A Generalized Quality-Ladder Growth Model with Patent Breadth: Quantifying the Effects of Blocking Patents on RD

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August 2007

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Abstract

Why is there so little R&D in the US? To quantify the effects of blocking patents on R&D, this paper firstly develops a tractable framework to model the transition dynamics of an economy with patent breadth and blocking patents in a generalized quality-ladder growth model. In this dynamic general-equilibrium setting, a dynamic distortion on capital accumulation that has been neglected by previous studies on patent policy is identified. Then, the model is applied to the aggregate data to quantify the extents of underinvestment in R&D and inefficiency arising from blocking patents. This numerical exercise suggests a number of findings. Firstly, the market economy underinvests in R&D so long as a non-negligible fraction of long-run TFP growth is driven by R&D. Secondly, eliminating blocking patents increases R&D by about two to six times and hence is an effective solution to the potential problem of R&D underinvestment. Finally, the effects of eliminating blocking patents on consumption in the long run and during the transition dynamics are considered.

Keywords**:** blocking patents, endogenous growth, intellectual property rights, patent breadth, R&D

JEL classification**:** O31, O34

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"Today, most basic and applied researchers are effectively standing on top of a huge pyramid… Of course, a pyramid can rise to far greater heights than could any one person... But what happens if, in order to scale the pyramid and place a new block on the top, a researcher must gain the permission of each person who previously placed a block in the pyramid, perhaps paying a royalty or tax to gain such permission? Would this system of intellectual property rights slow down the construction of the pyramid or limits its heights? … To complete the analogy, *blocking patents* play the role of the pyramid's building blocks." – Carl Shapiro (2001)

1. Introduction

What are the effects of blocking patents on research and development (R&D)? In an environment with only horizontal innovations, each invention is a different variety from each other. In this setting, a higher level of patent breadth increases the differentiability of each product that potentially results in a higher markup, a larger amount of monopolistic profits, and consequently, enhanced incentives for R&D. In a more complicated and realistic environment with sequential innovations, patent breadth takes the form of lagging breadth and leading breadth. Lagging breadth provides patent protection against imitation while leading breadth provides patent protection against subsequent innovations, which may infringe existing patents. A broadening of *leading* breadth may enhance or dampen the incentives for R&D depending on the extent of blocking patents, which is determined by the profit-sharing rule in patent pools.

To quantify the effects of blocking patents on R&D and consumption, this paper firstly develops a tractable framework to model the transition dynamics of an economy with patent breadth and blocking patents in a generalized quality-ladder growth model. In this dynamic general-equilibrium (DGE) setting, this paper analytically derives and identifies a *dynamic* distortionary effect on capital accumulation, which has been neglected by previous studies on patent policy focusing mostly on the *static* distortionary effect of markup pricing. Then, the model is applied to the aggregate data of the US's economy in order to quantify the extents of underinvestment in R&D and inefficiency arising from blocking patents.

The numerical exercise suggests a number of findings. Firstly, eliminating blocking patents increases the equilibrium amount of R&D spending by about two to six times. Secondly, the market economy underinvests in R&D relative to the first-best optimum so long as a non-negligible fraction of long-run total factor productivity (TFP) growth is driven by R&D. To understand this finding, the quality-ladder growth model involves multiple externalities in R&D: (a) a negative intratemporal congestion or duplication externality; (b) a positive or negative externality in intertemporal knowledge spillover; (c) the static consumer-surplus appropriability problem which is a positive externality; (d) the dynamic surplus appropriability problem in the form of sequential innovations which is a also a positive externality; (e) the business-stealing effect from creative destruction which is a negative externality; and (f) the negative effects of blocking patents on R&D in the case of suboptimal profit-sharing rules in patent pools. Given the existence of positive and negative externalities, whether the market economy over or under-invests in R&D depends crucially on the extents of intratemporal duplication and intertemporal spillover, which in turn are imputed from the balanced-growth condition between long-run TFP growth and R&D. Therefore, the larger is the fraction of long-run TFP growth driven by R&D, the more likely it is for the market economy to underinvest in R&D. Finally, the effects of eliminating blocking patents on consumption in the long run and during the transition dynamics are considered. When blocking patents are eliminated, the balanced-growth level of consumption increases significantly so long as a non-negligible fraction of TFP is driven by R&D. During the transition dynamics, the economy does not always experience a significant fall in consumption in response to the resource reallocation away from the production sector to the R&D sector. Over a range of parameters, upon eliminating blocking patents, consumption gradually rises towards the new balanced-growth path by reducing physical investment and temporarily running down the capital stock. This finding contrasts with Kwan and Lai (2003), whose model does not feature capital accumulation and hence predicts consumption losses from resource reallocation during the transition path.

Shapiro (2001) describes the current innovation process as a “dense web of overlapping intellectual property rights that a company must hack its way through in order to actually commercialize new technology”, and he refers to this web as a “patent thicket”. The current paper develops a quantitative framework to evaluate the effects of this patent thicket on R&D and provides an effective solution to the potential problem of R&D underinvestment identified by Jones and Williams (1998) and (2000). Jones and Williams (1998) develop a method to calculate the social rate of return to R&D based on endogenous-growth theory and find that the socially optimal amount of R&D spending is at least two to four times larger than the actual amount. Jones and Williams (2000) adopt a different approach by calibrating a variety-expanding growth model to the data and obtain a similar conclusion that there is underinvestment in R&D over a wide range of parameters.[1] The current paper follows this latter approach by calibrating a generalized quality-ladder growth model with patent breadth in sequential innovations to show that the potential problem of R&D underinvestment arises from the inefficiency of blocking patents and eliminating them can be an effective solution. Furthermore, the calibration exercise takes into consideration Comin's (2004) critique that long-run TFP growth may not be solely driven by R&D.

The current paper also complements the theoretical and qualitative studies on leading breadth from the patent-design literature,[2] such as Green and Scotchmer (1995), O'Donoghue *et al* (1998) and Hopenhayn *et al* (2006), by providing a quantitative DGE analysis using the aggregate data. O'Donoghue and Zweimuller (2004) is the first study that merges the patent-design and endogenous growth literatures to analyze the effects of patentability requirement, lagging and leading breadth on economic growth in a simple quality-ladder growth model. However, their focus was not in quantifying the effects of blocking patents on R&D. In addition, the current paper generalizes their model in a number of dimensions in order to perform a quantitative analysis on the transition dynamics. Goh and Olivier (2002) analyze the welfare effects of patent breadth in a two-sector variety-expanding growth model, and Grossman and Lai (2004) analyze the welfare effects of strengthening patent protection in developing countries as a result of the TRIPS agreement using a multi-country variety-expanding model. However, these studies do not analyze patent breadth in an environment with sequential innovations. Li (2001) analyzes the optimal policy mix

[1] Stokey (1995) also calibrates an R&D-growth model to examine the extents of R&D underinvestment in the market economy.
[2] The seminal work on optimal patent length is Nordhaus (1969). Some other recent studies on optimal patent design include Tandon (1982), Gilbert and Shapiro (1990), Klemperer (1990), O'Donoghue (1998), Hunt (1999) and Scotchmer (2004). Judd (1985) provides the first dynamic general equilibrium analysis on optimal patent length.

of R&D subsidy and lagging breadth in a quality-ladder model with endogenous step size, but he does not consider leading breadth. Furthermore, all the abovementioned studies are qualitatively oriented and do not feature capital accumulation so that the dynamic distortion is absent.

Laitner (1982) identifies in an exogenous growth model with overlapping generations of households that the existence of an oligopolistic sector and its resulting pure profits as financial assets creates both the usual static distortion and an additional dynamic distortion on capital accumulation due to the crowding out of households' portfolio space, and he finds that the latter is more significant than the former. The current paper extends this study to show that this dynamic distortion also plays an important role and through a different channel in an R&D-driven endogenous growth model in which both patents and physical capital are owned by households as financial assets.

In terms of quantitative analysis, this paper relates to Kwan and Lai (2003) and Chu (2007). Kwan and Lai (2003) numerically evaluate the effects of extending the effective lifetime of patent in the variety-expanding model originating from Romer (1990) and find substantial welfare gains despite the temporary consumption losses during the transition path in their model. Chu (2007) uses a generalized variety-expanding model and finds that whether or not an extension in the patent length is effective in stimulating R&D depends crucially on the patent-value depreciation rate. At the empirical range of patent-value depreciation rates estimated by previous studies, patent extension has only limited effects on R&D and thus social welfare. Therefore, Chu (2007) and the current paper together provide a comparison on the effectiveness of increasing patent length and eliminating blocking patents in solving the R&D underinvestment problem. The crucial difference between these two policy instruments arises because patent extension increases future monopolistic profits while eliminating blocking patents raises current monopolistic profits for the inventors.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 calibrates the model and numerically evaluates the effects of eliminating blocking patents. The final section concludes with some important caveats. Appendix I contains the proofs.

2. The Model

The model is a generalized version of Grossman and Helpman (1991) and Aghion and Howitt (1992). To prevent the model from overestimating the social benefits of R&D and hence the extents of R&D underinvestment, long-run TFP growth is assumed to be driven by R&D as well as an exogenous process as in Comin (2004). In order to perform a more realistic calibration, the model is further modified to include physical capital, which is a factor input for the production of intermediate goods and R&D, and the final goods can be used for consumption or investment in capital. Finally, the class of first-generation R&D-driven endogenous growth models, such as Grossman and Helpman (1991) and Aghion and Howitt (1992), exhibits scale effects and is inconsistent with the empirical evidence in Jones (1995a).[3] In the present model, scale effects are eliminated by assuming decreasing *individual* R&D productivity as in Segerstrom (1998), which becomes a semi-endogenous growth model.[4]

The various components of the model are presented in Sections 2.1–2.7, and the decentralized equilibrium is defined in Section 2.8. Section 2.9 summarizes the laws of motion that characterize the transition dynamics, and Section 2.10 analyzes the balanced-growth path. Section 2.11 derives the first-best optimal allocations.

2.1. Representative Household

The infinitely-lived representative household maximizes life-time utility that is a function of per-capita consumption c_t of the numeraire final goods and is assumed to have the iso-elastic form given by

(1) $$U = \int_0^{\infty} e^{-(\rho-n)t} \frac{c_t^{1-\sigma}}{1-\sigma} dt .$$

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[3] See, e.g. Jones (1999) for an excellent theoretical analysis on scale effects.

[4] In a semi-endogenous growth model, the balanced-growth rate is determined by the exogenous labor-force growth rate. An increase in the share of R&D factor inputs raises the *level* of the balanced growth path while holding the balanced-growth rate constant. Since increasing R&D has no long-run growth effect in this model, the estimated effects on consumption in the numerical exercises are likely to be more conservative than in other fully endogenous growth models.

$\sigma \geq 1$ is the inverse of the elasticity of intertemporal substitution. The household has $L_t = L_0 \exp(nt)$ members at time *t*. The population size at time 0 is normalized to one, and $n > 0$ is the exogenous population growth rate. ρ is the subjective discount rate. To ensure that lifetime utility is bounded, it is assumed that $\rho > n$. The household maximizes (1) subject to a sequence of budget constraints given by

(2) $$\dot{a}_t = a_t(r_t - n) + w_t - c_t .$$

Each member of the household inelastically supplies one unit of homogenous labor in each period to earn a real wage income w_t. a_t is the value of risk-free financial assets in the form of patents and physical capital owned by each household member, and r_t is the real rate of return on these assets. The familiar Euler equation derived from the intertemporal optimization is

(3) $$\dot{c}_t = c_t(r_t - \rho)/\sigma .$$

2.2. Final Goods

This sector is characterized by perfect competition, and the producers take both the output price and input prices as given. The production function for the final goods Y_t is a Cobb-Douglas aggregator of a continuum of differentiated quality-enhancing intermediate goods $X_t(j)$ for $j \in [0,1]$ given by

(4) $$Y_t = \exp\left(\int_0^1 \ln X_t(j)dj \right).$$ [5]

The familiar aggregate price index is

(5) $$P_t = \exp\left(\int_0^1 \ln P_t(j)dj \right) = 1,$$

[5] To maintain the analytical tractability of the aggregate conditions, a Cobb-Douglas aggregator instead of the more general CES aggregator is adopted. With the CES aggregator, it becomes very difficult to derive the aggregate conditions when there are both competitive and monopolistic industries in the intermediate-goods sector. Furthermore, computation of the transition dynamics becomes possible under the Cobb-Douglas aggregator. Although the arrival rate of innovations varies along the transitional path, a tractable form for the law of motion for aggregate technology can still be derived under the Cobb-Douglas aggregator but not under the CES aggregator.

and the demand curve for each variety of intermediate goods is

(6) $$P_t(j)X_t(j) = Y_t\,.$$

2.3. Intermediate Goods

There is a continuum of industries producing the differentiated quality-enhancing intermediate goods $X_t(j)$ for $j \in [0,1]$. A fraction $\theta \in [0,1)$ of the industries is characterized by perfect competition because innovations in these industries are assumed to be non-patentable. Each of the remaining industries is dominated by a temporary industry leader, who owns the patent for the latest R&D-driven technology for production. Without loss of generality, the industries are ordered such that industries $j' \in [0,\theta)$ are competitive and industries $j \in [\theta,1]$ are monopolistic. The production function in each industry has constant returns to scale in labor and capital inputs and is given by

(7) $$X_t(j) = z^{m_t(j)} Z_t K_{x,t}^{\alpha}(j) L_{x,t}^{1-\alpha}(j)$$

for $j \in [0,1]$. $K_{x,t}(j)$ and $L_{x,t}(j)$ are respectively the capital and labor inputs for producing intermediate-goods *j* at time *t*. $Z_t = Z_0 \exp(g_Z t)$ represents an exogenous process of productivity improvement that is common across all industries and is freely available to all producers. $z^{m_t(j)}$ is industry *j*'s level of R&D-driven technology, which is increasing over time through R&D investment and successful innovations. $z > 1$ is the exogenous step-size of a technological improvement arising from each innovation. $m_t(j)$, which is an integer, is the number of innovations that has occurred in industry *j* as of time *t*. The marginal cost of production in industry *j* is

(8) $$MC_t(j) = \frac{1}{z^{m_t(j)} Z_t}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha},$$

where R_t is the rental price of capital. The optimal price for the leaders in the monopolistic industries is a constant markup $\mu(z,\eta)$ over the marginal cost of production given by

(9) $$P_t(j) = \mu(z,\eta)MC_t(j)$$

for $j \in [\theta,1]$. The markup $\mu(z,\eta)$ is a function of the quality step size z and the level of patent breadth η (to be defined in Section 2.4). The competitive industries are characterized by competitive pricing such that

(10) $$P_t(j') = MC_t(j')$$

for $j' \in [0,\theta)$. The aggregate price level is

(11) $$P_t = \tilde{\mu}(z,\eta,\theta)MC_t,$$

where $\tilde{\mu}(z,\eta,\theta) \equiv \mu(z,\eta)^{1-\theta}$ is the aggregate markup in the economy. The aggregate marginal cost is

(12) $$MC_t = \exp\left(\int_0^1 \ln MC_t(j)dj\right).$$

2.4. Patent Breadth

Before providing the underlying derivations, this section firstly presents the Bertrand equilibrium price and the amount of monopolistic profits generated by an invention and captured by a patent pool under different levels of patent breadth, which is denoted by η.

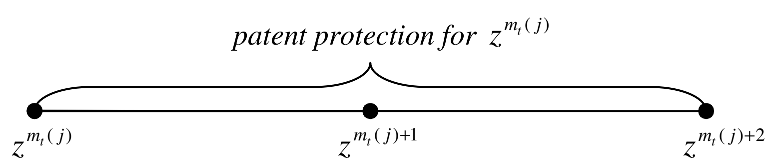
(13) $$P_t(j) = z^{\eta}MC_t(j)$$

(14) $$\pi_t(j) = (z^{\eta}-1)MC_t(j)X_t(j)$$

for $\eta \in \{1,2,3,...\}$ and $j \in [\theta,1]$. The expression for the equilibrium price is consistent with the seminal work of Gilbert and Shapiro's (1990) interpretation of "breadth as the ability of the patentee to raise price." A broader patent breadth corresponds to a larger η, and vice versa. Therefore, an increase in patent breadth *potentially* enhances the incentives for R&D by raising the amount of monopolistic profits generated by each invention but worsens the distortionary effects of markup pricing.

The patent-design literature has identified and analyzed two types of patent breadth in an environment with sequential innovations: (a) lagging breadth; and (b) leading breadth. In a standard quality-ladder growth model, lagging breadth (i.e. patent protection against imitation) is assumed to be complete while leading breadth (i.e. patent protection against subsequent innovations) is assumed to be zero. The following analysis focuses on non-zero leading breadth, and the formulation originates from O'Donoghue and Zweimuller (2004). A discussion of incomplete lagging breadth is in Appendix II.

The level of patent breadth $\eta = \eta_{lag} + \eta_{lead}$ can be decomposed into lagging breadth denoted by $\eta_{lag} \in (0,1]$ and leading breadth denoted by $\eta_{lead} \in \{0,1,2,...\}$. In the following, complete lagging breadth is assumed such that $\eta = 1 + \eta_{lead}$. Nonzero leading breadth protects patentholders against subsequent innovations and gives the patentholders property rights over future inventions. For example, if $\eta_{lead} = 1$, then the most recent innovation infringes the patent of the second-most recent inventor. If $\eta_{lead} = 2$, then the most recent innovation infringes the patents of the second-most and the third-most recent inventors, etc. The following diagram illustrates the concept of nonzero leading breadth with an example of leading breadth equal two.



Therefore, nonzero leading breadth facilitates the new industry leader and the previous inventors, whose patents are infringed, to consolidate market power through licensing agreements and the formation of a patent pool resulting in a higher markup.[6] The Bertrand equilibrium price with leading breadth is

$$P_t(j) = z^{1+\eta_{lead}} MC_t(j) \tag{15}$$

for $\eta_{lead} \in \{0,1,2,...\}$ and $j \in [\theta,1]$. Assumption 1 is *sufficient* to derive this equilibrium markup price.

[6] See, e.g. Gallini (2002) and O'Donoghue and Zweimuller (2004), for a discussion on market-power consolidation through licensing agreements.

Assumption 1: *An infringed patentholder cannot become the next industry leader while she is still covered by a licensing agreement in that industry.*[7]

Then, the total amount of monopolistic profits captured by the patent pool at time t is

(16) $$\pi_t(j) = (z^{1+\eta_{lead}} - 1)MC_t(j)X_t(j)$$

for $\eta_{lead} \in \{0,1,2,...\}$ and $j \in [\theta,1]$.

The share of profits obtained by each generation of patentholders in the patent pool depends on the profit-sharing rule (i.e. the terms in the licensing agreement). A stationary bargaining outcome is assumed to simplify the analysis.

Assumption 2: *The set of profit-sharing rule is symmetric across industries and is stationary. For each degree of leading breadth* $\eta_{lead} \in \{0,1,2,...\}$, *the profit-sharing rule is* $\sigma^{\eta_{lead}} = (\sigma_1,...,\sigma_\eta) \in [0,1]$, *where* σ_i *is the share of profits received by the i-th most recent inventor, and* $\sum_{i=1}^{\eta} \sigma_i = 1$.

Although the shares of profits and licensing fees eventually received by the owner of an invention are constant overtime, the present value of profits is determined by the actual profit-sharing rule. The two extreme cases are: (a) *complete frontloading* $\sigma^{\eta_{lead}} = (1,0,...,0)$; and (b) *complete backloading* $\sigma^{\eta_{lead}} = (0,0,...,1)$. Complete frontloading maximizes the incentives on R&D provided by leading breadth by maximizing the present value of profits received by an inventor. The opposite effect of blocking patents arises when profits are backloaded, and complete backloading maximizes this damaging

[7] The sufficiency of this assumption in determining the markup price is most easily understood with an example. Suppose leading breadth is one and lagging breadth is complete, the lower bound on the profit-maximizing markup is the square of z, which is the limit price from the collusion of the most recent and the second-most recent inventors against the third-most recent inventor, whose patent is not infringed upon by the most recent invention. In this example, the limit-pricing markup would be even larger if the third-most recent inventor happens to be the new industry leader. Continuing this reasoning, the markup could grow without bound; therefore, Assumption 1 is made to rule out this possibility. The empirical plausibility of this assumption is appealed to the existence of antitrust policy.

effect on the incentives for R&D. Section 2.7 derives the law of motion for the market value of ownership in patent pools for each generation of patentholders.

2.5. Aggregation

Define $A_t \equiv \exp\left(\int_0^1 m_t(j)dj \ln z\right)$ as the aggregate level of R&D-driven technology. Also, define total labor and capital inputs for production as $K_{x,t} = \int_0^1 K_{x,t}(j)dj$ and $L_{x,t} = \int_0^1 L_{x,t}(j)dj$ respectively.

Lemma 1: *The aggregate production function for the final goods is*

(17) $$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha},$$

where $\vartheta(\eta) \equiv (z^{\eta})^{\theta} / (z^{\eta}\theta + 1 - \theta)$ *is decreasing in* η *for* $\theta \in (0,1)$.

$\vartheta(\eta)$ represents the static distortionary effect of markup pricing. Markup pricing in the monopolistic industries distorts production towards the competitive industries and reduces the output of the final goods. Also, $\vartheta(\eta)$ is initially decreasing in θ and subsequently increasing with $\vartheta(\eta) = 1$ for $\theta \in \{0,1\}$. Therefore, at least over a range of parameters, the static distortionary effect becomes increasingly severe as the fraction of competitive industries increases.

The market-clearing condition for the final goods is

(18) $$Y_t = C_t + I_t,$$

where $C_t = L_t c_t$ is the aggregate consumption and I_t is the investment in physical capital. The factor payments for the final goods are

(19) $$Y_t = w_t L_{x,t} + R_t K_{x,t} + \pi_t.$$

$\pi_t = \int_\theta^1 \pi_t(j)dj$ is the total amount of monopolistic profits. Substituting (7) and (8) into (14) and then summing over all monopolistic industries yields

(20) $$\pi_t = (1-\theta)\left(\frac{z^\eta - 1}{z^\eta}\right)Y_t .$$

Therefore, the growth rate of monopolistic profits equals the growth rate of output. The amount of factor payments for labor and capital inputs in the intermediate-goods sector are respectively

(21) $$w_t L_{x,t} = (1-\alpha)\left(\frac{z^\eta \theta + 1 - \theta}{z^\eta}\right)Y_t ,$$

(22) $$R_t K_{x,t} = \alpha\left(\frac{z^\eta \theta + 1 - \theta}{z^\eta}\right)Y_t .$$

(22) shows that the markup drives a wedge between the marginal product of capital and its rental price. As will be shown below, this wedge creates a distortion on the rate of investment in physical capital. Finally, the correct value of gross domestic product (GDP) should include the amount of investment in R&D such that

(23) $$GDP_t = Y_t + w_t L_{r,t} + R_t K_{r,t} .$$ [8]

$L_{r,t}$ and $K_{r,t}$ are respectively the number of workers and the amount of capital for R&D.

2.6. Capital Accumulation

The market-clearing condition for physical capital is

(24) $$K_t = K_{x,t} + K_{r,t} .$$

K_t is the total amount of capital available in the economy at time t. The law of motion for capital is

[8] In the national income account, private spending in R&D is treated as an expenditure on intermediate goods. Therefore, the values of investment and GDP in the data are I_t and Y_t respectively. The Bureau of Economic Analysis and the National Science Foundation's R&D satellite account provides preliminary estimates on the effects of including R&D as an intangible asset in the national income accounts.

(25) $$\dot{K}_t = I_t - K_t\delta$$

δ is the rate of depreciation. The endogenous rate of investment in physical capital is

(26) $$i_t = (\dot{K}_t / K_t + \delta)K_t / Y_t$$

for all *t*. The no-arbitrage condition $r_t = R_t - \delta$ for the holding of capital and (22) imply that the capital-output ratio is

(27) $$\frac{K_t}{Y_t} = \frac{\alpha(z^{\eta}\theta + 1 - \theta)}{z^{\eta}(1 - s_{K,t})(r_t + \delta)}.$$

$s_{K,t}$ is the endogenous share of capital in the R&D sector. Substituting (27) into (26) yields

(28) $$i_t = \frac{\alpha(z^{\eta}\theta + 1 - \theta)}{z^{\eta}(1 - s_{K,t})}\left(\frac{\dot{K}_t / K_t + \delta}{r_t + \delta}\right).$$

In the Romer model, (skilled) labor is the only factor input for R&D (i.e. $s_{K,t} = 0$); therefore, the distortionary effect of markup pricing on the *steady-state* rate of investment is unambiguously negative (i.e. $\partial i / \partial \eta < 0$). In the current model, there is an opposing positive effect operating through the R&D share of capital. Intuitively, an increase in patent breadth potentially raises the private return on R&D and increases the R&D share of capital. Proposition 2 in Section 2.11 shows that the negative distortionary effect still dominates if the intermediate-goods sector is at least as capital intensive as the R&D sector.

2.7. R&D

$V_t(j)$ is the market value of the patent pool created by the most recent invention in industry $j \in [\theta, 1]$ at time *t* and is determined by the following no-arbitrage condition

(29) $$r_t V_t(j) = \pi_t(j) + \dot{V}_t(j) - \lambda_t V_t(j).$$

The first terms in the right is the flow profits generated by the patent pool at time *t*. The second term is the capital gain due to the growth in the amount of monopolistic profits. The third term is the expected value of capital loss due to creative destruction, and λ_t is the Poisson arrival rate of the next invention that creates a new patent pool. However, the incentives for R&D depend on the market value of the shares in patent pools obtained by an inventor. Denote $V_{i,t}(j)$ for $i \in \{1,...,\eta\}$ as the market value of ownership in patent pools for the *i*-th most recent inventor in industry $j \in [\theta,1]$.

Proposition 1: $V_{i,t}(j)$ *for* $i \in \{1,2,...,\eta\}$ *and* $j \in [\theta,1]$ *is determined by the following law of motion*

(30) $$r_t V_{i,t}(j) = \sigma_i \pi_t(j) + \dot{V}_{i,t}(j) + \lambda_t (V_{i+1,t}(j) - V_{i,t}(j)),$$

where $V_{\eta+1,t}(j) = 0$. *The no-arbitrage condition for* $V_{1,t}(j)$ *can be re-expressed as*

(31) $$V_{1,t}(j) = \pi_t(j)\left(\sum_{k=1}^{\eta} \sigma_k \lambda_t^{k-1}\left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}(j)/V_{i,t}(j)}\right)\right).$$

Assumption 4: *Innovation successes of the R&D entrepreneurs are randomly assigned to the industries in the intermediate-goods sector.*

The expected present value of an invention obtained by the most recent inventor at time *t* is

(32) $$V_{1,t} = \int_{\theta}^{1} V_{1,t}(j)dj = (1-\theta)\left(\frac{z^{\eta}-1}{z^{\eta}}\right)Y_t\left(\sum_{k=1}^{\eta} \sigma_k \lambda_t^{k-1}\left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}/V_{i,t}}\right)\right).$$

The arrival rate of an innovation success for an R&D entrepreneur $h \in [0,1]$ is a function of labor input $L_{r,t}(h)$ and capital input $K_{r,t}(h)$ given by

(33) $$\lambda_t(h) = \overline{\varphi}_t K_{r,t}^{\beta}(h) L_{r,t}^{1-\beta}(h).$$ [9]

$\overline{\varphi}_t$ is a productivity parameter that the entrepreneurs take as given. The expected profit from R&D is

(34) $$E_t[\pi_{r,t}(h)] = V_{1,t}\lambda_t(h) - w_t L_{r,t}(h) - R_t K_{r,t}(h).$$

[9] This specification nests the "knowledge-driven" specification in Romer (1990) as a special case with $\beta = 0$ and the "lab equipment" specification in River-Batiz and Romer (1991) as a special case with $\beta = \alpha$.

The first-order conditions are

(35) $$(1-\beta)V_{1,t}\overline{\varphi}_t(K_{r,t}(h)/L_{r,t}(h))^{\beta}=w_t\,,$$

(36) $$\beta V_{1,t}\overline{\varphi}_t(K_{r,t}(h)/L_{r,t}(h))^{\beta-1}=R_t\,.$$

To eliminate scale effects and capture various externalities, the *individual* R&D productivity parameter $\overline{\varphi}_t$ at time t is assumed to be decreasing in the level of R&D-driven technology A_t such that

(37) $$\overline{\varphi}_t=\frac{\varphi(K_{r,t}^{\beta}L_{r,t}^{1-\beta})^{\gamma-1}}{A_t^{1-\phi}}\,,$$

where $K_{r,t}=\int_0^1 K_{r,t}(h)dh$ and $L_{r,t}=\int_0^1 L_{r,t}(h)dh$. $\gamma\in(0,1]$ captures the intratemporal negative congestion or duplication externality or the so-called "stepping on toes" effects, and $\phi\in(-\infty,1)$ captures the externality of intertemporal knowledge spillovers.[10] Given that the arrival of innovations follows a Poisson process, the law of motion for R&D-driven technology is given by

(38) $$\dot{A}_t=A_t\lambda_t\ln z=A_t\overline{\varphi}_tK_{r,t}^{\beta}L_{r,t}^{1-\beta}\ln z=A_t^{\phi}(K_{r,t}^{\beta}L_{r,t}^{1-\beta})^{\gamma}\varphi\ln z\,.$$[11]

2.8. Decentralized Equilibrium

The analysis starts at $t=0$. The equilibrium is a sequence of prices $\{w_t,r_t,R_t,P_t(j),V_{1,t}\}_{t=0}^{\infty}$ and a sequence of allocations $\{a_t,c_t,I_t,Y_t,X_t(j),K_{x,t}(j),L_{x,t}(j),K_{r,t}(h),L_{r,t}(h),K_t,L_t\}_{t=0}^{\infty}$ such that they are

[10] This specification captures how semi-endogenous growth models eliminate scale effects as in Jones (1995b). $\phi\in(0,1)$ corresponds to the "standing on shoulder" effect, in which the *economy-wide* R&D productivity $A_q\overline{\varphi}$ increases as the level of R&D-driven technology increases (see the law of motion for R&D-driven technology). On the other hand, $\phi\in(-\infty,0)$ corresponds to the "fishing out" effect, in which early technology is relatively easy to develop and $A_q\overline{\varphi}$ decreases as the level of R&D-driven technology increases.

[11] This convenient expression is derived as $\ln A_t=\left(\int_0^1 m_t(j)dj\right)\ln z=\left(\int_0^t \lambda(\tau)d\tau\right)\ln z$; then, simple differentiation yields $\dot{A}_t/A_t=\lambda_t\ln z$.

consistent with the initial conditions $\{K_0, L_0, Z_0, A_0, \overline{\varphi}_0\}$ and their subsequent laws of motions. Also, in each period,

(a) the representative household chooses $\{a_t, c_t\}$ to maximize utility taking $\{w_t, r_t\}$ as given;

(b) the competitive firms in the final-goods sector choose $\{X_t(j)\}$ to maximize profits according to the production function taking $\{P_t(j)\}$ as given;

(c) each industry leader in the intermediate-goods sector chooses $\{P_t(j), K_{x,t}(j), L_{x,t}(j)\}$ to maximize profits according to the Bertrand price competition and the production function taking $\{R_t, w_t\}$ as given;

(d) the competitive firms in the intermediate-goods sector choose $\{K_{x,t}(j'), L_{x,t}(j')\}$ to maximize profits according to the production function taking $\{P_t(j'), R_t, w_t\}$ as given;

(e) each entrepreneur in the R&D sector chooses $\{K_{r,t}(h), L_{r,t}(h)\}$ to maximize profits according to the R&D production function taking $\{\overline{\varphi}_t, V_{1,t}, R_t, w_t\}$ as given;

(f) the market for the final-goods clears such that $Y_t = C_t + I_t$;

(g) the full employment of capital such that $K_t = K_{x,t} + K_{r,t}$; and

(h) the full employment of labors such that $L_t = L_{x,t} + L_{r,t}$.

2.9. Transition Dynamics

The transition dynamics of the decentralized equilibrium is characterized by the following differential equations. The capital stock is a predetermined variable and evolves according to

(39) $$\dot{K}_t = Y_t - C_t - K_t\delta .$$

R&D-driven technology is also a predetermined variable and evolves according to

(40) $$\dot{A}_t = A_t\lambda_t \ln z .$$

Consumption is a jump variable and evolves according to the Euler equation

$$\dot{c}_t = c_t(r_t - \rho)/\sigma \, . \tag{41}$$

The market value of ownership in patent pools is also a jump variable and evolves according to

$$\dot{V}_{i,t} = (r_t + \lambda_t)V_{i,t} - \lambda_t V_{i+1,t} - \sigma_i \pi_t \tag{42}$$

for $i \in \{1,2,...,\eta\}$ and $V_{\eta+1,t} = 0$.

At the aggregate level, the generalized quality-ladder model is similar to Jones's (1995b) model, whose dynamic properties have been investigated by a number of recent studies. For example, Arnold (2006) analytically derives the uniqueness and local stability of the steady state with certain parameter restrictions. Steger (2005) and Trimborn *et al* (2006) numerically evaluate the transition dynamics of the model. In summary, to solve the model numerically, I firstly transform $\{K_t, A_t, c_t, V_{i,t}\}$ in the differential equations into its stationary form,[12] and then, compute the transition path from the old steady state to the new one using the relaxation algorithm developed by Trimborn *et al* (2006).

2.10. Balanced-Growth Path

Equating the first-order conditions (21) and (35) and imposing the balanced-growth condition on R&D-driven technology

$$g_A = \overline{\varphi}_t L_{r,t}^{1-\beta} K_{r,t}^{\beta} \ln z \tag{43}$$

yield the steady-state R&D share of labor inputs given by

$$\frac{s_L}{1-s_L} = \frac{1-\beta}{1-\alpha}\left(\frac{\lambda}{r+\lambda-g_Y}\right)\frac{(z^\eta - 1)(1-\theta)}{z^\eta \theta + (1-\theta)} \nu(\sigma^{\eta_{lead}}) \, , \tag{44}$$

[12] Refer to Appendix III for the details.

where $\nu(\sigma^{\eta_{lead}}) \equiv \sum_{k=1}^{\eta} \sigma_k \left(\frac{\lambda}{r+\lambda-g_Y} \right)^{k-1} \in (0,1]$ is defined as the backloading discount factor. For example, in the case of complete frontloading, $\nu(\sigma^{\eta_{lead}}) = 1$. Similarly, solving (22), (36) and (43) yields the steady-state R&D share of capital inputs given by

(45) $$\frac{s_K}{1-s_K} = \frac{\beta}{\alpha} \left(\frac{\lambda}{r+\lambda-g_Y} \right) \frac{(z^{\eta}-1)(1-\theta)}{z^{\eta}\theta+(1-\theta)} \nu(\sigma^{\eta_{lead}}) .$$

On the balanced-growth path, c_t increases at a constant rate g_c, so that the steady-state real interest rate is

(46) $$r = \rho + g_c \sigma .$$

The balanced-growth rate of R&D technology g_A is related to the labor-force growth rate such that

(47) $$g_A = \frac{(K_{r,t}^{\beta} L_{r,t}^{1-\beta})^{\gamma}}{A_t^{1-\phi}} \varphi \ln z = \left(\frac{\gamma\beta}{1-\phi} \right) g_K + \left(\frac{\gamma(1-\beta)}{1-\phi} \right) n .$$

Then, the steady-state rate of creative destruction is $\lambda = g_A / \ln z$. The balanced-growth rates of other variables are given as follows. Given that the steady-state investment rate is constant, the balanced-growth rate of per capita consumption is

(48) $$g_c = g_Y - n .$$

From the aggregate production function (17), the balanced-growth rates of output and capital are

(49) $$g_Y = g_K = n + (g_A + g_Z)/(1-\alpha) .$$

Using (47) and (49), the balanced-growth rate of R&D-driven technology is determined by the exogenous labor-force growth rate n and productivity growth rate g_Z given by

(50) $$g_A = \left(\frac{1-\phi}{\gamma} - \frac{\beta}{1-\alpha} \right)^{-1} \left(n + \frac{\beta}{1-\alpha} g_Z \right).$$

Long-run TFP growth denoted by $g_{TFP} \equiv g_A + g_Z$ is empirically observed. For a given g_{TFP}, a higher value of g_Z implies a lower value of g_A as well as a lower calibrated value for $\gamma/(1-\phi)$ indicating smaller social benefits from R&D.

2.11. First-Best Optimal Allocations

This section firstly characterizes the socially optimal equilibrium rate of investment and R&D shares of labor and capital and then derives the dynamic distortion on capital accumulation.

Lemma 2: *The modified Golden-rule rate of investment on the balanced-growth path is*

$$i^* = \left(\alpha + \beta \frac{\gamma g_A}{\rho - n + (\sigma - 1)g_c + (1-\phi)g_A}\right)\frac{g_K + \delta}{\rho + g_c\sigma + \delta}, \tag{51}$$

and the socially optimal steady-state R&D shares of labor s_L^* *and capital* s_K^* *are respectively*

$$\begin{aligned} \frac{s_L^*}{1-s_L^*} &= \frac{1-\beta}{1-\alpha}\left(\frac{\gamma g_A}{\rho - n + (\sigma-1)g_c + (1-\phi)g_A}\right) \\ \neq \frac{s_L}{1-s_L} &= \frac{1-\beta}{1-\alpha}\left(\frac{\lambda}{\rho - n + (\sigma-1)g_c + \lambda}\right)\frac{(z^\eta - 1)(1-\theta)}{z^\eta\theta + 1 - \theta}\nu(\sigma^{\eta_{lead}}) \end{aligned}, \tag{52}$$

$$\begin{aligned} \frac{s_K^*}{1-s_K^*} &= \frac{\beta}{\alpha}\left(\frac{\gamma g_A}{\rho - n + (\sigma-1)g_c + (1-\phi)g_A}\right) \\ \neq \frac{s_K}{1-s_K} &= \frac{\beta}{\alpha}\left(\frac{\lambda}{\rho - n + (\sigma-1)g_c + \lambda}\right)\frac{(z^\eta - 1)(1-\theta)}{z^\eta\theta + 1 - \theta}\nu(\sigma^{\eta_{lead}}) \end{aligned}. \tag{53}$$

(52) and (53) indicate the various sources of R&D externalities: (a) the negative congestion externality $\gamma \in (0,1]$; (b) the positive or negative externality in intertemporal knowledge spillovers $\phi \in (-\infty,1)$; (c) the static consumer-surplus appropriability problem $(1-\theta)(z^\eta - 1)/z^\eta \in (0,1]$, which is a positive externality; (d) the markup distortion in driving a wedge of $(z^\eta\theta + 1 - \theta)/z^\eta \geq 1$ between the factor

payments for production inputs and their marginal products; (e) the positive externality of sequential innovations together with the negative externality of the business-stealing effect given by the difference between $g_A/(\rho-n+(\sigma-1)g_c+g_A)$ and $\lambda/(\rho-n+(\sigma-1)g_c+\lambda)$; and (f) the negative effects of blocking patents on R&D through the backloading discount factor $\nu(\sigma^{\eta_{lead}})\in(0,1]$. Given the existence of positive and negative externalities, it requires a numerical calibration to the data that will be performed in Section 3 to determine whether the market economy over- or under-invests in R&D.

If the market economy underinvests in R&D as also suggested by Jones and Williams (1998) and (2000), the government may want to increase patent breadth to reduce the extent of market failures. However, the following proposition states that even holding the effects of blocking patents constant, an increase in η mitigates the problem of R&D underinvestment at the costs of worsening the dynamic distortionary effect on capital accumulation in addition to increasing the static distortionary effect.

Proposition 2a: *The decentralized equilibrium rate of investment is below the socially optimal investment rate if either there is underinvestment in R&D or labor is the only factor input for R&D.*

Proposition 2b: *Holding the backloading discount factor ν constant, an increase in patent breadth leads to a reduction in the decentralized equilibrium rate of investment if the intermediate-goods sector is at least as capital intensive as the R&D sector.*

3. Calibration

Using the framework developed above, this section provides a quantitative assessment on the effects of eliminating blocking patents. Figure 1 shows that private spending on R&D in the US as a share of GDP has been rising sharply since the beginning of the 80's. Then, after a few years, the number of patents granted by the US Patent and Trademark Office also began to increase rapidly as shown in Figure 2. Given the patent policy changes in the 80's, the structural parameters are calibrated using long-run aggregate data of the US's economy from 1953 to 1980 to examine the extent of R&D underinvestment before these policy changes. The goal of this numerical exercise is to quantify the effects of eliminating blocking patents on R&D and consumption.

3.1. Backloading Discount Factor

The first step is to calibrate the structural parameters and the steady-state value of the backloading discount factor v. The average annual TFP growth rate g_{TFP} is 1.33%,[13] and the labor-force growth rate n is 1.94%.[14] The annual depreciation rate δ on physical capital and the household's discount rate are set to conventional values of 8% and 4% respectively. For the aggregate markup $\tilde{\mu} = \mu^{1-\theta}$, Laitner and Stolyarov (2004) estimate that $\tilde{\mu}$ is about 1.1 (i.e. a 10% markup) in the data. For a given $\tilde{\mu}$, each value of θ (i.e. the fraction of competitive industries in the intermediate-goods sector) corresponds to a unique value for the industry markup μ in monopolistic industries, and I will consider a wide range of values for $\theta \in \{0, 0.25, 0.5, 0.75\}$. A number of structural studies based on patent renewal models has estimated the arrival rate of innovations λ, and I will consider a reasonable range of values for $\lambda \in [0.04, 0.20]$.[15] For the capital intensity parameter in the R&D sector, I will set $\beta = \alpha$ as the benchmark case.[16]

For the remaining parameters $\{v, \alpha, \sigma\}$, the model provides three steady-state conditions for the calibration: (a) R&D as a share of GDP; (b) labor share; and (c) the rate of investment in physical capital.

$$\frac{wL_r + RK_r}{Y} = \left(\frac{\mu\theta + 1 - \theta}{\mu}\right)\left((1-\alpha)\frac{s_L}{1-s_L} + \alpha\frac{s_K}{1-s_K}\right), \tag{54}$$

[13] Multifactor productivity for the private non-farm business sector is obtained from the Bureau of Labor Statistics.

[14] The data on the annual average size of the labor force is obtained from the Bureau of Labor Statistics.

[15] For example, Lanjouw (1998) structurally estimate a patent renewal model using patent renewal data in a number of industries from Germany, and the estimated probability of obsolescence ranges 7% for computer patents to 12% for engine patents. Also, a conventional value for the rate of depreciation in patent value is about 15% (e.g. Pakes (1986)). In the current model, the patent-value depreciation rate is given by $\lambda - g_Y$, which implies that λ should be at least 15%. On the other hand, Caballero and Jaffe (2002) estimate a mean rate of creative destruction of about 4%.

[16] I have considered different plausible values for $\beta \in \{0, \alpha, 2\alpha, 3\alpha\}$ as a sensitivity analysis. The extent of R&D underinvestment and the effects of eliminating blocking patent and increasing patent breadth on long-run consumption are robust to these parameter changes.

(55) $$\frac{wL}{Y}=\frac{1-\alpha}{1-s_L}\left(\frac{\mu\theta+1-\theta}{\mu}\right),$$

(56) $$\frac{I}{Y}=\frac{\alpha(\mu\theta+1-\theta)}{\mu(1-s_K)}\left(\frac{n+g_{TFP}/(1-\alpha)+\delta}{\rho+\sigma g_{TFP}/(1-\alpha)+\delta}\right),$$

The average private spending on R&D as a share of GDP is 1.15%,[17] and the labor share is set to a conventional value of 0.7. The long-run ratio of business investment to non-housing GDP is 14%.[18]

Table 1 presents the calibrated values for the structural parameters along with the real interest rate $r=\rho+\sigma g_{TFP}/(1-\alpha)$ and the industry markup $\mu=(1.1)^{1/(1-\theta)}$ for $\theta\in\{0, 0.25, 0.5, 0.75\}$ and $\lambda\in[0.04, 0.20]$.

[insert Table 1 here]

Table 1 shows that the calibrated values for $\{\alpha,\sigma,r\}$ are invariant to different values of λ for a given value of θ. The calibrated value for the elasticity of intertemporal substitution (i.e. $1/\sigma$) is about 0.25, which is closed to the empirical estimates from econometric studies.[19] The implied real interest rate is about 11%, which is higher than the historical rate of return on the US's stock market, and this higher interest rate implies a lower optimal level of R&D spending and a higher steady-state value of the backloading discount factor. As a result, the model is less likely to overestimate the extent of R&D underinvestment and the degree of inefficiency from blocking patents. Re-expressing (55) into (58) shows that v decreases as λ increases.

(57) $$v=\frac{R\&D}{GDP}\Bigg/\frac{\lambda(1-\theta)(\mu-1)/\mu}{\rho-n+(\sigma-1)g_{TFP}/(1-\alpha)+\lambda}.$$

---

[17] The data is obtained from the National Science Foundation and the Bureau of Economic Analysis. R&D is net of federal spending, and GDP is net of government spending. The observations in the data series of R&D spending are missing for 1954 and 1955.

[18] Business investment refers to total private investment less investment in owner-occupied housing, and this data is obtained from Laitner and Stolyarov (2005).

[19] It is well-known that there is a discrepancy between the estimated elasticity of intertemporal substitution from dynamic macro models (closed to 1) and econometric studies (closed to 0). Guvenen (2006) shows that this discrepancy is due to the heterogeneity in households' preferences and wealth inequality. In short, the *average investor* has a high elasticity of intertemporal substitution while the *average consumer* has a much lower elasticity. Since my interest is on consumption, it is appropriate to calibrate the value of σ according to the average consumer.

Furthermore, the fact that the calibrated values of $\nu \in [0.169, 0.424]$ are very small suggests a severe degree of inefficiency from blocking patents in the economy. Therefore, eliminating blocking patents may be an effective method to stimulate R&D. After calibrating the externality parameters and computing the first-best level of R&D spending, the effects of eliminating blocking patents will be quantified.

3.2. Externality Parameters

The second step is to calibrate the values for the externality parameters γ (intratemporal duplication) and ϕ (intertemporal spillover). For each value of g_A, g_Z, n, α and β, the balanced-growth condition (50) determines a unique value for $\gamma/(1-\phi)$, which is sufficient to determine the new balanced-growth level of consumption. However, holding $\gamma/(1-\phi)$ constant, a larger γ implies a faster rate of convergence to the new balanced-growth path; therefore, it is important to consider different values of γ. As for the value of g_A, I will set $g_A = \xi g_{TFP}$ for $\xi \in [0,1]$. The parameter ξ captures the fraction of long-run TFP growth that is driven by R&D, and the remaining fraction is driven by the exogenous process Z_t such that $g_Z = (1-\xi) g_{TFP}$. Table 2 presents the calibrated values of ϕ for a subset of values for $\gamma \in [0.1, 1.0]$ and $\xi \in [0,1]$.

[insert Table 2 here]

Table 2 shows that the calibrated values for ϕ are very similar across different values of θ implying that the first-best level of R&D spending and the extent of R&D over- or underinvestment are about the same across different values of θ.

3.3. First-Best Level of R&D Spending

This section calculates the first-best level of R&D share $(1-\alpha)s_L^* /(1-s_L^*) + \alpha s_K^* /(1-s_K^*)$. Figure 3 plots the first-best R&D shares for $\gamma \in [0.1, 1.0]$ and $\xi \in [0,1]$.

[insert Figure 3 here]

Figure 3 shows that there was underinvestment in R&D prior to 1980 over a wide range of parameters unless γ and ξ are very small. Since it is difficult to determine the empirical value of ξ, I will leave it to the readers to decide on their preferred values and continue to present results for a range of parameters.

3.4. Eliminating Blocking patents

Given the calibrated structural parameters, this section quantifies the effects of eliminating blocking patents on R&D and consumption. Upon eliminating blocking patents (i.e. setting $\nu = 1$), the steady-state share of R&D given by $(wL_r + RK_r)/Y$ would increase substantially to the values in Table 3.

[insert Table 3 here]

In the following, the effect of eliminating blocking patents is firstly expressed in terms of the percentage change in the balanced-growth level of consumption per year. Along the balanced-growth path, per capita consumption increases at an exogenous rate g_c. Therefore, after dropping the exogenous growth path and some constant terms and solving for the balanced-growth path of R&D technology and steady-state capital-labor ratio, I derive the expression for the endogenous parts of long-run consumption as a function of the steady-state value of the backloading discount factor ν through the capital investment rate $i(\nu)$, and the R&D shares of capital and labor (where $s_r(\nu) = s_L(\nu) = s_K(\nu)$ because $\alpha = \beta$).

Lemma 3: For $\alpha = \beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

$$c_0(\nu) = \left(i(\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-i(\nu)) s_r(\nu)^{\frac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma}} (1-s_r(\nu))^{\frac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma}} \right). \tag{58}$$ [20]

[20] The proof in Appendix I also derives the expression for the general case in which $\alpha \neq \beta$.

Therefore, in the case of a change in ν, the percentage change in long-run consumption can be decomposed into four terms.

$$(59) \qquad \Delta \ln c_0(\nu) = \left(\begin{array}{l} \left(\dfrac{\alpha(1-\phi+\gamma)}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln i(\nu) + \Delta \ln(1-i(\nu)) + \\ \left(\dfrac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln s_r(\nu) + \left(\dfrac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma} \right) \Delta \ln(1-s_r(\nu)) \end{array} \right).$$[21]

Figure 4 shows that eliminating blocking patents should have a substantial positive effect on long-run consumption unless ξ is very small. Also, a back-of-the-envelope calculation shows that the change in consumption mostly comes from $(\gamma/((1-\alpha)(1-\phi)-\alpha\gamma))\Delta \ln s_r(\nu)$; in other words, other general-equilibrium effects only have secondary impacts on long-run consumption.

[insert Figure 4 here]

After examining the effect on long-run consumption, the next numerical exercise computes the entire growth path of consumption upon eliminating blocking patents. Figure 5a compares the transition path (in blue) of log consumption per capita with its original balanced-growth path (in red) and its new balanced-growth path (in green) for the following parameters $\{\xi,\gamma,\lambda,\theta,\delta\} = \{0.7, 0.7, 0.06, 0, 0.08\}$ to illustrate the transition dynamics. Then, I will discuss the effects of changing these parameter values.

[insert Figure 5a here]

Upon setting $\nu = \sigma_1 = 1$, consumption per capita gradually rises towards the new balanced growth path. Although factor inputs shift towards the R&D sector and the output of final goods drops as a result, the possibility of investing less and running down the capital stock enables consumption smoothing. To compare with previous studies, such as Kwan and Lai (2003), Figure 5b presents the transition dynamics for $\{\xi,\gamma,\lambda,\theta,\delta\} = \{0.7, 0.7, 0.06, 0, 1\}$ as an approximation to a model with no capital accumulation. In this case, the result is consistent with Kwan and Lai (2003) that consumption falls in response to the

[21] Note that the coefficients are determined by $\gamma/(1-\phi)$ rather than the individual values of γ and ϕ.

strengthening of patent protection. In this case, consumption falls by about 5% on impact and only recovers to its original growth path after 3 years.

[insert Figure 5b here]

To ensure the robustness of this finding, a sensitivity analysis has been performed for different values of ξ and γ. At a larger value of either ξ or γ, consumption increases by even more on impact. A larger ξ also implies a higher position of the new balanced-growth path. Holding ξ constant, a larger γ implies a faster rate of convergence. When both ξ and γ are small than 0.7, the household suffers consumption losses during the initial phase of the transition path. For example, Figure 5c presents the transition dynamics for $\{\xi, \gamma, \lambda, \theta, \delta\} = \{0.5, 0.5, 0.06, 0, 0.08\}$.

[insert Figure 5c here]

However, Figure 5d shows that when ξ is closed to one, γ could be as small as 0.5 without causing any short-run consumption losses.

[insert Figure 5d here]

In summary, reallocating resources from the production sector to the R&D sector does not always lead to short-run consumption losses. Finally, at a larger value of λ, the calibrated value for ν becomes smaller (see Table 1). This larger magnitude of the policy shock renders the algorithm unable to achieve convergence when ξ and γ are large. However, when the magnitude of the policy is small (e.g. ν increases from 0.5 to 1), convergence is always achieved.

4. Conclusion

This paper has attempted to accomplish three objectives. Firstly, it develops a tractable framework to model the transition dynamics of an economy with patent breadth and blocking patents in a generalized quality-ladder growth model. Secondly, it identifies a dynamic distortion on capital accumulation that has been neglected by previous studies on patent policy. Thirdly, it applies the model to the aggregate data to quantify the extents of underinvestment in R&D and inefficiency arising from blocking patents. The numerical exercise suggests the following findings. The market economy underinvests in R&D so long as a non-negligible fraction of long-run TFP growth is driven by R&D. Eliminating blocking patents increases R&D by about two to six times, and the resulting effects on consumption can be substantial.

However, the readers should interpret the numerical results with some important caveats in mind. The first caveat is that although the quality-ladder model has been generalized as an attempt to capture more realistic features of the economy, it is still an oversimplification of the real world. In particular, the finding of eliminating blocking patents having a substantial positive effect on consumption is based on the assumptions that a non-negligible fraction of long-run TFP growth is driven by R&D and the data on private R&D spending is not incorrectly measured by an order of magnitude. The validity of these assumptions remains as an empirical question. Therefore, the numerical results should be viewed as illustrative at best. The second caveat is that the representative-agent setting ignores the distributional consequences of broadening patent protection, and the efficiency-equity tradeoff should be carefully considered by policymakers.

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Appendix I: Proofs

Lemma 1: *The aggregate production function for the final goods is*

(a1) $$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha},$$

where $\vartheta(\eta) \equiv (z^{\eta})^{\theta} / (z^{\eta}\theta + 1 - \theta)$ *is decreasing in* η *for* $\theta \in (0,1)$.

Proof: Recall that the production function for the final goods is given by

(a2) $$Y_t = \exp\left(\int_0^1 \ln X_t(j) dj \right).$$

After substituting $X_t(j)$ for $j \in [0,1]$ into (a2), the aggregate production function becomes

(a3) $$Y_t = A_t Z_t \left(\frac{K_{x,t}}{L_{x,t}} \right)^{\alpha} L_{x,t}^{e},$$

where $L_{x,t}^{e}$ is defined as

(a4) $$L_{x,t}^{e} \equiv \exp\left(\int_0^1 \ln L_{x,t}(j) dj \right) \neq \left(\int_0^1 L_{x,t}(j) dj \right) = L_{x,t}$$

because of the competitive industries. Define $\vartheta(\eta)$ as the ratio of $L_{x,t}^{e}$ and $L_{x,t}$, which is given by

(a5) $$\vartheta(\eta) \equiv \frac{L_{x,t}^{e}}{L_{x,t}} = \frac{(z^{\eta})^{\theta}}{z^{\eta}\theta + 1 - \theta} \in (0,1)$$

for $\theta \in (0,1)$. $\vartheta(\eta)$ represents the static distortionary effect of markup pricing, and it enters the aggregate production function as

(a6) $$Y_t = \vartheta(\eta) A_t Z_t K_{x,t}^{\alpha} L_{x,t}^{1-\alpha}.$$

Finally, simple differentiation shows that for $\theta \in (0,1)$,

(a7) $$\frac{\partial \vartheta(\eta)}{\partial \eta} = -\frac{\theta(1-\theta)(z^{\eta} - 1)\ln z}{z^{\eta}\theta + 1 - \theta} \vartheta(\eta) < 0. \blacksquare$$

Proposition 1: *$V_{i,t}(j)$ for $i \in \{1,2,...,\eta\}$ and $j \in [\theta,1]$ is determined by the following law of motion*

(b1) $$r_t V_{i,t}(j) = \sigma_i \pi_t(j) + \dot{V}_{i,t}(j) + \lambda_t (V_{i+1,t}(j) - V_{i,t}(j)),$$

where $V_{\eta+1,t}(j) = 0$. The no-arbitrage condition for $V_{1,t}(j)$ can be re-expressed as

(b2) $$V_{1,t}(j) = \pi_t(j) \left(\sum_{k=1}^{\eta} \sigma_k \lambda_t^{k-1} \left(\prod_{i=1}^{k} \frac{1}{r_t + \lambda_t - \dot{V}_{i,t}(j) / V_{i,t}(j)} \right) \right).$$

Proof: The expected present value of the ownership in patent pools for the *i*-th most recent inventor in industry $j \in [\theta,1]$ is

(b3) $$V_{i,t}(j) = \int_t^{\infty} \left(\int_t^{s} \sigma_i \pi_x(j) \exp\left(-\int_t^{x} r_v dv \right) dx \right) f(s) ds + \int_t^{\infty} V_{i+1,s}(j) \exp\left(-\int_t^{s} r_v dv \right) f(s) ds,$$

where $f(s) = \lambda_s \exp\left(-\int_t^{s} \lambda_x dx \right)$ is the density function of *s* that is a random variable representing the time when the next innovation occurs and follows the Erlang distribution. The first term in $V_{i,t}(j)$ is the expected present value of monopolistic profits captured by the *i*-th most recent inventor in the current patent pool. The second term in $V_{i,t}(j)$ is the expected present value of the ownership in patent pools when the *i*-th most recent inventor becomes the *i*+1-th most recent inventor. Note that $V_{\eta+1,t}(j) = 0$ because the $\eta + 1$-th most recent inventor is no longer in any patent pool. In order to derive (b1), I differentiate (b3) with respect to *t*. To simplify notations, I firstly define a new function such that (b3) becomes

(b4) $$V_{i,t}(j) = \int_t^{\infty} g(t,s) ds,$$

where $g(t,s)=\left(\int_t^s \sigma_i\pi_x(j)\exp\left(-\int_t^x r_v dv\right)dx+V_{i+1,s}(j)\exp\left(-\int_t^s r_v dv\right)\right)f(s)$. Then, using the formula for differentiation under the integral sign,

(b5) $$\dot{V}_{i,t}(j)\equiv\frac{\partial V_{i,t}(j)}{\partial t}=-g(t,t)+\int_t^\infty\frac{\partial g(t,s)}{\partial t}ds,$$

where $g(t,t)=\lambda_t V_{i+1,s}(j)$, and

(b6) $$\frac{\partial g(t,s)}{\partial t}=\left((\lambda_t+r_t)\left(\int_t^s \sigma_i\pi_x(j)\exp\left(-\int_t^x r_v dv\right)dx+V_{i+1,s}(j)\exp\left(-\int_t^s r_v dv\right)\right)-\sigma_i\pi_t(j)\right)f(s).$$

After a few steps of mathematical manipulation, (b5) becomes

(b7) $$\dot{V}_{i,t}(j)=-\lambda_t V_{i+1,s}(j)+(\lambda_t+r_t)V_{i,t}(j)-\sigma_i\pi_t(j)\int_t^\infty f(s)ds.$$

Finally, after setting $\int_t^\infty f(s)ds=1$, (b7) becomes (b1).

Upon deriving (b1), each $V_{i,t}(j)$ for $i\in\{1,2,...,\eta\}$ can be rewritten as

(b8) $$V_{i,t}(j)=\frac{\sigma_i\pi_t(j)+\lambda_t V_{i+1,t}(j)}{r_t+\lambda_t-\dot{V}_{i,t}(j)/V_{i,t}(j)},$$

where $V_{\eta+1,t}(j)=0$. Recursive substitution shows that $V_{1,t}(j)$ can be re-expressed as (b2).■

Lemma 2: *The modified Golden-rule rate of investment on the balanced-growth path is*

(c1) $$i^*=\left(\alpha+\beta\frac{\gamma g_A}{\rho-n+(\sigma-1)g_c+(1-\phi)g_A}\right)\frac{g_K+\delta}{\rho+g_c\sigma+\delta},$$

and the socially optimal steady-state R&D shares of labor s_L^* *and capital* s_K^* *are respectively*

(c2) $$\frac{s_L^*}{1-s_L^*}=\frac{1-\beta}{1-\alpha}\left(\frac{\gamma g_A}{\rho-n+(\sigma-1)g_c+(1-\phi)g_A}\right),$$

(c3) $$\frac{s_K^*}{1-s_K^*} = \frac{\beta}{\alpha}\left(\frac{\gamma g_A}{\rho - n + (\sigma-1)g_c + (1-\phi)g_A}\right).$$

Proof: To derive the socially optimal rate of investment and R&D shares of labor and capital, the social planner chooses i_t, $s_{L,t}$ and $s_{K,t}$ to maximize

(c4) $$U = \int_0^\infty e^{-(\rho-n)t}\,\frac{((1-i_t)Y_t/L_t)^{1-\sigma}}{1-\sigma}\,dt$$

subject to: (a) the aggregate production function expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(c5) $$Y_t = A_t Z_t (1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha};$$

(b) the law of motion for capital expressed in terms of i_t given by

(c6) $$\dot{K}_t = i_t Y_t - K_t\delta;$$

and (c) the law of motion for R&D technology expressed in terms of $s_{L,t}$ and $s_{K,t}$ given by

(c7) $$\dot{A}_t = A_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma} K_t^{\beta\gamma} L_t^{(1-\beta)\gamma}\varphi \ln z.$$

The current-value Hamiltonian H is

(c8) $$\begin{aligned} H = {} & (1-\sigma)^{-1}\left(\frac{(1-i_t)A_t Z_t(1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}}{L_t}\right)^{1-\sigma} \\ & + \upsilon_{K,t}(i_t A_t Z_t (1-s_{K,t})^{\alpha}(1-s_{L,t})^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha} - K_t\delta) \\ & + \upsilon_{A,t}A_t^{\phi}(s_{K,t})^{\beta\gamma}(s_{L,t})^{(1-\beta)\gamma}K_t^{\beta\gamma}L_t^{(1-\beta)\gamma}\varphi \ln z \end{aligned}.$$

The first-order conditions are

(c9) $$H_i = -\frac{1}{(1-i_t)}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}Y_t = 0,$$

(c10) $$H_{s_L} = -\left(\frac{1-\alpha}{1-s_{L,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} - \upsilon_{K,t}\left(\frac{1-\alpha}{1-s_{L,t}}\right)i_t Y_t + \upsilon_{A,t}\left(\frac{(1-\beta)\gamma}{s_{L,t}}\right)\dot{A}_t = 0,$$

(c11) $$H_{s_K} = -\left(\frac{\alpha}{1-s_{K,t}}\right)\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} - \upsilon_{K,t}\left(\frac{\alpha}{1-s_{K,t}}\right)i_tY_t + \upsilon_{A,t}\left(\frac{\beta\gamma}{s_{K,t}}\right)\dot{A}_t = 0\,,$$

(c12) $$H_K = \frac{\alpha}{K_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\alpha\frac{i_tY_t}{K_t} - \delta\right) + \upsilon_{A,t}\left(\beta\gamma\frac{\dot{A}_t}{K_t}\right) = (\rho - n)\upsilon_{K,t} - \dot{\upsilon}_{K,t}\,,$$

(c13) $$H_A = \frac{1}{A_t}\left(\frac{(1-i_t)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t}\left(\frac{i_tY_t}{A_t}\right) + \upsilon_{A,t}\left(\phi\frac{\dot{A}_t}{A_t}\right) = (\rho - n)\upsilon_{A,t} - \dot{\upsilon}_{A,t}\,.$$

Note that the first-order conditions with respect to the co-state variables $\upsilon_{K,t}$ and $\upsilon_{A,t}$ yield the law of motions for capital and R&D technology. Then, imposing the balanced-growth conditions yields

(c14) $$H_i : \left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} = (1-i)\upsilon_{K,t}Y_t\,,$$

(c15) $$H_{s_L} : \gamma g_A A_t \upsilon_{A,t}\left(\frac{1-s_L}{s_L}\right) = \frac{1-\alpha}{1-\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} iY_t\right),$$

(c16) $$H_{s_K} : \gamma g_A A_t \upsilon_{A,t}\left(\frac{1-s_K}{s_K}\right) = \frac{\alpha}{\beta}\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_K iY_t\right),$$

(c17) $$H_K : \alpha\left(\left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} iY_t\right) + \beta\gamma g_A A_t \upsilon_{A,t} = (\rho + g_c\sigma + \delta)K_t\upsilon_{K,t}\,,$$

(c18) $$H_A : \left(\frac{(1-i)Y_t}{L_t}\right)^{1-\sigma} + \upsilon_{K,t} iY_t = (\rho - n + (\sigma-1)g_c + (1-\phi)g_A)A_t\upsilon_{A,t}\,.$$

Finally, solving (c14)-(c18) yields (c1)-(c3).■

Proposition 2a: *The decentralized equilibrium rate of investment is below the socially optimal investment rate if either there is underinvestment in R&D or labor is the only factor input for R&D.*

Proof: The socially optimal investment rate i^* is (d1) $$i^{*}=\left(\alpha+\beta\frac{\gamma g_{A}}{\rho-n+(\sigma-1)g_{c}+(1-\phi)g_{A}}\right)\frac{g_{K}+\delta}{\rho+g_{c}\sigma+\delta}.$$

The market equilibrium rate of investment i is

(d2) $$i=\frac{z^{\eta}\theta+1-\theta}{z^{\eta}}\left(\alpha+\beta\left(\frac{\lambda}{\rho-n+(\sigma-1)g_{c}+\lambda}\right)\frac{(z^{\eta}-1)(1-\theta)}{z^{\eta}\theta+1-\theta}v(\sigma^{\eta_{lead}})\right)\frac{g_{K}+\delta}{\rho+g_{c}\sigma+\delta}.$$

Therefore, either $\beta=0$ or the underinvestment in R&D such that $\frac{\gamma g_{A}}{\rho-n+(\sigma-1)g_{c}+(1-\phi)g_{A}}>$ $\left(\frac{\lambda}{\rho-n+(\sigma-1)g_{c}+\lambda}\right)\frac{(z^{\eta}-1)(1-\theta)}{z^{\eta}\theta+1-\theta}v(\sigma^{\eta_{lead}})$ is *sufficient* for $i^{*}>i$ because $(z^{\eta}\theta+1-\theta)/z^{\eta}<1$ for $\theta\in[0,1)$.■

Proposition 2b: *Holding the backloading discount factor v constant, an increase in patent breadth leads to a reduction in the decentralized equilibrium rate of investment if the intermediate-goods sector is at least as capital intensive as the R&D sector.*

Proof: Differentiating i with respect to η yields

(d3) $$\frac{\partial i}{\partial\eta}=-\frac{(1-\theta)\ln z}{z^{\eta}}\left(\alpha-\beta\frac{\lambda}{\rho-n+(\sigma-1)g_{c}+\lambda}v\right)\frac{g_{K}+\delta}{\rho+g_{c}\sigma+\delta}.$$

Since $\rho>n$ and $\sigma\geq1$, $\alpha\geq\beta$ is a *sufficient* condition for $\partial i/\partial\eta<0$.■

Lemma 3: For $\alpha=\beta$, the expression for the endogenous parts of consumption on the balanced-growth path is

(e1) $$c_{0}(v)=\left(i(v)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}}(1-i(v))s_{r}(v)^{\frac{\gamma}{(1-\alpha)(1-\phi)-\alpha\gamma}}(1-s_{r}(v))^{\frac{(1-\phi)}{(1-\alpha)(1-\phi)-\alpha\gamma}}\right).$$

Proof: The following derivation applies to the more general case in which α and β can be different. Without loss of generality, time is re-normalized such that time 0 is the first-period in which the economy reaches the balanced-growth path. The balanced-growth path of per capita consumption (in log) can be written as

(e2) $$\ln c_t = \ln c_0 + g_c t ,$$

where $g_c t$ represents the exogenous growth path of consumption because of the semi-endogenous growth formulation. The balanced-growth level of per capital consumption at time 0 is

(e3) $$c_0 = \tilde{\vartheta}(1-i)(1-s_K)^{\alpha}(1-s_L)^{1-\alpha} A_0 Z_0 \left(\frac{K_0}{L_0}\right)^{\alpha} ,$$

where Z_0 is normalized to one. The capital-labor ratio K_0 / L_0 and the level of R&D-driven technology A_0 at time 0 are respectively

(e4) $$\left(\frac{K_0}{L_0}\right)^{\alpha} = \left(\frac{\tilde{\vartheta} i\,(1-s_K)^{\alpha}(1-s_L)^{1-\alpha} A_0}{g_K + \delta}\right)^{\alpha/(1-\alpha)} ,$$

(e5) $$A_0 = \left(s_K^{\beta} s_L^{1-\beta}\left(\frac{K_0}{L_0}\right)^{\beta}\right)^{\gamma/(1-\phi)} \left(\frac{\varphi \ln z}{g_A}\right)^{1/(1-\phi)} .$$

After dropping the exogenous growth path and some constant terms, the expression for the endogenous parts of per capita consumption on the balanced-growth path that depends on $\tilde{\vartheta}(\eta)$, $i(\eta,\nu)$, $s_K(\eta,\nu)$ and $s_L(\eta,\nu)$ is

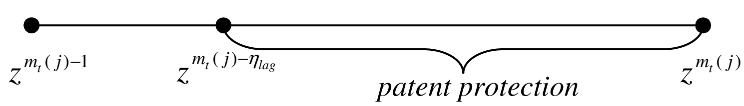
(e6) $$c_0(\eta,\nu) = \left(\begin{array}{l} \tilde{\vartheta}(\eta)^{1/(1-\alpha)} i(\eta,\nu)^{\frac{\alpha(1-\phi)+\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-i(\eta,\nu)) \\ s_K(\eta,\nu)^{\frac{\beta\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-s_K(\eta,\nu))^{\frac{\alpha(1-\phi)}{(1-\alpha)(1-\phi)-\beta\gamma}} \\ s_L(\eta,\nu)^{\frac{(1-\beta)\gamma}{(1-\alpha)(1-\phi)-\beta\gamma}} (1-s_L(\eta,\nu))^{\frac{(1-\alpha)(1-\phi)}{(1-\alpha)(1-\phi)-\beta\gamma}} \end{array}\right) .$$

Finally, after setting $\alpha = \beta$ and dropping $\tilde{\vartheta}(\eta)$, (e6) becomes (e1).■

Appendix II: Lagging Breath

Incomplete lagging breadth delimits a patentholder's property rights over her invention. The following formulation of lagging breadth originates from Li (2001). Assume zero leading breadth $\eta_{lead} = 0$ as in standard models for now. To reiterate, each invention is a quality improvement of z, and this production technology, once invented, becomes public knowledge to fulfill the disclosure requirement for obtaining a patent. In the case of complete lagging breadth, the patent for $m_t(j)$ allows the new industry leader to produce with any technology level $\in (z^{m_t(j)-1}, z^{m_t(j)}]$, but the profit-maximizing level is $z^{m_t(j)}$. The former industry leader, who holds the patent for $m_t(j)-1$, is also technologically feasible to upgrade its production process. However, to do so, she would infringe the patent of the new industry leader, and any licensing agreement would drive the licensee's profits to zero.

The parameter $\eta_{lag} \leq 1$ represents the degree of lagging breadth. In the special case of complete lagging breadth $\eta_{lag} = 1$, any technology level beyond $z^{m_t(j)-1}$ is protected by the patent for $m_t(j)$. In the case of incomplete lagging breadth $\eta_{lag} < 1$, only technology level beyond $z^{m_t(j)-\eta_{lag}}$ is protected. The following diagram illustrates the concept of incomplete lagging breadth.



In other words, although the invention is a quality improvement of z, the patent only protects part of this invention $z^{\eta_{lag}}$ against imitation. Therefore, with incomplete lagging breadth, the Bertrand equilibrium price is $P_t(j) = z^{\eta_{lag}} MC_t(j)$ and the amount of monopolistic profits is $\pi_t(j) = (z^{\eta_{lag}} - 1) MC_t(j) X_t(j)$ for $\eta_{lag} \in (0,1)$ and $j \in [\theta,1]$. Incomplete patent protection against imitation forces the industry leader to lower its markup. On one hand, incomplete lagging breadth reduces the distortionary effects of markup pricing; on the other hand, the reduced profit worsens the incentives for R&D due to incomplete property rights over an invention.

Appendix III: Transition Dynamics

This appendix provides the details of transforming the variables in equations (39) - (42) into their stationary forms for the purpose of computing the transition dynamics numerically. To simplify the analysis, the transformation is performed for the special case of $\alpha = \beta$. The Euler equation is

(f1) $$\dot{c}_t = c_t(r_t - \rho)/\sigma .$$

Define a stationary variable $\tilde{c}_t \equiv c_t/(A_t Z_t)^{1/(1-\alpha)}$, and its resulting law of motion is

(f2) $$\frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{1}{\sigma}(r_t - \rho) - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z) .$$

The law of motion for physical capital is

(f3) $$\dot{K}_t = Y_t - C_t - K_t\delta .$$

Define a stationary variable $k_t \equiv K_t/(L_t(A_t Z_t)^{1/(1-\alpha)})$, and its resulting law of motion is

(f4) $$\frac{\dot{k}_t}{k_t} = (1 - s_{r,t})\vartheta(\eta)k_t^{\alpha-1} - \frac{\tilde{c}_t}{k_t} - (\delta + n) - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z) .$$

The law of motion for R&D-driven technology is

(f5) $$\dot{A}_t = A_t\lambda_t \ln z .$$

Define a stationary variable $a_t \equiv k_t^{\alpha\gamma} A_t^{\alpha\gamma/(1-\alpha)-(1-\phi)} Z_t^{\alpha\gamma/(1-\alpha)} L_t^{\gamma}\varphi$, and its resulting law of motion is

(f6) $$\frac{\dot{a}_t}{a_t} = \alpha\gamma(1 - s_{r,t})\vartheta(\eta)k_t^{\alpha-1} - \alpha\gamma\frac{\tilde{c}_t}{k_t} - (1-\phi)\lambda_t \ln z + (n\gamma - \alpha\gamma(\delta + n)) .$$

The law of motion for the market value of ownership in patent pools is given by

(f7) $$\dot{V}_{i,t} = (r_t + \lambda_t)V_{i,t} - \lambda_t V_{i+1,t} - \sigma_i \pi_t$$

for $i \in \{1,2,...,\eta\}$ and $V_{\eta+1,t} = 0$. Define a stationary variable $\tilde{v}_{i,t} \equiv V_{i,t}/(L_t(A_t Z_t)^{1/(1-\alpha)})$, and its resulting law of motion is

(f8) $$\frac{\dot{\tilde{v}}_{i,t}}{\tilde{v}_{i,t}} \equiv (r_t + \lambda_t) - \lambda_t\frac{\tilde{v}_{i+1,t}}{\tilde{v}_{i,t}} - (1 - s_{r,t})\vartheta(\eta)\sigma_i(1-\theta)\left(\frac{\mu - 1}{\mu}\right)\frac{k_t^{\alpha}}{\tilde{v}_{i,t}} - n - \frac{1}{1-\alpha}(\lambda_t \ln z + g_Z)$$

for $i \in \{1,2,...,\eta\}$ and $\tilde{v}_{\eta+1,t} = 0$. To close this system of differential equations, the endogenous variables $\{r_t, s_{r,t}, \lambda_t\}$ are also expressed in terms of the four newly defined stationary variables. The interest rate is

(f9) $$r_t = \alpha\vartheta(\eta)k_t^{\alpha-1}(\mu\theta + 1 - \theta)/\mu - \delta.$$

From the first-order condition of the R&D sector, the share of factor inputs in R&D is

(f10) $$s_{r,t} = \frac{1}{k_t^{\alpha/(1-\gamma)}}\left(\frac{a_t\tilde{v}_t}{\vartheta(\eta)}\right)^{1/(1-\gamma)}\left(\frac{\mu}{\mu\theta + 1 - \theta}\right)^{1/(1-\gamma)}.$$

From the law of motion of R&D-driven technology, the Poisson arrival rate of innovations is

(f11) $$\lambda_t = s_{r,t}^{\gamma}a_t.$$

Finally, the steady-state values of the variables are

(f12) $$\lambda = g_A / \ln z,$$

(f13) $$\frac{s_r}{1-s_r} = \frac{\lambda v(\mu-1)(1-\theta)/(\mu\theta + (1-\theta))}{\rho - n + (\sigma-1)g_c + \lambda},$$

(f14) $$a = \lambda / s_r^{\gamma},$$

(f15) $$k = \left(\frac{\alpha\vartheta(\eta)(\mu\theta + 1 - \theta)}{\mu(\delta + \rho + \sigma(g_A + g_Z)/(1-\alpha))}\right)^{1/(1-\alpha)},$$

(f16) $$\tilde{c} = (1-s_r)\vartheta(\eta)k^{\alpha} - k\left(\delta + n + \frac{g_A + g_Z}{1-\alpha}\right),$$

(f17) $$\tilde{v}_i = \frac{k^{\alpha}(1-s_r)\vartheta(\eta)\sigma_i(1-\theta)(\mu-1) + \lambda\,\tilde{v}_{i+1}}{\alpha\vartheta(\eta)k^{\alpha-1}(\mu\theta + 1 - \theta) + \mu\left(\frac{1}{\ln z} - \frac{1}{1-\alpha}\right)g_A - \mu\left(\frac{g_Z}{1-\alpha} + \delta + n\right)}$$

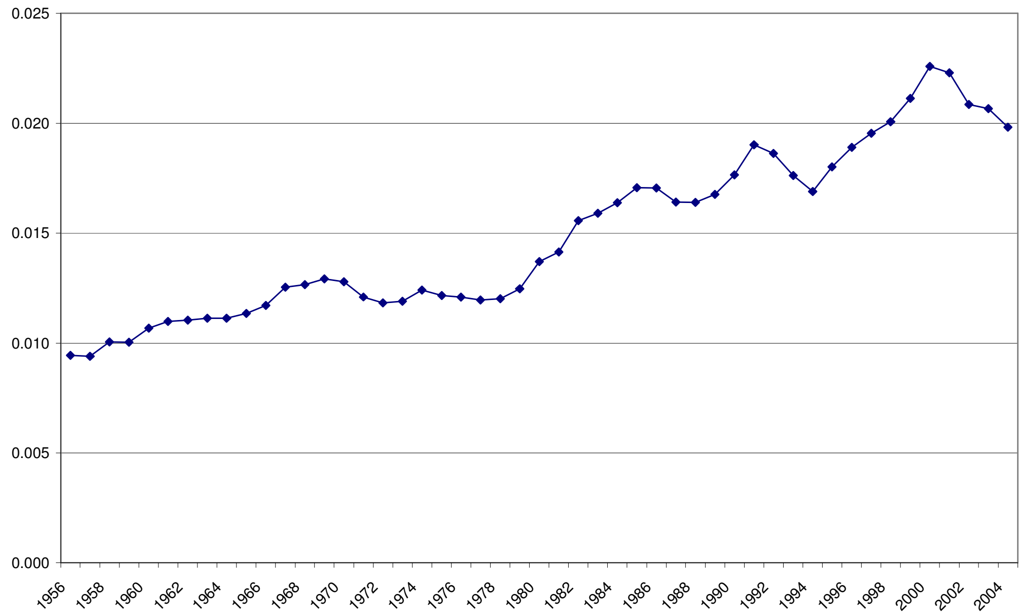
for $i \in \{1,2,...,\eta\}$ and $\tilde{v}_{\eta+1} = 0$. Note that upon eliminating blocking patents, the backloading discount factor v and the share of profits captured by the most recent inventor σ_1 become one.

Table 1a: Structural Parameters for θ = 0						Table 1b: Structural Parameters for θ = 0.25					
λ	ν	α	σ	r	μ	λ	ν	α	σ	r	μ
0.04	0.339	0.240	3.672	0.104	1.100	0.04	0.348	0.241	3.736	0.105	1.136
0.06	0.268	0.240	3.672	0.104	1.100	0.06	0.275	0.241	3.736	0.105	1.136
0.08	0.232	0.240	3.672	0.104	1.100	0.08	0.238	0.241	3.736	0.105	1.136
0.10	0.211	0.240	3.672	0.104	1.100	0.10	0.216	0.241	3.736	0.105	1.136
0.12	0.197	0.240	3.672	0.104	1.100	0.12	0.201	0.241	3.736	0.105	1.136
0.14	0.187	0.240	3.672	0.104	1.100	0.14	0.191	0.241	3.736	0.105	1.136
0.16	0.179	0.240	3.672	0.104	1.100	0.16	0.183	0.241	3.736	0.105	1.136
0.18	0.173	0.240	3.672	0.104	1.100	0.18	0.177	0.241	3.736	0.105	1.136
0.20	0.169	0.240	3.672	0.104	1.100	0.20	0.172	0.241	3.736	0.105	1.136
Table 1c: Structural Parameters for θ = 0.5						**Table 1d: Structural Parameters for θ = 0.75**					
λ	ν	α	σ	r	μ	λ	ν	α	σ	r	μ
0.04	0.366	0.243	3.859	0.108	1.210	0.04	0.424	0.249	4.197	0.114	1.464
0.06	0.288	0.243	3.859	0.108	1.210	0.06	0.331	0.249	4.197	0.114	1.464
0.08	0.249	0.243	3.859	0.108	1.210	0.08	0.285	0.249	4.197	0.114	1.464
0.10	0.226	0.243	3.859	0.108	1.210	0.10	0.257	0.249	4.197	0.114	1.464
0.12	0.210	0.243	3.859	0.108	1.210	0.12	0.238	0.249	4.197	0.114	1.464
0.14	0.199	0.243	3.859	0.108	1.210	0.14	0.225	0.249	4.197	0.114	1.464
0.16	0.191	0.243	3.859	0.108	1.210	0.16	0.215	0.249	4.197	0.114	1.464
0.18	0.184	0.243	3.859	0.108	1.210	0.18	0.207	0.249	4.197	0.114	1.464
0.20	0.179	0.243	3.859	0.108	1.210	0.20	0.201	0.249	4.197	0.114	1.464

Table 2a: ϕ for θ = 0						Table 2b: ϕ for θ = 0.25					
ξ / γ	0.2	0.4	0.6	0.8	1.0	ξ / γ	0.2	0.4	0.6	0.8	1.0
1.00	0.64	0.29	-0.07	-0.42	-0.78	1.00	0.64	0.29	-0.07	-0.42	-0.78
0.80	0.56	0.11	-0.33	-0.78	-1.22	0.80	0.56	0.11	-0.33	-0.78	-1.22
0.60	0.41	-0.18	-0.78	-1.37	-1.96	0.60	0.41	-0.19	-0.78	-1.37	-1.96
0.40	0.11	-0.78	-1.66	-2.55	-3.44	0.40	0.11	-0.78	-1.67	-2.56	-3.45
0.20	-0.78	-2.55	-4.33	-6.10	-7.88	0.20	-0.78	-2.56	-4.33	-6.11	-7.89
Table 2c: ϕ for θ = 0.5						**Table 2d: ϕ for θ = 0.75**					
ξ / γ	0.2	0.4	0.6	0.8	1.0	ξ / γ	0.2	0.4	0.6	0.8	1.0
1.00	0.64	0.29	-0.07	-0.43	-0.78	1.00	0.64	0.28	-0.08	-0.43	-0.79
0.80	0.55	0.11	-0.34	-0.78	-1.23	0.80	0.55	0.10	-0.34	-0.79	-1.24
0.60	0.41	-0.19	-0.78	-1.38	-1.97	0.60	0.40	-0.20	-0.79	-1.39	-1.99
0.40	0.11	-0.78	-1.67	-2.56	-3.46	0.40	0.10	-0.79	-1.69	-2.59	-3.48
0.20	-0.78	-2.56	-4.35	-6.13	-7.91	0.20	-0.79	-2.59	-4.38	-6.17	-7.96

Table 3: R&D Shares without Blocking Patents									
θ / λ	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
0.00	3.4%	4.3%	4.9%	5.4%	5.8%	6.1%	6.4%	6.6%	6.8%
0.25	3.3%	4.2%	4.8%	5.3%	5.7%	6.0%	6.3%	6.5%	6.7%
0.50	3.1%	4.0%	4.6%	5.1%	5.5%	5.8%	6.0%	6.2%	6.4%
0.75	2.7%	3.5%	4.0%	4.5%	4.8%	5.1%	5.3%	5.5%	5.7%

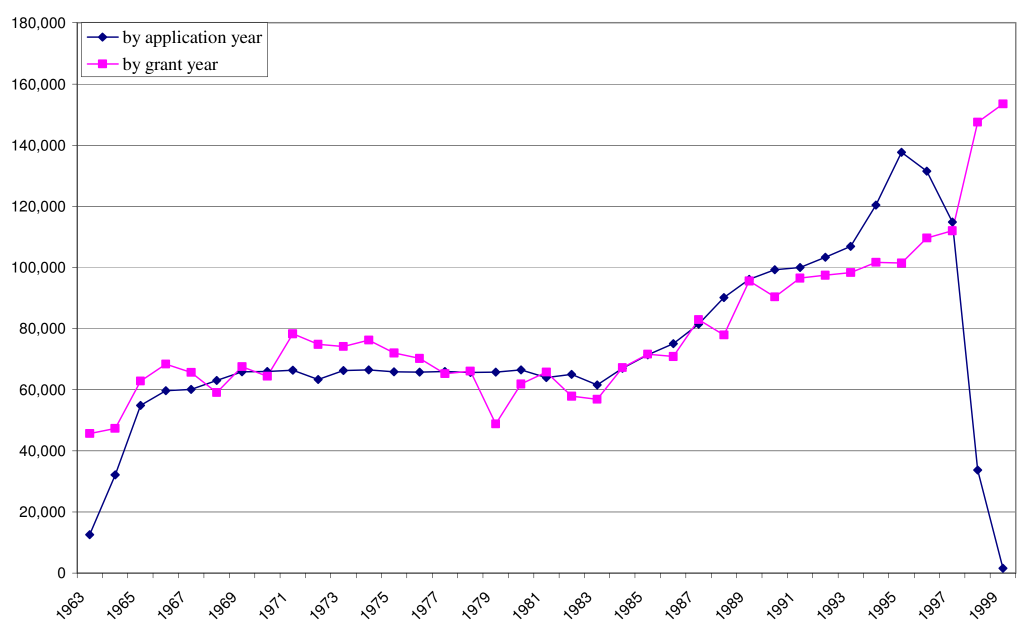
Figure 1: Private Spending on R&D as a Share of GDP



Data Sources: (a) Bureau of Economic Analysis: National Income and Product Accounts Tables; and (b) National Science Foundation: Division of Science Resources Statistics.

Footnote: R&D is net of federal spending, and GDP is net of government spending.

Figure 2: Number of Patents Granted



Data Source: Hall, Jaffe and Trajtenberg (2002): The NBER Patent Citation Data File.

Figure 3a: First-Best Optimal R&D Shares for θ = 0

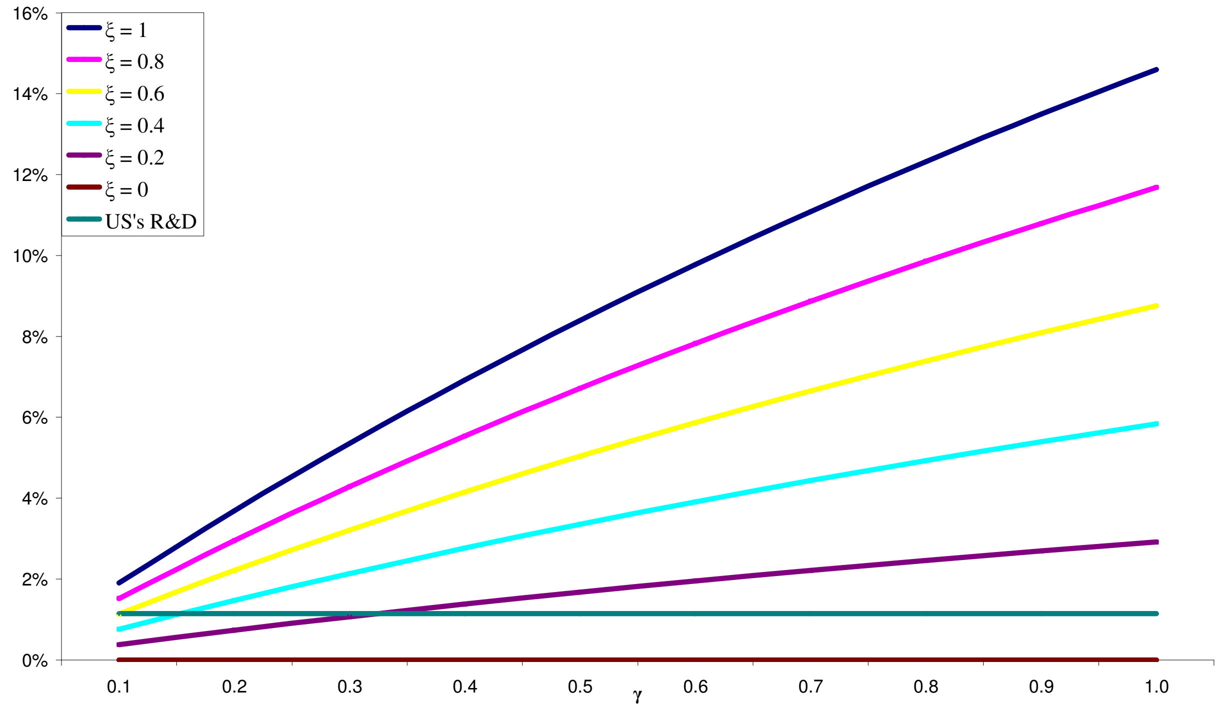


Figure 3b: First-Best Optimal R&D Shares for θ = 0.25

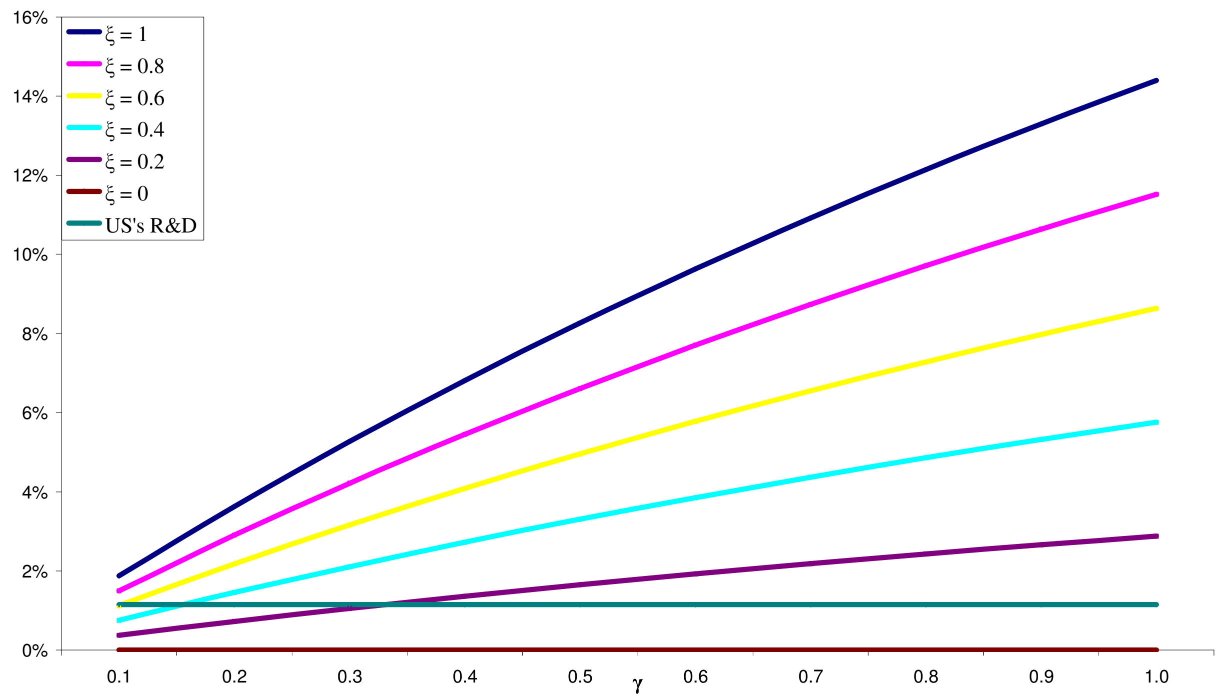


Figure 3c: First-Best Optimal R&D Shares for θ = 0.5

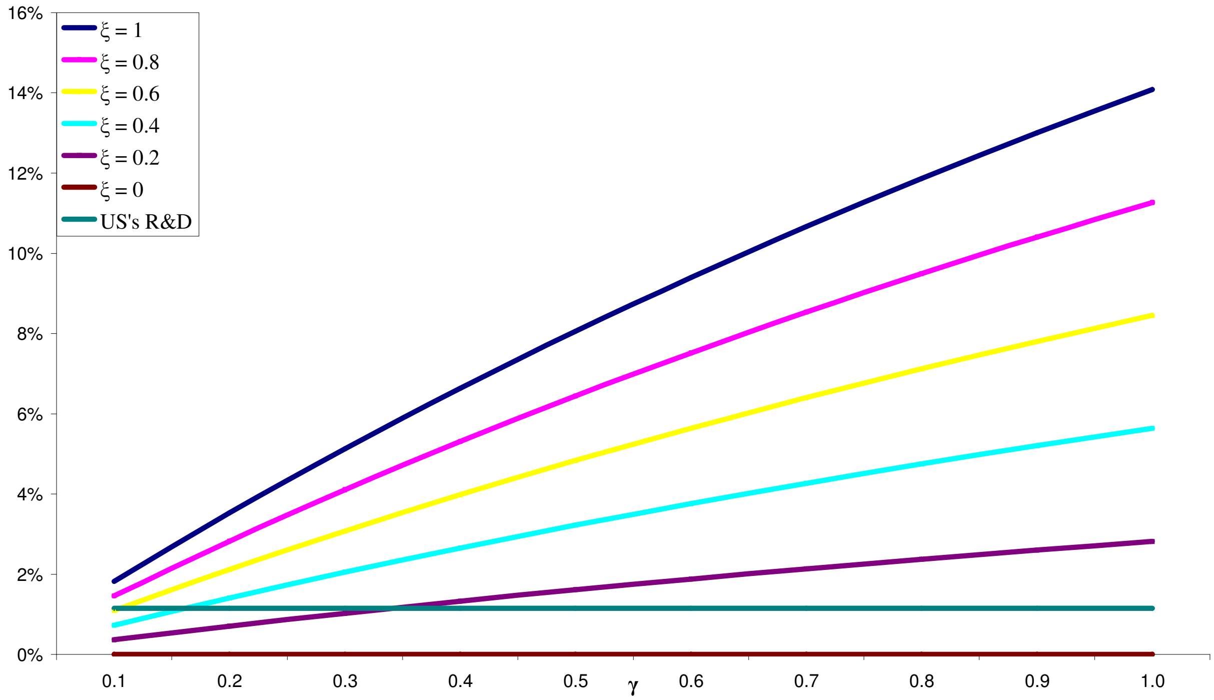


Figure 3d: First-Best Optimal R&D Shares for θ = 0.75

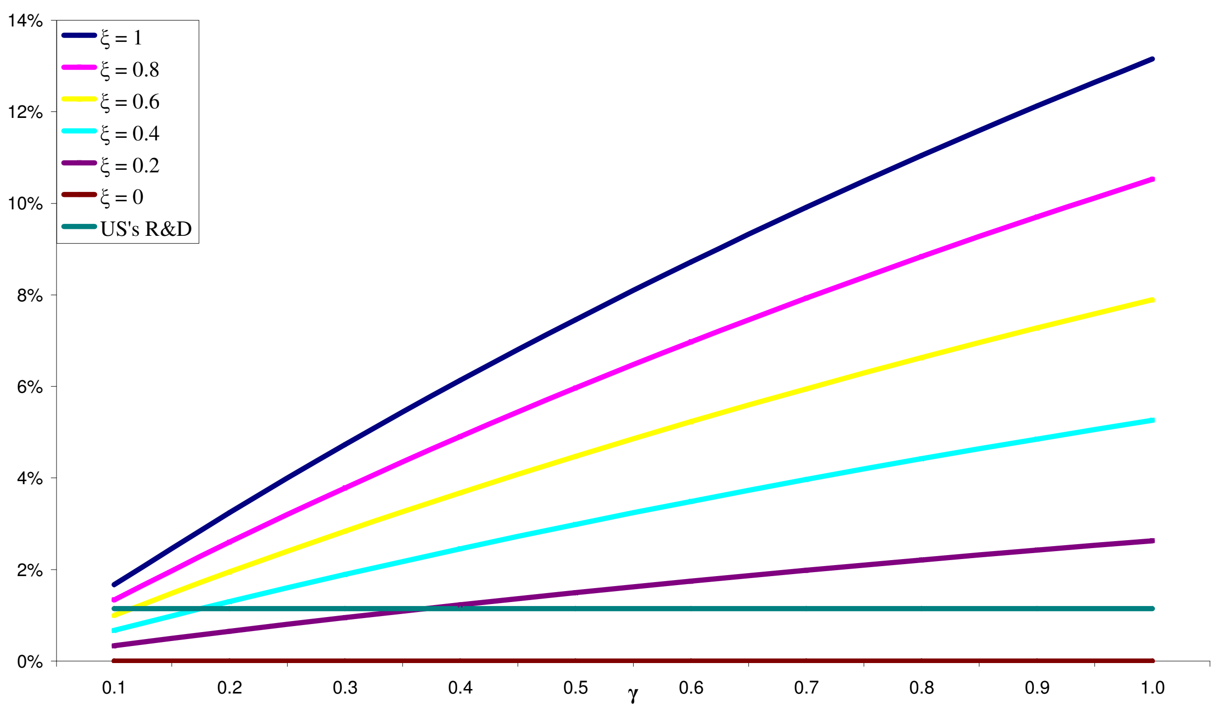


Figure 4a: Change in Long-Run Consumption from Eliminating Blocking Patents for θ = 0

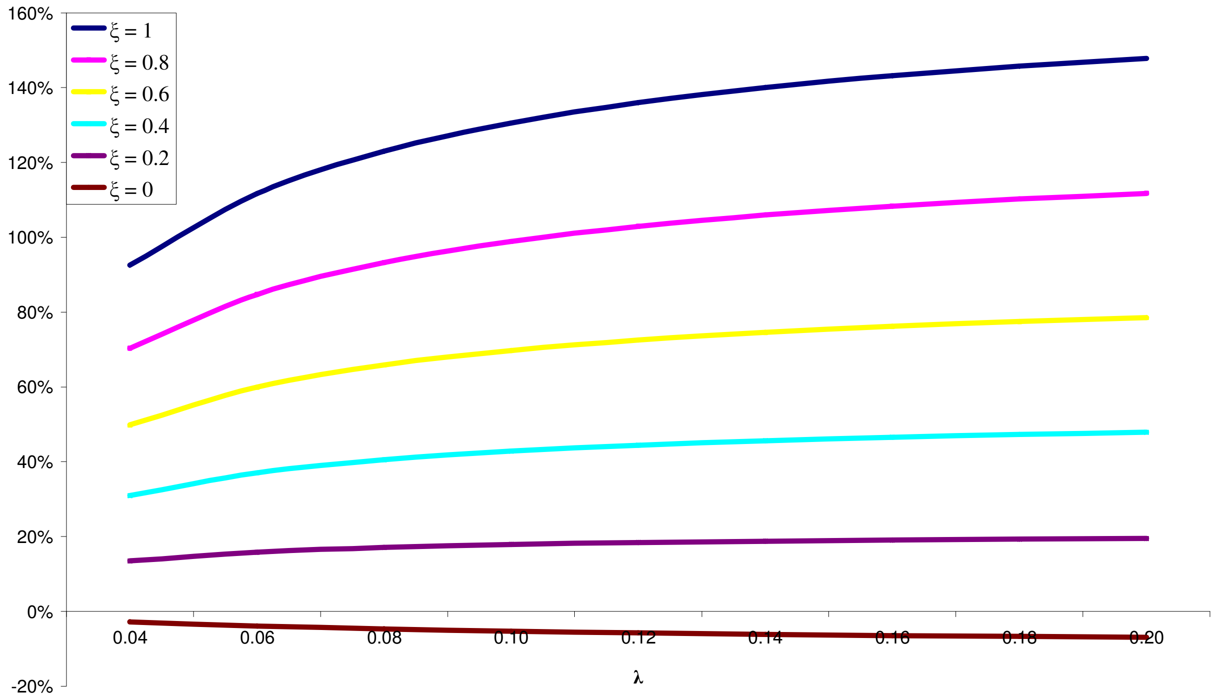


Figure 4b: Change in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.25

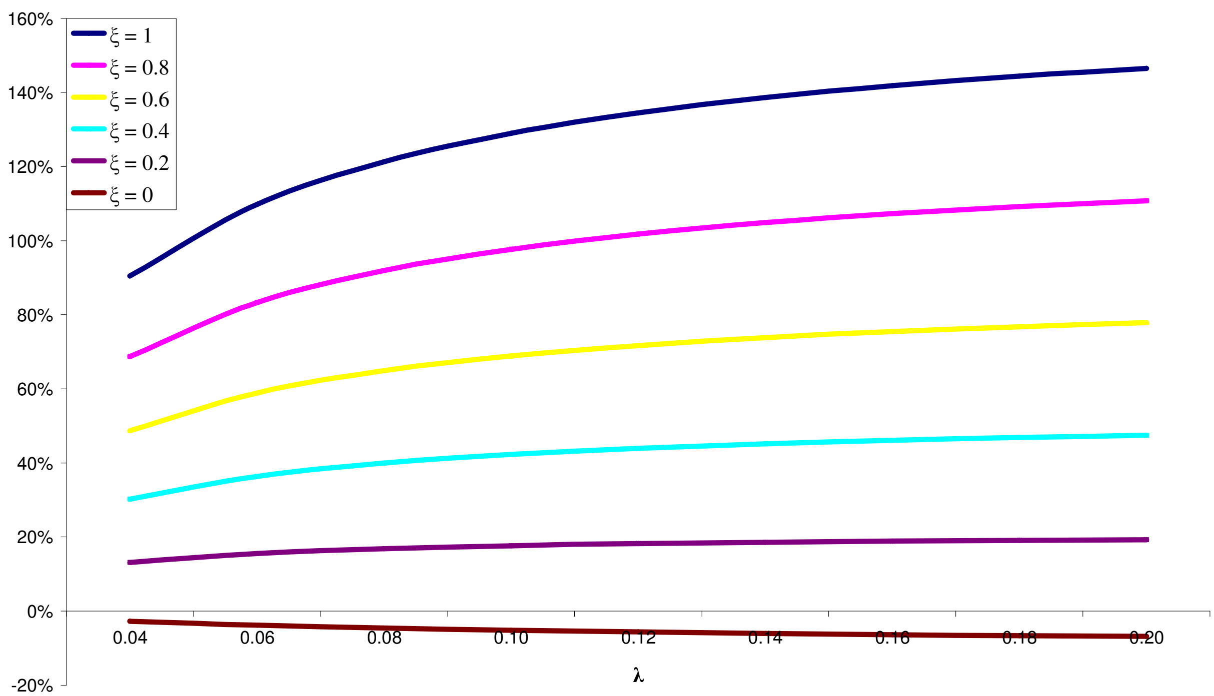


Figure 4c: Change in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.5

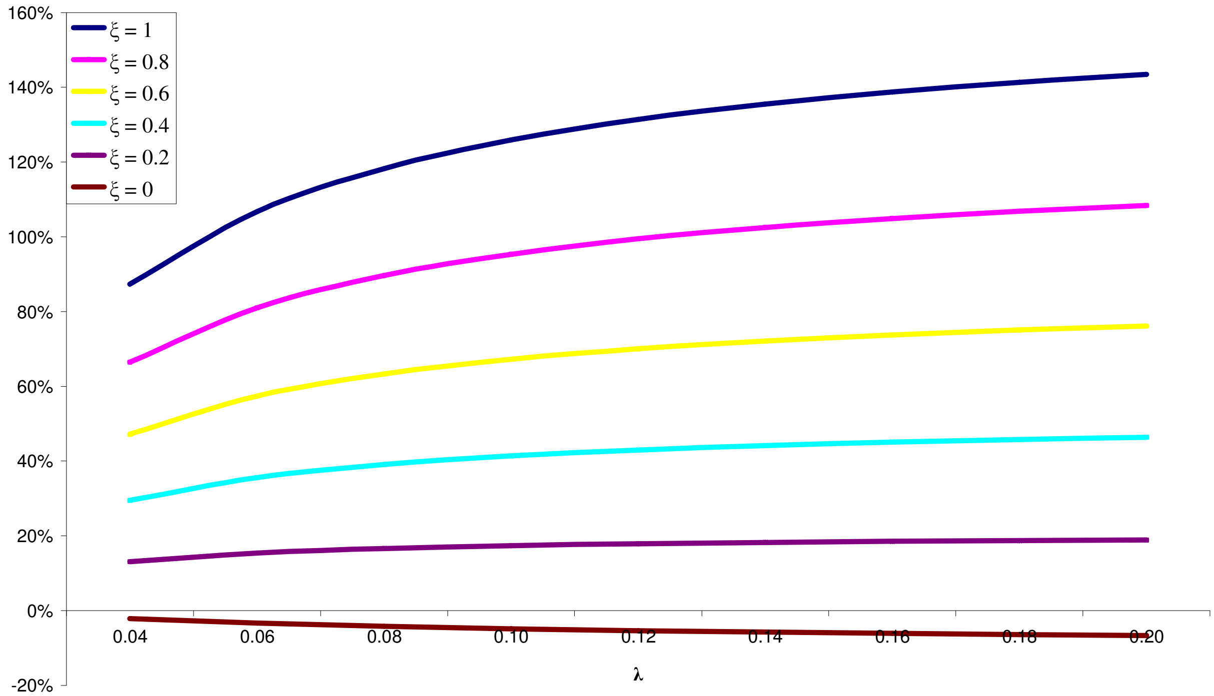


Figure 4d: Change in Long-Run Consumption from Eliminating Blocking Patents for θ = 0.75

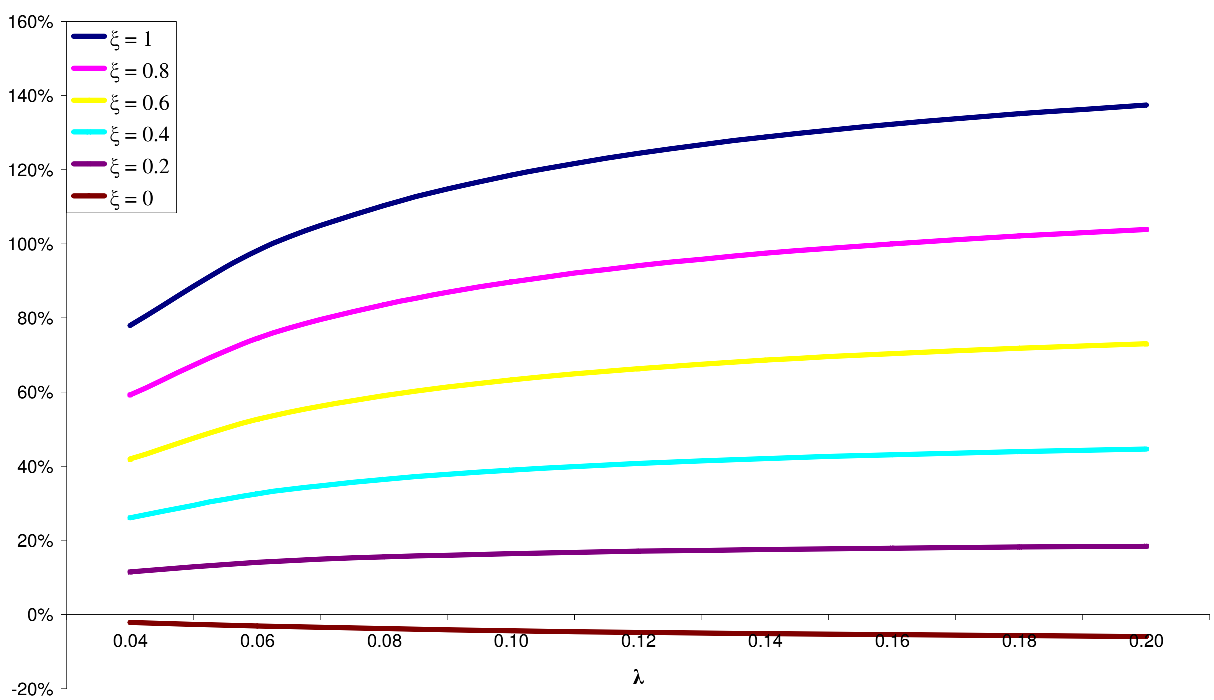


Figure 5a: Transition Dynamics of Consumption for ξ = γ = 0.7 with Partial Capital Depreciation

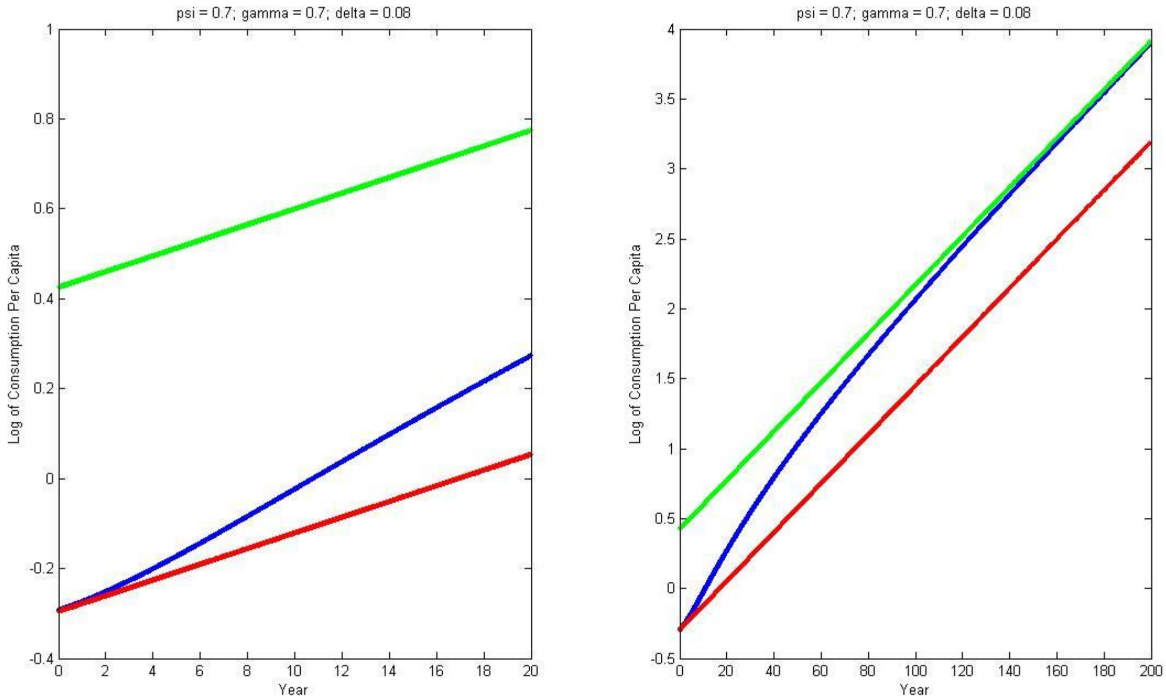


Figure 5b: Transition Dynamics of Consumption for ξ = γ = 0.7 with Complete Capital Depreciation

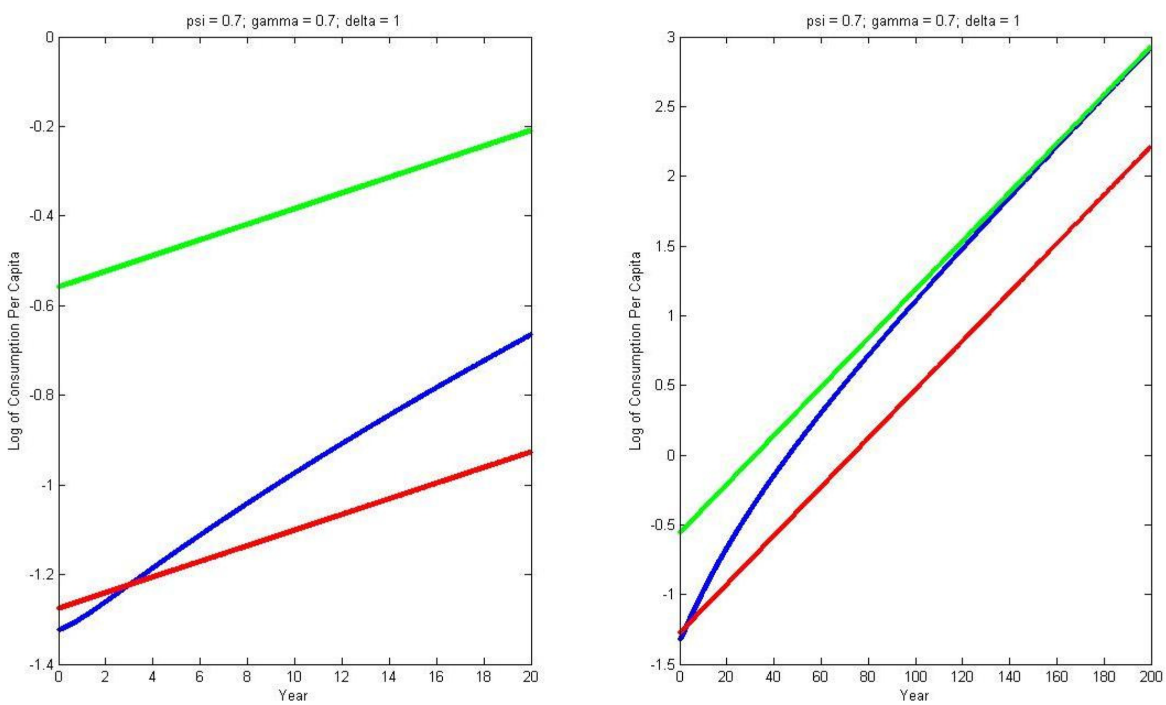


Figure 5c: Transition Dynamics of Consumption for ξ = γ = 0.5 with Partial Capital Depreciation

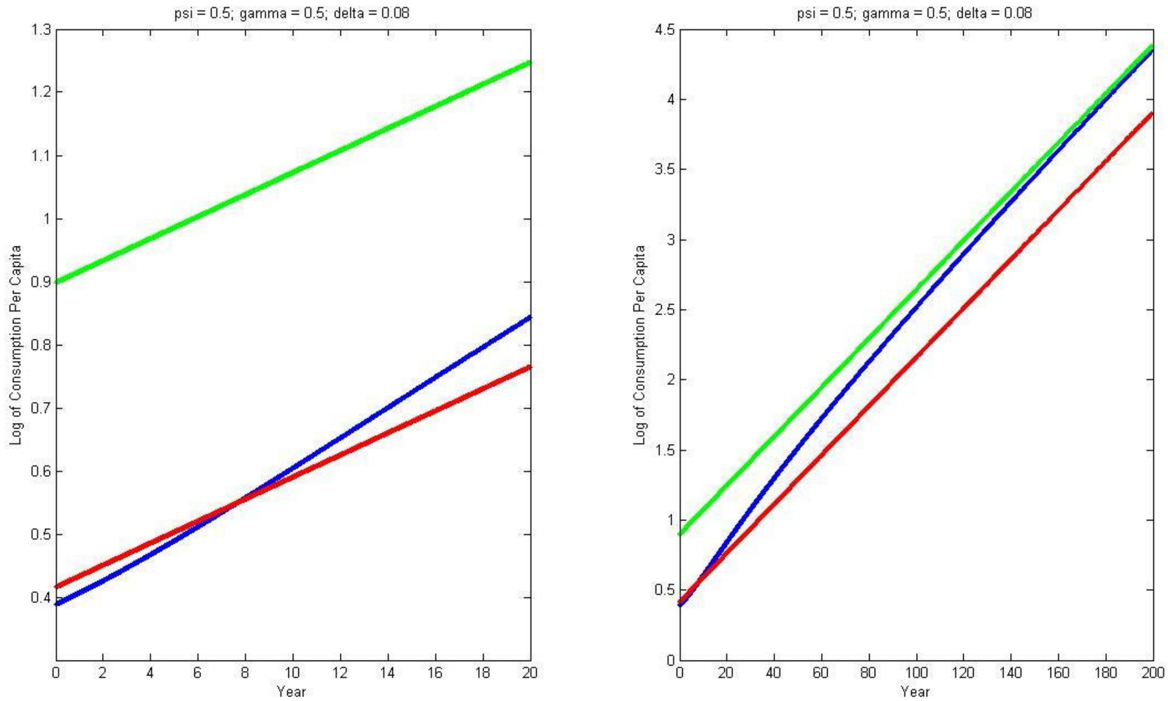


Figure 5d: Transition Dynamics of Consumption for ξ = 0.95 and γ = 0.5 with Partial Capital Depreciation