A Small Macroeconomic Model to Support Inflation Targeting in Israel

Argov, Eyal and Binyamini, Alon and Elkayam, David and Rozenshtrom, Irit

Bank of Israel

July 2007

Bank of Israel
Monetary Department

A Small Macroeconomic Model to Support Inflation Targeting In Israel

Eyal Argov

Alon Binyamini

David Elkayam

Irit Rozenshtrom

July 2007

Catalog # 3110107002/2
http://www.bankisrael.gov.il

Printed in Israel by Ayalon Printing Ltd.

The views expressed in this paper do not necessarily reflect Bank of Israel policy

Contents

Summary*

This study presents a small New Keynesian model of Israel's economy describing the relationships among the main variables relevant to the transmission mechanism of monetary policy. The model encompasses three structural equations – for inflation, the output gap, and the exchange rate – and an interest rate rule, that describes how the interest rate should be adjusted in order to eventually achieve the inflation target. The theory underlying the model is broadly in line with the current monetary theory prevailing in academia and central banks. The model is small, yet it includes the main channels through which the central bank's interest rate affects inflation in a small open economy. An important advantage of a small model is its ability to describe, clearly and simply, the interrelation between the main variables that are related to the transmission mechanism of the monetary policy, while maintaining theoretical coherence. The model presented is intended to serve as an operational tool that can aid policy makers to assess the interest rate path required to achieve the inflation target, but it can also serve others interested in monitoring monetary developments or in forecasting the evolution of the relevant variables. The model can also help to focus the monetary-policy discussion and enhance the communication between the various entities, both within and outside the central bank, involved in monetary policy and its outcomes. To improve the forecasting ability of the model, in formulating the equations we tried to strike a balance between the need to maintain economic logic and the need to obtain an adequate empirical description of the relation between the relevant variables. The parameters of the model were estimated using quarterly data of Israel's economy in the years 1992 to 2005.

---

* We would like to thank Akiva Offenbacher, Douglas Laxton, Michael Beenstock, Alex Ilek, Guy Segal and the participants of the seminars in the Monetary and Research Departments, whose comments contributed to the development of the model.

1. Introduction

In this study we formulate and estimate a small macroeconomic model which is intended *inter alia* to assist the central bank in assessing the interest rate that is required to achieve the inflation target. The model is quarterly and is the smallest model possible for describing the transmission mechanism of monetary policy in a small and open economy. The model contains four equations, which describe the determination of inflation, the output gap, the nominal exchange rate and the interest rate.

An important advantage of a small model is its ability to describe clearly and simply the interrelationships between the main variables related to the transmission mechanism while maintaining theoretical consistency. This simplicity and clarity helps to focus the discussion and improves the communication between the different entities interested in monetary policy and its outcomes both within and outside of the central bank.

We endeavored, when formulating the empirical model and estimating the parameters, to strike a balance between the desire to adhere to the pure economic theory and the desire to reach a good empirical description of the relationships between the relevant variables. A good fit of the model to the data is an important condition for its operative use – generating forecasts for short and medium terms of up to three years, examining policy alternatives and performing risk analysis. It should be emphasized, however, that every model is a simplistic description of a complex reality. Accordingly, the model can serve as only one of the tools employed in assessing the present and future state of the economy. There will always be some special circumstances and forces affecting the economy which are not expressed in the model. Therefore, one of the challenges when using the model is to integrate external information in simulations of the model.

In this study we obviously rely on models that were developed and formulated for other economies. The emphasis here is on paying special attention to the characteristics of the Israeli economy and the quality of the data available for Israel. In contrast to most of the studies of the subject worldwide, the emphasis here is on an empirical (classical) estimation of the model's parameters.

In the second section we will briefly describe the roles of monetary models under an inflation targeting regime. In the third section, we will provide an abbreviated verbal description of the model's equations and of the transmission mechanism of monetary policy according to the model (that is, we will detail the channels whereby the interest rate and exchange rate affect prices). The fourth section provides a detailed description of the model's equations and the economic thinking on which they are based. In the fifth section, we will examine the properties of the model (the dynamic elasticities), and in the sixth section we will demonstrate the use of the model for forecasting. In the seventh section, we will examine the empirical fit of the model (a dynamic simulation within the estimation period and a comparison of moments by means of a Monte Carlo simulation), and in the eighth section we will summarize the main findings and indicate directions for further research.

2. Roles of a monetary model under an inflation targeting regime

The principal task of monetary policy is to provide an anchor for inflation. The policy instrument, which the central bank uses for achieving this objective, is the interest rate.[1] The main problem facing monetary policy is to assess the rate of interest that is required to achieve the inflation target, under the constraints that will be detailed below. This is actually the task of most central banks in the Western world.

Implicit in the fact that central banks are imposed with the task of attaining the inflation target by means of the interest rate, is the assessment held by governments in the Western world that inflation can be stabilized via the use of this instrument. This assessment reflects the development of a theoretical framework that evolved over the past 20 years. This framework, the New Keynesian theory, describes the principal relationships characterizing the transmission mechanism of monetary policy – the mechanism that relates the interest rate and the inflation rate.[2]

A monetary model is a system of equations describing the transmission mechanism of monetary policy. Monetary models based on the New Keynesian theory have been developed at the majority of central banks in the Western world during the last decade. Although the models differ in the extent of their complexity, they have a number of common features:

1. The principal variables, essential for describing and understanding the transmission mechanism in a small and open economy, are inflation, the output gap,[3] the exchange rate and the interest rate.
2. Due to a temporary rigidity of prices and wages, changes in the nominal interest rate generate changes in the real interest rate, and these generate a temporary deviation of real variables from their trends. However, changes

[1] The interest rate on monetary loans to commercial banks or on the banks' deposits at the Bank of Israel.

[2] For a detailed presentation of this theory, see Svensson (2000), Walsh (2003) and Woodford (2003).

[3] The output gap is the gap in percent between actual output and potential output. Potential output in this context is a theoretical concept: the output that would have prevailed in a situation of price flexibility. In this study, it is estimated by means of the HP Filter method (as will be detailed later).

in the interest rate do not impact the trends of real variables such as the real interest rate, the real exchange rate, output and employment.

3. The economy (and inflation in particular) is subject to unexpected shocks. By constantly adjusting the interest rate in response to these shocks, relative stability in the pace of inflation can be achieved, meaning that it is possible over time to reduce the deviations of inflation from its target.
4. Monetary policy is the sole factor that eventually determines the inflation rate.[4] Suppose, for example, that policy-makers wish to reduce the pace of inflation. In such a case, an increase in the nominal interest rate leads to a temporary rise in the real interest rate and to a temporary decrease in real activity and in the real exchange rate. These have the effect of permanently reducing inflation (and the nominal interest rate). In other words, in the short run, a reduction in inflation has a real cost in terms of output and employment: for some time, the level of the latter two will be below their potential level. However, the effect on activity and employment is temporary, while the decline in inflation is permanent.

Because of the tradeoff, in the short run, between the deviations of inflation from the target and the deviations of output from its potential level, in most countries monetary authorities adopt a flexible inflation targeting policy. A flexible policy refers to policy-makers' attempts to achieve a balance between the objective of reducing the deviations of inflation from the target and the objective of reducing the deviations of output from its potential level. In most cases, policy-makers have an additional objective – to moderate the fluctuations in the interest rate, due to concern for financial stability. A

[4] This is assuming that a situation of fiscal dominance does not emerge. If this does happen, an interest rate hike could actually lead to a rise in inflation. A situation such as this, which effectively reflects a loss of control by monetary policy, could arise when the ratio of the government debt to GDP and/or the proportion of the debt in foreign currency to the total debt increases to a major extent (see for example Blanchard [2004]). Since the 1985 stabilization program, the probability of such a development in Israel has been low. (See for example Liviatan [2003]).

balance between these objectives (or constraints) implies that the response of the interest rate to shocks affecting inflation will be gradual, and hence the convergence of inflation to its target will also be gradual. This is in order to moderate fluctuations in output and the interest rate.

Given a model that describes the transmission mechanism, numerical values for the model's parameters, and the objectives of monetary policy (attainment of the inflation target while reducing the deviations of output from its trend and moderation of interest rate fluctuations) – it is possible to calculate the interest rate path that is required in order to keep inflation as close as possible to the target, subject to the previously mentioned constraints. This is the principal role of a monetary model, but not its only role.

A monetary model assists in analyzing the present condition of the economy and in identifying the shocks affecting it. Given an assessment regarding the present state of the economy, the next step is using the model to forecast the future developments of the relevant variables, such as the interest rate, the inflation rate, the exchange rate and the output gap. All this in a structural and consistent manner.

Another important application of the model is the examination of policy alternatives and the analysis of risks. The model can be used to assess the appropriate policy action required in response to different shocks that may hit the economy.

Finally, a monetary model assists in focusing the discussion and improving the communication between the different entities interested in monetary policy and its results, both within and outside of the central bank. As an example, the central bank is required to publish a semi-annual inflation report, which details the considerations behind the policy conducted during the preceding months. A monetary model could serve as an important tool for facilitating a clear and consistent analysis of recent developments in the economy, and thereby contribute to increased communication and credibility.

3. An overview of the model (the transmission mechanism)

The model presented below belongs to the class of New Keynesian models. In these models, under an inflation targeting regime, the role of monetary policy is clearly defined: to provide the economy with an anchor for inflation and for inflation expectations. In these models, if monetary policy functions properly (as will be defined in more details below), the economy converges to an equilibrium in which real variables converge to their potential values (as detailed below), inflation converges to its target, and the other nominal variables adjust in accordance with the inflation target.[5]

It is also assumed that due to nominal rigidities, changes in the nominal interest rate have short run real effects. Changes in the nominal interest rate are reflected, in the short run, by changes in the real interest rate, and these affect the real (and nominal) exchange rate, real activity and employment. However, these effects are only short-term in nature. In the medium run (one to three years) and the long run, the impact of these rigidities is exhausted, and the real variables revert to their potential levels, which are determined by supply factors that are not affected by monetary policy.

For these reasons, all the real variables in the model are expressed in terms of deviations from their trends (hereinafter: "gaps"). As stated, this formulation is based on the perception whereby nominal variables in general and monetary policy in particular have no effect on the trends of real

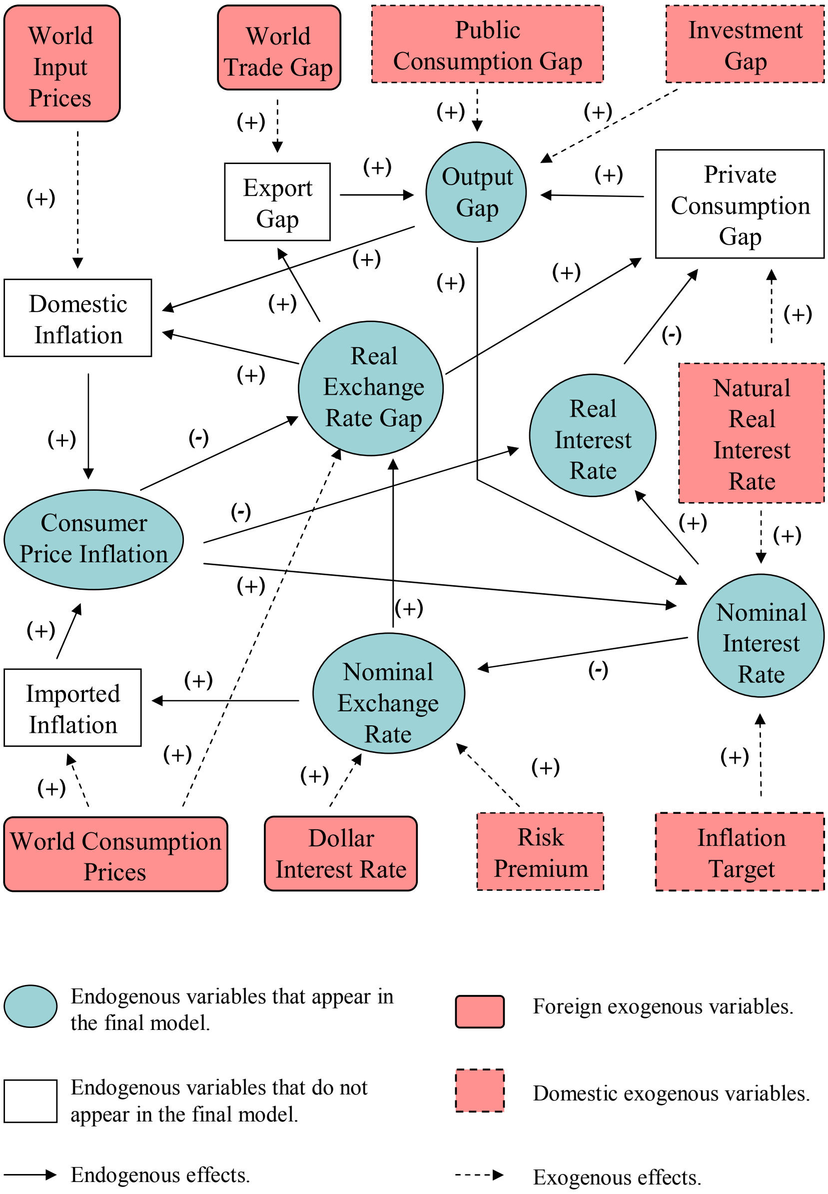
[5] As stated, this is on condition that a situation of fiscal dominance does not emerge.

variables. It should be noted that the present model is not intended to provide an explanation for the trends in the real variables. For example, the model does not explain the growth rate of potential output, the natural interest rate and the trend in the real exchange rate. These values are determined outside the model. However, changes in the nominal interest rate do, as stated, have the effect of creating deviations of real variables from their trends, and these deviations affect the inflation rate (as detailed later).

Before we present the model equations, we will describe the nature of the economy to which the model relates. We assume a small and open economy (the Israeli economy) with four main groups of agents: households, firms, the rest of the world (households and firms) and the central bank.[6] Households (which are identical) derive utility from two types of goods – goods that are produced in the domestic market, whose price is determined by domestic firms, and imported goods, whose price is determined abroad. Households save in order to allocate consumption over time, and can invest their savings in domestic assets bearing a sheqel interest rate or in foreign assets bearing a dollar interest rate. The allocation of their savings between domestic and foreign assets is determined according to the interest rates in Israel and abroad, the expected depreciation in the exchange rate and the relative risk of domestic assets.

[6] The government is not included here because its function in a model that describes inflation is limited. Due to the existence of a law that prohibits the government to print money, the government does not exert a direct effect on the money supply in the economy, which is determined solely by the central bank. The government can finance its deficit only by means of loans from Israeli residents or from nonresidents. An increase in the deficit and the government debt will be reflected by a higher real interest rate, and this will affect the trend in other real variables such as investment and the pace of growth. The increase can also affect the level of the (nominal and real) interest rate that will be necessary in order to attain the inflation target, but not the pace of inflation that will eventually prevail. Government demand directed at the business sector is reflected in the model in the output gap equation (see later).

Figure 3.1: Schematic Diagram of the Model



Households abroad import goods from different countries. The goods imported from each country, and in particular from Israel, are not identical. The exporting country exerts an effect on the price of its exports, and demand from abroad for Israeli exports is dependent on the price of Israeli exports, which is related to the prices of the goods produced in Israel, relative to the prices of other countries' exports.

The firms in the model act under monopolistic competition. This means that there are many firms, each of which manufactures a product that is unique to some extent, with the result that it can determine the price of its product.[7] Each firm employs domestic inputs and imported inputs, and offers its products to the domestic market (to households) and to markets abroad (exports).

The model is aggregative. It contains four dynamic behavioral equations that are derived from the solutions to the optimization problems of households and firms together with a rule for the central bank's interest rate. A schematic diagram of the model is presented in Figure 3.1.

The first equation describes CPI inflation as a function of inflation expectations, the output gap, the real exchange rate gap and the changes in the (sheqel) price of imported goods. Inflation expectations reflect the effect of price rigidity in the short run. As a result of this rigidity, when a firm revises its price, it takes into account the fact that its price will not be readjusted for a while, hence the price decision in the present is affected by the expectations for future inflation. The output gap reflects the effect of aggregate demand on prices. An increase in aggregate demand leads to a rise in prices resulting from the increased production costs deriving from a rise in the demand for factors of production (labor, capital and raw materials), and from (monopolistic)

---

[7] The extent of the demand for the product of a single firm is determined according to its price relative to the other firms' prices.

firms’ tendency to exploit an increased demand in order to increase their profit rate. An increase in the nominal exchange rate or in world prices has the direct effect of increasing the prices of imported goods in the consumption basket. Such an increase also has an indirect effect on the prices of goods that are produced in the domestic market. This is because it increases the demand for domestic goods and for domestic factors of production (a substitution effect), leading to an increase in their prices (as detailed in the next section).

The second equation describes the deviations of output from its trend (the output gap). The output gap is affected by the real interest rate gap, which is the difference between the (ex-ante) real interest rate[8] and the natural real interest rate, by the real exchange rate gap,[9] by the world trade gap, which represents world demand, by the domestic public consumption gap, and by the domestic investment gap.

Assuming temporary price rigidity, a rise in the nominal interest rate leads, in the initial stage, to an increase in the real interest rate, and this has the effect of reducing demand for consumption in the present (inter-temporal substitution effect), implying a downward deviation of consumption from the trend and with it a downward deviation from the trend of business-sector output. A rise in the nominal interest rate also has the effect of reducing the nominal exchange rate (see below). This decrease is also reflected in the initial stage by a downward deviation of the real exchange rate from its trend. By increasing the dollar price of the domestic economy's exports and reducing the relative price of imports, the decrease in the real exchange rate leads to a decline in world and domestic demand for domestically produced goods[10] (all in terms of deviations).

---

[8] The Bank of Israel interest rate *less* inflation expectations.

[9] The deviations of the real exchange rate from its trend.

[10] It is assumed that in the initial stage, the price of Israeli exports in sheqel terms remains unchanged. If the nominal exchange rate falls, the price increases in dollar terms. From the

A rise in the interest rate obviously affects investment as well. The present model lacks an equation for investments. We intend to address this matter in the future.

The third equation describes the dynamic relationship between the nominal exchange rate of the sheqel against the dollar, and the differential between the short-term dollar interest rate and the domestic nominal interest rate. A rise in the domestic interest rate increases the attractiveness of domestic assets compared to foreign assets: investors will want to convert dollars to sheqels, and the exchange rate will appreciate.

The fourth equation describes the manner in which the central bank adjusts the nominal interest rate in order to ultimately attain the inflation target. Appearing in this equation are long-term factors – the inflation target and the natural interest rate – and short-term factors: the difference between inflation expectations and the inflation target, and the output gap. According to the interest rate equation, the adjustment of the interest rate to changing conditions (reflected in the inflation gap and the output gap) is gradual rather than immediate. This means that the interest rate is adjusted in a manner whereby in response to shocks hitting the economy, the reversion of inflation to the target and of current output to potential output will be gradual.[11]

Before proceeding to describe the transmission mechanism of monetary policy, it should be noted that the quantity of money does not appear explicitly

---

worldwide perspective, an increase in prices leads to substitution with other goods which the world imports and thereby to a drop in the demand for goods from Israel.

[11] The interest rate is adjusted in a manner whereby generally within a term of two years the output gap zeroes and inflation is expected to revert to its target. For this reason, the forecasts for inflation or for the output gap are usually not the interesting product of monetary planning, as these are expected to revert to their prescribed paths within a certain period. Rather, the major interesting product of monetary planning is the forecast or assessment regarding the future course of the Bank of Israel interest rate which will be necessary in order to attain the inflation target and to ensure that output reverts to its trend.

in the model, although it does exist in the background.[12] As sole supplier of the monetary base to the economy, the central bank can determine either its amount or its price (the nominal interest rate). For important reasons, the discussion of which is outside the scope of this study,[13] most central banks in the Western world have chosen to use the price of money as the policy instrument. The determination of a particular level for the interest rate implies that the central bank is willing to supply the public (usually via the commercial banks) the entire amount of money which the public wishes to hold at this particular interest rate. Put differently, in a regime where the interest rate is the monetary policy instrument, the supply of money is determined by the public's demand. Theoretically, and on the assumption that a stable relationship exists between the interest rate and the demand for money, it would have been possible to select the money supply as a policy variable, and to formulate a response rule for the money supply which would lead to the convergence of inflation to the target. In such a case, the money supply would have appeared explicitly in the model, and the interest rate would have remained in the background.

The transmission mechanism in the model

In the model, a rise in the nominal interest rate has the effect of reducing inflation in a number of channels:

1. The direct channel of private consumption. Assuming temporary price rigidity, a rise in the nominal interest rate results in an increase in the real interest rate. This leads to a decline in the demand for private consumption in the present, due to inter-temporal substitution, and

---

[12] More precisely, money (rather than a specific amount of it) exists in the background. For a discussion of the role of monetary aggregates in the transmission mechanism in the New Keynesian model see for example Woodford (2006).

[13] See for example Clarida et al. (1999).

consequently to a decline in the demand for business-sector output and to a decrease in prices.

2. The exchange rate channel. A rise in the interest rate increases the relative attractiveness of holding sheqel as compared to dollar assets. The public will therefore tend to convert dollars to sheqels, and the exchange rate will appreciate. An appreciation has a deflationary effect through four channels:

a. The price of consumption from imports will fall. Since this price is part of the consumer price index, the index will fall as well.

b. A decrease in the price of consumption from imports implies a decrease in the relative price of imports (relative to the prices of domestically produced goods). This increases the demand for consumption from imports and reduces the demand for consumption from domestic production due to the intra-temporal substitution between consumption from domestic production and consumption from imports. The decrease in domestic demand leads to a decline in the domestic component of the consumer price index and thereby to a decline in the index.

c. An appreciation also has the effect of reducing world demand for Israeli exports. Suppose that initially, the price of Israeli exports in sheqel terms remains unchanged. As a result of the exchange rate appreciation, the price in dollar terms increases. A relative reduction in the dollar price of other countries' exports leads to a drop in world demand for Israeli exports. The demand for domestic production falls, and the price of the domestic component of the CPI falls with it.

d. An appreciation reduces the price of imported inputs. This reduces the cost of domestic production, and the price of the domestic component of the CPI falls with it.

A rise in the interest rate leads to a decline in inflation, in the output gap and in the exchange rate. Assuming that at the starting point the inflation gap and the output gap were zero, these gaps have become negative and under the interest rate rule, the interest rate must be cut. At the next stage, the interest rate will be reduced gradually, and inflation, the output gap and the real exchange rate will gradually revert to their rates at the starting point.

Another important channel through which the interest rate affects prices is the expectations channel. We assume that expectations are rational,[14] meaning that they develop in a manner consistent with the model. As will be explained later, the effect of expectations in the model is reflected by an increase in the speed at which monetary policy impacts prices.

It should be noted at this stage that the formulation of the model does not enforce an immediate pass-through from the exchange rate to the import prices in the domestic market. The fact that we lack data on the prices in the domestic market of the relevant imported products, makes it difficult to address this matter. We will describe how we cope with these difficulties later.

4. The model's equations

A detailed discussion of the theoretical framework of the model's equations is presented in a previous study, by Argov and Elkayam (2006), which also contains references to relevant sources. In the present study we employ the equations developed there, with certain adjustments that are intended to simplify the equations and enhance their fit to the data of the Israeli economy. All this was done without losing the theoretical basis of the model. Below we

[14] More precisely, as will be detailed later, we assume that at least part of the public have rational expectations.

will present the model's equations and the corresponding estimated coefficients. Further explanation for the particular specification that was chosen and a presentation of the estimation results are contained in Appendices A and B.

4.a The inflation equation

The inflation target in Israel is defined upon the consumer price index. For the purpose of analyzing consumer prices in an open economy, it is common to regard the consumer price index as comprised of two main components: the price index of goods and services that are produced in the domestic market (p^h) and the price index of imported goods and services (p^f). The consumer price index (p^c) is a weighted average of these two components, that is: [15]

(4.1) $p^c_t = w^f p^f_t + (1 - w^f)\, p^h_t$

where w^f is the weight of the import component in the consumer price index.

It should be realized that while data on the general index (p^c) obviously exist, there are no data on the two previously mentioned components, which are unobservable variables. We will subsequently describe more extensively how we attempted to overcome this difficulty. Here, we note that the estimate obtained for the weight of the imported goods and services in the general index (w^f) is 0.45. This estimate is derived from the effect of exchange rate depreciations on the consumer price index inflation (as detailed later). At a first glance, this weight appears to be high. As an example, the proportion of direct imports of consumer goods in total private consumption is only about 10 percent. However, there are other import components that are not included

[15] All the variables in equation (4.1) are in log terms.

in this measurement, such as certain energy components and overseas travel.[16] Similarly, the housing component (which accounts for about 20 percent of the index) is heavily affected by the exchange rate of the sheqel against the dollar. In the previously mentioned partition therefore, it probably belongs to the component of imported products.[17]

For domestic prices, we assumed the following equation:

$$\pi^h_t = a_{ld}\,\pi^h_{t+1} + (1 - a_{ld})\,\pi^h_{t\text{-}1} + a_y\,(0.5\,y_t + 0.5\,y_{t\text{-}1}) + a_{zf}\,(p^{zf}_t - p^h_t) \tag{4.2}$$

The estimates obtained for the parameters are:

$a_{ld} = 0.53 \quad a_y = 0.06 \quad a_{zf} = 0.06$

where $\pi^h_t = p^h_t - p^h_{t\text{-}1}$ is the inflation in the prices of domestic goods, y_t is the business-sector output gap and p^{zf}_t are the prices of the imported inputs used for domestic production.[18] The variable π^h_{t+1} represents the expectations at time t for inflation at time t+1.

A detailed derivation of an equation similar to this, based on microeconomic foundations, is presented in a previous study, by Argov and Elkayam (2006). The theory of equations of this type for a closed economy is presented in Woodford (2003), and for an open economy in Svensson (2000).

---

[16] Although a number of imported energy components are included in the category of imported inputs to production, the added value which some of them acquire in the transition from the importers to the final consumers is very small. As an example, the vehicle fuel and oil component (weighted at 4.7 percent in the index) is effectively an imported product. The same applies to oil and diesel for heating (weighted at 0.4 percent) and a large proportion of gas and service fees (which are weighted at 0.6 percent). Another component that is effectively an imported service is the component of expenditure on overseas travel (which is weighted at 4.2 percent). It is reasonable to assume that there are other components which are effectively direct imports, and exclusive of the housing component (see below), we asses that their weight in the index is around 6 percent.

[17] Since the housing component is not an import component and is not a tradable product either, and due to the high degree of correlation between it and the exchange rate against the dollar, it would be desirable to compile a separate equation for it. We hope to do this in the future.

[18] $(p^{zf}_t - p^h_t)$ represents a price ratio. Like the output gap, this expression is also in terms of the deviation from its natural trend.

The underlying concept behind the effect of inflation expectations on current inflation is the assumption (currently accepted as fact) that firms incur a cost when changing their price. Accordingly, when producers set their price they take into account the fact that for a certain period in the future, they will refrain from changing their price. The extent of the change at the time of change is therefore affected by the increase in prices expected during the time in which the producer believes that he will refrain from changing his price. These considerations lead to an equation in which inflation is a function of inflation expectations, and where the coefficient of the latter equals 1 ($a_{ld} = 1$).

The majority of empirical studies worldwide find that inflation is affected by lagged inflation as well. This finding may reflect indexation arrangements, or the existence of adaptive elements or a learning process in the formation of expectations. In such cases, the coefficient of lagged inflation ($1 - a_{ld}$) reflects the extent of indexation to inflation or the proportion of those with adaptive expectations among the public. The restriction stating that the coefficients of inflation expectations and of the lag amount to 1 is of special significance, in that at long-term equilibrium (namely, in a state where *inter alia* the output gap and the price ratio gap are zero), any rate of inflation fulfills the requirements of the equation. The actual inflation rate that will prevail will obviously depend on the inflation target. Given the inflation target, the central bank adjusts the interest rate (see below), thereby affecting the output gap and the nominal and real exchange rates, so as to attain the desired target. As expected in accordance with the theory, when we estimated the equation with no restrictions, the sum of the relevant coefficients we obtained was very close to 1.

The larger is the coefficient of inflation expectations (a_{ld}), the more rapid will be the reaction of inflation to changes in monetary policy.[19] The greater

[19] By means of their impact on the output gap and on the price ratio gap ($p^{zf}{}_t - p^h{}_t$).

the credibility of monetary policy, the larger will be the coefficient which we would expect to obtain. A value of less than 0.5 is not uncommon. In our study, we obtained a value of 0.53, which is indicative of a relatively rapid response of prices.

The extent of the change in prices is also a function of the extent of demand for output relative to its trend (the output gap). It is assumed that firms operate in a market of monopolistic competition and each of them can therefore affect the price it charges. A growth in demand has the effect of increasing firms' demand for factors of production (labor, capital and intermediates) thereby increasing their cost, and the increase in marginal cost leads in turn to a rise in the price of the final good. In addition, firms may exploit a growth in demand to increase their price and with it their rate of profit – the difference between price and marginal cost (the markup). The coefficient of the output gap (a_y) reflects these effects on prices. In the empirical estimation, we found that the output gap affects prices both contemporaneously and with a lag of a quarter.[20] Walsh (2003)[21] cites a number of studies on the US economy where a coefficient of between 0.01 and 0.1 was estimated. Leitemo (2006b) estimated an equation for the UK and obtained a coefficient of 0.07. In our study, a coefficient of 0.06 was obtained, which is definitely a respectable place in the middle.[22]

Another factor affecting domestic prices in an open economy is the price of imported production inputs relative to the price of domestic output. An

[20] In the empirical estimation we found that the coefficient of the effect of the output gap on prices is not stable. For some periods the contemporaneous effect is stronger, while for others the lagged effect is stronger. For the final equation we put in the average effect.

[21] Chapter 11, page 527.

[22] The comparison with other studies is not simple, because the specification of the equations in each study differ. Nevertheless, we shall note that Caputo (2004) obtained a coefficient of 0.067 from data on the Chilean economy and Lopez (2003) obtained a coefficient of 0.027 from data on the Columbian economy. These two authors estimated the coefficient with respect to inflation of the CPI. In our study, the analogous coefficient is 0.033 (0.55*0.06).

increase in the price of imported inputs implies an increase in production costs – both directly and through a substitution effect, which increases the prices of non-imported inputs – that leads to an increase in the price of domestic output. The size of the coefficient of the relative price of imported inputs ($p^{zf}_t - p^h_t$) is *inter alia* a function of the proportion of imports in inputs. When estimating the equation here, we obtained a value of 0.06 for this coefficient. Worldwide studies usually obtain a coefficient of less than 0.1,[23] although the comparison is not simple due *inter alia* to the fact that in most cases, a distinction is not made in the empirical estimation between the prices of imported inputs and the prices of imported consumer goods.

The relative price of production inputs ($p^{zf}_t - p^h_t$) can be written as the sum of two expressions: the price of imported inputs relative to the prices of imported consumer goods ($p^{zf}_t - p^f_t$), and the price of consumption from imports relative to domestic prices ($p^f_t - p^h_t$). There is an advantage to this breakdown when building a forecast and running a simulation, because the first expression is an exogenous variable[24] (the price ratio determined abroad) and the second expression is a central endogenous variable in the model, which appears in the output gap equation as well (see below).

We will assume for a moment that p^f (and therefore also π^f) is measurable. By means of a number of algebraic operations with equations (4.1) and (4.2), which are detailed in Appendix A, the inflation equation of consumer prices can be expressed in terms of measurable variables as follows:

[23] Leitemo (2006b) and Caputo (2004) for example, obtained a coefficient of less than 0.05. Svensson (2000) assumed a coefficient of 0.3, although this is an assumption rather than an estimate or a calibration.

[24] Assuming similar rates of pass-through in the prices of imported consumer goods and imported inputs.

$$
\begin{aligned}
(4.2')\quad \pi^c_t &= a_{ld}\,\pi^c_{t+1} + (1 - a_{ld})\,\pi^c_{t\text{-}1} + (1 - w^f)\,a_y\,(0.5\,y_t + 0.5\,y_{t\text{-}1}) \\
&\quad + a_{zf}\,[(1 - w^f)\,(p^{zf}_t - p^f_t) + (p^f_t - p^c_t)] \\
&\quad + w^f\,[\pi^f_t - a_{ld}\,\pi^f_{t+1} - (1 - a_{ld})\,\pi^f_{t\text{-}1}]
\end{aligned}
$$

As stated, we do not have data on the price which the consumer pays in the domestic market for imported consumer goods (p^f) although we do have data on the dollar price paid by the importer. We will denote this latter price as p^{*f} and will denote by e the exchange rate of the sheqel against the dollar (both of them in log). The assumption of immediate pass-through to domestic prices implies $p^f_t = e_t + p^{*f}_t$. Usually, it is reasonable to assume that the pass-through is gradual rather than immediate. We assumed a gradual pass-through of the following form:

$$(4.3)\quad p^f_t = \alpha_1\,(e_{t+1} + p^{*f}_{t+1}) + \alpha_2\,(e_t + p^{*f}_t) + (1 - \alpha_1 - \alpha_2)\,(e_{t\text{-}1} + p^{*f}_{t\text{-}1})$$

The estimates obtained for the parameters are:

$\alpha_1 = 0.20 \qquad \alpha_2 = 0.63$

According to equation (4.3), the price of imported consumer goods in the domestic market is adjusted (by the importer) partly to the expected cost in the future, partly to the contemporaneous cost and partly to the cost in the previous quarter.

By Substituting (p^f) and (π^f) in equation (4.2') by the right hand side of (4.3), and applying a number of algebraic operations as detailed in Appendix A, the following equation is obtained for the inflation in the consumer price index, in terms of measurable variables:

$$(4.4)\quad \pi^c_t = a_{ld}\,\pi^c_{t+1} + (1 - a_{ld})\,\pi^c_{t-1} + (1 - w^f)\,a_y\,(0.5\,y_t + 0.5\,y_{t-1})$$
$$+ a_{zf}\,[(1 - w^f)\,up^{*zf}_t + q_t]$$
$$+ w^f\,[\alpha_1\,dep^{*f}_{t+1} + \alpha_2\,dep^{*f}_t + (1 - \alpha_1 - \alpha_2)\,dep^{*f}_{t-1}]$$

where

$$dep^{*f}_t = (\Delta e_t + \Delta p^{*f}_t) - a_{ld}\,(\Delta e_{t+1} + \Delta p^{*f}_{t+1}) - (1 - a_{ld})\,(\Delta e_{t-1} + \Delta p^{*f}_{t-1})$$

$\pi^c_t = p^c_t - p^c_{t-1}$ is inflation in the consumer price index, $q_t = e_t + p^{*f}_t - p^c_t$ is the real exchange rate gap, $up^{*zf}_t = p^{*zf}_t - p^{*f}_t$ is the difference between the world price of imported inputs (p^{*zf}) and the world price of imported consumer goods (p^{*f}) and Δ is the difference operator.[25] Initial examination showed that with respect to the price ratios ($p^f_t - p^c_t$) and ($p^{zf}_t - p^f_t$), the results are similar if either a gradual or immediate pass-through is assumed. For reasons of simplicity therefore, we assumed immediate pass-through. Of course, with respect to imported inflation rate (π^f_t), we assumed a gradual pass-through. Equation (4.4) is the model's inflation equation. The estimates are presented in Appendix B.

4.b The output gap equation

The output gap equation describes the excess demand of the relevant entities (the public, the government and abroad) for the output of the business sector. It will be recalled that the output gap (y_t) is the difference between actual output and potential output, the latter being defined as the output that would have prevailed under flexible prices and wages. In practice, potential output is measured via the trend of the actual output.

[25] It should be realized that the values of the coefficients presented above refer to the equation where all the variables are expressed in similar terms (quarterly or annual).

Behind the output gap equation are equations derived from the optimization of domestic households and foreign households, as well as the identity of national accounts.[26] Domestic households consume domestically manufactured and imported goods. These households are able to invest in interest-bearing sheqel assets or interest-bearing dollar assets. The Euler equation (4.5) below describes how domestic households allocate their total consumption (c_t) over time as a function of the real interest rate:

(4.5) $c_t = c_{t+1} - (1 / \sigma)\, 0.25\, (r_t - r^n_t)$

where ($r_t = i_t - 4*\pi^c_{t+1}$) is the annualized real interest rate, (r^n_t) is the natural real interest rate,[27] and σ is the inverse of the inter-temporal elasticity of substitution in consumption. According to this equation, the higher is the current interest rate, the lower will be the contemporaneous level of consumption (c_t) compared with the next period (c_{t+1}). This is because the higher is the current interest rate, the greater will be the future return on the forgoing of consumption in the present (that is, an increase in saving), making it possible to increase future consumption at a higher rate.

Equation (4.6) describes the allocation of consumption between domestic production and imports as a function of relative price (which is related to the real exchange rate):

(4.6) $c^h_t = c_t - \eta\, (p^h_t - p^c_t)$

where c^h_t is consumption from domestic production and η is the elasticity of substitution between consumption from imports and consumption from

[26] We will present here the equations forming the basis for the output gap equation. For a precise derivation of these equations, see Argov and Elkayam (2006).

[27] The natural interest rate is a concept analogous to potential output – the real interest rate that would have prevailed in a state of flexible prices and wages. In this study, we estimated it via the real yield to maturity on 5 to 10 year CPI-indexed bonds.

domestic production. The logic in this equation is quite simple: the proportion of consumption from domestic production to total consumption will fall as its price (relative to the general price of consumption) increases.

Just as with the allocation of households' consumption between domestic production and imports, foreign households distribute their imports between Israel and other countries. Equation (4.7) describes this as a function of the relative price of Israeli exports:[28]

$$(4.7)\quad x^h{}_t = y^*{}_t - \eta\,[(p^h - e_t) - p^{*f}{}_t] = y^*{}_t + \eta\,(e_t + p^{*f}{}_t - p^h)$$

where $x^h{}_t$ is Israeli exports and $y^*{}_t$ is world trade.[29]

The national accounts identity links consumption from domestic production (c^h), public consumption[30] (g^h), investment (inv^h) and exports (x^h) to total production of the business sector (all in value added terms and in the form of gaps):

$$(4.8)\quad y_t = \gamma_c\, c^h{}_t + \gamma_g\, g^h{}_t + \gamma_{inv}\, inv^h{}_t + \gamma_x\, x^h{}_t$$

where γ_i is the proportion in total product of component i ($i = c, g, inv, x$).

We will substitute (4.6) – (4.8) in equation (4.5), using the identity of the consumer price index. Assuming immediate pass-through in the prices of consumption ($p^f{}_t = e_t + p^{*f}{}_t$) we obtain:[31]

$$(4.9)\quad y_t = y_{t+1} + b'_r\, 0.25\,(r_t - r^n{}_t) + b'_q\,(q_t - q_{t+1})$$
$$+ b'_{y*}\,(y^*{}_t - y^*{}_{t+1}) + b'_{inv}\,(inv^h{}_t - inv^h{}_{t+1}) + b'_g\,(g^h{}_t - g^h{}_{t+1})$$

[28] $p^h{}_t - e_t$ is the price of Israeli exports, meaning the prices of goods from domestic production in dollar terms, and $p^{*f}{}_t$ is the price of world imports. It should be noted that the latter is also the relevant price of imports of consumer goods to Israel.

[29] For the purpose of simplicity, we assume that the elasticity of substitution in countries worldwide between Israeli exports and other goods is equal to the elasticity of substitution in (4.6).

[30] Here we relate to the procurement (from the business sector) component.

[31] It transpired in the estimation that in the output gap equation, as in the inflation equation, forcing immediate pass-through in relative prices does not affect the results.

where:
$b'_r = \gamma_c / \sigma \quad b'_q = \eta\,(\gamma_c\, w^f + \gamma_x) / (1 - w^f) \quad b'_{y*} = \gamma_x \quad b'_{inv} = \gamma_{inv} \quad b'_g = \gamma_g$

This equation describes the development of the output gap as a function of the expected output gap and other variables which exert an effect via their present and expected values. In the majority of empirical studies worldwide, it was found that the output gap is also affected by its lags and by the lags of other variables. Such an effect can be attributed to consumption habits and/or the existence of adjustment costs, and/or the existence of adaptive expectations among at least part of the public. One way to integrate a lag of the output gap in the equation is to assume that the above equation is correct for a desired output gap[32] (which we will denote as y'_t). We will assume that the actual gap gradually converges to the desired gap, according to the following equation:[33]

$$(4.10) \quad y_t = b_{yld}\, y'_t + (1 - b_{yld})\, y_{t-1}$$

By substituting (4.9) into (4.10), an output gap equation is obtained in which the lagged output gap appears in addition to the factors appearing in equation (4.9). In an initial estimation, it was found that apart from the lag in the output gap, a lag of the real interest rate gap also exerts an effect. Accordingly, the final output gap equation is the following:

$$\begin{aligned}(4.11) \quad y_t = {} & b_{yld}\, y_{t+1} + (1 - b_{yld})\, y_{t-1} - b_r\, 0.25\, [0.5\,(r_t - r^n_t) + 0.5\,(r_{t-1} - r^n_{t-1})] \\ & + b_q\,(q_t - q_{t+1}) + b_{y*}\,(y^*_t - y^*_{t+1}) + b_{inv}\,(inv^h_t - inv^h_{t+1}) \\ & + b_g\,(g^h_t - g^h_{t+1})\end{aligned}$$

[32] Namely the output gap that would have been obtained in the absence of one or more of the previously mentioned impeding factors.

[33] This approach is employed by Svensson (2000), for example.

where:

$$b_r = b_{yld}*b'_r \quad b_q = b_{yld}*b'_q \quad b_{y*} = b_{yld}*b'_{y*} \quad b_{inv} = b_{yld}*b'_{inv} \quad b_g = b_{yld}*b'_g$$

The estimates obtained for the parameters are:

$$b_{yld} = 0.80 \quad b_r = 0.45 \quad b_q = 0.24 \quad b_{y*} = 0.22 \quad b_{inv} = 0.12 \quad b_g = 0.22$$

According to the equation, the output gap is negatively affected by the real interest rate gap ($r - r^n$), contemporaneously and with a lag, and positively affected by the real exchange rate gap (q). An increase in the real interest rate has the effect of reducing the demand for business-sector product in the present compared with the subsequent period and the previous period. (This is for reasons of inter-temporal substitution in consumption.) An increase in the real exchange rate reduces the relative price of Israeli exports as perceived by buyers worldwide, and increases the relative price of imports, thereby increasing world demand and domestic demand for domestic production. Other factors affecting the output gap are the expected output gap (for consumption smoothing reasons), the lagged output gap, the world demand gap (y^*), which affects demand for domestic exports, the gap in government demand for business-sector product (g^h) and the investment gap (inv^h).

In most economies, the effects of the monetary policy on the output gap is gradual, which is reflected *inter alia* by a positive coefficient (usually greater than 0.5) for the output gap lag.[34] Here, a coefficient of 0.20 was obtained, indicating that in our case the effect of monetary policy is more rapid than elsewhere. The interest rate gap coefficient[35] (which is largely a function of the inter-temporal elasticity of substitution in consumption) is similar in size

---

[34] As an example, Lopez (2003) obtained a coefficient of 0.89 for Columbia, Caputo (2004) obtained 0.55 for Chile and Leitemo (2006b) obtained 0.47 for the UK.

[35] The interest rate in the equation is in annual terms while the other variables are in quarterly terms. As part of the coefficient, therefore, the interest rate gap is multiplied by 0.25.

to that in other economies.[36] With respect to the coefficient of the real exchange rate, the variability between different studies is considerable.[37] The coefficients of world trade, the procurement component of domestic public consumption and investment are mainly a function of the proportions of these factors in total product.[38] Since investment is affected by the interest rate, a separate equation should be formulated for it. We are planning to address this matter in the future.

4.c The nominal exchange rate equation

The starting point for the exchange rate equation is the uncovered interest parity equation (UIP) whereby the exchange rate (e_t) is a function of the expected exchange rate in the subsequent period (e^{exp}_{t+1}) plus the differential between the interest rate abroad and the interest rate in the domestic market, adjusted for the domestic currency risk:

$$(4.12) \quad e_t = e^{exp}_{t+1} + 0.25\,(i^*_t - i_t + rp_t)$$

where i_t is the sheqel interest rate, i^*_t is the dollar interest rate, and rp_t is the exchange rate risk premium. (Since the interest rates are in annual terms, the coefficient of the nominal interest rate differential is 0.25.) This equation is actually obtained together with equation (4.5), and reflects a first order condition in the solution of households' optimization problem with respect to the allocation of households' savings between domestic and foreign assets.[39]

---

36 We obtained a value of 0.45, Leitemo (2006b) obtained a value of 0.28 for the UK and Lopez (2003) obtained a value of 0.67 for Columbia. Svensson (2000) assessed this value at 0.35.

37 We obtained a coefficient of 0.24 as compared to 0.11 in Leitemo (2006b), 0.016 in Caputo (2004) and 0.002 in Lopez (2003). Svensson (2000) assessed this value at 0.30.

38 The coefficient of world trade is related to the proportion of exports in output. From domestic public consumption, we took the procurement component, which represents demand for business sector product.

39 See for example Argov and Elkayam (2006).

Many studies worldwide find that the extent of this equation's fit to the data is low.[40] One possible reason for this, that is mentioned in the literature, is the presence of large variability in the risk premium, which is obviously an unobservable variable. Another possibility suggested is that the expectations regarding the future exchange rate are not rational. We will assume that these expectations are rational with a gradual adaptation[41] as follows:

(4.13) $e^{exp}_{t+1} = c_{ld}\, e_{t+1} + (1 - c_{ld})\, e^{exp}_{t}$

We will substitute (4.12) into (4.13) to obtain:

(4.14) $e_t = c_{ld}\, e_{t+1} + (1 - c_{ld})\, e_{t-1}$
$+ 0.25\, [(i^*_t - i_t) - (1 - c_{ld})\, (i^*_{t-1} - i_{t-1}) + (rp_t - (1 - c_{ld})\, rp_{t-1})]$

A value of $c_{ld} = 0.45$ was obtained in the estimation, implying that slightly less than half the public form expectations rationally. (With respect to inflation, we found that slightly more than half – 0.55 – have rational expectations.) The results of the equation estimation are detailed in Appendix B.

4.d Monetary policy (a rule for the interest rate)

The objective of monetary policy is to minimize the deviations of inflation from the target and the deviations of output from its potential. The policy instrument used for this purpose is the central bank's interest rate. At every point in time, shocks occur that cause inflation to deviate from the target and output to deviate from potential output. Hence, in order to attain the monetary policy objective, a continues adjustment of the central bank's interest rate, in response to these shocks, is necessary. Since large and frequent changes in the

---

[40] The exchange rate in a small and open economy is subject to large shocks. This is one of the reasons that led to a regime of intervention in the foreign exchange market that characterized many countries in the past.

[41] See for example Leitemo and Soderstorm (2005b).

interest rate could undermine financial stability, a secondary objective (or maybe an additional constraint) of monetary policy is to moderate the changes in the interest rate.

In principle, it would be desirable to formulate a loss function and by means of this function and on the basis of the model, to derive the optimal interest rate rule. At this stage, we chose to formulate an interest rate rule similar to the one common in many other models.[42] We assume an (extended) forward-looking Taylor rule, of the following form:

(4.15) $i_t = (1 - d_{lag})\ \{r^n{}_t + \pi_t^{target} + d_\pi\ (E\pi^c{}_t - \pi_t^{target}) + d_y\ y_t\} + d_{lag}\ i_{t\text{-}1}$

where:

$$E\pi^c{}_t = p^c{}_{t+3} - p^c{}_{t\text{-}1}$$

$$d_{lag} = 0.6 \quad d_\pi = 1.5 \quad d_y = 0.5$$

According to this rule, the interest rate (i_t) is adjusted with respect to three factors: the differential between inflation expectations for the next four quarters ($E\pi^c{}_t$) and the inflation target (π_t^{target}) which is defined in annual terms, the output gap and the level of the interest rate in the previous quarter.

This kind of rule is called a "Forecast-Based Rule" and is an extension of the original Taylor rule – Taylor (1993). In the original rule, the interest rate is adjusted with respect to past inflation while here, the interest rate is adjusted to the inflation expected in the coming year. It transpires that such a rule is robust to model uncertainty, that is, the rule performs well in different models.[43] This is because the inflation expectations derived from the model

[42] The rule below is similar to the one formulated for Israel by Epstein et al. (2006). For the various characteristics of such kind of rule see Batini and Haldane (1999), Batini et al. (2003), Levin et al. (2003) and Leitemo (2006b).

[43] Levin et al. (2003) analyze the characteristics of robust forecast based rules and conclude that for robustness the rule should: respond to a short horizon forecast (up to one year) of a

contain all the information relevant to the future development of inflation – on the assumption, naturally, of rational expectations. This rule is commonly used also due to the fact that the response to a deviation of inflation expectations from the target helps in anchoring these expectations, and thereby enhances the credibility of monetary policy. It should be noted however that alertness is necessary when using this rule, because cases may arise in which its use could impair economic stability.[44]

Since inflation expectations are *inter alia* a function of the current interest rate, the above rule is actually an implicit equation that is solved together with the other equations of the model. For this reason, the equation cannot explicitly reflect the behavior of the central bank, and constitutes merely an attempt to simulate its conduct. An interesting question in this respect is the extent to which the past activity of the Bank of Israel can be characterized by means of such an equation. When we attempted to estimate this equation with *ex-post* inflation figures, as an approximation to expectations, the results obtained were not good.[45] This may be due to the difficulty in identifying, in a relatively small sample, the effect of inflation expectations, that are presumably embodied in future (ex-post) inflation data. Support for this possibility can be obtained from the fact that when inflation expectations derived from capital market data are used as a proxy for expected inflation, significant estimates are obtained with the correct signs and at orders of magnitude close to those expected (see Appendix B). However, significant results are also obtained when instead of expectations, inflation in the previous four quarters is used. These results raise important and interesting questions

---

smoothed measure of inflation, and incorporate response to current output gap lagged interest rate.

[44] See for example Leitemo (2006b) and Levin et al. (2006).

[45] The coefficients of both the inflation gap and the output gap were not found to be significant.

regarding the nature of the inflation expectations derived from different sources and the pattern of behavior characterizing the conduct of monetary policy. The answers to these questions are beyond the scope of this study.

The above equation is, as stated, the reaction function with which we chose to close the model, for the purpose of generating forecasts and examining alternative scenarios. In order to examine the extent of the model's fit to the data (Section 7) however, we used an empirical reaction function. This function is similar in its characteristics to the previously-mentioned equation, with one exception: instead of expected inflation in the next four quarters, we used inflation in the previous four quarters (see Appendix B for details). The coefficients obtained for this equation are:

$$d_{lag} = 0.80 \quad d_{\pi} = 1.70 \quad d_{y} = 0.36$$

When we replaced actual inflation by inflation expectations data, that are derived from the capital market, a slightly lower coefficient for the interest rate lag is obtained (0.73), together with a slightly higher coefficient for the inflation gap (2.04) and a lower coefficient for the output gap of 0.12 (see Appendix B). For the policy rule used for forecasting – that is, equation (4.15) above – we selected the generally accepted coefficients recommended by Taylor in the original rule: we selected a coefficient of 1.5 for the inflation gap, and a coefficient of 0.5 for the output gap. Since the inflation gap coefficient selected is lower than those obtained from the estimation, we reduced the extent of the interest rate smoothing and chose an interest rate lag coefficient of 0.6.

4.e The complete model

We collect below the model equations with the estimated parameters. Subsequently, for the sake of convenience, we collect also the list of variables.

The inflation equation:

$$\text{(F.1)}\quad \pi^c_t = a_{ld}\,\pi^c_{t+1} + (1 - a_{ld})\,\pi^c_{t\text{-}1} + (1 - w^f)\,a_y\,(0.5\,y_t + 0.5\,y_{t\text{-}1}) + a_{zf}\,[q_t + (1 - w^f)\,up^{*zf}_t] + w^f\,[\alpha_1\,dep^{*f}_{t+1} + \alpha_2\,dep^{*f}_t + (1 - \alpha_1 - \alpha_2)\,dep^{*f}_{t\text{-}1}]$$

where: $dep^{*f}_t = (\Delta e_t + \Delta p^{*f}_t) - a_{ld}\,(\Delta e_{t+1} + \Delta p^{*f}_{t+1}) - (1 - a_{ld})\,(\Delta e_{t\text{-}1} + \Delta p^{*f}_{t\text{-}1})$

The output gap equation:

$$\text{(F.2)}\quad y_t = b_{yld}\,y_{t+1} + (1 - b_{yld})\,y_{t\text{-}1} - \mathrm{b_r}\,0.25\,[0.5\,(r_t - r^n_t) + 0.5\,(r_{t\text{-}1} - r^n_{t\text{-}1})] + b_q\,(q_t - q_{t+1}) + b_{y*}\,(y^*_t - y^*_{t+1}) + b_{inv}\,(inv^h_t - inv^h_{t+1}) + b_g\,(g^h_t - g^h_{t+1})$$

The exchange rate equation:

$$\text{(F.3)}\quad e_t = c_{ld}\,e_{t+1} + (1 - c_{ld})\,e_{t\text{-}1} + 0.25\,[(i^*_t - i_t) - (1 - c_{ld})\,(i^*_{t\text{-}1} - i_{t\text{-}1}) + (rp_t - (1 - c_{ld})\,rp_{t\text{-}1})]$$

The nominal interest rate equation

$$\text{(F.4)}\quad i_t = (1 - d_{lag})\,\{r^n_t + \pi_t^{target} + d_\pi\,(E\pi^c_t - \pi_t^{target}) + d_y\,y_t\} + d_{lag}\,i_{t\text{-}1}$$

where: $E\pi^c_t = (\pi^c_t + \pi^c_{t+1} + \pi^c_{t+2} + \pi^c_{t+3})$

The model is closed by means of an identity for the real exchange rate (F.5), assuming that the equilibrium real exchange rate is fixed, and definitions for the nominal depreciation (F.6) and the real interest rate (F.7):

$$\text{(F.5)}\quad q_t = q_{t\text{-}1} + \Delta p^{*f}_t + \Delta e_t - \pi^c_t$$

$$\text{(F.6)}\quad \Delta e_t = e_t - e_{t\text{-}1}$$

$$\text{(F.7)}\quad r_t = i_t - 4{*}\pi^c_{t+1}$$

Table 4.1: Collection of Model Parameter Values

Eq. (F.1)	$a_{ld} = 0.53$	$a_y = 0.06$	$a_{zf} = 0.06$	$w^f = 0.45$	$\alpha_1 = 0.20$	$\alpha_2 = 0.63$
Eq. (F.2)	$b_{yld} = 0.80$	$b_r = 0.45$	$b_q = 0.24$	$b_{y*} = 0.22$	$b_{inv} = 0.12$	$b_g = 0.22$
Eq. (F.3)	$c_{ld} = 0.45$					
Eq. (F.4)	$d_{lag} = 0.6$	$d_\pi = 1.5$	$d_y = 0.5$			

List of the model's variables

a. The endogenous variables:

$\pi^c{}_t$ − rate of change in the consumer price index.

y_t − output gap.

e_t − log of the nominal exchange rate.

i_t − nominal interest rate (annualized).

b. The identities:

q_t − real exchange rate gap.

Δe_t − depreciation in the nominal exchange rate.

r_t − ex-ante real interest rate (annualized).

c. The exogenous variables:

$\Delta p^{*f}{}_t$ − rate of change in world prices of imported consumer goods.

$y^*{}_t$ − world trade gap.

$inv^h{}_t$ − investment gap.

$g^h{}_t$ − public consumption gap.

$i^*{}_t$ − dollar interest rate (annualized).

rp_t − exchange rate risk premium (annualized).

$r^n{}_t$ − natural real interest rate (annualized).

π_t^{target} − inflation target (annualized).

$up^{*zf}{}_t$ − gap of the ratio between world price of imported inputs and prices of imported consumer goods.

5. Model properties - impulse response functions

This chapter describes the impulse response functions for the main endogenous variables of the model, i.e., the way these variables develop over time in response to various shocks that the economy may undergo. This is a conventional way of examining the characteristics of macroeconomic models. We examine the responses of the inflation rate, the interest rate, output, the nominal exchange rate, and the real exchange rate (all in terms of deviations from long-run equilibrium values) to an exogenous shock that affects each of the following variables: monetary policy (the Bank of Israel interest rate), the exchange rate, the output gap, the inflation rate, the prices of imported inputs (e.g., oil prices), and the dollar interest rate (alternatively, the risk premium).[46] In each case, we use long-run equilibrium as our starting point in order to isolate the effect of the specific shock at issue. At the beginning of the simulation, the relevant variable is affected by an exogenous shock (of one-quarter duration only) of a magnitude of 1 percentage point, and we examine the development of the endogenous variables from this point onward – until the long-run equilibrium is restored.[47] The examination is both qualitative (shape of response) and quantitative (intensity of response and time that lapses until equilibrium is restored).

The impulse response functions are described in Figures 5.1–5.6. The tables that correspond to the figures show the response values during the year following the shock and two years after the shock. The variables in the figures

[46] Throughout this article, we use the expression "shock to" in the sense of "a shock that affects...."

[47] A shock to one of the endogenous variables in the model (Bank of Israel interest rate, the exchange rate, inflation, or the output gap) is manifested in the residual of the equation that corresponds to this variable. The residual acquires the value of 1 percentage point for the period of the shock only, and maintains the value of 0 for all other periods. When the variable at issue is exogenous (the dollar interest rate or import price ratio), the shock is manifested in a deviation of this variable from its equilibrium value during the shock period only.

and tables are expressed in terms of deviations from their long-run equilibrium values. Deviations of rates of interest, inflation, and depreciation are expressed in annualized percentage points. Output gaps, the real exchange rate, and the ratio of imported input prices to imported consumption prices are expressed in terms of ordinary percents.

The results described below pertain to shocks of a 1-percentage-point magnitude.[48] Since the model is linear, the responses to an *x*-percentage-point shock are simply the products of the responses to a 1-percentage-point shock multiplied by *x*. It should be borne in mind, however, that since the linear model is derived from a log-linear approximation of the equations of the underlying theoretical model around the steady state, it is valid only in a close vicinity of the steady state; therefore, it is not designed to describe responses to large shocks.

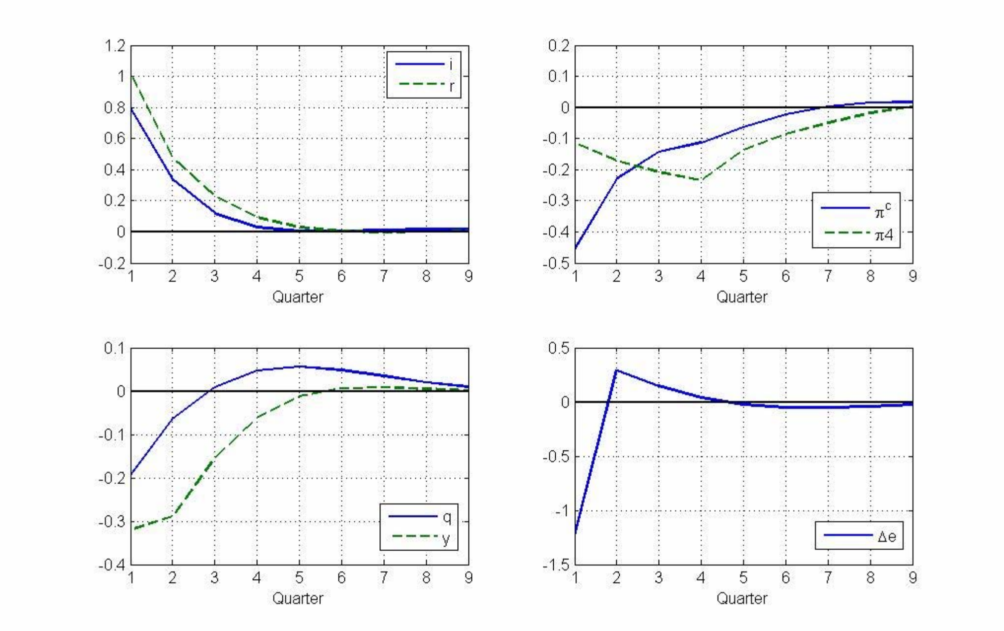
5a. Impulse response functions relating to a 1-percentage-point shock to the Bank of Israel interest rate

Figure 5.1 and Table 5.1 describe the development of the relevant variables in response to a 1-percentage-point shock to the Bank of Israel interest rate. Shocks that affect monetary-policy decisions may originate in various random factors that are not expressed in the interest-rate rule, or in errors in measuring the variables that influence policy.

As the figures show, the shock induces a hike in the **nominal interest rate** (by 0.8 percentage points: the decrease in the output gap and in inflation expectations mitigates the effect of the original shock). Due to price rigidity in the short run, the rise in the nominal interest rate generates a rise in the **real interest rate** as well. The increase in the Bank of Israel interest rate causes the

[48] Another common practice is to study a shock of a magnitude of one (estimated) standard deviation in each case.

Figure 5.1: Impulse Response Functions relating to a 1-Percentage-Point Shock to Bank of Israel Interest Rate



π^c – inflation in quarter
$\pi 4$ – inflation in past four quarters
Δe – nominal exchange-rate depreciation

i – nominal interest rate
r – real interest rate
q – real exchange-rate gap
y – output gap

The variables are expressed in terms of deviations from long-run equilibrium. Rates of inflation, interest rates, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

Table 5.1: Impulse Response Functions relating to a 1-Percentage-Point Shock to Bank of Israel Interest Rate

Deviation from equilibrium*

Quarter	1	2	3	4	8	Convergence
Nominal interest rate	0.8	0.3	0.1	0.0	0.0	0.0
Inflation in quarter	-0.5	-0.2	-0.1	-0.1	0.0	0.0
Inflation in past 4 quarters	-0.1	-0.2	-0.2	-0.2	0.0	0.0
Nominal exchange-rate depreciation	-1.2	0.3	0.1	0.0	0.0	0.0
Real interest rate	1.0	0.5	0.2	0.1	0.0	0.0
Output gap	-0.3	-0.3	-0.2	-0.1	0.0	0.0
Real exchange-rate gap	-0.2	-0.1	0.0	0.0	0.0	0.0

* Deviation – in percentage points. Rates of interest, inflation, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

sheqel/dollar exchange rate to decline immediately and non-recurrently (by 1.2 percent in annual terms); this, in turn, manifests itself in **real appreciation**. The increase in the real interest rate and the real appreciation reduce the **output gap** (by 0.3 percentage points).[49] The declines of the exchange rate and the output gap cause the **inflation rate** to fall (by about 0.5 percent in annual terms).

The existence of inertia in the interest-rate rule (reflecting the motive of interest smoothing by the central bank) manifests itself in a gradual decrease in the interest rate (deviation), which remains positive for about a year despite the negative deviations of inflation and the output gap during that time. As the figures show, the inflation rate and the output gap gradually converge after falling due to the aforementioned shock to the interest rate, so that their deviations remain negative until convergence occurs about a year and a half after the shock. As for the behavior of the exchange rate, after the appreciation during the shock period, moderate depreciation occurs starting in the next quarter, induced by the downward path of the interest rate.[50] The interest-rate shock results in a cumulative loss of 0.2 percent of output in the year after the shock and a 0.25-percentage-point decline in inflation during that year.

Examining the behavior of inflation and the output gap, we see that the most vigorous response of inflation and the output gap occurs during the period of the interest-rate shock. Empirical studies worldwide have found that inflation and output respond to shocks in a hump-shaped pattern: the strongest response takes place not immediately but several quarters after the shock.

[49] Most (more than two-thirds) of the decline in the output gap during the period of the shock stems from the effect of the output gap in the succeeding period (the effect of expectations).

[50] In the exchange-rate equation, the exchange rate is a positive function of the interest rate spread (the difference between the dollar interest rate plus a risk premium and the sheqel interest rate) and the *change* in this spread (and also a function of the lagged exchange rate and the rate expected in the next period). As a result, a *declining* path of the domestic interest rate has a pro-depreciation effect.

Accordingly, attempts to develop models that generate similar responses have been made (e.g., Christiano, Eichenbaum, and Evans [2005], hereinafter: CEE).[51] Notably, however, CEE concern themselves with a closed economy; in a small and open economy such as Israel's, where the exchange rate has a perceptible contributory effect on rapid transmission from the interest rate to inflation, a large immediate response, at least of the inflation rate, may be more reasonable. Indeed, the steep immediate decline in the inflation rate, observable in the upper right-hand figure, traces largely to the immediate appreciation occasioned by the upturn in the interest rate. We should also note that the rapid transmission mechanism of monetary policy also stems from the large coefficients of the expectation variables in the inflation and output gap equations.

5b. Impulse response functions relating to a 1 percent (annualized) shock to the nominal exchange rate

Figure 5.2 and Table 5.2 describe the responses of the variables following a non-recurrent 1 percent shock to the nominal exchange rate.[52]

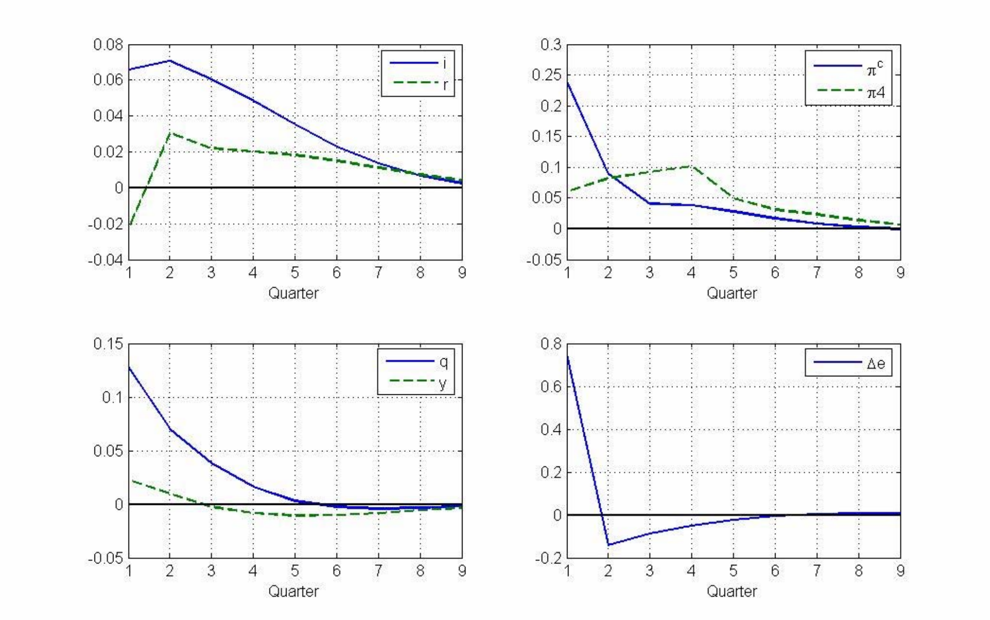
As Figure 5.2 shows, the shock to the **exchange rate** is reflected in 0.75 percent annualized depreciation. (The immediate response of the interest rate mitigates the original shock.) The depreciation triggers an immediate 0.25-percentage-point increase in **inflation,** thus, one-third of the depreciation

---

[51] To obtain inertia in inflation, it is customary to assume indexation to past inflation; to obtain inertia in output, one assumes habit in consumption and adjustment costs associated with changes in the level of investment in capital stock.

[52] The discussion here pertains to a shock to the exchange rate of 1 percent in annual terms (an increase of 0.25 percent in the average quarterly exchange rate). To assess the reasonability of the *intensity* of the responses, it is more convenient to refer to a 10 percent annualized depreciation–close to the standard deviation of the residuals in the estimated exchange-rate equation. Since the model is linear, one may obtain the responses to a 10 percent shock by multiplying the responses to a 1 percent shock by 10. For a dollar exchange rate of around 4.5 sheqels, a 1 percent annualized depreciation reflects an increase of about 0.01 sheqels in the average dollar exchange rate per quarter.

Figure 5.2: Impulse Response Functions relating to a 1 Percent (Annualized) Shock to the Sheqel/Dollar Exchange Rate



π^c – inflation in quarter
$\pi4$ – inflation in past four quarters
Δe – nominal exchange-rate depreciation
i – nominal interest rate
r – real interest rate
q – real exchange-rate gap
y – output gap

The variables are expressed in terms of deviations from long-run equilibrium. Rates of inflation, interest rates, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

Table 5.2: Impulse Response Functions relating to a 1 Percent (Annualized) Shock to the Sheqel/Dollar Exchange Rate

Deviation from equilibrium*

Quarter	1	2	3	4	8	Convergence
Nominal interest rate	0.07	0.07	0.06	0.05	0.01	0.00
Inflation in quarter	0.24	0.09	0.04	0.04	0.00	0.00
Inflation in past 4 quarters	0.06	0.08	0.09	0.10	0.01	0.00
Nominal exchange-rate depreciation	0.75	-0.14	-0.09	-0.05	0.01	0.00
Real interest rate	-0.02	0.03	0.02	0.02	0.01	0.00
Output gap	0.02	0.01	0.00	-0.01	-0.01	0.00
Real exchange-rate gap	0.13	0.07	0.04	0.02	0.00	0.00

* Deviation – in percentage points. Rates of interest, inflation, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

passes through to inflation immediately. Monetary policy responds to the upturn in one-year-ahead inflation expectations by raising the **interest rate** (by 0.07 percentage points, i.e., a 0.7-percentage-point increase in response to a 10 percent annualized depreciation). The response of monetary policy, reflecting the wish to avoid sharp changes in the interest rate, allows the **real interest rate** to decline temporarily (by 0.02 percentage points during the shock period). In the succeeding quarter, however, the real interest rate rises and its deviations remain positive until convergence occurs about two years after the shock. The nominal depreciation is also reflected in **real depreciation,** albeit milder than the nominal depreciation (0.5 percent in annual terms) due to the upturn in inflation. The **output gap** widens (to 0.02 percent) mainly due to the increase in the real exchange rate.

The deviations of inflation remain positive, i.e., the inflation rate exceeds the 2 percent target, for about two years. The nominal and real interest rates also surpass their equilibrium levels for two years after the shock. The positive deviations of the interest rate lead to appreciation. The combination of nominal appreciation and positive deviations of the inflation rate cause the real exchange rate to decline, so that it converges to its equilibrium level five quarters after the exchange-rate shock.

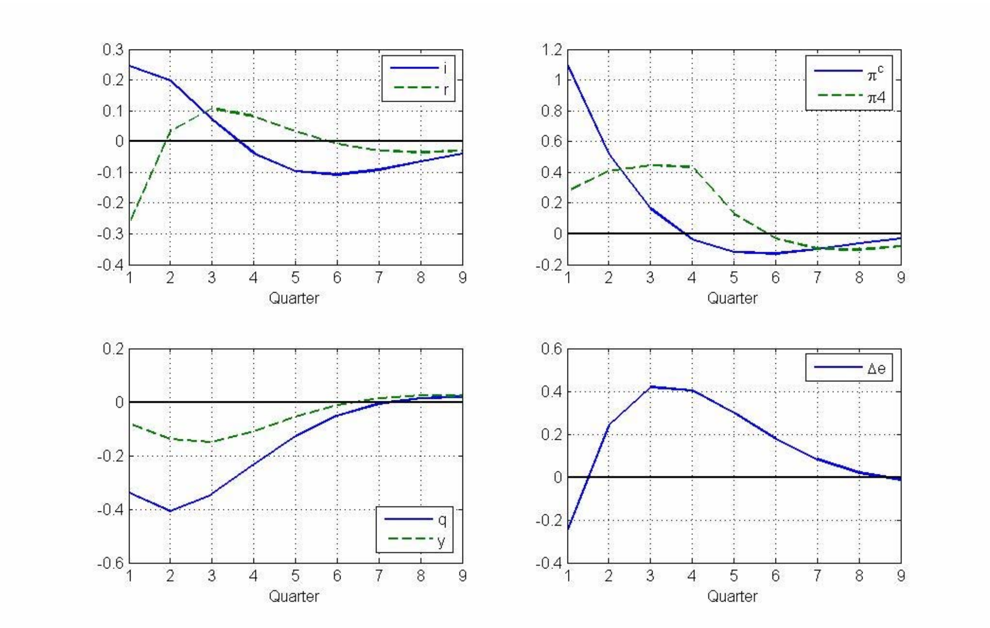
5c. Impulse response functions relating to a 1-percentage-point shock to inflation (per quarter, annualized)

A shock to the inflation rate reflects price increases (or decreases) occasioned by shocks to firms' production costs or monopoly power. (The greater the monopoly power, the higher the price that the producer charges beyond his (marginal) production costs, i.e., the larger the markup.)

As Figure 5.3 shows, the shock causes the **inflation rate** to rise by 1.1 percentage points (slightly greater than the original shock, due to the effect of expectations for inflation in the succeeding quarter). In response to the upturn in inflation expectations, the **nominal interest rate** rises (by 0.25 percentage points). The **real short-term interest rate**, perversely, declines because the interest-smoothing motive mitigates the increase in the nominal interest rate, but it already surpasses equilibrium in the succeeding quarter. The increase in the nominal interest rate induces **appreciation** (by 0.25 percent in annual terms), and the nominal appreciation, coupled with the upturn in inflation, causes **real appreciation** (by 0.35 percent, i.e., 1.4 percent in annual terms). The low real exchange rate and positive deviations of real interest (starting in the second quarter) induce a negative **output gap**.

In the aftermath of the inflation shock, the inflation rate remains above-target for three quarters. However, from one year after the shock until convergence to the target, the inflation deviations are actually negative due to the negative deviations of output and the real exchange rate. The nominal interest rate, which also rose in response to the shock to inflation, also slips below equilibrium a year later and remains under equilibrium until convergence occurs about two and a half years after the shock. Negative deviations of nominal interest (and the decline in the interest rate after its initial upturn) induce depreciation starting in the second quarter after the shock. (The entire process results in cumulative depreciation of 0.3 percent.) Nominal depreciation is needed in order to return the real exchange rate to equilibrium after the price shock drove it down. The negative deviations of the inflation rate starting one year after the shock also abet the increase in the real exchange rate.

Figure 5.3: Impulse Response Functions relating to a 1-Percentage-Point (Annualized) Shock to Inflation Rate



π^c – inflation in quarter
$\pi 4$ – inflation in past four quarters
Δe – nominal exchange-rate depreciation

i – nominal interest rate
r – real interest rate
q – real exchange-rate gap
y – output gap

The variables are expressed in terms of deviations from long-run equilibrium. Rates of inflation, interest rates, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

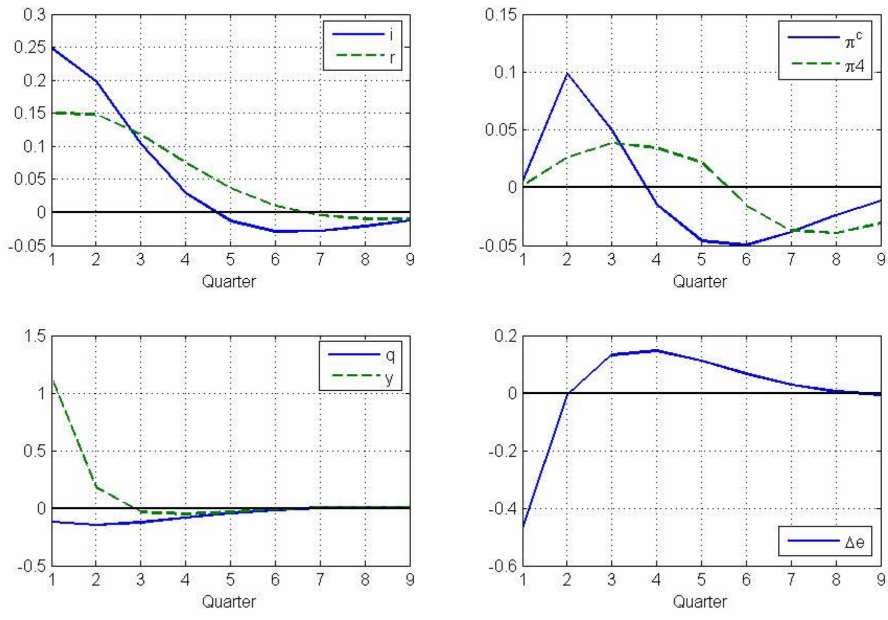
Table 5.3: Impulse Response Functions relating to a 1-Percentage-Point (Annualized) Shock to Inflation Rate

Deviation from equilibrium*

Quarter	1	2	3	4	8	Convergence
Nominal interest rate	0.2	0.2	0.1	0.0	-0.1	0.0
Inflation in quarter	1.1	0.5	0.2	0.0	-0.1	0.0
Inflation in past 4 quarters	0.3	0.4	0.4	0.4	-0.1	0.0
Nominal exchange-rate depreciation	-0.3	0.2	0.4	0.4	0.0	0.0
Real interest rate	-0.3	0.0	0.1	0.1	0.0	0.0
Output gap	-0.1	-0.1	-0.2	-0.1	0.0	0.0
Real exchange-rate gap	-0.3	-0.4	-0.3	-0.2	0.0	0.0

* Deviation – in percentage points. Rates of interest, inflation, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

Figure 5.4: Impulse Response Functions relating to a 1-Percentage-Point Shock to the Output Gap



π^c – inflation in quarter	i – nominal interest rate
$\pi 4$ – inflation in past four quarters	r – real interest rate
Δe – nominal exchange-rate depreciation	q – real exchange-rate gap
	y – output gap

The variables are expressed in terms of deviations from long-run equilibrium. Rates of inflation, interest rates, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

Table 5.4: Impulse Response Functions relating to a 1-Percentage-Point Shock to the Output Gap

Deviation from equilibrium*

Quarter	1	2	3	4	8	Convergence
Nominal interest rate	0.2	0.2	0.1	0.0	0.0	0.0
Inflation in quarter	0.0	0.1	0.0	0.0	0.0	0.0
Inflation in past 4 quarters	0.0	0.0	0.0	0.0	0.0	0.0
Nominal exchange-rate depreciation	-0.5	0.0	0.1	0.1	0.0	0.0
Real interest rate	0.2	0.1	0.1	0.1	0.0	0.0
Output gap	1.1	0.2	0.0	-0.1	0.0	0.0
Real exchange-rate gap	-0.1	-0.1	-0.1	-0.1	0.0	0.0

* Deviation – in percentage points. Rates of interest, inflation, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

5d. Impulse response functions relating to a 1-percentage-point shock to the output gap

A positive shock to the output gap expresses an increase in demand for output that may originate in an upturn in public consumption or an exogenous increase in export or private-consumption demand.[53]

As Figure 5.4 and Table 5.4 show, the **output gap** widens by 1.1 percentage points during the shock period. (The rate of increase exceeds the original shock due to the effect of expectations for the output gap in the succeeding period.)[54] The **nominal interest rate** rises by 0.25 percentage points in response to the positive output gap; consequently, the **real interest rate** also climbs (by 0.15 percentage points). The upturn in interest causes **appreciation** (i.e., the exchange rate falls) of nearly 0.5 percent in annual terms. The nominal appreciation is manifested in a similar rate of **real appreciation.** The deviation of **inflation** approximates zero during the shock period due to the contrasting effects of the positive output gap and the appreciation occasioned by the response of monetary policy.

One may see that output converges rapidly after the shock that it absorbed, owing to the relatively low coefficient of the lagged output gap in the output gap equation.[55] We may recall that a more gradual convergence of the output gap was found in response to a shock to the interest rate or to inflation.

---

[53] A conventional way of modeling the latter is by inserting a shock to households' preferences, i.e., to their utility function.

[54] The output gap widens at a rate greater than the original shock due to the effect of expectations for the output gap in the succeeding period, whereas the output gap in the succeeding period widens under the influence of the shock during the current period, via the lagged-gap factor. In other words, the positive effect on the current output gap of both the lagged output gap and the expected output gap in the next quarter generate a mechanism that amplifies the original shock to output. This mechanism is also at work in the inflation equation. Since the sum of the coefficients of the lagged variable and the expected variable is 1, the closer the coefficients are to .5, the stronger the effect.

[55] Examination of the main factors that affect the behavior of the output gap shows that the significant developments in the gap trace to the direct effect of the exogenous shock, whereas the effect of monetary policy, via the real interest rate and the real exchange rate, is small. In

The nominal interest rate remains above equilibrium for about a year (due to interest smoothing) and slips slightly below equilibrium in the successive year. The declining path of the interest rate and its subsequent negative deviations cause the sheqel to depreciate until it converges to a constant exchange rate approximately two years after the shock to the output gap.

Thus far, we have examined impulse response functions relating to shocks to the endogenous variables of the model: interest rate, exchange rate, inflation, and the output gap. Next we ask how the endogenous variables respond to changes in variables that are exogenous to the model – prices of imported inputs (e.g., oil prices) and the dollar interest rate (or Israel's risk premium).

5e. Impulse response functions relating to a 1 percent shock to imported-input prices

A 1 percent shock (in quarterly terms) to prices of imported inputs (e.g., oil) is reflected in the model in a 1-percentage-point shock to the gap in the ratio of imported-input prices to imported consumption prices (in quarterly terms).[56] Here, as in the previous exercises, we assume that the shock is short-lived, so that the ratio of imported-input prices to imported-consumption prices returns to its equilibrium level in the succeeding quarter.

The ratio of imported-input prices to imported-consumption prices appears as a factor in the inflation equation. Thus, the direct effect of the shock is borne by inflation and the effects on the other variables flow from it. A 1-

---

other words, the rapid convergence of the output gap is brought on not by an aggressive monetary policy that seeks to stabilize the output gap but rather by the intrinsic dynamics of the output gap as a result of the shock.

[56] Note that the real exchange-rate gap (which relates to domestic and foreign consumption prices) does not absorb a shock concurrently. Thus, a shock to the price ratio of imported inputs and imported consumption reflects a shock to imported-input prices but no shock to imported consumption prices.

percentage-point shock (in quarterly terms) to this variable brings on a 0.15-percentage-point annualized increase in **inflation** during that quarter. Hence, the impulse response functions of the various variables are identical to those obtained for a shock to inflation that raises the inflation rate by 0.15 percentage points in annual terms. (See above: impulse response functions relating to inflation.)

5.f Impulse response functions relating to a 1-percentage-point shock to the dollar interest rate (or the risk premium)[57]

We now examine the response of the economy to a shock to the dollar interest rate. Note that when we examined a shock to an endogenous variable in the model, we assumed that it was non-recurrent and had no inertia. However, the variable that absorbed the shock developed in subsequent periods as prescribed by the model; in particular, it did not return to equilibrium immediately. In contrast, if we assume a non-recurrent shock to the dollar interest rate – an exogenous variable in the model – we would then assume that the dollar interest rate already returns to its original value by the next quarter. Such an assumption does not stand to reason; we know that there is some inertia in monetary policy;[58] therefore, such an exercise does not easily lend itself to an economic interpretation. Accordingly, we also examine, in Subsection (b), the case of a shock to the dollar interest rate on the assumption that the behavior of this interest rate does include some inertia.

The premise of a non-recurrent shock may be more reasonable if we interpret the event as a shock to the risk premium.[59] In such an interpretation,

[57] The two shocks are equivalent because the dollar interest rate and the risk premium appear in one form only: the sum of both.

[58] Including US monetary policy.

[59] In this sense, a shock to the dollar interest rate is not equivalent to a shock to the risk premium – if we assume that they have different degrees of inertia. Thus far, we assumed no inertia in the shocks.

however, we implicitly assume that investors expect the risk premium to return to its prior level by the next quarter. Accordingly, the effect of such a shock is relatively minor, as we will see below.

a. Impulse response functions relating to a non-recurrent 1-percentage-point shock to the dollar interest rate or to the risk premium

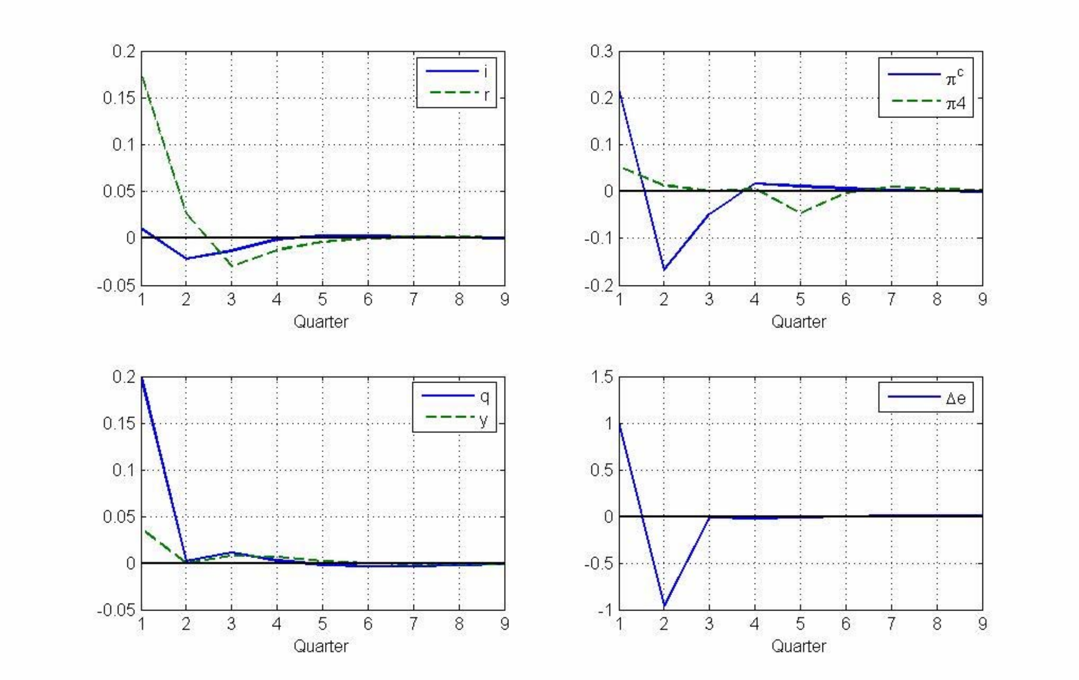
An increase in the dollar interest rate or the risk premium directly affects the sheqel/dollar exchange rate and, via it, the other variables. The verbal description that follows treats this shock as an interest-rate shock.

As Figure 5.5 shows, the non-recurrent shock triggers an abrupt change in the **(nominal) exchange rate**: (annualized) depreciation of about 1 percent in the period of the shock, an appreciation of a similar rate in the succeeding quarter, and a return to a constant exchange-rate in the quarter afterward. Thus, the exchange rate rises by 1 percent (annualized) during the period of shock to the dollar interest rate, due to the sheqel/dollar interest differential during the quarter, and returns to its original level in the succeeding quarter, when the interest spread is eliminated.[60]

The behavior of **inflation** is mainly a result of the changes in the exchange rate. Thus, the deviation of inflation is positive during the period of the shock (0.2 percentage points) and negative in the succeeding quarter. The deviations in inflation during the past four quarters are minuscule. (In particular, the inflation deviation during the year after the shock is nil.)

[60] The exchange rate is also affected by the expected exchange rate in the next quarter. (See exchange-rate equation 4.14.) The deviation of the exchange rate in the quarter following the shock to the dollar interest rate is zero due to the offsetting effects of the lagged dollar interest rate and the lagged exchange rate. Note that in the exchange-rate equation derived from the standard UIP condition (assuming purely rational expectations), the lagged components do not appear and the exchange-rate deviation in the succeeding quarter, when the interest spread has been closed, is zero.

Figure 5.5: Impulse Response Functions relating to a 1-Percentage-Point Shock to Dollar Interest Rate



π^c – inflation in quarter
$\pi 4$ – inflation in past four quarters
Δe – nominal exchange-rate depreciation

i – nominal interest rate
r – real interest rate
q – real exchange-rate gap
y – output gap

The variables are expressed in terms of deviations from long-run equilibrium. Rates of inflation, interest rates, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

Table 5.5: Impulse Response Functions relating to a 1-Percentage-Point Shock to Dollar Interest Rate

Deviation from equilibrium*

Quarter	1	2	3	4	8	Convergence
Nominal interest rate	0.0	0.0	0.0	0.0	0.0	0.0
Inflation in quarter	0.2	-0.2	0.0	0.0	0.0	0.0
Inflation in past 4 quarters	0.1	0.0	0.0	0.0	0.0	0.0
Nominal exchange-rate depreciation	1.0	-1.0	0.0	0.0	0.0	0.0
Real interest rate	0.2	0.0	0.0	0.0	0.0	0.0
Output gap	0.0	0.0	0.0	0.0	0.0	0.0
Real exchange-rate gap	0.2	0.0	0.0	0.0	0.0	0.0

* Deviation – in percentage points. Rates of interest, inflation, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

Since the temporary shock to the dollar interest rate has a negligible effect on inflation in the coming year and on the output gap, it has almost no effect on the **Bank of Israel's interest rate**.

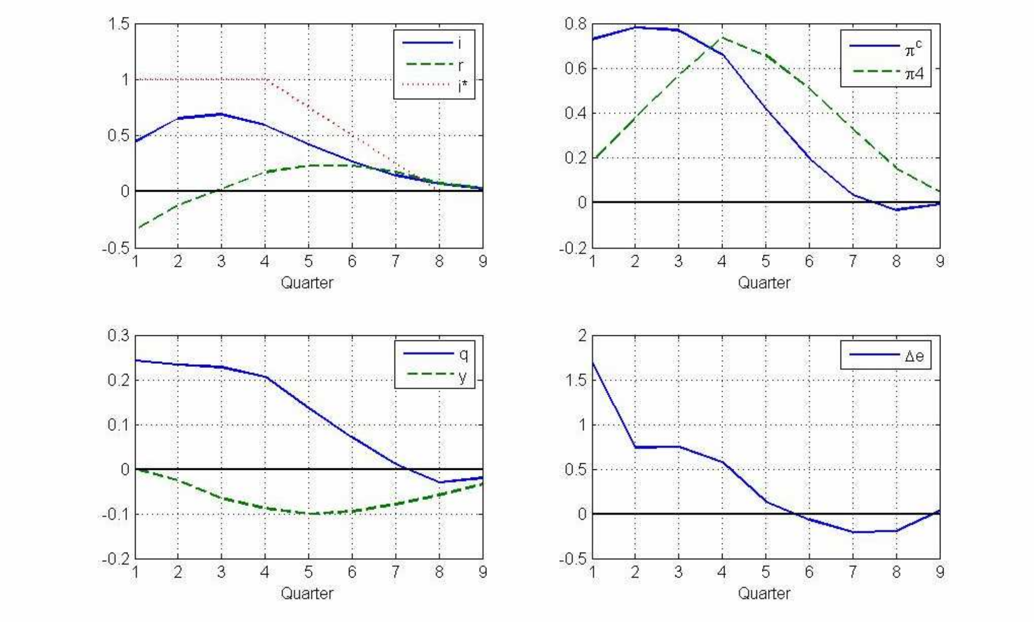
The increase in the real exchange rate during the shock period and the upturn in real interest have contrasting effects on the **output gap**: the gap widens very slightly during the shock period and falls back to the vicinity of zero in the succeeding quarter.

b. Impulse response functions relating to a persistent shock to the dollar interest rate

The source of the acute dynamic of change in the exchange rate, as stated, is the brief (one-period) duration of the shock to the dollar interest rate. In reality, of course, the rate behaves with some inertia. Therefore, we also examined the effect on the economy of a protracted shock to the dollar interest rate, in which the rate remains 1 percentage point above equilibrium during four quarters and then gradually returns to equilibrium. The behavior of the variables in the aftermath of these shocks to the dollar interest rate is shown in Figure 5.6.

In response to the shocks to the dollar interest rate, the sheqel **nominal interest rate** rises gradually (due to the interest-smoothing motive) – by 0.4 percentage points immediately and by another increment in the next two quarters as well. As the increase in sheqel interest is smaller than the increase in the dollar interest rate, the **sheqel depreciates** immediately by 1.7 percent in annual terms. The exchange rate continues to rise during the coming year as a result of the protracted interest spread. The nominal depreciation is also reflected in **real depreciation.** Due to the higher exchange rate, **inflation** rises by 0.7–0.8 percentage points (annualized) in the course of the year following the shock. Later, inflation rates subside gradually until convergence occurs.

Figure 5.6: Impulse Response Functions relating to a Persistent Shock to dollar Interest Rate



π^c – inflation in quarter
$\pi4$ – inflation in past four quarters
Δe – nominal exchange-rate depreciation
q – real exchange-rate gap
i – nominal interest rate
r – real interest rate
i* – dollar interest rate
y – output gap

The variables are expressed in terms of deviations from long-run equilibrium. Rates of inflation, interest rates, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.

Table 5.6: Impulse Response Functions relating to a Persistent Shock to dollar Interest Rate**

Deviation from equilibrium*

Quarter	1	2	3	4	8	Convergence
Nominal interest rate	0.4	0.7	0.7	0.6	0.1	0.0
Inflation in quarter	0.7	0.8	0.8	0.7	0.0	0.0
Inflation in past 4 quarters	0.2	0.4	0.6	0.7	0.2	0.0
Nominal exchange-rate depreciation	1.7	0.7	0.7	0.6	-0.2	0.0
Real interest rate	-0.3	-0.1	0.0	0.2	0.1	0.0
Output gap	0.0	0.0	-0.1	-0.1	-0.1	0.0
Real exchange-rate gap	0.2	0.2	0.2	0.2	0.0	0.0
Dollar interest rate	1.0	1.0	1.0	1.0	0.0	0.0

* Deviation – in percentage points. Rates of interest, inflation, and nominal depreciation – in annualized percentage points. Output gap and real exchange-rate gap – in terms of ordinary percents.
** A 1-percentage-point shock to the dollar interest rate over four quarters and gradual convergence (over an additional year).

The slower upturn in nominal interest, than in inflation, is reflected in negative deviations of the **real interest rate** in the first two quarters, followed by more than a year of deviations above equilibrium. Consequently, the **output gap** remains negative for about two years.

6. Use of the model for forecasting and simulations

In this section we explain in detail, and demonstrate by example, how the model may be used to generate forecasts and perform simulations. The simulations may be helpful in estimating the intensity of inflation pressures and characterizing their origin. Based on past developments in the economy and predicted developments in the exogenous variables, the model derives an interest-rate path that assures the attainment of the inflation target within one to two years.

Obviously, the model does not offer a precise description of reality; instead, it provides a simplified presentation of important relationships in the transmission mechanism of monetary policy. Therefore, the forecasting and simulations of the model neither express a full analysis of the economy nor provide an unequivocal prescription for policy management. However, the model is unique in one sense (relative to other tools or indicators): it refers directly, transparently, and consistently to the underlying factors that (in our view) determine inflation. Accordingly, the results of the simulation can be analyzed. One may ask, for example, why inflation would be expected to rise or fall, why the interest rate should be raised or lowered, and so on. Furthermore, a model – unlike other tools – allows us to examine various scenarios and alternatives. For example, inflation expectations as derived from the capital markets are an important indicator that the Bank of Israel examines before it makes interest decisions.[61] While the interest rate that corresponds to these expectations may be derived – for example, from the term structure of Treasury-bill (Makam) yields, one cannot identify the underlying factors that cause the capital market to entertain these expectations. Furthermore, one cannot derive inflation expectations assuming an alternative interest-rate path or a given shock to the economy (e.g., an increase in oil prices). A model such as ours is unique in that it is able to answer this kind of questions.

The forecast and the simulations are generated in several stages. First, the latest available data have to be gathered (data updating). Then, the assumptions that will underlie the simulation must be determined (Phases A and B below). Next, a baseline scenario is generated on the basis of the assumptions made in the previous phases (Phase C). Finally, several alternative scenarios are examined (Phase D).

Below we describe these phases in detail. To illustrate the process, we interject at each phase, as an example, a simulation that was carried out in the middle of September 2006 (after the release of the August CPI). The example is presented in indented italics in order to distinguish it from the main description of how the forecast is prepared. In this section, as in the previous one, we express not only the interest-rate figures but also the rates of inflation and depreciation in annual terms.

Phase A: Updating the data of the endogenous variables

Before we perform the simulation, we need to collect data on the endogenous and exogenous variables up to the quarter preceding the beginning of the simulation. These data constitute the initial state of the simulation. One may understand their role, for example, by observing inflation equation (4.4). One

[61] One year ahead expectations for inflation, as derived from the capital markets, are calculated by the difference between the (nominal) yield to maturity of an un-indexed one-year bond (Makam) and the real yield to maturity of a one-year CPI-indexed bond.

of the factors in the equation is lagged inflation (π^c_{t-1}). Thus, the inflation rate at the initial state affects the derived inflation rate in the first quarter of the simulation.

At the beginning of this phase, we need to determine the quarter in which the simulation will begin. The choice is not trivial because at the time the forecast is produced, some of the data are known in real time (e.g., nominal exchange rate and interest rates) while other statistics are published at a lag of several months. (GDP data, for example, are released at a two-month lag.) For the most part, we begin the simulation in the following quarter. Therefore, when updating the data (for both endogenous and exogenous variables), we have to make some assumptions that will allow us to "close" the quarter preceding the beginning of the simulation.[62]

In the example that follows, the simulation was performed in the middle of September 2006. Since the interest rate for the entire third quarter was already known, we chose to begin the simulation in the fourth quarter.

In what follows we describe the sources of data for the four endogenous variables of the model:

Inflation (π^c_t) is the quarterly percent change in the Consumer Price Index (annualized). Adjustment for seasonality is made on the basis of seasonal factors that are estimated in the inflation equation. The CPI is reported at a two-week lag. To close the quarter preceding the beginning of the simulation, we must fill in up to two CPIs that were not published. We do this by availing

[62] For example, if at the end of September 2006 we wish to perform a simulation starting from the fourth quarter of 2006, we must make an assumption about the September CPI (which is not published until October 15) – in order to determine the rate of inflation in the third quarter, which is a part of the initial state for the simulation.

ourselves of professional forecasters' outlooks and a monthly model developed by the Bank of Israel's Monetary Department.[63]

In the example, we assumed that the September CPI (not published at the time that the forecast was performed) declined by 0.2 percent. The assumption was based on the aforementioned Monetary Department monthly model. Following this assumption, we found that inflation in the third quarter (annualized, seasonally adjusted) was 1.1 percent.

Nominal sheqel/dollar exchange rate (e_t) from which the (annualized) nominal rate of depreciation is also derived. We assume that there is no seasonality in the evolution of the exchange rate. The rate is reported in real time. To close a quarter, we need to make an assumption about the behavior of the rate up to one or one-half months ahead. For the most part, we assume that it will remain at the average level that it had established in recent weeks.

In the example, we assumed an average dollar exchange rate in the second half of September 2006 of 4.39 sheqels. (In the first half of September, it fluctuated in the 4.36– 4.40 sheqels range.) Accordingly, we found that appreciation continued in the third quarter of 2006 much as it had in the second quarter (9.6 percent in the third quarter as against 13.2 percent in the second quarter, both in annual terms). These appreciations explain the relatively low inflation rate in the third quarter of 2006.

The output gap (y_t) is based on domestic gross product of the business sector in constant prices. Initial output data are reported at a two-month lag. Accordingly, the closing of a quarter for the initial state is based on the Bank

[63] See Ilek (2007).

of Israel's growth outlooks. We calculate the gap using the HP filter[64] as follows:

$$y_t = [\log(Y_t) - \mathrm{HP}(\log(Y_t))] * 100$$

where Y_t is business-sector output in constant prices.

To surmount the end-of-sample problem of the HP filter, we extend the output series, prior to applying the HP filter, on the basis of real development outlooks by the Bank of Israel's Research Department and judgment assumptions. By so doing, we dampen the effect of the latest output data on the output gap estimate at the initial state of the simulation.

In the example: By September, National Accounts data had been released for the first half of 2006 only. We "extended" the output series on the assumption that business-sector product would grow by 4.5 percent during the entire year and by 4.0 percent in subsequent years. By using the HP filter, we found that the output gap in the third quarter of 2006 was –0.3 percent.[65]

Nominal interest rate (i_t) – We use the Bank of Israel's (effective) nominal interest rate, which is known in real time. Occasionally, in order to close a quarter before the simulation begins, we need to make a judgmental assumption about the interest rate for the next month.

The nominal interest rate, as stated, was known for the entire third quarter of 2006 and rested on average at 5.4 percent.

---

[64] Hodrick-Prescott filter.

[65] Meaning that business-sector product in this quarter was 0.3 percent below its potential level.

Phase B: Updating the data on the exogenous variables and establishing their future paths

As with the endogenous variables, we need to gather data about the exogenous variables up to the beginning of the simulation. In addition, we must make assumptions about their paths throughout the simulation period. Thus, each forecast actually hinges on specific paths of development of the exogenous variables.

Below we describe the sources of data for the ten exogenous variables in the model and explain how we determine their path:

Change in the world price of imported consumption goods (Δp^{*f}_t) is based on the change in the price index of imported consumption goods (in dollar terms), published by the Central Bureau of Statistics as part of Israel's foreign-trade statistics. Since the data are released at an average lag of five months (!), we already have to base ourselves on assumptions (as opposed to published data) in determining the initial state of the simulation. To accomplish this, we use auxiliary equations that help us to predict the index change in view of data reported on a current basis, including the behavior of global foreign exchange rates (dollar/euro and dollar/yen).

Future trading in forex abroad allows us to derive the foreign markets' expectations of developments in cross-currency exchange rates. With the help of these expectations and the auxiliary equation, we produce a forecast for several quarters ahead of the change in the world price of imported consumption goods. Farther on, we assume that the rate of change in import prices will converge toward the long-term inflation rate abroad (which we estimate at 2.0 percent).

The most recent import prices reported pertain to the first quarter of 2006. The auxiliary equation shows that due to dollar depreciation against many currencies in the second quarter of 2006, the prices of many goods increased in *dollar terms. (Prices of imported goods are usually denominated in the currency of the country from which they are imported. When the dollar depreciates against these currencies, the dollar price of the goods rises.) Therefore, we assumed that the index of imported consumption-goods prices rose at a rather hefty 6.0 percent (annualized) rate in the second quarter of 2006. We used the auxiliary equation to derive the expected change in the index up to the second quarter of 2007. From the third quarter of 2007, we assumed an annual rate of change of 2 percent per quarter.*

Gap (deviation from trend) of the relative price of imported inputs for domestic production (up^{*zf}_{t}) – First we calculate the ratio of the weighted average of the index of imported inputs and investment goods prices to the index of imported consumption goods prices (obtained from the Central Bureau of Statistics foreign-trade statistics). We use the HP filter to derive the gap, as we did for the output gap, as described above.

The problem of lag in the release of foreign-trade data also exists in regard to this variable. Therefore, as with the prices of imported consumption goods, we use auxiliary equations to predict the change in the index of import prices of inputs and investment goods. These equations make use, in addition to global exchange rates, of prices of commodities that are reported on a current basis, especially oil, and that are traded in future contracts.

We use the auxiliary equations mainly to estimate the value of the variable at issue at the initial state of the forecast, and to estimate how it will develop to a range of several quarters ahead. After several quarters, we use judgment to determine the pace of the narrowing of the gap.

In the example, prices of imported inputs were published up to the first quarter of 2006. In the second quarter of 2006, oil prices rose by a steep 60 percent (annualized). Furthermore, dollar depreciation against other world *currencies during that quarter caused the dollar price of imported goods to rise. Accordingly, we used the auxiliary equation to determine that input prices climbed by 12.6 percent (annualized) in the second quarter of 2006. We performed a similar analysis for the remaining quarters up to the middle of 2007. By using the HP filter and the aforementioned assumptions, we calculated the gap in the relative price of imported inputs (relative to the prices of imported consumption goods) at 2.7 percent in the third quarter. Applying judgment, we assumed that this gap would contract in the fourth quarter to 1.4 percent (it being found that the foreign financial markets expected oil prices to decline) and would fall to zero in the first quarter of 2007. Thus, we assumed that the high level of input prices still implies some built-in pressure on domestic prices. Examining this, we found that this pressure contributed 0.2 percent to inflation in the first four quarters of the forecast.*

The world trade gap (y^*_t) is based on real import data for the industrialized countries, published by the IMF (the IFS database). The gap is derived by the use of the HP filter, much as the output gap is derived.

The world trade data are reported at an average lag of half a year! To establish the initial state of the simulation and estimate the future path of the gap, we avail ourselves of auxiliary equations, the IMF outlooks, and estimates based on judgment.

In the example, world trade data were known up to the first quarter of 2006. According to the IMF outlooks and our auxiliary equations, global trade was expected to grow in 2006 at a healthy 5.6–6.4 percent pace. Thus, we derived a positive world trade gap of 0.4 percent at the initial state. We expected the gap to widen to 0.9 percent in early 2007 and then to converge slowly toward zero.

Dollar interest rate (i^*_t) – We use the one-month LIBID dollar interest rate, which is reported on a current basis. We derive the expected path of the dollar interest rate from the yield curve of US Treasury bonds. After four to eight quarters, we apply judgment to converge the interest rate to its long-term value (estimated today at 4.5 percent: a natural interest rate of 2.5 percent plus 2.0 percent long-term inflation).

The average dollar interest rate in the third quarter of 2006 was 5.2 percent. Trading in foreign capital markets showed that the rate was expected to begin declining gradually in the first quarter of 2007 and to settle at 4.7 percent in the middle of 2008. Farther on, we assumed convergence to 4.5 percent – its long-term level.

The natural real interest rate (r^n_t) – To approximate the natural real interest rate for the estimation of the parameters of the model, we used the forward real yield derived from five- to ten-year Galil indexed bonds. However, since there are empirical reasons to assume that the natural interest rate has been lower in recent years, we apply judgment in assuming the initial conditions and the future path of the natural interest rate.

In the example, we assumed a natural interest rate of 3.0 percent throughout the simulation period.

Exchange-rate risk premium (rp_t) – To estimate the exchange-rate equation, we assume a constant risk premium. Estimating the equation for the period 1997–2005 yields an estimate of a 5.8 percent risk premium. When we shorten the estimation period to 1999–2005, the risk-premium estimate decreases to 3.6 percent. There is empirical reason to assume that in the last few years it has been even lower; hence we apply judgment in assuming the initial conditions and future path of the risk premium.

In the example, we assumed an exchange-rate risk premium of 0.5 percent throughout the simulation. Support for this assumption may be found in Hecht and Pompushko (2006), whose calculations set the risk premium at 0.5–1.0 percent since April 2006.

Inflation target (π_t^{target}) – Since 2003, the midpoint of the inflation-target range has been 2.0 percent.

The long-term rates of dollar interest and real natural interest, the risk premium, and the inflation target determine the long-term rates of inflation, nominal interest, and exchange-rate depreciation. To make sure that the model will converge, we must assume that in the long run the dollar interest rate plus the risk premium will equal the domestic natural real interest rate plus the inflation target: $i^* + rp = r^n + \pi^{target}$.

This assumption expresses the theoretical principle that in the long run, absent inflation differentials, the nominal interest spread will reflect the risk premium.

The investment gap ($inv^h{}_t$) is based on the gross investment component of the National Accounts data. We obtain the initial conditions of the investment gap by using the HP filter, similarly to the output gap. The future path of the investment gap is determined by judgment, assisted by the Bank of Israel Research Department's short-term forecasts of real economic developments.

In the example, by using the HP filter we found that the gap was (–8.5) percent at the initial state. Since the estimate of this gap is unstable, we assumed that it would be 0 from the beginning of the simulation.

Public-consumption gap (g^h_t) – Since it is business-sector product that we are interested in, we base ourselves on the domestic-procurements component of public consumption in the National Accounts data. The initial conditions for the public-consumption gap, like the output gap, are derived by means of the HP filter. The future path of the public-consumption gap is determined by judgment, assisted by the Bank of Israel's Research Department's short-term forecasts of real economic developments.

In the example, the HP filter shows that the gap was 1.6 percent at the initial state (third quarter of 2006). We assumed that the gap would remain slightly positive in the first quarters of the simulation because of government expenditure for post-war rehabilitation of northern Israel. From the second quarter of 2007, it will zero out.

The equilibrium real exchange rate, like the output gap, is derived by means of the HP filter. The equilibrium real exchange rate[66] is needed in order to determine the real exchange-rate gap at the initial state. In generating the forecast, we assume that the equilibrium real exchange rate would remain constant throughout the simulation period.

In the example, the real exchange rate fell due to the nominal appreciation of the sheqel against the dollar. By using the HP filter, we found that the real exchange-rate gap in the third quarter of 2006 was (–2.4) percent, i.e., the rate was below its trend.

[66] The real exchange rate in the model ($e_t + p^{*f}_t - p^c_t$) is based on the index of prices of imported consumption goods, the nominal sheqel/dollar exchange rate, and the Consumer Price Index.

Phase C: Generating the forecast

After filling in the data for the initial state (for the quarters preceding the beginning of the simulation), and assuming the future paths of the exogenous variables, we may draw up a forecast. This forecast, called the "baseline scenario," is based on the most plausible assumptions about the future behavior of the exogenous variables.

The model contains several forward-looking variables (expectations). For example, in the exchange-rate equation (4.14), the rate in the current quarter is explained (among other things) by the rate that is expected in the next quarter. Theoretically, one may apply several approaches in dealing with these variables when performing simulations. One possibility is to assume that expectations are adaptive, i.e., that the expected exchange rate is equal to the past rate. Another possibility is to formulate an equation that describes how the expectations are formed, i.e., to explain the development of the expectations by means of other variables in the model. We chose the rational-expectations approach, which posits that the future exchange rate will equal the expected rate plus a random error. If one assumes zero error (no new shocks), the model will resolve by adding a condition – that the expected rate for the next period will be identical to the rate derived by the model for the next quarter. (The literature refers to this assumption as "model-consistent expectations.") Here another difficulty emerges: one of the exogenous variables that affect the exchange rate in a given quarter is the dollar interest rate in that quarter. Therefore, the dollar interest rate that prevails in the next quarter will affect the exchange rate in the next quarter. To derive (rational) expectations about the exchange rate in the next quarter, one must assume the current level of expectations about the dollar interest rate in the next quarter. The same pertains to the remaining exogenous variables. In producing the forecast, we assume that the expectations about the behavior of the exogenous variables are consistent with the paths that we described in the previous sub-section – i.e., that the future development of the exogenous variables, in accordance with the foregoing assumptions, is foreknown and manifested in the creation of (rational) expectations about the endogenous variables.

When we generate the baseline scenario, we focus on the forecasted path of the endogenous variables: the nominal interest rate, inflation, the exchange rate, and the output gap. Since the model is closed by means of an interest rule that assures convergence of inflation toward the target, the most important variable in the forecast is the derived path of the nominal interest rate. To demonstrate this point, let us assume that inflationary pressures exist. An interest rate hike that we derive from the model may leave the derived inflation rate close to the target. If we inspect the derived inflation only, we may get the misleading impression that there are no inflation pressures. Examining the derived path of the interest rate, however, will reveal the existence of inflation pressures that are being restrained by the rising interest-rate path.

After examining the numerical results of the simulation, particularly the paths of the interest rate and inflation, we analyze the major factors underlying the simulation results. Underlying factors may lie at the initial state of the variables or along the future paths of the exogenous variables.

Table 6.1 describes the actual development of the relevant variables and their paths as derived from the illustrative simulation. The table reports inflation, depreciation, and interest rates in annual terms and gaps in terms of (regular) percents.

The results of the simulation show that the interest rate will edge downward to 5.2 percent in the fourth quarter of 2006 and to 5.0 percent in the first quarter of 2007, that inflation in the next four quarters will be 2.2 percent, and that the sheqel/dollar exchange rate will climb gradually to 4.50 sheqels one year ahead.

Table 6.1: Assumptions and Derived Paths of Main Variables in the Baseline Scenario
(Shaded cells are actual data)

	Inflation in quarter, seasonaly adjusted	Inflation in past 4 quarters	Nominal exchange rate depreciation	Sheqel / dollar exchange rate	Real exchnage rate gap	Import price-ratio (inputs / consumption)	Business sector output gap	Ex-ante real interest rate	BoI interest rate	Dollar interest rate
	π^c	$\pi 4$	Δe	e	q	up^{*z}	y	r	i	i^*
2005:4	3.4	2.6	10.4	4.65	1.5	2.1	2.2		4.1	4.1
2006:1	1.9	3.2	1.8	4.67	1.7	2.8	3.9		4.7	4.4
2006:2	2.3	3.6	-13.2	4.51	0.0	2.7	4.1		5.2	5.0
2006:3	1.1	2.2	-9.6	4.40	-2.4	2.7	-0.3	3.7	5.4	5.2
2006:4	2.7	2.0	2.8	4.43	-2.0	1.4	-0.4	2.9	5.2	5.3
2007:1	2.5	2.2	2.6	4.45	-1.6	0.9	-0.2	3.3	5.0	5.2
2007:2	1.8	2.1	2.2	4.48	-1.2	0.8	-0.2	3.2	4.7	5.1
2007:3	1.7	2.2	1.7	4.50	-0.7	0.4	0.0	3.1	4.6	5.0
2007:4	1.6	1.9	1.1	4.51	-0.3	0.2	0.1	3.0	4.6	4.9
2008:1	1.7	1.7	0.6	4.52	-0.1	0.0	0.1	3.0	4.6	4.8
2008:2	1.7	1.7	0.2	4.52	0.0	0.0	0.1	2.9	4.7	4.7
2008:3	1.8	1.7	0.0	4.52	0.0	0.0	0.1	2.9	4.7	4.6
2008:4	1.9	1.8	-0.1	4.52	0.0	0.0	0.1	2.9	4.8	4.5

At first glance, since inflation is close to the midpoint of the target range and the interest rate is gradually returning to equilibrium (5.0 percent), one may infer that there are no expansionary or contractionary pressures – but this is not so. Analysis of the simulation reveals that opposing forces are at work. On the one hand, there was nominal and real appreciation in the second and third quarters of 2006; the resulting negative real exchange-rate gap at the initial state is the main factor of restraint in the model – the factor from which the decline in the nominal interest rate is derived. On the other hand, several expansionary factors keep inflation from falling under the midpoint of the target range:

a. *It is true, under the assumptions of the baseline scenario, that the dollar interest rate is no longer rising (after two years of steady increase) but it remains over its long-term plateau during most of the simulation as it converges to its equilibrium level slowly. Since the simulation derives a decrease in the domestic nominal interest rate, the sheqel is expected to depreciate moderately, contributing to an upturn in inflation.*
b. *As we noted in our presentation of the assumptions about imported-input prices, these prices still imply moderate upward pressure on domestic prices.*

c. *At the initial state (third quarter of 2006), the inflation rate was 1.1 percent. With the help of Equation 4.1 (the CPI identity) and Equation 4.3 (inflation of import prices), one may separate the CPI inflation into its two components: domestic inflation ($\pi^h{}_t$) and imported inflation ($\pi^f{}_t$). This shows us that while imported inflation was strongly negative (–6.7 percent) due to the exchange-rate appreciation that occurred, domestic inflation was high (7.4 percent). As Equation 4.2 (the domestic-inflation equation) shows, lagged domestic inflation affects present domestic inflation. Therefore, the high domestic inflation rate obtained in the third quarter of 2006 is an expansionary factor in the determination of inflation from the fourth quarter onward.*

Phase D: Examining alternative scenarios

The main power of models lies not in producing forecasts but in the possibility of using them to examine the implications of possible exogenous developments on the economy (i.e., on the endogenous variables). Therefore, it is important to present several alternatives to the baseline scenario that reflect the degree of uncertainty (or risk) with respect to its realization.

Alternatives may pertain to the future path of exogenous variables (e.g., a different path of the dollar interest rate) or to shocks to endogenous variables (e.g., a shock that brings on exchange-rate depreciation).

To illustrate this, let us examine two alternatives to the foregoing baseline scenario.

In Alternative A*, we assume that the dollar interest rate will continue rising in 2007 until it hits 5.75 percent. Thus, this alternative pertains to the future path of an exogenous variable. The remaining assumptions and the initial state are identical to the baseline scenario. Table 6.2 shows the results of the simulation.*

In this alternative, due to the higher path of the dollar interest rate, no interest rate decrease is derived until the second quarter of 2007, the exchange-rate depreciation is stronger, and inflation is higher (2.7 percent in the next four quarters, compared with 2.2 percent in the baseline scenario).

This scenario shows that a continued increase in the dollar interest rate will delay the reduction in the Bank of Israel rate and contribute to an upturn in inflation.

Table 6.2: Assumptions and Derived Paths of Main Variables in Alternative A (Shaded cells are actual data)

	Inflation in quarter, seasonaly adjusted	Inflation in past 4 quarters	Nominal exchange rate depreciation	Sheqel / dollar exchange rate	Real exchnage rate gap	Import price-ratio (inputs / consumption)	Business sector output gap	Ex-ante real interest rate	BoI interest rate	Dollar interest rate
	π^c	$\pi 4$	Δe	e	q	up^{*z}	y	r	i	i^*
2005:4	3.4	2.6	10.4	4.65	1.5	2.1	2.2		4.1	4.1
2006:1	1.9	3.2	1.8	4.67	1.7	2.8	3.9		4.7	4.4
2006:2	2.3	3.6	-13.2	4.51	0.0	2.7	4.1		5.2	5.0
2006:3	1.1	2.2	-9.6	4.40	-2.4	2.7	-0.3	3.7	5.4	5.2
2006:4	3.1	2.1	3.4	4.43	-2.0	1.4	-0.4	2.6	5.5	5.5
2007:1	3.0	2.4	3.4	4.47	-1.5	0.9	-0.3	3.2	5.5	5.8
2007:2	2.4	2.4	2.9	4.50	-1.1	0.8	-0.3	3.2	5.3	5.8
2007:3	2.2	2.7	2.2	4.53	-0.6	0.4	-0.1	3.2	5.1	5.8
2007:4	2.0	2.4	1.3	4.54	-0.2	0.2	0.0	3.1	5.0	5.5
2008:1	2.0	2.2	0.8	4.55	0.0	0.0	0.1	3.1	5.0	5.3
2008:2	2.0	2.1	0.4	4.55	0.1	0.0	0.1	3.1	4.9	5.1
2008:3	2.0	2.0	0.0	4.55	0.1	0.0	0.0	3.1	4.9	4.9
2008:4	1.9	2.0	-0.2	4.55	0.1	0.0	0.0	3.0	4.9	4.7

In Alternative B*, we examined the implications of an exogenous shock to the nominal exchange rate. Let us assume that an exogenous shock in the fourth quarter lowers the sheqel/dollar exchange rate to 4.30 sheqels (8.4 percent appreciation in annual terms). In this alternative, we examine an implication of a shock to an endogenous variable. Table 6.3 presents the results of the simulation.*

The nominal appreciation lowers import prices in the Consumer Price Index and widens (in absolute terms) the negative real exchange-rate gap. The table shows that this appreciation may drive inflation below the lower bound of the target range and may bring the interest rate down to as low as 3.9 percent in the middle of 2007.

Table 6.3: Assumptions and Derived Paths of Main Variables in Alternative B (Shaded cells are actual data)

	Inflation in quarter, seasonaly adjusted	Inflation in past 4 quarters	Nominal exchange rate depreciation	Sheqel / dollar exchange rate	Real exchnage rate gap	Import price-ratio (inputs / consumption)	Business sector output gap	Ex-ante real interest rate	BoI interest rate	Dollar interest rate
	π^c	$\pi4$	Δe	e	q	up^{*z}	y	r	i	i^*
2005:4	3.4	2.6	10.4	4.65	1.5	2.1	2.2		4.1	4.1
2006:1	1.9	3.2	1.8	4.67	1.7	2.8	3.9		4.7	4.4
2006:2	2.3	3.6	-13.2	4.51	0.0	2.7	4.1		5.2	5.0
2006:3	1.1	2.2	-9.6	4.40	-2.4	2.7	-0.3	3.7	5.4	5.2
2006:4	-0.8	1.1	-8.4	4.30	-3.9	1.4	-0.7	3.2	4.3	5.3
2007:1	1.2	1.0	4.7	4.35	-2.7	0.9	-0.4	2.8	4.0	5.2
2007:2	1.2	0.7	3.5	4.39	-1.8	0.8	-0.2	2.9	3.9	5.1
2007:3	1.1	0.7	2.4	4.41	-0.9	0.4	0.1	2.8	3.9	5.0
2007:4	1.2	1.2	1.5	4.43	-0.4	0.2	0.3	2.7	4.1	4.9
2008:1	1.4	1.2	0.7	4.44	-0.1	0.0	0.3	2.7	4.3	4.8
2008:2	1.6	1.3	0.2	4.44	0.1	0.0	0.3	2.8	4.5	4.7
2008:3	1.8	1.5	-0.1	4.44	0.1	0.0	0.2	2.8	4.6	4.6
2008:4	1.9	1.7	-0.2	4.44	0.1	0.0	0.1	2.9	4.7	4.5

A clarification is needed about the intensity of the response of the variables in this alternative. The shock that we assumed is a nominal one, i.e., it lacks the capacity to change the real exchange rate over time. In the long run, then, the entire appreciation should be passed through to the Consumer Price Index. (Note that the interest rate cut leads to depreciation starting in 2007 and, by so doing, holds back a little of the pressure for a decrease in inflation.)

7. Testing the model's "goodness-of-fit"

The previous section described the primary use of the model as a tool in the conduct of monetary policy. This involves simulating a baseline scenario, for the forecast of the endogenous variables and simulations of alternative scenarios in order to assess the relevant risks. In this section, we will carry out two tests of the model’s goodness-of-fit, in order to identify weak points, that need to be improved and taken into account when using the model. The first test involves an in-sample dynamic simulation of the model, which tests it's ability to reproduce the evolution of the endogenous variables (the inflation, exchange rate depreciation, the output gap and the interest rate) given the ex-post evolution of the exogenous variables. The second test compares the moments derived from Monte Carlo simulations of the model (hereinafter MC simulations)[67] to those appearing in the data. The simulations in both tests relate to the sample period beginning in the first quarter of 1999 (following the stabilization of inflation).

Before presenting the methodology and findings for the two tests, three modifications of the model, which are required in order to carry out the tests, will be discussed: the handling of the exogenous variables, the employment of an empirical interest rate rule and assumptions regarding convergence.

Specifying a law of motion for the exogenous variables

Both tests require the solution of a rational expectations model. That is, the model must be transformed from its structural form (as presented in Section 4) to its reduced form, in which each variable is dependent only on state variables, i.e. lagged variables and contemporaneous shocks. With regard to the exogenous variables, it is assumed that their future paths are unknown (which is in fact the case) since otherwise their future values would affect the solution of the model in the present. This assumption requires an attempt to partition each exogenous variable into its expected and unexpected components. In order to do this, an assumption will be made regarding the process generating the exogenous variables, including their shocks. We assume an autoregressive process, which makes it possible to calculate expectations conditional on the information available up until the point in time at which the expectations are formed.

[67] Monte Carlo simulations is a method of simulating the behavior of systems with stochastic variables. Random values for the stochastic variables are drawn by the computer from a distribution defined by the user.

The general form of a (linear) structural model with rational expectations is as follows:

$$(7.1) \quad Y_t = E_t \sum_{i=-L}^{F} H_i * Y_{t+i} + U_t$$

where Y_t is a vector of the n model variables (endogenous, exogenous and identities), U_t is a vector of structural shocks, H_i are matrices of order n x n and E_t is the mathematical expectation operator conditional on the information set at period t. As indicated by Equation (7.1), each of the variables can be influenced both by the lags and by the expectations regarding future values of all the variables in the system (where L is the longest lag and F is the longest lead).

In the reduced form of the model, all the variables at time t (Y_t) are functions of the lags in Y_t and the shocks (U_t):

$$(7.2) \quad Y_t = \sum_{i=1}^{L} A_i * Y_{t-i} + BU_t$$

In order to obtain the reduced form, A_i and B must be identified so that the model represented by (7.2) be made equivalent to the one represented by (7.1). The methodology, for transforming a (linear) structural model with rational expectations into a reduced form representation, is described in Blanchard and Kahn (1980).

In order to solve the model, it must be augmented by specifications for the dynamic processes of the exogenous variables. (Note that the vector of variables Y_t includes all the system's variables, including the exogenous ones.) As already mentioned, an autoregressive equation is estimated for each exogenous variable using OLS for the period 1994–2005. Following are the results:[68]

The gap (deviation from trend) of the **ratio between the world price of imported inputs and the price of imported consumption goods** (foreign trade data):

$$(7.3)\quad up_t^{*z} = \underset{(0.08)}{0.77} \cdot up_{t-1}^{*z} + \underset{(0.14)}{0.27} \cdot \left(up_{t-1}^{*z} - up_{t-2}^{*z}\right) + \varepsilon_t^z$$

$$\mathrm{R}^2 = 0.73 \qquad \mathrm{S.E.} = 1.54 \qquad \mathrm{D.W.} = 2.09 \quad \mathrm{Sample: 94q1 - 05q4}$$

As an approximation to the **natural real interest rate's** dynamics (though not to its level!) an equation was estimated for the law of motion of the implied 5 to 10 year forward yield to maturity of CPI-indexed "Galil" bonds:

$$(7.4)\qquad r_t^n = \underset{(0.08)}{0.78} \cdot r_{t-1}^n + \left(1 - 0.78\right) \cdot \underset{(0.22)}{4.5} + \varepsilon_t^{r^n}$$

$$\mathrm{R}^2 = 0.64 \qquad \mathrm{S.E.} = 0.36 \quad \mathrm{D.W.} = 1.91 \quad \mathrm{Sample: 94q1 - 05q4}$$

In order to specify the law of motion for the **dollar interest rate**, the following equation was estimated for the short-term dollar LIBID interest rate:

$$(7.5)\qquad i_t^* = \left(1 - 0.95\right) \cdot \underset{(0.85)}{4.3} + \underset{(0.03)}{0.95} \cdot i_{t-1}^* + \underset{(0.10)}{0.74} \cdot \left(i_{t-1}^* - i_{t-2}^*\right) + \varepsilon_t^{i^*}$$

$$\mathrm{R}^2 = 0.97 \qquad \mathrm{S.E.} = 0.32 \qquad \mathrm{D.W.} = 2.17 \quad \mathrm{Sample: 94q1 - 05q4}$$

The following equation was estimated for the exchange rate's **risk premium** (the risk premium in this estimation was calculated according to the method proposed by Hecht and Pomposhko, 2006):

$$(7.6)\qquad rp_t = \underset{(0.09)}{0.81} \cdot rp_{t-1} + (1 - 0.81) \cdot \underset{(0.45)}{2.14} + \varepsilon_t^{rp}$$

$$\mathrm{R}^2 = 0.67 \qquad \mathrm{S.E.} = 0.58 \qquad \mathrm{D.W.} = 1.65 \quad \mathrm{Sample: 94q3 - 05q4}$$

---

[68] In this section, interest rates, inflation and depreciation rates are expressed in annual terms.

The **world trade gap** (imports of the industrialized nations, IFS data):

$$(7.7)\qquad y_t^* = \underset{(0.08)}{0.75}\cdot y_{t-1}^* + \underset{(0.14)}{0.38}\cdot\left(y_{t-1}^* - y_{t-2}^*\right) + \varepsilon_t^{y^*}$$

$R^2 = 0.72$ S.E. $= 1.58$ D.W. $= 2.18$ Sample: 94q2 - 05q4

The **investment gap**:

$$(7.8)\qquad inv_t^h = \underset{(0.14)}{0.25}\cdot inv_{t-1}^h + \varepsilon_t^{inv}$$

$R^2 = 0.06$ S.E. $= 9.08$ D.W. $= 1.98$ Sample: 94q2 - 05q4

The **gap in the domestic-procurement component of public consumption**:

$$(7.9)\qquad g_t^h = \underset{(0.12)}{0.23}\cdot g_{t-1}^h + \underset{(0.11)}{0.23}\cdot g_{t-2}^h + \varepsilon_t^g$$

$R^2 = 0.18$ S.E. $= 2.87$ D.W. $= 2.32$ Sample: 94q2 - 05q4

The system includes two additional exogenous equations which were not estimated: the **annualized quarterly change in the dollar price of imported consumption goods** and the **inflation target**.[69] Equations (7.10) and (7.11) were specified for these two variables, respectively:

$$(7.10)\qquad \Delta p_t^{*f} = 2 + \varepsilon_t^{\Delta p^{*f}}$$

$$(7.11)\qquad \pi_t^{target} = 0.85\cdot\pi_{t-1}^{target} + (1-0.85)\cdot 2 + \varepsilon_t^{\pi^{target}}$$

Finally, the **identity of the real exchange rate gap** (F.5) was augmented, in order to include shocks to the equilibrium level of the real exchange rate:

$$(7.12)\qquad q_t = q_{t-1} + 0.25*\left(\Delta p_t^{*f} + \Delta e_t - \pi_t^c\right) - \varepsilon_t^q$$

[69] The equation for the inflation target is only relevant for the within-sample dynamic simulation. It reflects the fact that in 1999 the mid-point of the inflation target range was 4.0% and was reduced gradually to 2.0% by 2003. In the comparison of the moments obtained from the MC simulations, we assume a constant inflation target.

Employing an estimated interest rate rule:

In routine implementation of the model, we employ a forecast-based interest rate rule according to the rationale described in Section 4.d. As mentioned there, the attempt to estimate such a rule was unsuccessful. Since our goal is to test the model's goodness-of-fit as an integrated system, we should employ the rule which the Central Bank actually used, rather than a desirable one. The tests, that are described below, were carried out using the following estimated rule:

$$(7.13)\quad i_t = 0.2 \cdot \left\{ r_t^n + \pi_t^{target} + 1.7 \cdot \left(\pi 4_t - \pi_t^{target} \right) + 0.4 \cdot 0.5 \cdot \left(y_t + y_{t-1} \right) \right\} + (1 - 0.2) \cdot i_{t-1} + \varepsilon_t^i$$

where $\pi 4_t$ is the average rate of inflation during the previous four quarters. Unlike the interest rate rule employed in routine implementation of the model – which responds to **expected** inflation – here the interest rate responds to **ex-post** inflation. (For additional details, including a description of the estimation, see Section 4.d in the body of the paper and Section 4 in Appendix 2.)

Additional assumptions required for the convergence of the simulation:

The estimated equations (7.3) – (7.9) were adjusted so that the exogenous variables would converge to the following long-run values:

- The rate of change in import prices converges to 2.0 percent.
- The natural interest rate converges to 3.0 percent.
- The dollar interest rate converges to 4.5 percent.
- The inflation target converges to 2.0 percent.
- The risk premium converges to 0.5 percent.

The addition of the equations for the exogenous variables, to those for the model's endogenous variables and identities, enables us to solve for the rational expectations or, in other words, to transform the model from a structural representation (as represented by Equation 7.1) into a system of functions of state variable (as represented by Equation 7.2).

Having discussed the augmentations common to both tests, we now present the methodology and findings for each of the tests individually.

7.a. In-sample dynamic simulation

This section presents a comparison between the paths of the endogenous variables derived from a dynamic simulation of the model and their in sample realizations. The goal is to test the ability of the model to replicate the paths of the endogenous variables, given the dynamic processes and the shocks that characterized the evolution of the exogenous variables.

Methodology

In order to simulate a model that contains rational expectations, the exogenous variables must be decomposed into their expected component (which affects the system before it occurs) and their unexpected component (which affects the system only after its realization). In order to do so, the dynamic processes of the exogenous variables were specified, as described above. For example, imported energy prices are exogenous variables in the model. Had energy prices been treated, in the simulation, as if their path was known in advance, then the increase in energy prices that occurred in 2005, would have already affected the forecast in 1999 (the year in which the simulation begins). Obviously, it is not reasonable to assume that in 1999 the market foresaw the 2005 significant increase in energy prices. Therefore, in carrying out the simulation, only the expected component of the future evolution of the exogenous variables should be taken into account. As already mentioned, we estimate the expected component using the forecast derived from an auto-regression for each of the exogenous variables. If the residuals of the estimated equation are not serially correlated, then the estimate of the expected component is unbiased.

With regard to the endogenous variables, the simulation is dynamic. That is, the results of the forecast, obtained in each period for these variables, serve as the input for the forecast of the following period. In contrast, the simulation for the exogenous variables is static: the actual values, in each period, serve as the input for the following period (rather than the simulation results).

Formally stated, transformation of the model into the form represented by Equation (7.2) enables the implementation of dynamic simulations, where the vector of exogenous variables y_t, is composed of two components: $y_t = E_{t-1}(y_t) + u_t$; where $E_{t-1}(y_t)$ is the component that was expected in period $t-1$ and u_t is a vector of shocks identified only upon realization. We treat the endogenous variables in a similar manner, with one exception: we assume their shocks are nil, even after they have actually occurred.[70]

Before presenting the findings, it is important to mention a shortcoming in a test of this sort. The credibility of its results depend on a reasonable identification of the expected and unexpected components of the exogenous variables, at each point in time. It is only a working assumption that the generating process of the exogenous variables is autoregressive; in some cases it is a reasonable assumption, in others, it is not. Reliable estimation of the expected and unexpected components requires individual treatment of each exogenous variable, in each of the simulation's periods.[71] Here, it is arbitrarily assumed, as is commonly done in the literature, that the exogenous variables are characterized by auto-regressive generating processes.

---

[70] For a more detailed explanation of the concept and the implementation of the simulation, see Appendix 3.

[71] Or at least a richer modeling of the exogenous data's generating process.

Figure 7.1: Dynamic Simulation of the Model for the Period 1999.1 to 2006.2
Endogenous variables: inflation, nominal depreciation, output gap and interest rate

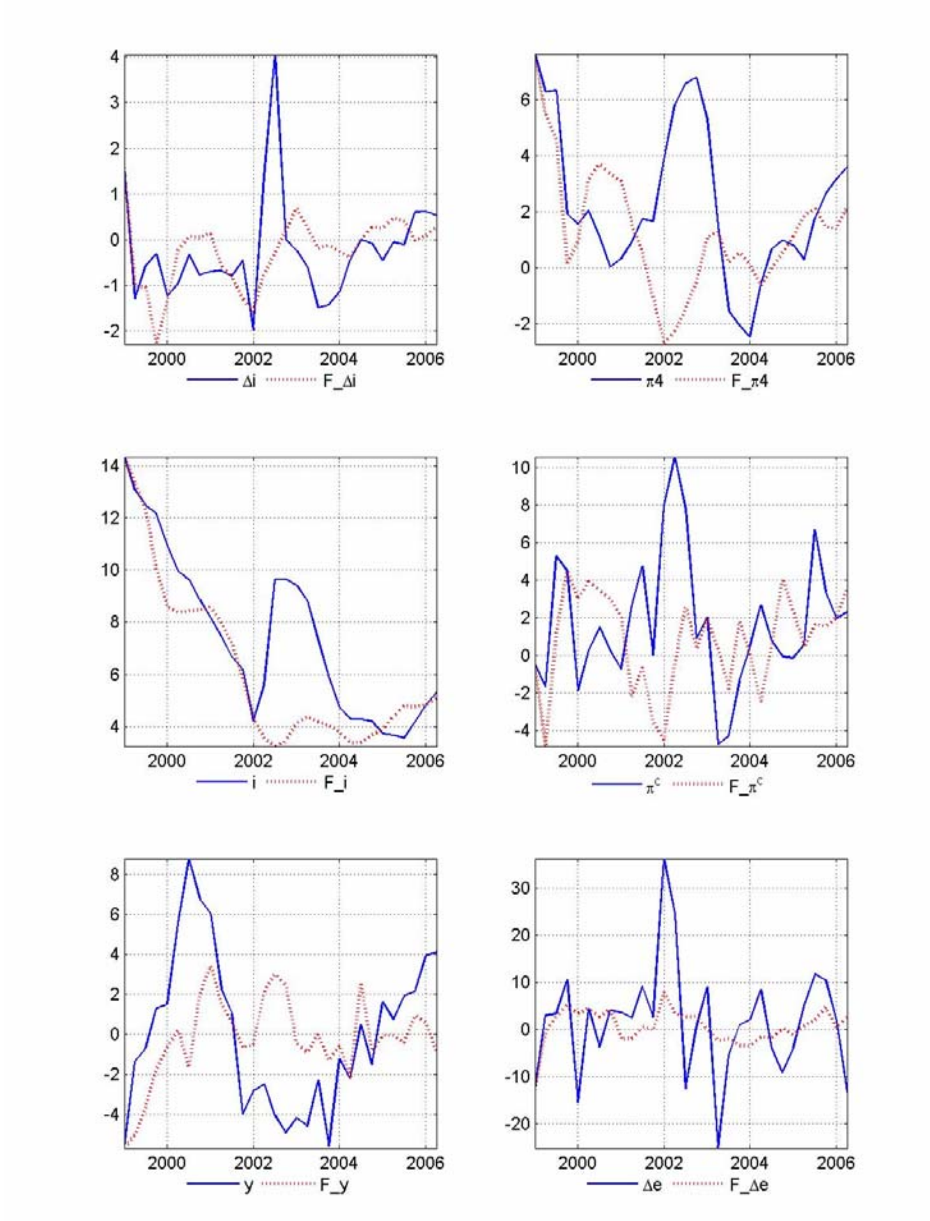
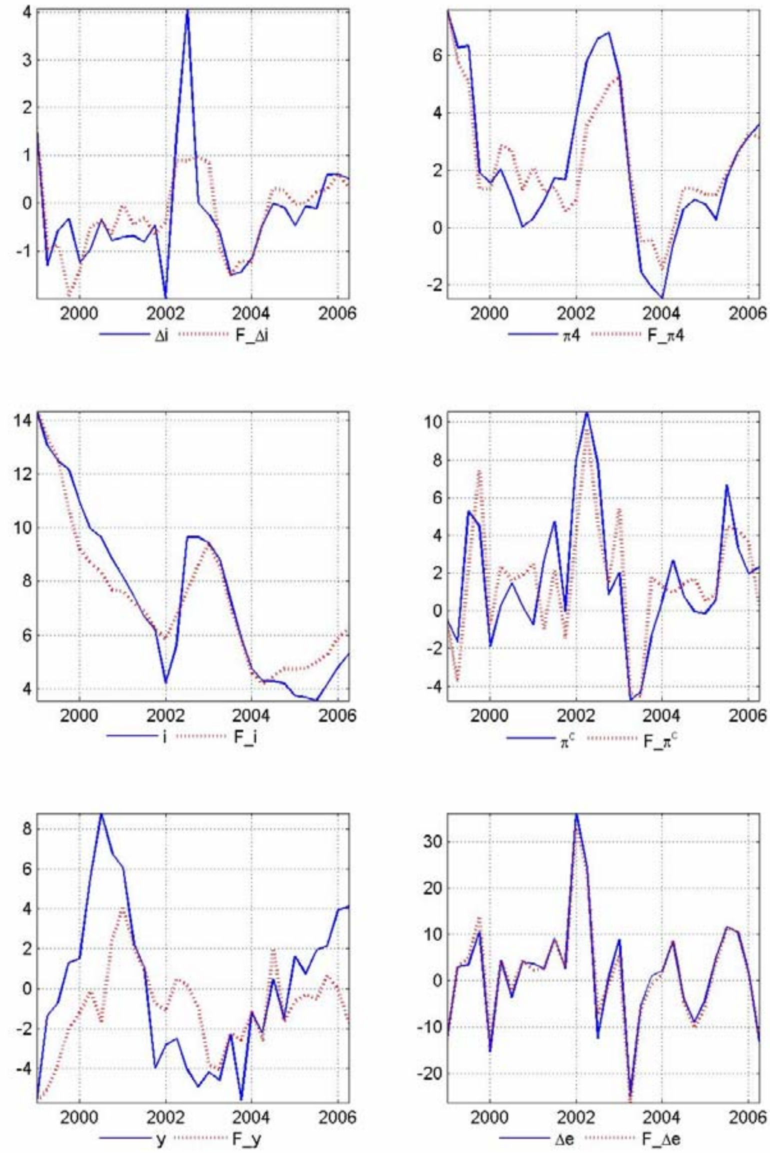


Figure 7.2: Dynamic Simulation of the Model for the Period 1999.1 to 2006.2
Endogenous variables: inflation, output gap and interest rate



Findings

Figure 7.1 compares the actual paths of the endogenous variables, to those forecasted by the dynamic simulation for the period 1999.1–2006.2 (labeled as F_). The diagrams show that while the **trend** of each variable is tracked to a reasonable degree of accuracy by the model, the variables' **higher frequency volatility** is not explained to a satisfactory extent. Thus, for example, the model does not forecast the increases in inflation and the interest rate in 2002. Furthermore, the equation for the exchange rate suffers from the lowest explanatory power. Thus, the model produces a smooth path for the nominal depreciation rate, while in reality it was characterized by a high degree of volatility.

The model explains the nominal exchange rate through the expected exchange rate and the nominal interest rate differential between Israel and the US (with the addition of a risk premium). The low explanatory power of the nominal depreciation in the dynamic simulation indicates, that interest rate differentials provide only a partial, and relatively small, explanation of the fluctuations in the exchange rate. The sharp depreciation, which took place at the beginning of 2002, is not predicted by the model and therefore neither are the high rates of inflation which prevailed in 2002, nor the increases in the rate of interest which began in that year.[72]

In the following simulation, the exchange rate will be treated as an exogenous variable though only in a particular sense; that is, we assume that, as in the case of the exogenous variables, the shocks in the exchange rate equation are known (from the period in which they are realized) while the **expected** path of the exchange rate (which affects

[72] It is worth mentioning that the depreciation which occurred at that time was primarily the result of the large deviation in monetary policy in December 2001, which was manifested in an unexpected reduction of 2 percentage points in the Bank of Israel interest rate.

the other endogenous variables through expectations) continues to be endogenous, i.e. derived from the exchange rate equation. The results of this simulation are presented in Figure 7.2.

The improvement in the explanatory power of all the variables is clearly evident: most of the fluctuations in inflation, the interest rate and the output gap are now "captured" by the dynamic simulation. The improvement in this simulation is evidence of the significant influence of the nominal exchange rate on inflation and the interest rate in Israel, particularly during the years 2002 and 2003 when the exchange rate was characterized by significant changes.

7.b. Stochastic simulations – comparison of second moments

One of the methods of evaluating the goodness-of-fit of a dynamic model, is to examine the extent to which data, produced by MC simulations, can reproduce the joint distributions of the variables; particularly their coefficients of correlation (both the autocorrelation and cross-correlation coefficients). This method is also used for calibration and even for estimation (SMM – Simulated Method of Moments). The comparison, of observed moments to those generated by MC simulation, is of particular interest in the case that the estimation criterion is not the minimization of the distance between the two types of moments.[73]

In general, the relevant moments for testing vary according to the model and the purpose for which it is used. In the case of dynamic models, we are interested in testing moments that express dynamic relations, such as the cross-correlation coefficients (which measure the correlation between

[73] The equations of the model were estimated using GMM, which has a different estimation criterion.

variables at two different points in time). An example would be the coefficient of correlation between the rate of inflation and the interest rate lagged by four quarters. The objective of the test is to evaluate the dynamic relations between the variables, over periods that are longer than one quarter, and the extent to which the model captures these relations. This test complements the impression received from the dynamic simulation presented in the previous section, where we could examine the quality of the model's forecasts given a **particular** path for the exogenous variables (i.e. the actual one, which was also used for the estimation). In contrast, the present test reproduces the joint distribution of the data, given the dynamic **characteristics** of the exogenous variables rather than their **path**.

The test is similar to that presented in Smets and Wouters (2003) for the European economy.[74] In this test, the characteristics of the data, produced by the simulation of the model, are compared to the observed quarterly data for the Israeli economy during the period 1999.1 to 2006.2 (hereinafter the comparison period).[75] The test determines whether the moments observed in the data are significantly different from the theoretical moments derived from the model. This is done through the estimation of confidence intervals for the moments derived from the model.[76]

---

[74] They derive and estimate a monetary DSGE (Dynamic Stochastic General Equilibrium) model for the European economy.

[75] The moments observed in the data were calculated for the period beginning after inflation was brought under control. The reason for this is the desire to avoid upward bias in the correlation coefficients, as a result of the downward trends in the data during the disinflation era.

[76] In contrast, Rotemberg and Woodford (1998) present a visual comparison of the moments observed in the data to the theoretical moments derived from the model (rather than to the confidence intervals of the estimates for these moments). The theoretical moments were not calculated analytically; rather, they were calculated from an asymptotic simulated series (of 5000 periods). The theoretical moments of Rotemberg and Woodford (1998) are analogous to the center of the confidence interval in the Smets and Wouters (2003) approach, which was employed here.

Methodology

The confidence interval is estimated in a number of steps:

a. The standard deviation of the model's equation residuals are estimated for the comparison period.[77]

b. Thirty values are drawn for each residual (for thirty quarters in the simulation period) using normal distributions with the estimated standard deviations.

c. Using the drawn residuals, artificial time series are generated by simulating the model in its reduced form. This step produces a time series of thirty periods for each of the model's variables (which is the length of the comparison period).

d. Using the series obtained, the relevant moments are calculated.

e. Steps (b) to (d) are repeated 1,000 times, so that 1,000 estimates are collected for each moment.

f. The 90% confidence interval is estimated, for each moment, by omitting the 50 lowest and 50 highest estimates.

Findings

The results are presented in Figure 7.3. The moments presented are the coefficients of correlation, both cross-correlation and autocorrelation, up to the fifth lag, for each of the endogenous variables: the nominal depreciation (Δe), the inflation (π^c), the nominal interest rate (i) and the output gap (y).

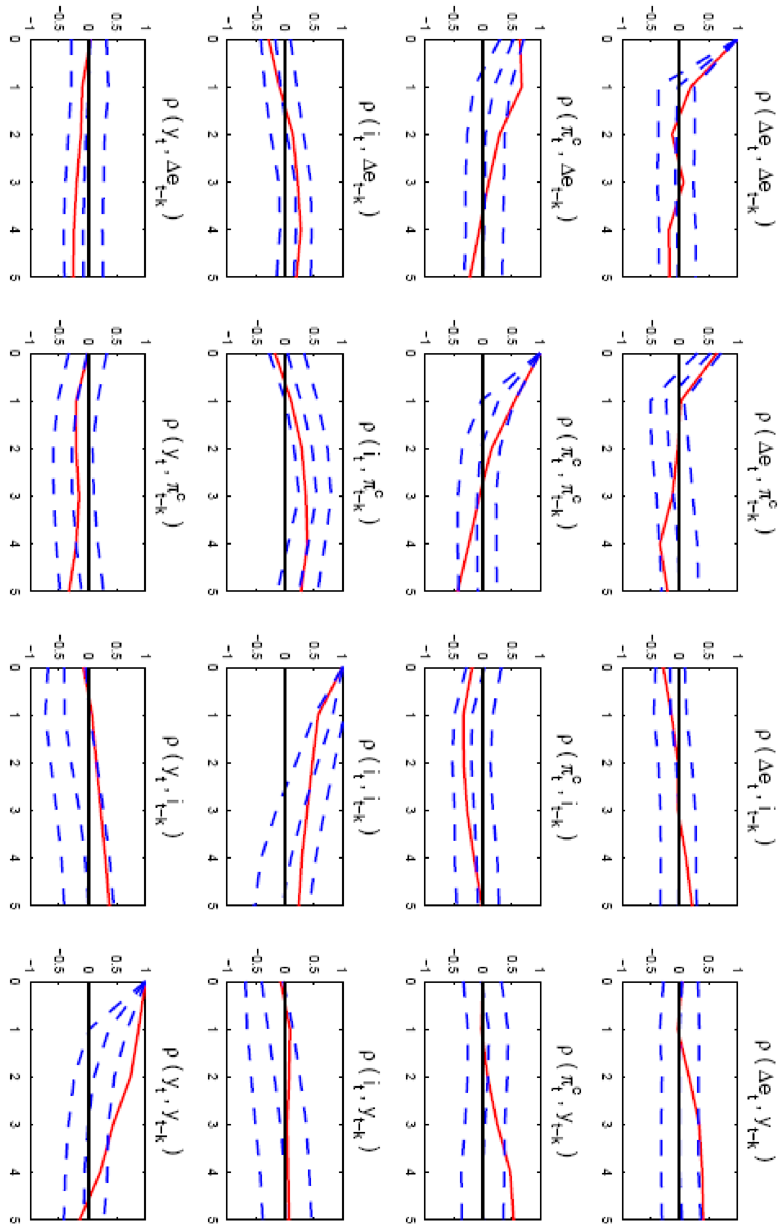
---

[77] Including the equations for the exogenous variables that were presented at the beginning of this section.

Figure 7.3: Actual Coefficients of Correlation and the Model-Based Confidence Intervals

Solid line: Empirical coefficients of correlation

Dotted line: model-based 90% confidence interval



The vertical axis of each diagram represents the correlation coefficient between the variable x_t and the variable z_{t-k}; That is, between two variables separated by k periods. The horizontal axis represents k. Thus, for example, the far-left diagram in the second row, titled $\rho(\pi^c_{t,}\ \Delta e_{t-k})$, presents the coefficients of cross-correlation between the rate of inflation (π^c) and each of the first five lags of the nominal depreciation (Δe), as well as its contemporaneous rate.

Figure 7.3 shows that in most cases the observed moments are located within the confidence interval, meaning that in general the model successfully reproduces the serial and cross-correlation functions, that characterize the endogenous variables in the model.

The first row of the figure presents the coefficients of correlation between the depreciation rate and each of the other variables, contemporaneously and for up to five lags. It is worth pointing out that the model "captures" fairly accurately the nature of the relations – both contemporaneous and lagged – between the depreciation rate and inflation. The contemporaneous correlation between the depreciation rate and inflation is high, while the correlation between the depreciation rate and lagged inflation is negative. According to the model, this is explained by the fact that a depreciation leads to price increases and therefore the contemporaneous correlation is positive. At a later stage, the interest rate increases, in accordance with the interest rate rule, which acts to lower the exchange rate and thus results in a negative correlation between the depreciation rate and lagged inflation. The correlation between the depreciation and the interest rate, both contemporaneous and lagged, is also captured fairly well. On the other hand, the model does not reproduce the positive correlation between the lagged output gap (starting from the second lag) and the depreciation rate. We are unable to explain this correlation (which may very well be a random phenomenon).

The second row presents the cross-correlation coefficients between inflation and the other variables, both contemporaneous and for up to five lags. It can be seen that the data exhibits a large and positive degree of correlation between inflation and contemporaneous and lagged depreciation rates. The model is able to reproduce these relations but not the magnitude with which they appear in the data. In other words, though the model reproduces the direction of the pass-through from the exchange rate to prices quite accurately, the data displays a stronger and more prolonged pass-through effect. We are unable to offer a complete explanation for this phenomenon, though one possibility is that the effect of the exchange rate on the housing component, and through it on the CPI, is underestimated.[78] As can be seen from the diagrams, the model reproduces the serial correlation (that is, the inertia) in inflation quite accurately, as well as the relation between inflation and the lagged interest rate. In contrast, the model does not reproduce the positive correlation observed in the data between inflation and the lagged output gap (third to fifth lag). It is worth mentioning that, during estimation, we noticed this relation, but decided to ignore it in the final specification of the inflation equation.[79]

The third row presents the coefficients of correlation between the interest rate and the other variables, both contemporaneous and for up to five lags. As can be seen from the diagrams, the model reproduces fairly well the correlation between the interest rate and both lagged depreciation and inflation rates. Thus, the interest rate is negatively correlated contemporaneously with

[78] The housing component is particularly influenced by the sheqel/dollar exchange rate, especially since 1999 when the methodology of measuring it was changed. However, the model does not relate to this component separately. In the future, we intend to specify a separate equation for the housing component.

[79] We assume (though without a great deal of evidence at this stage) that this is the result of the inability to obtain a satisfactorily reliable estimate of the output gap. It is a known phenomenon in the literature, that the estimation of potential output using the HP filter method creates a dynamic of lags in the effect of the estimated output gap on inflation.

depreciation and inflation, though positively correlated with the lags of these two variables. This phenomenon, and the fact that the model successfully reproduces it, is consistent with the conjecture that during the comparison period, monetary policy reacted to inflation that had already occurred.[80] The model also reasonably reproduces the inertia in the interest rate and the relation between the output gap (and its lags) and the interest rate.

The fourth row presents the correlation coefficient between the output gap and the other variables, both contemporaneously and for up to five lags. As shown by the diagrams, the model quite accurately reproduces the negative correlation between lagged inflation and the output gap. On the other hand, it is unable to reproduce the correlation between the output gap and the interest rate (and its lags). According to the model, there is a negative correlation between the output gap and the interest rate (and its lags) while in the data there is no contemporaneous correlation and the lagged correlation is positive. (The observed moment is located on the upper bound of the confidence interval.) It is also worth mentioning that the model reproduces the inertia in the output gap, though its magnitude is smaller than in the data.

We would emphasize that the comparison of moments as presented here is evidence of correlation but not of causality. Care must be taken while drawing conclusions from the correlations, with respect to the direction of causality due to, among other things, the existence of the expectations channel. Nonetheless, one cannot ignore the strong relations between the rate of inflation and its own lags, and between it and the nominal depreciation (both contemporaneously and with lags) that appear in the data and are also "captured" by the model (as can be seen from the second row of Figure 7.3). However, while inflation can be forecasted using its own lags, or those of

[80] Using MC simulation with a forward-looking interest rate rule in the form of Equation (9), it was found that most of the positive correlation is between the interest rate and contemporaneous inflation, rather than lagged inflation.

other variables such as the interest rate, the nominal exchange rate is harder to forecast, since it is neither correlated with its own lags, nor with those of the other endogenous variables (see the first row of diagrams in Figure 7.3). **The graphic presentation indicates that the model does indeed provide whatever explanatory power (with respect to the exchange rate) that can be obtained from the joint distributions of the endogenous variables (both contemporaneous and lagged).** Thus, for example, it is possible to identify a relation between the depreciation and the interest rate, both contemporaneously and lagged by one quarter.

It is worth pointing out a shortcoming in this kind of test. The confidence intervals of the moment estimators broaden as the level of uncertainty with respect to the moments increases, Thus, increasing the chance that the observed moments will fall within them. Therefore, attention should be paid to the width of the confidence interval, as well as to its path. For example, the **contemporaneous** relation between the interest rate and the depreciation (which does not appear in the interest rate rule) is negative; however, the relation between the interest rate and **lagged** depreciation rate becomes increasingly stronger and reaches a peak at the third lag (at which it is already positive). This pattern characterizes both the confidence interval and the observed moment and therefore can be viewed as relatively reliable. In contrast, the path of the coefficients of autocorrelation for inflation that is observed in the data, does not resemble the one derived from the model and therefore the conclusions in this case do not have as strong a foundation. (This is also the case for the autocorrelation coefficients of the interest rate.)

In summary, the results, of the two tests presented here, show that the exchange rate has a significant influence on prices, though its evolution is difficult to explain and predict. If the exchange rate is treated as a kind of exogenous variable, the model reproduces the historical evolution of the endogenous variables fairly well. One of the implications of this finding is that the planning of monetary policy should take into account a number of alternative scenarios, regarding possible evolutions of the nominal exchange rate.

8. Conclusions and future research directions

In this study, we specified and estimated a New Keynesian model for the Israeli economy. Classical estimation[81] of models like this one is often unsuccessful. In our case, classical estimation techniques did produce fairly good results that are consistent with those obtained for other economies.

One of the main findings of the study is the importance of the exchange rate in the Israeli economy's transmission mechanism. The exchange rate affects inflation both directly and through its influence on the output gap. In addition, the direct influence of the exchange rate on inflation, and its effect on the output gap, are stronger than in other economies. The sensitivity of inflation and the output gap to the exchange rate, and to the exogenous shocks that characterize it, is manifested in their relatively large fluctuations, and as a result also in the fluctuations of the nominal interest rate (through the interest rate rule) and the real interest rate.[82]

[81] The term "classical estimation" is used here to include methods such as Maximum Likelihood, OLS, 2SLS and GMM, as opposed to calibration or Bayesian estimation.

[82] It is worth emphasizing that the sensitivity of the exchange rate to various developments, as well as its high level of volatility, is not unique to the Israeli economy. The variance of the exchange rate in Israel is no higher than that in other economies; however, the effect of changes in the exchange rate is larger than in other economies.

On the other hand, the intensity of the exchange rate channel increases the influence of monetary policy on inflation. Thus, changes in the interest rate rapidly affect prices via the exchange rate.

The estimation of the inflation equation showed that there is a rapid pass-through, from prices abroad and from the exchange rate, to import prices in the domestic economy. About 20 percent, of a one-quarter-ahead expected depreciation,[83] is transmitted immediately into higher import prices. Once a depreciation has occurred, some 60 percent of it is transmitted into higher import prices in the same quarter, while about 20 percent are transmitted in the following quarter. Monte Carlo simulations show that the model reproduces the high correlation between inflation and contemporaneous and lagged depreciation rates fairly well, though with a certain degree of underestimation.

The estimation showed that the weight of imported goods and services in the CPI is about 0.45. This estimate is higher than the one obtained for other open economies (where it is about 0.3) and reflects the strong influence of the exchange rate on the housing component (which accounts for about 20 percent of the CPI). This phenomenon, in which the price of a component that is not imported (and is not even tradable) is almost completely linked to the sheqel/dollar exchange rate, is, to the best of our knowledge, unique to the Israeli economy – a legacy from the era of hyper-inflation that prevailed during the years 1978 to 1985. In order to better understand and forecast inflation and to reliably estimate the import component of the CPI, it would be worthwhile separating the housing component from the CPI and estimating a separate equation for it.

Also in the case of the output gap equation, it was found that the exchange rate (in this case, the real exchange rate) has a larger effect than that found in similar studies of other economies. This finding is further evidence of the

[83] Or an increase in world prices.

Israeli economy's degree of openness and its sensitivity to exogenous global shocks.

As mentioned above, the exchange rate has a major influence on the system but is subject to large shocks that can't be forecasted. Thus, for example, the dynamic simulation of the model shows that the model is unable to reproduce most of the fluctuations in the exchange rate and, as a result, its ability to reproduce the evolution of the other variables is also limited. On the other hand, in a simulation that takes into account the actual evolution of the exchange rate, the model quite successfully reproduces the evolution of inflation and the interest rate during the sample period.

The estimation of the inflation equation shows that there is inertia in the inflation process, but to a smaller degree than in other economies. A relatively large coefficient for inflation expectations (in the inflation equation) indicates that monetary policy has a relatively rapid effect on inflation via expectations.

The effect of the output gap on inflation was found to be close to that in other economies. However, it was found that the coefficient of the output gap, and the timing of its effect, is sensitive to changes in the estimation period. In the final specification of the inflation equation, we chose to include the output gap as a two quarters moving average.

The estimation of the output gap equation yielded a high coefficient for the expected output gap, which indicates a low degree of inertia both in comparison to findings for other economies and in comparison to the degree of inertia in inflation. The timing of the influence of the interest rate on the output gap was found to be somewhat sensitive to the estimation period and in the final equation we chose to include a moving average of the interest rate in the current and preceding quarters. The estimated influence of the interest rate on the output gap is consistent with the estimates obtained for other economies.

In most economies, a relatively long lag is found in the influence of the interest rate on inflation. In contrast, our study found that the transmission mechanism from the interest rate to inflation in the Israeli economy is rapid, even in comparison to other small open economies. This finding is a result of two factors: the first is the immediate effect of the interest rate on the exchange rate and the strong and rapid effect of a depreciation on inflation; and the second is the high coefficients obtained for the expectation variables in the equations for inflation and the output gap. The strong influence of expectations in these two equations is consistent with the New Keynesian theory, which emphasizes the importance of the public's expectations in the transmission mechanism.

The output and relative price gaps were estimated in this study using the HP filter, which is a common statistical technique. This was done primarily for reasons of convenience and the lack of simple alternative methods. It is likely that the measurement errors in these variables, assuming they are not systematic, do not significantly affect the quality of the parameter estimates. However, these gaps also serve as the starting point of the forecast. In this case it was found that the results of the forecast are sensitive to the estimate of the real exchange rate gap at the initial state. When producing a forecast, especially during periods in which there has been a major change in the nominal exchange rate, it is difficult to identify the trend in the real exchange rate and, as a result, the relevant gap. Therefore, it is important to develop tools that will help identify the trend in the real exchange rate. It would also be worthwhile improving the estimate of the trend in output, although in this case the forecast is less sensitive to the estimate at the initial state.

The model in its present form is, as already mentioned, a small one which includes all the variables required to describe the transmission mechanism of monetary policy in a small open economy. Possible improvement and expansion of the model can take a number of directions:

- Specification of a separate equation for the housing component, which is influenced to a significant extent by the sheqel/dollar exchange rate. The primary inflation equation will then relate to the CPI excluding the housing component (which accounts for 20 percent of the index).[84]
- Derivation of an optimal interest rate rule, which is derived from minimizing a loss function subject to the model.
- Estimation of potential output, the natural rate of interest and the trend of the real exchange rate using a structural model or auxiliary equations.
- Specification of the labor market within the model, with the possibility of including wage rigidity in addition to price rigidities. This requires, among other things, the specification of a wage equation and a change in the characterization of the price equation.
- Endogenization of the investment component: a relatively simple possibility is to omit this term from the right-hand side of the output gap equation while attempting to improve the characterization of the equation (by adding variables and/or lags). Another, more complicated, possibility is to specify an equation for the value added of investment. In this case, one of the difficulties is to estimate the relevant stock of capital.

[84] The possibility of separating out the component of gasoline and car oil (which accounts for 5 percent of the CPI) should also be considered. This component is essentially an imported good but its price is not included in the world price of consumption goods (but rather in the price of imported production inputs). Alternatively, the world price of imported consumption goods could be adjusted.

Appendix 1: Derivation of the inflation equation

For purposes of discussion, we decompose the CPI into two main components: the index of prices for domestically-produced goods and services and the index of prices for imported goods and services:

(A.1) $p^c_t = w^f p^f_t + (1 - w^f)\, p^h_t$

where:

p^c – (log of) the CPI;

p^f – (log of) the price index for imported goods and services;

p^h – (log of) the price index for domestically-produced goods and services.

First-differencing yields the following equation:

(A.2) $\pi^c_t = w^f \pi^f_t + (1 - w^f)\, \pi^h_t$

where:

π^c – CPI inflation;

π^f – Inflation in the price index for imported goods and services;

π^h – Inflation in the price index for domestically-produced goods and services.

In what follows, we will deal with each component separately.

Domestic price equation (π^h)

It is assumed that there are a large number of firms operating in monopolistic competition. Each firm produces a good that is differentiated to some extent from the others and therefore the firm can influence the price of its own product. In addition, it is assumed that a producer incurs a cost in order to change its price and therefore price adjustment is not a continuous process.

The question of whether to change the price of the product and by how much is dependent on two considerations: the cost of the change and the distance from the desired price (in other words, the price that would be set if the process were costless).

The firm uses domestic inputs (labor and intermediate inputs) and imported inputs in the production process. The two types of inputs are not perfect substitutes and therefore the firm must decide on their appropriate combination. An increase in the price of imported inputs in the domestic market acts to increase production costs and therefore affects domestic prices, both directly and through the substitution effect which increases the price of domestic inputs.

The considerations discussed above lead to the following equation for the domestic inflation rate.

(A.3) $\pi^h{}_t = \pi^h{}_{t+1} + a_y y_t + a_{zf} (p^{zf}{}_t - p^h{}_t)$

where:

y_t – business sector output gap

$p^{zf}{}_t$ – prices of imported inputs.

This is the conventional equation used for the analysis of domestic prices in an open economy[85] and is an expansion of the commonly-used approach to a closed economy. The relative price of imported inputs is added in the case of an open economy.

The price ratio $(p^{zf}{}_t - p^h{}_t)$ can be written as a sum of two terms as follows:

(A.4) $(p^{zf}{}_t - p^h{}_t) = (p^{zf}{}_t - p^f{}_t) + (p^f{}_t - p^h{}_t)$

[85] For a detailed derivation, see, for example, the appendix in Svensson (1998).

This formulation has an advantage in simulations since the first term is exogenous[86] and the second is an important endogenous variable that also appears in the output gap equation. The price ratio ($p^f_t - p^h_t$) can also be written in terms of the CPI (p^c) (for which data is available) by using Equation (A.1) in the following manner:

(A.5) $(p^f_t - p^h_t) = p^f_t - (p^c_t - w^f p^f_t) / (1 - w^f) = (p^f_t - p^c_t) / (1 - w^f)$

By substituting (A.4) and (A.5) into (A.3), we obtain:

(A.6) $\pi^h_t = \pi^h_{t+1} + a_y y_t + a_{zf} [(p^{zf}_t - p^f_t) + (p^f_t - p^c_t) / (1 - w^f)]$

We now assume that some individuals form their expectations rationally while the rest form them adaptively. This means that instead of π^h_{t+1}, we substitute the term $a_{ld}\, \pi^h_{t+1} + (1 - a_{ld})\, \pi^h_{t-1}$. Since we found (in the estimation) that the lagged output gap also influences inflation, it was added in order to obtain the following equation for the rate of change in the domestic component of the CPI:[87]

(A.7) $\pi^h_t = a_{ld}\, \pi^h_{t+1} + (1 - a_{ld})\, \pi^h_{t-1} + a_y (0.5\, y_t + 0.5\, y_{t-1})$
$+ a_{zf} [(p^{zf}_t - p^f_t) + (p^f_t - p^c_t) / (1 - w^f)]$

Recall that both p^h and p^f are unobservable. We continue in two steps: first, we assume that p^f is observable. Thus, we solve for π^h_t in Equation (A.2), substitute it into (A.7) and, following some algebraic manipulations, obtain the equation in terms of overall CPI inflation:

[86] This is true if an immediate pass-through is assumed from the exchange rate and from world prices to prices in the domestic market, or if at least the same speed of pass-through is assumed for both components.

[87] Hereinafter the variable x_{t+i} denotes the rational expectations of variable x in period $t+i$ based on the information known up until (and including) period t.

(A.8) $\pi^c_t = a_{ld}\, \pi^c_{t+1} + (1 - a_{ld})\, \pi^c_{t-1} + (1 - w^f)\, a_y\, (0.5\, y_t + 0.5\, y_{t-1})$
$+ a_{zf}\, [(1 - w^f)\, (p^{zf}_t - p^f_t) + (p^f_t - p^c_t)]$
$+ w^f\, [\pi^f_t - a_{ld}\, \pi^f_{t+1} - (1 - a_{ld})\, \pi^f_{t-1}]$

The equation for import prices (π^f)

As mentioned earlier, we do not possess data on the prices paid by the consumer in the domestic market for imported consumer goods (p^f). However, we do have data on the dollar prices paid by importers. We denote this price as p^{*f} and denote the sheqel/dollar exchange rate as e (both variables are expressed in log form). If we assume immediate pass-through to domestic prices, we obtain $p^f_t = e_t + p^{*f}_t$. In general, it is reasonable to assume that the pass-through is not immediate, but rather is a gradual process. We assume, therefore, a gradual pass-through mechanism of the following form:

(A.9) $p^f_t = \alpha_1\, (e_{t+1} + p^{*f}_{t+1}) + \alpha_2\, (e_t + p^{*f}_t) + (1 - \alpha_1 - \alpha_2)\, (e_{t-1} + p^{*f}_{t-1})$

where:

p_t^{*f} – (log of) the world price of imported consumption goods (in dollar terms).

e_t – (log of) the nominal sheqel/dollar exchange rate.

First differencing this equation yields the inflation component related to imports:

(A.10) $\pi^f_t = \alpha_1\, (\Delta e_{t+1} + \Delta p^{*f}_{t+1}) + \alpha_2\, (\Delta e_t + \Delta p^{*f}_t) + (1- \alpha_1 - \alpha_2)\, (e_{t-1} + \Delta p^{*f}_{t-1})$

where:

Δ is the difference operator.

Derivation of the equation for CPI inflation (π^c)

In the last step, we substitute (A.10) into the second square brackets of Equation (A.8). Based on empirical considerations,[88] we impose an immediate pass-through mechanism on the price ratios ($p^{zf}_t - p^f_t$) and ($p^f_t - p^c_t$) or, in other words, we substitute $p^f_t = e_t + p^{*f}_t$ and $p^{zf}_t = e_t + p^{*zf}_t$ into the equation. Following these manipulations, we obtain the equation for CPI inflation in terms of observable variables:

$$\text{(A.11)} \quad \pi^c_t = a_{ld}\, \pi^c_{t+1} + (1 - a_{ld})\, \pi^c_{t\text{-}1} + (1 - w^f)\, a_y\, (0.5\, y_t + 0.5\, y_{t\text{-}1})$$
$$+ a_{zf}\, [(1 - w^f)\, up^{*z}_t + q_t]$$
$$+ w^f\, [\alpha_1\, dep^{*f}_{t+1} + \alpha_2\, dep^{*f}_t + (1 - \alpha_1 - \alpha_2)\, dep^{*f}_{t\text{-}1}]$$

where:

$$dep^{*f}_t = (\Delta e_t + \Delta p^{*f}_t) - a_{ld}\, (\Delta e_{t+1} + \Delta p^{*f}_{t+1}) - (1 - a_{ld})\, (\Delta e_{t\text{-}1} + \Delta p^{*f}_{t\text{-}1})$$

q_t – real exchange rate gap ($p^{*f}_t + e_t - p^c_t$);
up^{*z}_t – the gap of input prices ($p^{*zf}_t - p^{*f}_t$).

Appendix 2: Estimation of the Model's Equations

1. The inflation equation

The final equation estimated for the model is Equation (A.11) in Appendix 1. The equation was estimated using GMM. The results are presented in Table B.1 below:

[88] A pre-test showed that with regard to the price ratios above, there is no difference in the results between the assumptions of gradual and immediate pass-through.

Table B.1 – Estimation of the Inflation Equation (Equation A.11)

Parameter	Value	t-statistic	Parameter	Value	t-statistic
a_{ld}	0.527	12.3	w^f	0.445	5.9
a_y	0.063	1.9	α_1	0.203	4.8
a_{zf}	0.058	2.6	α_2	0.630	16.8
R^2 = 0.799 S.E. = 3.007 Jstat = 0.20 D.W. = 2.58 Sample = 1992Q1-2005Q3					

Comments:

a. The gap variables were estimated using HP filter. For details, see Appendix 4.

b. All the variables were multiplied by 4 in order to express them in annual terms.

c. The variables were seasonally adjusted. The equation was estimated with dummy variables for the seasonality of inflation (not reported).

d. In order to estimate the equation, we used the following instrumental variables (in addition to the constant and the dummy variables): two lags of inflation, the output gap, the nominal depreciation, the nominal interest rate and the real exchange rate gap; as well as the gap in the input price ratio, the change in the world price of imported production inputs, the change in the world price of imported consumption goods and the change in the price of world trade – all of which were included with their contemporaneous value and with two lags.

2. The output gap equation

$$(4.11)\quad y_t = b_{yld}\, y_{t+1} + (1 - b_{yld})\, y_{t-1} - b_r\, 0.25\, [0.5\,(r_t - r^n_t) + 0.5\,(r_{t-1} - r^n_{t-1})] + b_q\,(q_t - q_{t+1}) + b_{y*}\,(y^*_t - y^*_{t+1}) + b_{inv}\,(inv^h_t - inv^h_{t+1}) + b_g\,(g^h_t - g^h_{t+1})$$

We estimated the output gap equation using GMM. The results are presented in Table B.2 below:

Table B.2 – Estimation of the Output Gap Equation (Equation 4.11)

Parameter	Value	t-statistic	Parameter	Value	t-statistic
b_{yld}	0.802	21.1	b_{y*}	0.223	2.6
b_r	0.448	5.7	b_{inv}	0.121	6.1
b_q	0.238	6.3	b_g	0.221	8.8
R^2 = 0.642 S.E. = 2.318 Jstat = 0.249 D.W. = 2.51 Sample = 1992Q1-2005Q3					

Comments:

a. The gap variables were estimated using HP filter. For details, see Appendix 4.

b. The real interest rate spread was multiplied by 0.25 since the interest rate is expressed in annual terms, while the rest of the variables are expressed in quarterly terms.

c. All of the explanatory variables, apart from the real interest rate, were seasonally adjusted. The equation was estimated with seasonal dummy variables (not reported).

d. As an approximation for the natural interest rate, r^n_t, we used the implied 5 to 10 year forward yield to maturity of CPI-indexed "Galil" bonds.

e. As an approximation for expected inflation (which is part of the real interest rate), we used CPI inflation, excluding fruits and vegetables.

f. In addition to the constant and seasonal dummy variables, we used the following instrumental variables: two lags of the output gap, the real exchange rate gap, CPI inflation excluding fruits and vegetables, the investment gap, nominal depreciation and the nominal interest rate; as well as the world trade gap, the change in the world price of imported consumption goods, the gap of public sector procurement, the implied 5 to 10 year forward real yield, the change in world prices of imported production inputs, the dollar interest rate and inflation in the US – which were included with their contemporaneous value and with two lags.

3. The exchange rate equation

In order to estimate the exchange rate equation (4.14), we assumed a constant risk premium:

(B.1) $e_t = c_{ld}\, 0.25\, rp + c_{ld}\, e_{t+1} + (1 - c_{ld})\, e_{t-1}$
$+ 0.25\, (i^*_t - i_t) - (1 - c_{ld})\, 0.25\, (i^*_{t-1} - i_{t-1})$

This equation was estimated using GMM. The results of the estimation are presented in Table B.3 below:

Table B.3 – Estimation of the Exchange Rate Equation (Equation B.1)

Parameter	Value	t-statistic	Parameter	Value	t-statistic
rp	5.845	5.47	c_{ld}	0.452	11.82
$R^2 = 0.453$ S.E. = 2.488 Jstat = 0.240 D.W. = 2.84 Sample = 1997Q3-2005Q4					

Comments:

a. The nominal interest rate differential and the parameter which captures the average risk premium were multiplied by 0.25 in order to express the whole equation in quarterly terms.

b. For purposes of estimation, e_{t-1} was moved to the left-hand side of the equation so that the dependent variable became the quarterly change in the nominal exchange rate.

c. For purposes of estimation, we used the following instrumental variables (in addition to the constant): four lags of the exchange rate, the nominal interest rate, the dollar interest rate and CPI inflation.

4. The interest rate equation

The interest rate equation estimated for the simulations is as follows:

(7.13) $i_t = (1 - d_{lag})\, \{r^n_t + \pi_t^{target} + d_\pi\, (\pi 4_t - \pi_t^{target}) + d_y\, 0.5\, (y_t + y_{t-1})\}$
$+ d_{lag}\, i_{t-1}$

where: $\pi 4_t = (\pi^c_t + \pi^c_{t-1} + \pi^c_{t-2} + \pi^c_{t-3})$

This equation was estimated using GMM. The estimation results are presented in Table B.4 below:

Table B.4 – Estimation of the Interest Rate Equation (Equation 7.13)

Parameter	Value	t-statistic	Parameter	Value	t-statistic
d_π	1.706	3.2	d_{lag}	0.794	8.2
d_y	0.359	8.2			
$R^2 = 0.956$ S.E. = 0.962 Jstat = 0.198 D.W. = 1.87 Sample = 1994Q3-2005Q3					

Comment:

We used the following instrumental variables: a constant, the implied forward real yield for 5 to 10 years, the inflation target, two lags of the output gap, the nominal interest rate and the exchange rate. In addition, we used the change in the world price of imported consumption goods and the change in the price of world trade (contemporaneously and with two lags).

When ex-post inflation ($\pi 4_t$) is replaced by market-based expectations, the following estimates are obtained:

Table B.5 – Estimation of the Interest Rate Equation (Equation 7.13) with Market-Based Expectations

Parameter	Value	t-statistic	Parameter	Value	t-statistic
d_π	2.039	9.9	d_{lag}	0.726	13.5
d_y	0.125	1.6			
$R^2 = 0.949$ S.E. = 1.036 Jstat = 0.223 D.W. = 1.78 Sample = 1994Q3-2005Q3					

Appendix 3: The principles of dynamic simulations Using a rational expectations model

In this appendix, we will describe the methodology employed in the dynamic simulation in Section 7.1. For purposes of illustration and without loss of generality, we will explain the methodology using a very simple model. In this model, we assume that there are two variables – one endogenous and one exogenous. The endogenous variable is domestic CPI inflation (π_t), while the exogenous variable is world inflation (π^*_t). The inflation equation is thus as follows:

(C.1) $$\pi_t = a \cdot E_t \pi_{t+1} + b \cdot \pi_{t-1} + c \cdot \pi_t^* + \varepsilon_t^\pi$$

In other words, inflation depends on its expected value in the next period,[89] on inflation in the previous period and on world inflation, in addition to a structural shock to inflation (ε_t^π). We do not claim any particular economic meaning for this equation – it is simply being used for purposes of demonstration.

If a is equal to zero so that expectations do not appear in the equation, then the method for carrying out a within-sample dynamic simulation would be straightforward: given π_0 and π^*_t for each t=1…T, inflation derived from the dynamic simulation ($\hat{\pi}_t$) is a function of the inflation derived for the previous quarter, as well as contemporaneous world inflation:

(C.2) $$\hat{\pi}_t = b \cdot \hat{\pi}_{t-1} + c \cdot \pi_t^*$$

However, when the model does include expectations (a>0), particularly rational expectations, a problem arises since expectations are not measurable.

[89] $E_t x_{t+i}$ is the expectation of variable x in period $t+i$ based on the information known up until period t.

One possible solution is to carry out a deterministic simulation in which the actual values of world inflation are perfectly foresighted. In this case, we solve the following T equations for $\hat{\pi}_1 ... \hat{\pi}_t ... \hat{\pi}_T$:

$$\hat{\pi}_1 = a \cdot \hat{\pi}_2 + b \cdot \pi_0 + c \cdot \pi_1^*$$
$$\hat{\pi}_2 = a \cdot \hat{\pi}_3 + b \cdot \hat{\pi}_1 + c \cdot \pi_2^*$$
$$\vdots$$
$$\hat{\pi}_{T-1} = a \cdot \hat{\pi}_T + b \cdot \hat{\pi}_{T-2} + c \cdot \pi_{T-1}^*$$
$$\hat{\pi}_T = a \cdot \overline{\pi} + b \cdot \hat{\pi}_{T-1} + c \cdot \pi_T^*$$

Given inflation at the initial state (π_0) and long-run inflation ($\overline{\pi}$), the equations have a unique solution.[90] This simulation assumes that the entire path of the exogenous variable (π^*) is known ahead of time and therefore it is accounted for by the expectations of inflation already from the first quarter of the simulation. However, this is a problematic assumption. In terms of our model, we cannot assume that the exogenous shocks to the price of oil in 2005 were implicit in the expectations of inflation already in 1999.

The obvious solution is to carry out a non-deterministic simulation in which the exogenous variables are not known with certainty ahead of time. In order to do so, the rational expectations must be solved for. Namely, the reduced form of the model must be used, in which each variable is dependent only on the model's lagged variables and on contemporaneous shocks (ε). The transformation to the reduced form requires an equation for each variable in the model, including the exogenous ones.

We specify an autoregressive equation of the following form for the exogenous variable (π^*_t):

(C.3) $\pi_t^* = d \cdot \pi_{t-1}^* + \varepsilon_t^{\pi^*}$

[90] Alternatively, instead of $\overline{\pi}$, actual inflation at T+1 can be used on the condition that it is known.

The purpose of this equation, which can be estimated using standard methods, is to calculate the expectations for the exogenous variable in future periods. Thus, for example, the expectation of the exogenous variable in the subsequent period is as follows:

$$E_t\pi^*_{t+1} = E_t(d \cdot \pi^*_t + \varepsilon^{\pi^*}_{t+1}) = d \cdot \pi^*_{\mathrm{t}}$$

Equations (C.1) and (C.3) make it possible to solve for the rational expectations and to express the model in its reduced form, where the scalars *A*, *B*, *C* and *D* are functions of the structural parameters *a*, *b*, *c* and *d*:

(C.4) $\pi_t = A\pi_{t-1} + B\pi^*_{t\text{-}1} + C\varepsilon^{\pi}_t + D\varepsilon^{\pi^*}_t$

(C.5) $\pi^*_t = d \cdot \pi^*_{t\text{-}1} + \varepsilon^{\pi^*}_t$

The methodology for solving a model with rational expectations and the transformation to the reduced form is presented in Blanchard and Kahn (1980).

We are now interested in carrying out a simulation of the endogenous variables, given the actual evolution of the exogenous variables. Therefore, in the first step, we use data on domestic inflation (π_t) and world inflation (${\pi^*}_t$) in order to derive the in-sample structural shocks (ε^{π}_t) and ($\varepsilon^{\pi^*}_t$) from equations (C.4) and (C.5). In the second step, we carry out a dynamic simulation of the reduced form using the shocks from the equations of the exogenous variables only. This means that for each period t=1...T, we derive ($\hat{\pi}_t$) and ($\hat{\pi}^*_t$) as follows:

(C.6) $\hat{\pi}_t = A\hat{\pi}_{t-1} + B\hat{\pi}^*_{t\text{-}1} + 0 \cdot \varepsilon^{\pi}_t + D\varepsilon^{\pi^*}_{\mathrm{t}}$

(C.7) $\hat{\pi}^*_t = d \cdot \hat{\pi}^*_{t\text{-}1} + \varepsilon^{\pi^*}_t$

Notice that the derived exogenous variable is identical to the actual one ($\hat{\pi}_t^* = \pi_t^*$) so that the system can also be written in the following form:

(C.8) $\hat{\pi}_t = A\hat{\pi}_{t-1} + B\pi_{t\text{-}1}^* + 0 \cdot \varepsilon_t^{\pi} + D\varepsilon_t^{\pi^*}$

(C.9) $\pi_t^* = d \cdot \pi_{t\text{-}1}^* + \varepsilon_t^{\pi^*}$

In the dynamic simulation, inflation in each period is influenced by the actual exogenous variable. Furthermore, inflation is dependent on expected inflation (where the future shocks are unknown). If we add the structural shock for inflation (ε_t^{π}), then the dynamic simulation will exactly produce actual inflation. Therefore, the comparison of the results of the above simulation (without ε_t^{π}) to actual inflation indicates the share of the unexplained component in the inflation equation (C.1). The closer that simulation-derived inflation is to actual inflation, particularly if it does not diverge from it, the more important is the role of the explanatory variables (lagged inflation, expected inflation and world inflation) in explaining inflation.

Appendix 4: Description of the data

In estimating the model we used three types of data: gaps, rates of change and interest rates. This appendix will describe each type and its source. In general, rates of change and interest rates are annualized, while the gaps are expressed in ordinary quarterly terms. This is the case for estimation, simulation, calculation of the impulse response functions and the testing of goodness-of-fit. An exception is the presentation of estimates for the inflation equation

(4.4), in which the reported parameters relate to an estimation in which all the variables are expressed in annual terms (in other words, the output gap and the relative price gap are multiplied by 4).

1. Gaps

All the gaps were estimated using the HP filter. In what follows, we denote the trend of series X calculated using the HP filter as HP(X). The gap of the original series Z (for example, business output) is denoted by z (the output gap) and is estimated as follows:

(D.1) $$z = \left[\log(Z) - \mathrm{HP}(\log(Z))\right] * 100$$

The term in square brackets is the rate of deviation of Z from its trend. Multiplying by 100 expresses the gap in percent.[91]

Following is a description of the gaps included in the model:

y – the output gap; based on business sector output in constant prices (Y). The trend was estimated using data starting from 1968.1.

g^h – the public consumption gap; based on the procurement of the public sector in constant prices (G^h). The trend was estimated using data starting from 1964.1.

inv^h – the investment gap; based on gross investment in constant prices (INV^h). The trend was estimated using data starting from 1964.1.

y^* – the world trade gap; based on the imports of the industrialized countries in constant prices (Y^*). The trend was estimated using data starting in 1960.1. The source of the data is the IFS database.

q – the real exchange rate gap; based on the real exchange rate (Q) which is calculated as follows:

[91] In estimating the inflation equation, this term was multiplied by 4 in order to express it in annual terms.

(D.2) $Q = \dfrac{P^{*f} \cdot E}{P^{c}}$

where P^c is the consumer price index (CPI), E is the sheqel/dollar nominal exchange rate and P^{*f} is the Fisher price index for imported consumption goods (in dollar terms). The trend was estimated using data starting from 1991.1.

up^{*zf} – the gap of the world price of imported inputs relative to the price of imported consumption goods; based on the price ratio calculated as follows:

(D.3) $UP^{*zf} = \dfrac{P^{*zf}}{P^{*f}}$

where P^{*zf} is the weighted average of the Fisher price index for imported production inputs and investment goods (in dollar terms). The weights are based on the relative weights of imported production inputs and investment goods. (The weight of production inputs is 0.75.) P^{*f} is the Fisher price index for imported consumption goods (in dollar terms). The trend was estimated using data starting from 1991.1.

2. Rates of change

The percentage rate of change in variable X, denoted by Δx (or π), is calculated as follows:

(D.4) $\Delta x = (x - x_{-1}) * 100 * 4$

where:

(D.5) $x = \log(X)$

Multiplying by 100 transforms the rate of change into percent and multiplying by 4 expresses the change in annual terms.

Following is a description of the rate-of-change variables included in the model:

π^c – CPI inflation; based on the consumer price index.

Δe – the nominal depreciation; based on the nominal sheqel/dollar exchange rate.

Δp^{*f} – rate of change in the world price of imported consumption goods; based on the Fisher price index for imported consumption goods (in dollar terms).

Δp^{y*} - rate of change in the price of world trade (serves only as an instrumental variable); based on the price index for the imports of the industrialized countries. Source: the IFS database.

π^{*us} – inflation in the US (serves only as an instrumental variable); based on the US CPI. Source: the IFS database.

3. Interest rates

The interest rates appear in annual terms. i is the effective Bank of Israel interest rate and i^* is the LIBID monthly dollar interest rate. i^{*us} is the interest rate in the US and is used only as an instrumental variable. The average forward real yield to maturity for 5 to 10 years, denoted in the equations as r^n, is a proxy for the equilibrium real interest rate. It is derived from the trade in CPI-indexed government bonds ("Galil").

The expectations variables also require explanation. Whenever a variable appears in the estimated equation with a subscript t+1 or t+2, which is meant to represent the rational expectations of the variable based on the information available up until period t, we used the actual realization of the variable in period t+1 or t+2, respectively. An exception is expected inflation as it appears in the output gap equation (as part of the real interest rate) where we used the actual period $t+1$ CPI inflation *excluding fruits and vegetables,* since it is freer of unexpected shocks that are not known in period t.

References

Adolfson, M. (2001). *Monetary policy with incomplete exchange rate pass-through*, Sveriges Riksbank Working Paper No. 127.

Adolfson, M. (2002). *Incomplete exchange rate pass-through and simple monetary policy rules*, Sveriges Riksbank Working Paper No. 136.

Argov, E. and D. Elkayam (2006). *An Estimated New Keynesian Model for a Small Open Economy: an Application for Israel*, Bank of Israel, Monetary Department Discussion Paper 2006.2. (Hebrew).

Batini, N. and A. Haldane (1999). "Forward-looking rules for monetary policy", In: Taylor, J.B. (ed.), *Monetary Policy Rules*, The University of Chicago Press, Chapter 4, pp. 157-201.

Batini, N., R. Harrison and S.P. Millard (2003). "Monetary policy rules for an open economy", *Journal of Economic Dynamics and Control* 27 (11-12), 2059-2094.

Blanchard, O. and C. Kahn (1980). "The Solution of Linear Difference Models under Rational Expectations", *Econometrica* 48, No.5, 1305-1312.

Blanchard, O. (2004). *Fiscal dominance and inflation targeting: lessons from Brazil*, NBER Working paper No. 10389 (March).

Calvo, G.A. (1983). "Staggered pricing in a utility-maximizing framework", *Journal of Monetary Economics* 12, 383-398.

Caputo, R. (2004). *External shocks and monetary policy: does it pay to respond to exchange rate deviations?*, Central Bank of Chile.

Christiano J., M. Eichenbaum and C. Evans (2005). "Nominal rigidities and the dynamic effects of a shock to monetary policy", *Journal of Political Economy* 113 (1), 1-45.

Clarida, R., J. Gali and M. Gertler (1999). "The science of monetary policy a New Keynesian perspective", *Journal of Economic Literature* 37, 1661-1707.

Clarida R., J. Gali and M. Gertler (2001). "Optimal monetary policy in open versus closed economies: an integrated approach", *American Economic Review* 91 (2), 248-252.

Clarida R., J. Gali and M. Gertler (2002). "A simple framework for international monetary policy analysis", *Journal of Monetary Economics* 49, 879-904.

Corsetti, G. and P. Pesenti (2005). "International dimensions of optimal monetary policy", *Journal of Monetary Economics* 52, 281-305.

Elkayam, D. (2001). *A model for monetary policy under inflation targeting: The case of Israel*, Bank of Israel, Monetary Department Discussion Paper 2001.03.

Epstein, N., P. Karam, D. Laxton and D. Rose (2006). "A simple forecasting and policy analysis system for Israel: structure and applications", In: Haas, R. (ed.), *Israel: Selected Issues*, IMF Country Report No. 06/121.

Fuhrer, J.C. (2000). "Habit formation in consumption and its implications for monetary-policy models", *American Economic Review* 90 (3), 367-390.

Fuhrer, J.C. and G.R. Moore (1995). "Inflation persistence", *Quarterly Journal of Economics* 110, 127-159.

Gali, J. and T. Monacelli (2005). "Monetary policy and exchange rate volatility in a small open economy" *Review of Economic Studies* 72 (3), 707-734.

Gali, J. and M. Gertler (1999). "Inflation dynamics: a structural econometric analysis", *Journal of Monetary Economics* 44, 195-222.

Ghironi, F. and A. Rebucci (2002). *Monetary rules for emerging market economies*, IMF Working Paper wp/02/34.

Guender, A. (2003). *Optimal discretionary monetary policy in the open economy: Choosing between CPI and domestic inflation as target variables*, Bank of Finland Discussion Papers 12/2003.

Hecht, Y. and H. Pompushko (2006). *Rnd*, Bank of Israel, Monetary Department Discussion Paper 2006.01. (Hebrew).

Husebo, T.A., S. McCaw, K. Olsen and O. Roisland (2004). *A small calibrated macromodel to support inflation targeting at Norges Bank*, Norges Bank Staff Memo 3/2004.

Ilek, Alex (2007). *Aggregation versus disaggregation – what can we learn from it?*, Bank of Israel, Monetary Department Discussion Paper 2007.02.

Leitemo, K. and Ø. Røisland (2000). *The choice of monetary policy regime for small open economies*, Norges Bank Working Paper No. 5/2000.

Leitemo, K. and U. Söderström (2005a). *Robust monetary policy in a small open economy*, CEPR Discussion Paper 5071.

Leitemo, K. and U. Söderström (2005b). "Simple monetary policy rules and exchange rate uncertainty", *Journal of International Money and Finance* 24 (3), 481-507.

Leitemo, K. (2006a). "Open-economy inflation forecast targeting", *German Economic Review* 7 (1), 35-64.

Leitemo, K. (2006b). "Targeting inflation by forecast feedback rules in small open economies", *Journal of Economic Dynamics and Control* 30 (3), 393-413.

Levin, A., V. Wieland and J. Williams (2003). "The performance of forecast-based monetary policy rules under model uncertainty", *American Economic Review* 93 (3), 622-645.

Levine, P., P. McAdam and J. Pearlman (2006). *Inflation forecast-based-rules and indeterminacy, a puzzle and a resolution*, European Central Bank Working Paper No. 643.

Linde, J., M. Nessen and U. Söderström (2004). *Monetary policy in an estimated open-economy model with imperfect pass-through*, Sveriges Riksbank Working Paper No. 167.

Liviatan, N. (2003). *Fiscal dominance and monetary policy dominance in the Israeli monetary experience*, Bank of Israel, Research Department Discussion paper 03.17 (November).

Lopez, P. (2003). *Efficient policy rule for Inflation targeting in Columbia*, Central bank of Columbia.

McCallum, B.T. and E. Nelson (1999). "Nominal income targeting in an open-economy optimizing model", *Journal of Monetary Economics* 43 (3), 553-578.

McCallum, B.T. and E. Nelson (2000). "Monetary policy for an open economy: an alternative framework with optimizing agents and sticky prices", *Oxford Review of Economic Policy* 16 (4), 74-91.

Monacelli, T. (2005). "Monetary policy in a low pass-through environment", *Journal of Money, Credit and Banking* 37 (6), 1047-1066.

Parrado, E. (2004). *Inflation targeting and exchange rate rules in an open economy*, IMF working paper wp/04/21.

Rotemberg, J. and M. Woodford (1998). *An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version*, NBER Technical Working Paper 233.

Smets, F. and R. Wouters (2002). "Openness, imperfect exchange rate pass-through and monetary policy", *Journal of Monetary Economics* 49, 947-981.

Smets, F. and R. Wouters (2003). "An estimated stochastic dynamic general equilibrium model of the euro area", *Journal of the European Economic Association* 1 (5), 1123-1175.

Stein, R. (2003). *Estimation of Expected Exchange-Rate Change Using Forward Call Options*, Bank of Israel, Monetary Department Discussion Paper 2003.2.

Svensson, L.E.O. (1999). *Open-economy inflation targeting*, NBER Working Paper 6545 (May).

Svensson, L.E.O. (2000). "Open-economy inflation targeting", *Journal of International Economics* 50, 155-183.

Svensson, L.E.O. (2003). "What is wrong with Taylor rules? Using judgment in monetary policy through targeting rules", *Journal of Economic Literature* 41, 426-477.

Taylor, J.B. (1993). "Discretion versus policy rules in practice", *Carnegie-Rochester conference series on public policy* 39, 195-214.

Walsh, C. (2003). *Monetary theory and policy*, MIT Press, Cambridge MA, Second edition.

Woodford, M. (2003). *Interest and Prices Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton, NJ.

Woodford, M. (2006). How important is money in the conduct of monetary policy? Presented at the ECB central banking conference: *The role of money: money and monetary policy in the twenty-first century*, (November).