ne}^h - R_*^h)}{\partial\mu} = \frac{1+\theta^h}{(1+\mu\theta^h)^2}\left[\frac{1}{\frac{1-\alpha}{1-\alpha+\theta^h}\left(1+\frac{\varphi^h}{\rho}\right)\ln z^h - 1} - \frac{1}{\frac{1}{1+\theta^h}\left(1+\frac{\varphi^h}{\rho}\right)\ln z^h - 1}\right] > 0.$$

In other words, the wedge between the Nash-equilibrium and optimal nominal interest rates is monotonically increasing in the market power of firms. This result differs from Arseneau (2007), who shows that a larger market power of firms tends to reduce the wedge between

[19] In an unpublished appendix (see Appendix B), we derive the first-best allocations of R&D labor in countries h and f and show that the parameter restrictions on these optimal R&D labors to be positive also imply $\Psi^h > 1$ and $\Psi^f > 1$.

the Nash-equilibrium and optimal nominal interest rates. The different implications between the two studies are due to the different CIA constraints. We have analyzed a CIA constraint on R&D investment, whereas Arseneau (2007) analyzes a CIA constraint on consumption. In the next subsection, we show that our model also delivers the insight of Arseneau (2007) under a CIA constraint on consumption, and we will explain the difference between the two CIA constraints then.

4.3 Elastic labor supply with CIA on consumption only

In this subsection, we consider the case of elastic labor supply with the CIA constraint on consumption. However, we remove the CIA constraint on R&D. In this case, (33) becomes

$$L_r^h = (\mu - 1)L_x^h - \frac{\rho}{\varphi^h}. \tag{64}$$

Solving (34), (35), (36) and (64) yields the equilibrium labor allocations in country h, and we can follow the same procedure to solve for the equilibrium allocations in country f. In this case, the equilibrium allocations of $\{L_r^h, L_x^{h,h}, L^h, L_r^f, L_x^{h,f}\}$ are given by

$$L_r^h = \frac{(\mu-1)/\mu}{1+\theta^h(1+\xi^h R^h)}\left(1+\frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \tag{65}$$

$$L_x^{h,h} = \frac{(1-\alpha)/\mu}{1+\theta^h(1+\xi^h R^h)}\left(1+\frac{\rho}{\varphi^h}\right), \tag{66}$$

$$L^h = 1 - \frac{\theta^h(1+\xi^h R^h)}{1+\theta^h(1+\xi^h R^h)}\left(1+\frac{\rho}{\varphi^h}\right), \tag{67}$$

$$L_r^f = \frac{(\mu-1)/\mu}{1+\theta^f(1+\xi^f R^f)}\left(1+\frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \tag{68}$$

$$L_x^{h,f} = \frac{\alpha/\mu}{1+\theta^f(1+\xi^f R^f)}\left(1+\frac{\rho}{\varphi^f}\right). \tag{69}$$

Substituting (65)-(69) into U^h in (47) and the analogous conditions in country f into U^f, we find that the cooperative governments' optimization problem yields

$$\frac{\partial(U^h + U^f)}{\partial R^h} < 0$$

for all values of $R^h \geq 0$. Therefore, the optimal nominal interest rates are $\{R_*^h, R_*^f\} = \{0, 0\}$ implying that the Friedman rule[20] is always optimal under the CIA constraint on consumption.

As for the Nash-equilibrium nominal interest rate, we differentiate U^h with respect to R^h and find that

$$\frac{\partial U^h}{\partial R^h} = -\frac{\theta^h \xi^h (1-\alpha)\left(1+\rho/\varphi^h\right)}{\mu\left[1+\theta^h\left(1+\xi^h R^h\right)\right]^2}\left[\frac{(1-\alpha)\left(1+\xi^h R^h\right)-1}{L_x^{h,h}\left(1+\xi^h R^h\right)} + \frac{(\mu-1)\,\varphi^h}{\rho}\ln z^h\right], \tag{70}$$

[20] In his seminal study, Friedman (1969) argues that the optimal nominal interest rate is zero.

where $L_x^{h,h}$ is given by (66). Suppose $\mu \to 1$. In this case, $\partial U^h / \partial R^h > 0$ when $R^h = 0$. In other words, the government has incentives to choose a strictly positive nominal interest rate. This effect captures the important result of an inflationary bias due to monetary policy competition in Cooley and Quadrini (2003). Equation (70) also shows that a larger markup μ would reduce this inflationary bias capturing the dampening effect of monopolistic distortion raised by Arseneau (2007). If the markup μ becomes sufficiently large, then $\partial U^h / \partial R^h < 0$ for all values of $R^h \geq 0$. In this case, the Nash-equilibrium nominal interest rate would be also zero. We summarize these results in Proposition 6.

Proposition 6 *Under elastic labor supply with only a CIA constraint on consumption, the optimal nominal interest rates are always zero. If the markup $\mu \to 1$, then the Nash-equilibrium nominal interest rate in country h is $R_{ne}^h = \alpha / [\xi^h (1-\alpha)] > 0$, which is increasing in α. As the markup μ increases, the Nash-equilibrium nominal interest rates decrease. If the markup μ is sufficiently large, then the Nash-equilibrium nominal interest rates would also be zero.*

Proof. See Appendix A. ■

Proposition 6 together with the analysis in the previous subsection show that the market power of firms has very different implications on the inflationary bias from monetary policy competition. The difference between the CIA constraint on consumption and the CIA constraint on R&D is that under the latter, an increase in the nominal interest rate leads to a reallocation of labor from R&D to production. As a result, a positive nominal interest rate would be chosen by the government if the business-stealing effect measured by the markup μ is strong. In contrast, under the CIA constraint on consumption, this reallocation effect is absent because an increase in the nominal interest rate decreases both R&D and production labors. Given that the economy features an undersupply of labor due to the distortion of monopolistic competition, the government would avoid setting too high a nominal interest rate that worsens this monopolistic distortion.

4.4 Quantitative analysis

In this subsection, we provide a numerical illustration on the growth and welfare effects of monetary policy across countries. We consider the general case with elastic labor supply and both CIA constraints on R&D and consumption. We calibrate the model using aggregate data from 1999 to 2007[21] in the Euro Area and the UK, which are two relatively open economies. To fix notation, we consider the UK as the home country h and the Euro Area as the foreign country f. To make this quantitative analysis more realistic, we introduce R&D subsidies $\{s^h, s^f\}$ into the model.[22] In this case, the free-entry condition of the R&D sector in country h becomes $V_t^h \lambda_t^h = (1 - s^h)(1 + R^h) W_t^h L_{r,t}^h$. The R&D subsidies in each country are financed by a distortionary labor-income tax subject to a balanced-budget condition

[21] We do not include data from 2008 onwards due to the international financial and debt crises.

[22] Setting $\{s^h, s^f\} = \{0, 0\}$ would strengthen our numerical results.

given by $s^h(1+R^h)W_t^h L_{r,t}^h = \tau^h W_t^h L_t^h$,[23] where τ^h is the labor-income tax rate in country h.[24]

The two-country model features the following set of parameters $\{\alpha, \rho, \mu, s^h, s^f, z^h, z^f, \varphi^h, \varphi^f, \theta^h, \theta^f, \xi^h, \xi^f, R^h, R^f\}$. Given the calibrated parameter values, we then perform a counterfactual policy experiment on the effects of decreasing the nominal interest rate R^f in the Euro Area on economic growth and social welfare in the two economies. The average value of imports from the Euro Area to the UK as a percentage of GDP in the UK is 14.9%,[25] and we use this empirical moment to calibrate the parameter α. As for the discount rate, we consider a standard value of $\rho = 0.03$. As for the markup, we consider a conservative value of $\mu = 1.03$, which is slightly less than the lower-bound value of empirical estimates reported in Jones and Williams (2000).[26] Before 2008, the effective rate of R&D subsidies in the UK was about 15%.[27] The effective rates of R&D subsidies vary substantially across countries in the Euro Area,[28] so we also consider 15% for the rate of R&D subsidies s^f in the foreign economy. The average growth rates of total factor productivity in the UK and the Euro Area are respectively 0.9% and 0.8%,[29] and we use these values to calibrate the step-size parameters $\{z^h, z^f\}$. The average value of R&D expenditures as a percentage of GDP in the UK and the Euro Area are respectively 1.8% and 1.9%,[30] and we use these values to calibrate the R&D productivity parameters $\{\varphi^h, \varphi^f\}$. As for the leisure parameters $\{\theta^h, \theta^f\}$, we calibrate them by setting the supply of labor $\{L^h, L^f\}$ to a standard value of 0.33. The ratios of M1 to GDP in the UK and the Euro Area are respectively 0.541 and 0.361,[31] and we use these values to calibrate the consumption-CIA parameters $\{\xi^h, \xi^f\}$. The average inflation rates in the UK and the Euro Area are respectively 2.1% and 2.8%,[32] and we use these values to calibrate $\{R^h, R^f\}$. We report the parameter values in Table 1.

Table 1: Calibrated parameter values

α	μ	ρ	s^h	s^f	z^h	z^f	φ^h	φ^f	θ^h	θ^f	ξ^h	ξ^f	R^h	R^f
0.149	1.03	0.03	0.15	0.15	1.20	1.15	7.29	8.07	1.95	1.97	0.52	0.34	0.060	0.066

The policy experiments are as follows. We lower the nominal interest rate R^f in the Euro Area and examine its effects on economic growth $\{g^h, g^f\}$ and social welfare $\{U^h, U^f\}$ in the two economies. We consider two policy objectives. First, we lower R^f to achieve price stability (i.e., a zero inflation rate) in the Euro Area. Second, we lower R^f to achieve the

[23] We do not consider the seigniorage revenue as a source of financing for R&D subsidies because the European Central Bank is banned from printing money to finance government expenditures.

[24] We present the equilibrium conditions in an unpublished appendix (see Appendix C).

[25] Data source: The OECD Database.

[26] Considering a larger markup would increase the welfare gains from lowering the nominal interest rate.

[27] Before 2008, firms can reduce their taxable income by 150% of their R&D expenditures. Given a corporate tax rate of 30% at that time, the government was effectively providing an R&D subsidy of 15%.

[28] For example, Warda (2009) provide data on the R&D subsidy rates in OECD countries in 2008. The subsidy rates in Spain and France were 35% and over 40% respectively, whereas the subsidy rates in Italy and Germany were respectively 12% and close to 0%.

[29] Data source: The Conference Board Total Economy Database.

[30] Data source: Eurostat (European Commission).

[31] Data source: Bank of England Statistics Database and ECB Statistical Data Warehouse.

[32] Data source: The OECD Database.

Friedman rule (i.e., a zero nominal interest rate) in the Euro Area. We report the numerical results in Table 2[33] for our benchmark value of $\alpha = 0.149$ as well as two other values of α to be discussed below. Under our benchmark value of $\alpha = 0.149$, a decrease in the nominal interest rate R^f to achieve price stability in the Euro Area leads to a welfare gain of 0.979% of consumption per year in the Euro Area and a welfare gain of 0.287% of consumption per year in the UK, which is a nonnegligible cross-country welfare effect of monetary policy. If we consider a larger reduction in the nominal interest rate R^f to achieve the Friedman rule (i.e., decreasing the nominal interest rate from 6.6% to 0%) in the Euro Area, the welfare gains are even more significant at 2.447% of consumption in the Euro Area and 0.706% of consumption in the UK. The magnitude of this cross-country welfare effect is increasing in the importance of foreign goods in domestic consumption (i.e., the value of α), which in turn determines the volume of trade between the two economies. In the data, the volume of trade between the UK and the Euro Area has been rising. The value of imports from the Euro Area as a percentage of GDP in the UK rises from 14.2% in 1999 to 17.9% in 2007. We vary α to match these values while holding other parameter values constant. At the value of $\alpha = 0.142$ in 1999, the cross-country welfare effect of decreasing the nominal interest rate to achieve the Friedman rule is 0.673% of consumption in the UK. At the value of $\alpha = 0.179$ in 2007, the cross-country welfare effect increases to 0.849% of consumption in the UK. Therefore, given increasing trade between the two economies, we conjecture that the cross-country welfare effects of monetary policy will become even stronger in the future.

Table 2: Growth and welfare effects of R^f

α	0.142	0.149	0.179	α	0.142	0.149	0.179
Price stability				Friedman rule			
Δg^h	0.005%	0.006%	0.007%	Δg^h	0.014%	0.015%	0.018%
Δg^f	0.035%	0.034%	0.033%	Δg^f	0.086%	0.085%	0.082%
ΔU^h	0.273%	0.287%	0.344%	ΔU^h	0.673%	0.706%	0.849%
ΔU^f	0.992%	0.979%	0.920%	ΔU^f	2.481%	2.447%	2.302%

5 Conclusion

In this study, we have analyzed the growth and welfare effects of monetary policy in an open-economy version of the Schumpeterian growth model with CIA constraints on consumption and R&D investment. We find that economic growth and social welfare in the domestic economy are affected by both domestic and foreign monetary policies. Furthermore, the cross-country welfare effects of monetary policy are quantitatively significant and strengthening overtime due to the rising importance of international trade. Our analysis is based on a first-generation R&D-based growth model that features the scale effect, under which a growing population leads to a rising growth rate.[34] We could allow for population growth

[33] Changes in the growth rate are expressed as changes in percentage points. Welfare gains are expressed as the usual equivalent variation in consumption.

[34] See Jones (1999) and Laincz and Peretto (2006) for a discussion of the scale effect in R&D-based growth models.

and remove the scale effect by introducing a dilution effect as in Chu and Cozzi (2013), and our results would be robust to this modification. Nevertheless, it could be fruitful to explore the effects of monetary policy in other vintages of the Schumpeterian growth model, such as the semi-endogenous growth version of the Schumpeterian model in Segerstrom (1998) and the second-generation Schumpeterian growth model in Peretto (1998) and Howitt (1999). We leave these interesting extensions to future research.

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Appendix A

Proof of Proposition 1. Taking the log of the free-entry condition $\varphi^h V_t^h = (1+R^h)W_t^h$ in the R&D sector and then differentiating it with respect to t yield

$$\frac{\dot{V}_t^h}{V_t^h} = \frac{\dot{W}_t^h}{W_t^h}. \tag{A1}$$

Substituting (23) and then (5) into (A1) yields

$$\rho - \frac{\dot{\eta}_t^h}{\eta_t^h} + \lambda_t^h - \frac{\omega_t^h}{V_t^h} = \frac{\dot{W}_t^h}{W_t^h}, \tag{A2}$$

where $\lambda_t^h = \varphi^h L_{r,t}^h$ from (25). Taking the log of (3) and differentiating with respect to t yield

$$-\frac{\dot{\eta}_t^h}{\eta_t^h} = \frac{\dot{P}_t^h}{P_t^h} + \frac{\dot{C}_t^h}{C_t^h}. \tag{A3}$$

Substituting the balanced-trade condition $P_t^{h,f} Y_t^{h,f} = \mathcal{E}_t P_t^{f,h} Y_t^{f,h}$ into (21) and (22) yields

$$\omega_t^h = \frac{\mu-1}{\mu}\left(P_t^{h,h} Y_t^{h,h} + P_t^{h,f} Y_t^{h,f}\right) = \frac{\mu-1}{\mu} P_t^h C_t^h, \tag{A4}$$

$$W_t^h L_{x,t}^h = \frac{1}{\mu}\left(P_t^{h,h} Y_t^{h,h} + P_t^{h,f} Y_t^{h,f}\right) = \frac{1}{\mu} P_t^h C_t^h, \tag{A5}$$

where the second equality of (A4) and (A5) applies (7) and (8). Taking the log of (A5) and differentiating it with respect to t yield

$$\frac{\dot{W}_t^h}{W_t^h} = \frac{\dot{P}_t^h}{P_t^h} + \frac{\dot{C}_t^h}{C_t^h} - \frac{\dot{L}_{x,t}^h}{L_{x,t}^h}. \tag{A6}$$

Substituting (A3) and (A6) into (A2) and then rearranging terms yield

$$\frac{\dot{L}_{x,t}^h}{L_{x,t}^h} = \frac{\omega_t^h}{V_t^h} - \varphi^h L_{r,t}^h - \rho. \tag{A7}$$

Substituting $\varphi^h V_t^h = (1+R^h)W_t^h$, (A4) and (A5) into (A7) yields

$$\frac{1}{\varphi^h}\frac{\dot{L}_{x,t}^h}{L_{x,t}^h} = \frac{\mu-1}{1+R^h} L_{x,t}^h - L_{r,t}^h - \frac{\rho}{\varphi^h}, \tag{A8}$$

where $L_{r,t}^h = L_t^h - L_{x,t}^h$ from the resource constraint. Substituting (A5) into (4) yields

$$L_t^h = 1 - \mu\theta^h(1+\xi^h R^h) L_{x,t}^h. \tag{A9}$$

Substituting (A9) into the resource constraint on labor yields

$$L_{r,t}^h = 1 - \mu\theta^h(1+\xi^h R^h) L_{x,t}^h - L_{x,t}^h. \tag{A10}$$

Substituting (A10) into (A8) and then rearranging terms yield

$$\frac{1}{\varphi^h}\frac{\dot{L}^h_{x,t}}{L^h_{x,t}} = \left[\frac{\mu + R^h}{1 + R^h} + \mu\theta^h(1 + \xi^h R^h)\right] L^h_{x,t} - \left(1 + \frac{\rho}{\varphi^h}\right), \tag{A11}$$

which is a one-dimensional differential equation in $L^h_{x,t}$. It is easy to see that the dynamics of $L^h_{x,t}$ is characterized by saddle-point stability such that $L^h_{x,t}$ must jump to its interior steady-state value, which in turn implies that L^h_t and $L^h_{r,t}$ also jump to their steady-state values according to (A9) and (A10). An analogous proof would show that labor allocations in country f also jump to their steady-state values. ■

Proof of Proposition 2. Setting $\dot{L}^h_{x,t} = 0$ in (A11) yields the steady-state value of $L^h_{x,t}$ given by

$$L^h_x = \frac{1 + R^h}{\mu + R^h + \mu\theta^h(1 + \xi^h R^h)(1 + R^h)}\left(1 + \frac{\rho}{\varphi^h}\right). \tag{A12}$$

Substituting (A12) into (A10) and rearranging terms yield the steady-state value of $L^h_{r,t}$ in (37). Similarly, substituting (A12) into (A9) yields the steady-state value of L^h_t in (40). Finally, we combine $L^{h,h}_x + L^{f,h}_x = L^h_x$ and (36) to derive the steady-state values of $L^{h,h}_x$ and $L^{f,h}_x$ in (38) and (39). ■

Proof of Proposition 4. The analogous expression of (47) for U^f is given by

$$\rho U^f = (1-\alpha)\ln L^{f,f}_x + \alpha \ln L^{f,h}_x + \frac{(1-\alpha)(\varphi^f \ln z^f)}{\rho} L^f_r + \frac{\alpha(\varphi^h \ln z^h)}{\rho} L^h_r + \theta^f \ln(1 - L^f). \tag{A13}$$

The analogous expressions of (37)-(40) in country f are

$$L^f_r = \frac{\mu - 1}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)}\left(1 + \frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \tag{A14}$$

$$L^{f,f}_x = (1-\alpha)L^f_x = \frac{(1-\alpha)(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)}\left(1 + \frac{\rho}{\varphi^f}\right), \tag{A15}$$

$$L^{h,f}_x = \alpha L^f_x = \frac{\alpha(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)}\left(1 + \frac{\rho}{\varphi^f}\right), \tag{A16}$$

$$L^f = 1 - \frac{\mu\theta^f(1 + \xi^f R^f)(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)}\left(1 + \frac{\rho}{\varphi^f}\right). \tag{A17}$$

Under inelastic labor supply, we set $\theta^h = \theta^f = 0$ in (37)-(40) and (A14)-(A17). Then, we substitute the resulting expressions into $U^h + U^f$ from (47) and (A13) and differentiate it with respect to $\{R^h, R^f\}$ to obtain the optimal nominal interest rates given by

$$R^h_* = \max\left\{\frac{\mu - \Omega^h}{\Omega^h - 1}, 0\right\}, \tag{A18}$$

$$R_*^f = \max\left\{\frac{\mu - \Omega^f}{\Omega^f - 1}, 0\right\}, \tag{A19}$$

where $\Omega^h \equiv (1+\varphi^h/\rho)\ln z^h > 1$ and $\Omega^f \equiv (1+\varphi^f/\rho)\ln z^f > 1$. Therefore, $\{R_*^h, R_*^f\} = \{R_{ne}^h, R_{ne}^f\}$ in (53) and (54). As for the comparative statics, R_*^h is decreasing in Ω^h, which in turn is increasing in φ^h and z^h but decreasing in ρ. Similarly, R_*^f is decreasing in Ω^f, which in turn is increasing in φ^f and z^f but decreasing in ρ. ■

Proof of Proposition 5. In the absence of the CIA constraint on consumption, we set $\xi^h = \xi^f = 0$ in (37)-(40) and (A14)-(A17). The government in country h chooses R^h to maximize the welfare of the representative household in country h. We substitute (37), (38), (40), (A14) and (A16) under $\xi^h = \xi^f = 0$ into U^h in (47) and then differentiate it with respect to R^h to obtain the Nash-equilibrium nominal interest rate R_{ne}^h in country h given by (60). Similarly, the government in country f chooses R^f to maximize the welfare of the representative household in country f. We substitute (A14), (A15), (A17), (37) and (39) under $\xi^h = \xi^f = 0$ into U^f in (A13) and then differentiate it with respect to R^f to obtain the Nash-equilibrium nominal interest rate R_{ne}^f in country f given by (61). As for the comparative statics, R_{ne}^h in (60) is decreasing in Φ^h, which in turn is decreasing in α; therefore, R_{ne}^h is increasing in α. Similarly, R_{ne}^f in (61) is decreasing in Φ^f, which in turn is also decreasing in α; therefore, R_{ne}^f is also increasing in α. The cooperative governments choose $\{R^h, R^f\}$ to maximize the welfare of both domestic and foreign households. We set $\xi^h = \xi^f = 0$ in (37)-(40) and (A14)-(A17) and substitute the resulting expressions into $U^h + U^f$ from (47) and (A13). Then, we differentiate $U^h + U^f$ with respect to $\{R^h, R^f\}$ to obtain the optimal nominal interest rates given by (62) and (63), in which Ψ^h and Ψ^f are both independent of α. ■

Proof of Proposition 6. In the absence of the CIA constraint on R&D, the equilibrium labor allocations $\{L_r^h, L_x^{h,h}, L^h, L_r^f, L_x^{h,f}\}$ are given by (65)-(69). For the Nash-equilibrium nominal interest rate R_{ne}^h in country h, we substitute (65)-(69) into U^h in (47) and then differentiate it with respect to R^h. We find that

$$\frac{\partial U^h}{\partial R^h} = -\frac{\theta^h \xi^h (1-\alpha)\left(1+\rho/\varphi^h\right)}{\mu\left[1+\theta^h\left(1+\xi^h R^h\right)\right]^2}\left[\frac{(1-\alpha)\left(1+\xi^h R^h\right)-1}{L_x^{h,h}\left(1+\xi^h R^h\right)} + \frac{(\mu-1)\varphi^h}{\rho}\ln z^h\right]. \tag{A20}$$

Equation (A20) shows that as $\mu \to 1$, the Nash-equilibrium nominal interest rate is $R_{ne}^h = \alpha/[\xi^h(1-\alpha)] > 0$, which is increasing in α. Furthermore, if μ is sufficiently large, then $\partial U^h/\partial R^h < 0$ for all values of R^h; in this case, $R_{ne}^h = 0$. The analogous expressions of (65)-(69) for $\{L_r^f, L_x^{f,f}, L^f, L_r^h, L_x^{f,h}\}$ are

$$L_r^f = \frac{(\mu-1)/\mu}{1+\theta^f(1+\xi^f R^f)}\left(1+\frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \tag{A21}$$

$$L_x^{f,f} = \frac{(1-\alpha)/\mu}{1+\theta^f(1+\xi^f R^f)}\left(1+\frac{\rho}{\varphi^f}\right), \tag{A22}$$

$$L^f = 1 - \frac{\theta^f(1+\xi^f R^f)}{1+\theta^f(1+\xi^f R^f)}\left(1+\frac{\rho}{\varphi^f}\right), \tag{A23}$$

$$L_r^h = \frac{(\mu-1)/\mu}{1+\theta^h(1+\xi^h R^h)}\left(1+\frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \tag{A24}$$

$$L_x^{f,h} = \frac{\alpha/\mu}{1+\theta^h(1+\xi^h R^h)}\left(1+\frac{\rho}{\varphi^h}\right). \tag{A25}$$

For the Nash-equilibrium nominal interest rate R_{ne}^f in country f, we substitute (A21)-(A25) into U^f in (A13) and differentiate it with respect to R^f. We find that

$$\frac{\partial U^f}{\partial R^f} = -\frac{\theta^f \xi^f (1-\alpha)\left(1+\rho/\varphi^f\right)}{\mu\left[1+\theta^f\left(1+\xi^f R^f\right)\right]^2}\left[\frac{(1-\alpha)\left(1+\xi^f R^f\right)-1}{L_x^{f,f}\left(1+\xi^f R^f\right)} + \frac{(\mu-1)\varphi^f}{\rho}\ln z^f\right]. \tag{A26}$$

Equation (A26) shows that as $\mu \to 1$, the Nash-equilibrium nominal interest rate is $R_{ne}^f = \alpha/[\xi^f(1-\alpha)] > 0$, which is increasing in α. Furthermore, if μ is sufficiently large, then $\partial U^f/\partial R^f < 0$ for all values of R^f; in this case, $R_{ne}^f = 0$. As for the optimal nominal interest rates $\{R_*^h, R_*^f\}$, we add U^h from (47) and U^f from (A13) and substitute (65)-(69) and (A21)-(A25) into $U^h + U^f$. Then, we differentiate $U^h + U^f$ with respect to $\{R^h, R^f\}$ and find that

$$\frac{\partial(U^h+U^f)}{\partial R^h} = -\theta^h\xi^h\left[\frac{\xi^h R^h}{\left[1+\theta^h\left(1+\xi^h R^h\right)\right]\left(1+\xi^h R^h\right)} + \frac{(\mu-1)\left(1+\varphi^h/\rho\right)\ln z^h}{\mu\left[1+\theta^h\left(1+\xi^h R^h\right)\right]^2}\right] < 0, \tag{A27}$$

$$\frac{\partial(U^h+U^f)}{\partial R^f} = -\theta^f\xi^f\left[\frac{\xi^f R^f}{\left[1+\theta^f\left(1+\xi^f R^f\right)\right]\left(1+\xi^f R^f\right)} + \frac{(\mu-1)\left(1+\varphi^f/\rho\right)\ln z^f}{\mu\left[1+\theta^f\left(1+\xi^f R^f\right)\right]^2}\right] < 0. \tag{A28}$$

Therefore, given the zero lower bound on the nominal interest rates, it must be the case that $\{R_*^h, R_*^f\} = \{0, 0\}$. ■

Appendix B: Not for publication

In this appendix, we show the conditions under which $\{\Omega^h, \Omega^f, \Psi^h, \Psi^f, \Phi^h, \Phi^f\} > 1$. First, we derive the first-best allocations of labor. Combining (47) and (A13) yields

$$\begin{aligned}\rho(U^h + U^f) &= (1-\alpha)(\ln L_x^{h,h} + \ln L_x^{f,f}) + \alpha(\ln L_x^{h,f} + \ln L_x^{f,h}) + \frac{\varphi^h \ln z^h}{\rho} L_r^h + \frac{\varphi^f \ln z^f}{\rho} L_r^f \\ &\quad + \theta^h \ln(1-L^h) + \theta^f \ln(1-L^f). \qquad \text{(B1)}\end{aligned}$$

The social planner maximizes (B1) subject to the following resource constraints:

$$L_x^{h,h} + L_x^{f,h} + L_r^h = L^h, \qquad \text{(B2)}$$

$$L_x^{f,f} + L_x^{h,f} + L_r^f = L^f. \qquad \text{(B3)}$$

The first-best optimal allocations of R&D labor and labor supply in country h are

$$L_r^h = 1 - \frac{\rho(1+\theta^h)}{\varphi^h \ln z^h} > 0, \qquad \text{(B4)}$$

The parameter restriction on $L_r^h > 0$ implies $\Psi^h > 1$. To see this result, from (63),

$$\Psi^h > 1 \Leftrightarrow \left(1 + \frac{\varphi^h}{\rho}\right) \frac{\ln z^h}{1+\theta^h} > 1. \qquad \text{(B5)}$$

From (B4), we see that

$$L_r^h > 0 \Leftrightarrow \left(\frac{\varphi^h}{\rho}\right) \frac{\ln z^h}{1+\theta^h} > 1. \qquad \text{(B6)}$$

Therefore, if $L_r^h > 0$, then $\Psi^h > 1$. An analogous procedure in country f would show that if $L_r^f > 0$, then $\Psi^f > 1$. In the case of elastic labor supply, we have

$$\Omega^h > 1 \Leftrightarrow \left(1 + \frac{\varphi^h}{\rho}\right) \ln z^h > 1, \qquad \text{(B7)}$$

$$L_r^h|_{\theta^h=0} > 0 \Leftrightarrow \left(\frac{\varphi^h}{\rho}\right) \ln z^h > 1. \qquad \text{(B8)}$$

Therefore, if $L_r^h|_{\theta^h=0} > 0$, then $\Omega^h > 1$. An analogous procedure for country f would show that if $L_r^f|_{\theta^f=0} > 0$, then $\Omega^f > 1$.

In the rest of this appendix, we consider the following alternative maximization problem: a social planner representing only country h unilaterally maximizes

$$\rho U^h = (1-\alpha)\ln L_x^{h,h} + \alpha \ln L_x^{h,f} + \frac{(1-\alpha)(\varphi^h \ln z^h)}{\rho} L_r^h + \frac{\alpha(\varphi^f \ln z^f)}{\rho} L_r^f + \theta^h \ln(1-L^h) \qquad \text{(B9)}$$

subject to (B2) and the equilibrium condition in (36), which ensures that $L_x^{f,h}$ and $L_x^{h,f}$ are positive. In this case, the planner's allocation of R&D labor in country h is

$$L_r^h = 1 - \frac{\rho(1-\alpha+\theta^h)}{(1-\alpha)(\varphi^h \ln z^h)} > 0. \qquad \text{(B10)}$$

The parameter restriction on $L_r^h > 0$ implies $\Phi^h > 1$. To see this result, from (60),

$$\Phi^h > 1 \Leftrightarrow \frac{1-\alpha}{1-\alpha+\theta^h}\left(1+\frac{\varphi^h}{\rho}\right)\ln z^h > 1. \tag{B11}$$

From (B10), we see that

$$L_r^h > 0 \Leftrightarrow \frac{1-\alpha}{1-\alpha+\theta^h}\left(\frac{\varphi^h}{\rho}\right)\ln z^h > 1. \tag{B12}$$

Therefore, if $L_r^h > 0$, then $\Phi^h > 1$. An analogous procedure in country f would show that if $L_r^f > 0$, then $\Phi^f > 1$.

Appendix C: Not for publication

In this appendix, we present the equilibrium conditions in the presence of R&D subsidies financed by a distortionary wage-income tax in each country. In this case, (33)-(36) become

$$L_r^h = \frac{\mu - 1}{(1 - s^h)(1 + R^h)} L_x^h - \frac{\rho}{\varphi^h}, \tag{C1}$$

$$L^h = 1 - \frac{\mu\theta^h(1 + \xi^h R^h)L_x^h}{1 - \tau^h}, \tag{C2}$$

$$L_x^{h,h} + L_x^{f,h} + L_r^h = L^h, \tag{C3}$$

$$L_x^{f,h} = \frac{\alpha}{1 - \alpha} L_x^{h,h}. \tag{C4}$$

To close this system of equations, we use the following balanced-budget condition:

$$s^h(1 + R^h)L_r^h = \tau^h L^h. \tag{C5}$$

Therefore, we can solve the five endogenous variables $\{L_r^h, L^h, L_x^{h,h}, L_x^{f,h}, \tau^h\}$ using (C1)-(C5). There is a set of analogous equations in country f.