# A comparison of public and private schools in Spain using robust nonparametric frontier methods

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## Abstract

This paper uses an innovative approach to evaluate educational performance of Spanish students in PISA 2009. Our purpose is to decompose their overall inefficiency between different components with a special focus on studying the differences between public and state subsidized private schools. We use a technique inspired by the non-parametric Free Disposal Hull (FDH) and the application of robust order-*m* models, which allow us to mitigate the influence of outliers and the *curse of dimensionality*. Subsequently, we adopt a metafrontier framework to assess each student relative to the own group best practice frontier (students in the same school) and to different frontiers constructed from the best practices of different types of schools. The results show that state-subsidised private schools outperform public schools, although the differences between them are significantly reduced once we control for the type of students enrolled in both type of centres.

*Key words: Education, Efficiency, Multilevel Modelling, Free Disposal Hull.*

*JEL codes: C14, H41, I21*

## 1. INTRODUCTION

Since the pioneer study of Coleman *et al.* (1982), the debate about the performance of private and public schools has become one of the main topics of research in a wide range of educational contexts (Rouse and Barrow, 2009). In general terms, it is widely assumed that private schools are likely to perform better than public schools because market competition forces them to achieve a more efficient use of resources (Friedman and Friedman, 1981; Chubb and Moe, 1990; Hoxby 2003). However, empirical studies comparing both, public and private schools, need to control for differences in the personal and socio-economic background of students as well as the potential self-selection bias that can arise because more informed and motivated parents are more likely to apply to better schools (Mayston, 2003; Tamm, 2008; Burgess and Briggs, 2010).

The conclusions reached in the vast literature devoted to this issue are mixed. Some studies find that private schools do better, even after controlling for the aforementioned factors (Jiménez *et al.*, 1991; Toma, 1996; Altonji *et al.*, 2005; Dronkers and Roberts, 2008; Annand *et al.*, 2009; Dronkers and Avram, 2010; Kim, 2011), although those differences are reduced or disappear when those variables are taken into account (Williams and Carpenter, 1991; Goldhaber, 1996; Sander, 1996; McEwan and Carnoy, 2000; McEwan, 2001; Hsieh and Urquiola, 2006; Chudgar and Quin, 2012) or even public schools can outperform private ones (Bifulco and Ladd, 2006; Newhouse and Beegle, 2006).

This paper contributes to the above literature by applying a new method to estimate the differences in efficiency between public and private schools. In this sense, it must be noted that the educational system in Spain represents a relevant case study, since two types of schools compete for public funds: public and state-subsidized private schools[1]. The former are managed by public authorities while the latter are owned and managed directly or indirectly by a private non-government organization (mainly Catholic entities)[2]. This scheme aims at allowing parents to freely design their preferred school and, indirectly, stimulating competition among schools to

[1] There are also private government-independent schools, controlled by non-government organizations, which are mainly funded through student fees. However, in this paper, we focus only on the publicly financed schools.

[2] According to the regulation, these institutions can only benefit from government subsidies if they fulfill some requirements, such as providing education free of charge, maintaining a certain rate of pupil-teacher ratio, teaching the official curriculum and not allowing any type of discrimination among students in their admission processes. See Mancebon and Muñiz (2008) or Mancebon *et al.* (2012) for details.

improve their performance. In this context, the comparison between their levels of efficiency becomes extremely attractive.

In fact, the recent literature provides some empirical studies focused on this comparison using Spanish data with different methodological approaches, although the findings are still inconclusive. Hence, Mancebon and Muñiz (2008) do not find significant differences after using an extension of Silva Portela and Thanassoulis (2001) proposal. The same conclusion is reached by Calero and Escardibul (2007) using multilevel analysis and Perelman and Santin (2011) using parametric stochastic distance functions. Mancebon *et al*. (2012) obtain even better results for public schools combining the use of multilevel analysis with the same extension of DEA. In contrast, Crespo *et al*. (2013) conclude that, after applying a propensity score matching technique to correct the potential bias, students attending state-subsidized private schools perform significantly better than students from public schools.

In this paper, we combine the application of two recently developed nonparametric methods to estimate the efficiency of both types of schools. Firstly, we use the order-*m* partial frontiers approach (Cazals *et al*., 2002) in order to avoid some of the main drawbacks of the nonparametric methods, such as the high impact of atypical observations or the bias that can arise when the evaluated units (students) are grouped into groups (schools) of different size (Zhang and Bartels, 1998). This approach consists of using only part of the sample (*m* observations) to determine efficiency scores, thus it mitigates the impact of outliers and potential errors in data and assures the same size for the reference set, avoiding the *curse of dimensionality* that systematically pursues the traditional nonparametric estimations (Daraio and Simar, 2007). Secondly, in order to assess the performance of both types of schools we adopt the metafrontier framework, developed by Battese and Rao (2002), Battese *et al*. (2004) and O´Donnell *et al*. (2008). This method allows us to assess each student relative to their own group (meaning, students attending the same type of schools) and, secondly, to the overall metafrontier, constructed from the best practices of both types of schools.

De Witte *et al.* (2010) were pioneers in using those methods to assess the performance of a sample of British secondary schools, although they only evaluate public centres. Cherchye *et al*. (2010) also used a robust nonparametric approach to assess educational efficiency of Flemish pupils attending public and private primary schools, although their comparison between different types of schools is based on stochastic dominance criteria. De Witte and Kortelainen (2013) use the partial order-*m* approach to estimate the efficiency of Dutch pupils in PISA, but their focus is placed on the identification of exogenous variables affecting the performance of students and not on comparing public and private schools. Finally, Thieme *et al.* (2013) represents the only previous study in which both approaches employed in this paper are combined to assess the performance of students in primary education in Chile, although they do not consider the managerial decomposition between public and private schools. Therefore, this paper represents the first combined application of both methods using data from secondary schools. In particular, we analyse the performance of Spanish students in PISA 2009, which provides a wide volume of data regarding multiple factors that can affect the performance at student and school level.

One of the main advantages of this paper is the possibility of working with student level data, which facilitates the interpretation of the results and assist in the estimation of the multiple factors affecting the performance of students (Summers and Wolf, 1977; Hanushek *et al.*, 1996). Furthermore, the measurement of efficiency at student level allows considering separately student's own socioeconomic background and their schoolmates´ one (the so-called *peer-group effect*), two inputs which cannot be simultaneously included with aggregated data (Santín, 2006).

The rest of the paper is organized as follows. Section 2 presents the methods used to estimate students´ efficiency and separate the school effect. Section 3 describes the main characteristics of the dataset and the criteria followed to select the variables included in the analysis. Section 4 discusses the main results. Finally, section 5 provides concluding remarks.

## 2. METHODOLOGY

### *2.1. The deterministic model*

The definition of the production technology that a student uses to acquire knowledge is a very difficult task. The only thing that we know is that pupils transform a set of inputs $x(x \in \Re_+^q)$ such as their own capabilities or their parental background into heterogeneous outputs $y(y \in \Re_+^q)$, usually represented by their results in standardized test scores. This can be represented by equation (1):

$$\psi = \{(x, y) \in \Re_+^{p+q} \mid x \text{ can produce } y \} \quad (1)$$

Given that the production set cannot be observed, some assumptions are required such as the free disposability of inputs and outputs and the feasibility of all the combinations of those variables. In order to estimate the relative efficiency of each student, we need to constitute a frontier that represents the best performing students. This boundary set is characterized by the following expression:

$$\theta\psi = \{(x, y) \in \psi \mid (\theta x, y) \notin \psi, \forall 0 < \theta < 1, (x, \lambda y) \notin \psi, \forall \lambda > 1 \} \tag{2}$$

According to this definition, the efficient students will be part of the frontier, while the inefficiency of those that do not belong to the frontier can be measured using equation (3) for input orientation or equation (4) for output orientation. However, in this paper we will focus on the latter option, since in the educational context the goal of the pupil is to achieve the best feasible results.

$$\theta(x, y) = \inf\{\theta \mid (\theta x, y) \in \psi\} \tag{3}$$

$$\theta(x, y) = \sup\{\lambda \mid (x, \lambda y) \in \psi\} \tag{4}$$

A procedure to measure the relative inefficiency scores $\theta$ and $\lambda$ is represented by nonparametric techniques, represented by Data Envelopment Analysis –DEA– (Charnes *et al*., 1978) and Free Disposal Hull –FDH– (Deprins *et al*., 1984). This approach is based on mathematical programming and does not require the imposition of a determined form on the production function. Both DEA and FDH estimate the technology set $\psi$ by the smallest set $\hat{\psi}$ that envelops the observed data, but FDH differs from DEA in its removal of the convexity assumption:

$$\hat{\psi}_{FDH} = \{(x, y) \in \Re_{+}^{p+q} \mid y \leq y_i ; x \geq x_i ; i = 1, \ldots, n \} \tag{5}$$

In practical terms, this implies that each unit (student) is compared only to other existing unit (student), and that it cannot be evaluated against any convex combinations of efficient units. As a result, the FDH frontier can be considered even more flexible than DEA, since there are even fewer required assumptions.

Although DEA is more popular among researchers in the field of education, in our study we opt for using FDH because it has higher flexibility, it has comparatively superior asymptotic properties (Park *et al*., 2000; Simar and Wilson, 2000) and it ensures that all reference units are real[3]. The output oriented efficiency score ( $\hat{\theta}_{FDH}$ ) of an observation can be obtained by solving the mixed integer linear programming problem in equation (6):

$$\hat{\theta}_{FDH} = \max\left\{\lambda \middle| \lambda y \leq \sum_{i=1}^{N} \gamma_i y_i ; x \geq \sum_{i=1}^{N} \gamma_i x_i ; \sum_{i=1}^{N} \gamma_i = 1; \gamma_i \in \{0,1\}; i = 1,....,n \right\} \tag{6}$$

where $\hat{\theta}_{FDH} = 1$ denotes an efficient pupil, while $\hat{\theta}_{FDH} > 1$ implies that the pupil is inefficient. However, this nonparametric approach, as well as DEA, presents some significant shortcomings that should be born in mind when using nonparametric methods to assess efficiency at student level: (1) statistical inference is not possible due to its deterministic nature; (2) it is very sensitive to the presence of outliers and measurement errors in data; (3) it experiences dimensionality problems due to their slow convergence rates. In the next sections, we explain some approaches that can be used in order to overcome these limitations.

### *2.2. The robust approach*

The first attempts to improve the robustness of nonparametric methods were the works of Kneipp *et al.* (1998) and Simar and Wilson (2000). Subsequently, Cazals *et al*. (2002) introduced the robust order-*m* estimation. This approach is related to the FDH estimator, but instead of constructing a full frontier, it creates a partial frontier that envelops only *m* (≥1) observations randomly drawn from the sample. This procedure is repeated *B* times resulting in multiple measures ( $\hat{\theta}_{mi}^{1},..., \hat{\theta}_{mi}^{B}$ ) from which the final order-*m* efficiency measure is computed as the simple mean ( $\hat{\theta}_{mi}$ ). Specifically, the order-*m* efficiency score is derived from equation (7):

$$\theta_m = E\left[ \min_{i=1,..,m} \left\{ \max_{j=1,...,p} \left( \frac{x_i^j}{x^j} \right) \right\} \middle| y_i \geq y \right] \tag{7}$$

where the $\rho$-dimensional random variables $x_i,\ldots,x_m$ are drawn randomly and repeatedly from the conditional distribution of X given $y_i \geq y$. This estimator allows us to compare the efficiency of an observation with that of *m* potential units that have a production larger or equal to *y*. As it does not include all the observations, it is less sensitive to outliers, extreme values or noise in

[3] Oliveira and Santos (2005) also use this approach to assess efficiency in the educational context.

the data. As $m$ increases, the expected order-$m$ estimator tends to the FDH efficiency score ( $\hat{\theta}_{FDH}$ ). For acceptable $m$ values, normally the efficiency scores will present values higher than unity, which indicates that students are inefficient, as outputs can be increased without modifying the level of inputs. When $\hat{\theta} < 1$, the student can be labelled as super-efficient, since the order-$m$ frontier exhibits lower levels of outputs than the student under analysis. This is not possible in the traditional nonparametric framework where by construction $\hat{\theta} \geq 1$.

Moreover, this approach allows us to avoid the problem of bias that can arise when we compare groups of units on a different size, which is the case in our application with schools, since the mean level of efficiency generally depends on the existing observations in each school (Zhang and Bartels, 1998). This problem can be reduced by using the same $m$ parameter for every school, which implies assuming that the performance of every student is compared to the same number of students independently of the number of students in his/her school. In our case, we determine the value of $m$ that equals the size of the smallest school in the data set (20), since it fits better in the metafrontier framework (see below). The main advantage of a lower trimming value $m$ is the reduced sensibility to outlying observations in the sample, although it also implies that the probability of drawing the evaluated observation is rather low and, consequently, we will observe more super-efficient observations.

### *2.3. The metafrontier approach*

Independently of the method employed to estimate the efficiency coefficients, we need to bear in mind that part of those estimates derives from the environmental situation of the school they attend. Therefore, results obtained with frontier models need to control for this heterogeneity in order to give significance to the results.

For that purpose, we adapt the metafrontier approach developed by Battese and Rao (2002), Battese *et al*. (2004) and O`Donnell *et al.* (2008) to deal with a hierarchical structure in data, which is typical in the educational context, where students (level 1) are nested within schools (level 2). This approach measures the efficiency of units relative to separate best practice frontiers and allows us to decompose the performance of each student into a part attributable to the school and a part attributable to his/her skills. If there are $K$ schools, each having their own technology and environmental factors, a metafrontier is defined as the boundary of the unrestricted technology set. Hence, the metafrontier envelops each of the separate group frontiers. Separetely, the local efficiency of the student with regards to the type of school he is involved in is measured relative to the $n_k$ observations in the school sample:

$$\theta^k(x_k, y_k) = \inf\left\{\theta^k \middle| (\theta^k x_k, y_k) \in \psi^k\right\} \tag{8}$$

where the technology set for group $k$ is defined as

$$\psi^k = \left\{(x_k, y_k) \in \Re_+^{p+q} \;\middle|\; x_k \text{ can produce } y_k\right\} \tag{9}$$

If all the schools have potential access to the same environment, all the observations can be pooled and students can be evaluated relative to the same standards. Thus, the metafrontier can be represented by the technology set defined by:

$$\psi = \left\{(x, y) \in \Re_+^{p+q} \;\middle|\; x \text{ can produce } y\right\} \tag{10}$$

This approach is basically an extension of the proposal by Silva Portela and Thanassoulis (2001) and Thanassoulis and Silva Portela (2002) to decompose the effect of school from students´ inefficiency as well as to distinguish between the types of funding regime under which the school operates. Given the purpose of this paper, we are more interested in the latter issue, although we will take into account both aspects in the metafrontier framework. Thus, in a first step two different types of frontiers are defined: the local frontier specific for each school, which can be interpreted as the student-within-school efficiency and the overall frontier, which represents the student-within-all-schools efficiency. According to this definition, the distance to the local frontier depends only on the student´s efficiency (*STE*) whereas the distance separating the local and the overall frontier can be interpreted as the school efficiency (*SCE*). This can be illustrated in Figure 1, where the efficiency level of a student $c$ depends on the level of the output achieved ($y_c$) using his input endowment ($x_c$). This student is inefficient, since there are students in the same school obtaining better results ($y´$) with the same amount of inputs ($x_c$). The student effect can be defined by the ratio between the local potential output divided by the actual output ($STE = \alpha´ = y´/y_c$). When this student is compared with the metafrontier, the overall efficiency can be defined as $OE = \alpha´´ = y´´/y_c$. From those two measures of efficiency, the school effect can be automatically derived as $SCE_1 = y´´/y´ = OE/STE$. In summary, the global efficiency can be decomposed in two effects: $OE = STE_1 \text{ x } SCE_1$ (*model 1*) (Thanassoulis and Silva Portela, 2002).

However, in order to represent adequately the heterogeneity that exists in each school, the metafrontier needs to consider not only student data, but also additional variables representing the characteristics of students attending each school (Thieme *et al*., 2013). If we do not consider these variables, we are implicitly assuming that all the schools are operating with the most favourable environment, which would not be real in many cases. Therefore, it is possible to improve the definition of the school effect ($SCE_2 = y_1/y' \geq 1$) by considering additional variables. In particular, we incorporate information about an additional variable: the socioeconomic status of students enrolled in the same school, i.e., the so-called peer effect[4].

The consideration of this variable is based on the assumption that the composition of schools and classrooms is not random, since it typically reflects neighbourhood characteristics and therefore the family background of students. The existing literature has used a wide variety of approaches to identify the peer effect (Mc Ewan, 2003; Lefgren, 2004; Lavy *et al*., 2012), which is usually identified by some variable related to student's classmates. However, we construct a variable based on the average of the schoolmates´ socioeconomic characteristics due to the lack of data at class level in the PISA dataset.

Specifically, we have estimated a new model (*model 2*) that allows us to expand the decomposition of the overall efficiency: $OE = STE_2 \ x \ SCE_2 \ x \ PEE_2$ , where the peer effect can be defined as $PEE = \alpha_1 = y''/y_1$. This decomposition is represented in Figure 2, where the metafrontier 2 considers the characteristics of students corresponding to the school under analysis, while metafrontier 1 assumes that the school has the optimal level of environmental factors.

Finally, as we are interested in the effect of the school type (public and private), the metafrontier needs to be separated between the two types. Silva Portela and Thanassoulis (2001), based on the previous work of Charnes *et al*. (1981) to decompose the overall efficiency, defined two components: managerial and programme inefficiency. Thus, this approach allows us to distinguish between inefficiency attributable to the institutional characteristics of the school where the students are enrolled and those attributable to the management regime under which it is operating. Indeed, the different formal rules Spanish public and private schools are subject to may influence their relative performance. As an example, let us pay attention on how it is regulated the teachers' contracts. On the one hand, in

[4] The extension to incorporate additional variables representing the school environment would be straightforward.

the public schools the director has no capacity to decide which profile of teachers should be hired (civil servants have stable position and short term contracts are offered to other teachers without considering the directors' opinion). On the other, state-subsidized private schools have more flexibility, since the director can manage the process to hire new teachers. Although it is not granted in advance, directors of private schools can take advantage of this flexibility and configure a more homogeneous and focused staff what, finally, can improve the school's performance. Nevertheless, public schools traditionally have more qualified teachers with better pedagogical skills and more experience, although the lack of positive incentives can influence their behavior and reduce the enthusiasm. An additional factor, coming from the human resources literature but very important in the education sector, is that the long term horizon in public schools can influence a positive compromise of the teachers with the strategic goals of the school (López-Torres and Prior, 2013). Summing up, the question on how interferes the institutional form of the school on the levels of its efficiency has not an obvious answer, as competing forces can play a role to modify the net effect.

In order to estimate this new frontier (metafrontier 3), we need to divide the whole sample of schools into two different subsamples, thus students are only compared among others attending the same type of school (public or private). Therefore, the previous school effect is decomposed into two different effects: the type of school ($SCT_3 = \alpha_2 = y_1/y_2$) and the new school effect ($SCE_3 = y_2/y' \geq 1$) (Figure 3). Once we have defined these new components of the overall inefficiency, it is straightforward to decompose it between the student effect, the type of school effect, the *net* school effect and the peer effect: $OE = STE_3 \times SCT_3 \times SCE_3 \times PEE_3$ *(model 3).*

## 3. DATA AND VARIABLES

In this study we assess the performance of Spanish students in PISA (Program for International Student Assessment) data set in 2009. This sample comprises more than 25,000 students who are enrolled in almost 900 schools, among which we can distinguish three different typologies: public, state-subsidized private and pure private schools. As we are only interested in comparing the performance of schools with a public funding, we excluded totally private schools from the sample[5]. Likewise, we also removed schools were the number of available observations did not reach a minimum number of students (20). As a result, our sample consists of 22,317 students belonging to 737 schools, among which two thirds are public and one third are state-subsidized

[5] Following the same criterion used in Crespo *et al*. (2013), private schools are classified as state-subsidized schools if they receive more than 50 % of their funding from public authorities.

schools (33%). Table 1 provides information on both students and schools included in the sample.

Table 1. Number of students and schools in the sample

| Type of school | Students | | Schools | |
|---|---|---|---|---|
| | Number | % | Number | % |
| Public | 14847 | 66.5 | 487 | 66.1 |
| Private state-subsidized | 7470 | 33.5 | 250 | 33.9 |
| Total | 22317 | 100 | 737 | 100 |

With regard to the selection of variables, we follow the same approach used in Mancebon and Muñiz (2008), where a restrictive efficiency notion is estimated taking into account the relationship between the academic results obtained by students and their socio-economic background, since this indicator fulfils the requirements of being continuous and have high positive correlation with outcome variables (e.g. Coleman *et al.*, 1966; Goldhaber and Brewer, 1997; Hanushek, 2003). According to this criterion, we evaluate whether the student is making the most of their potential ability, using his/her socioeconomic background as a proxy for this concept, or his/her performance is below the expected level, i.e., the student-within-school inefficiency.

The results obtained by students in the three competences evaluated in PISA, mathematics, reading comprehension and sciences are used as output indicators. These results are not expressed by only one value, but by five denominated *plausible values* randomly obtained from the distribution function of test results derived from the answers in each test (Rasch 1960, 1980), which can be interpreted as the representation of the ability range for each student (Mislevy *et al*., 1992; Wu and Adams, 2002). Although PISA analysts recommend to use all of them to obtain more consistent estimations, in our analysis we use the mean value of those five plausible values, since the robustness of results is guaranteed by the use of the order-*m* approach, which reduces the impact of measurement error by drawing repeatedly (B times) observations from the sample.

The input is measured by students' socioeconomic background (*ESCS*), an index of economic, social and cultural status of students created by PISA analysts that captures a range of aspects of a student's family and home background that combines information on parents' education and occupations and home possessions. The first variable is the higher educational level of any of the student's parents according to the *International*

*Standard Classification of Education* (ISCED, OECD, 1999). The second variable is the highest labour occupation of any of the student's parents according to the International Socio-economic Index of Occupational Status (ISEI, Ganzeboom *et al*., 1992). The third variable is an index of educational possessions related to household economy. Given that this continuous indicator presented negative values[6], the original values have been rescaled. As a result, the variable fulfils the requirement of isotonicity (i.e., *ceteris paribus,* more input implies equal or higher level of output) preserving the desirable property of translation invariance (Cooper, Seiford and Tone, 2007).

Finally, we have selected a variable to include information about the characteristics of students in each school. This variable is represented by schoolmates' background, i.e., the so-called *peer-group* effect (Patrinos, 1995). It is defined as the average level in the variable ESCS of students from the same school, whose theoretical ground lies in the fact that the level of knowledge that can be achieved by a student depends directly on the characteristics of his/her schoolmates.

Table 2 reports a comparison between the values of public and private state-subsidized schools in the four selected variables at student level (three outpus and one input) as well as the indicator representing the type of students in each school. We can observe that private schools obtain higher values in all the outcome variables. However those differences can be explained by the "higher quality" of pupils attending each type of school, represented by their socio economic index.

Table 2. Descriptive statistics of variables included in the analysis

| | **Public Schools** | | **State subsidized Private Schools** | |
|---|---|---|---|---|
| **Variables** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** |
| **Outputs** | | | | |
| Mathematic scores | 479.545 | 91.049 | 512.271 | 81.048 |
| Reading scores | 472.620 | 88.386 | 508.133 | 76.287 |
| Science scores | 481.199 | 88.370 | 511.838 | 76.236 |
| **Inputs** | | | | |
| Index of economic. social and cultural status (ESCS) | 5.861 | 1.026 | 6.409 | 0.989 |
| Average index of economic, social and cultural status (ESCS_mean) | 5.859 | 0.424 | 6.378 | 0.507 |

[6] The values of the *PISA index of economic, social and cultural status* were standardized to a mean of zero for the total population of students in OECD countries, with each country given equal weight.

## 4. RESULTS

In this section we present the results obtained using the robust order-*m* approach to estimate the efficiency levels ( $\hat{\theta}_{mi}$ ) in the three models. First, we estimate model 1, which only allows us to separate the school effect from overall efficiency calculated for each student. Secondly, we estimate two different metafrontiers in order to isolate the school effect from other components of inefficiency. Thus, model 2 determines the importance of the peer effect and how this can reduce the school effect and, subsequently, in model 3 we consider the different institutional rules affecting public and private schools. Therefore, in model 3 the school effect will appear as a residual, after isolating the impact of the peers and the institutional effects.

In our analysis, we use an output orientation, since both the individual students and the schools are attempting to maximize their attainment. As we mentioned previously, we select the value 20 for the *m* parameter and we use 200 bootstrap iterations (B) for statistical inference. The estimation of metafrontiers 1 and 2 uses the whole sample, while the estimation of metafrontier requires the division of the sample between public and state subsized private schools. Finally, the decomposition of inefficiency between different effects (STE, SCE, SCT and PEE) requires the estimation of one partial frontier for each of 737 schools.

Table 3 reports the summary statistics of the estimated scores for model 1 in all the schools. The overall efficiency ($\alpha''$) for each student in the sample has a mean value of 1.1793, which indicates that if all students would perform as efficiently as the best practice students, the test scores could increase on average by 18%[7]. It must be pointed out that, according to model 1, most of the inefficiency found is attributable to the student effect (1.1279 on average), substantially higher than the school effect (1.0461 on average). Moreover, it is worth noting that some pupils have a performance score below 1. These super-efficient students are performing better than the average of the ($m = 20$) students they are benchmarked with. According to the data shown in Table 4, we can observe that there are significant divergences between public (1.2017) and private centres (1.1350). The level of inefficiency attributable to the student is similar in both types of centres, which means that most part of the differences on the overall inefficiency depends on the school effect. According to the structure of model 1, the effect attributable to the schools is obtained by dividing the overall efficiency score ($\alpha''$) by the student effect ($\alpha'$). Table 3 indicates that the mean value of this effect is 4.6%, although behind these

[7] This percentage would be higher (22.5%) if we only consider the inefficient students.

values it is possible to detect that public schools are more inefficient than private schools (5.87% vs. 2.11% in Table 4). Those differences are significant according to the value of the Mann-Whitney nonparametric test applied to these values.

Regarding this point, the results obtained for model 2 are especially informative, because they allow us to distinguish which part of the school inefficiency can be explained by the socioeconomic characteristics of peers attending the same school. The results reported in Table 5 show that the importance of this factor is not too relevant if we consider the whole sample; however, the comparison between public and private schools (Table 6) alert us about the importance of this factor to explain the inefficiency of students attending public centres (3.63%), since it represents more than 60% of the initial average score attributed to the school effect (5.87%). In contrast, this factor has a residual impact on private schools (0.09%), since most of them are actually facing a favourable environment as we were assuming in model 1. Moreover, in public schools there is also a significant part of the inefficiency that depends on the school the student is enrolled (2.16%), while this component has a lower impact in private schools (1.27%).

Finally, the values of the school type effect calculated in model 3 have a mean value very close to 1, with a very low standard deviation for the whole sample (Table 7). This evidence shows that, once we have taken into account the type of students attending to each type of schools, the effect attributable to the type of school on inefficiency is almost inexistent (0.997). Therefore, the remaining divergences in terms of inefficiency depend on the residual factor, i.e., those variables representing the characteristics of schools that have not been included in the analysis (*school effect*).

Although the importance of the school type effect is low, the comparison between public and state subsidized private schools (Table 8) allows us to identify that the average score assigned to this effect is lower in private schools than in public schools, which indicates that the former (where the average level of the variable ESCS is higher) outperform the latter. This evidence is confirmed by the existence of significant differences in the mean values corresponding to the two subsamples[8].

[8] Differences between both types of schools are also significant at 1% level, according to the values of the Mann-Whitney test.

Table 3. Decomposition of overall efficiency between student and school effect (Model 1)

| Inefficiency component | Mean | Std. Dev. | Minimum | 5% | 1st quartile | Median | 3rd quartile | 95% | Maximum | Mean (inefficient) | Std. Dev. (inefficient) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Overall efficiency, OE (α'') | 1.1793 | 0.2174 | 0.7227 | 0.9348 | 1.0346 | 1.1331 | 1.2734 | 1.5747 | 8.6560 | 1.2255 | 0.0015 |
| Student Effect, $STE_1$ (α') | 1.1279 | 0.1686 | 0.7736 | 0.9737 | 1.0000 | 1.0796 | 1.1998 | 1.4533 | 5.9110 | 1.2310 | 0.0017 |
| School Effect, $SCE_1$ (α''/α') | 1.0461 | 0.1136 | 0.7833 | 0.9207 | 0.9759 | 1.0228 | 1.0871 | 1.2499 | 4.0391 | 1.2390 | 0.0019 |

Table 4. Decomposition of overall efficiency between student and school effect by type of school (Model 1)

| Inefficiency component | Public schools | | | | State-subsidized schools | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Minimum | Maximum | Mean | Std. Dev. | Minimum | Maximum |
| Overall efficiency, OE (α'') | 1.2017* | 0.2356 | 0.7227 | 8.6560 | 1.1350* | 0.1673 | 0.7973 | 2.2838 |
| Student Effect, $STE_1$ (α') | 1.1357* | 0.1808 | 0.7736 | 5.9110 | 1.1123* | 0.1400 | 0.9004 | 2.2638 |
| School Effect, $SCE_1$ (α''/α') | 1.0587* | 0.1214 | 0.7833 | 4.0391 | 1.0211* | 0.0913 | 0.7972 | 1.9784 |

*Test statistics significant at 1% level. (non-parametric Mann–Whitney test)

Table 5. Decomposition of overall efficiency between student, peer and school effect (Model 2)

| Inefficiency component | Mean | Std. Dev. | Minimum | 5% | 1st quartile | Median | 3rd quartile | 95% | Maximum | Mean (inefficient) | Std. Dev. (inefficient) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Overall efficiency, OE (α'') | 1.1793 | 0.2174 | 0.7227 | 0.9348 | 1.0346 | 1.1331 | 1.2734 | 1.5747 | 8.6560 | 1.2255 | 0.0015 |
| Student Effect, $STE_2$ (α') | 1.1279 | 0.1686 | 0.7736 | 0.9737 | 1.0000 | 1.0796 | 1.1998 | 1.4533 | 5.9110 | 1.1830 | 0.0013 |
| School Effect, $SCE_2$ ($\alpha_1$/α') | 1.0281 | 0.1085 | 0.7735 | 0.9099 | 0.9617 | 1.0062 | 1.0661 | 1.2243 | 4.1379 | 1.0935 | 0.0009 |
| Peer Effect, $PEE_2$(α''/$\alpha_1$) | 1.0174 | 0.0264 | 0.9304 | 0.9939 | 1.0042 | 1.0125 | 1.0230 | 1.0532 | 1.5843 | 1.0213 | 0.0002 |

Table 6. Decomposition of overall efficiency between student, school-type and school effect by type of school (Model 2)

| Inefficiency component | Public schools | | | | State-subsidised schools | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Minimum | Maximum | Mean | Std. Dev. | Minimum | Maximum |
| Overall efficiency, OE ($\alpha''$) | 1.2017* | 0.2356 | 0.7227 | 8.6560 | 1.1350* | 0.1673 | 0.7973 | 2.2838 |
| Student Effect, $STE_2$ ($\alpha'$) | 1.1357* | 0.1808 | 0.7736 | 5.9110 | 1.1123* | 0.1400 | 0.9004 | 2.2638 |
| School Effect, $SCE_2$ ($\alpha_1/\alpha'$) | 1.0216* | 0.0302 | 0.9304 | 1.5843 | 1.0127* | 0.0902 | 0.7999 | 1.9721 |
| Peer Effect, $PEE_2$ ($\alpha''/\alpha_1$) | 1.0363* | 0.1158 | 0.7735 | 4.1379 | 1.0090* | 0.0131 | 0.9754 | 1.1527 |

*Test statistics significant at 1% level. (non-parametric Mann–Whitney test)

Table 7. Decomposition of overall efficiency between student, school-type, peer and school effect (Model 3)

| Inefficiency component | Mean | Std. Dev. | Minimum | 5% | 1st quartile | Median | 3rd quartile | 95% | Maximum | Mean (inefficient) | Std. Dev. (inefficient) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Overall efficiency, OE ($\alpha''$) | 1.1793 | 0.2174 | 0.7227 | 0.9348 | 1.0346 | 1.1331 | 1.2734 | 1.5747 | 8.6560 | 1.2255 | 0.0015 |
| Student Effect, $STE_3$ ($\alpha'$) | 1.1279 | 0.1686 | 0.7736 | 0.9737 | 1.0000 | 1.0796 | 1.1998 | 1.4533 | 5.9110 | 1.1830 | 0.0013 |
| School Effect, $SCE_3$ ($\alpha_2/\alpha'$) | 1.0307 | 0.1081 | 0.7817 | 0.9130 | 0.9651 | 1.0094 | 1.0684 | 1.2255 | 4.0816 | 1.0930 | 0.0010 |
| School-type Effect, $SCT_3$ ($\alpha_1/\alpha_2$) | 0.9976 | 0.0136 | 0.8440 | 0.9780 | 0.9903 | 0.9980 | 1.0051 | 1.0152 | 1.3668 | 1.0083 | 0.0001 |
| Peer Effect, $PEE_3$ ($\alpha''/\alpha_1$) | 1.0174 | 0.0264 | 0.9304 | 0.9939 | 1.0042 | 1.0125 | 1.0230 | 1.0532 | 1.5843 | 1.0213 | 0.0002 |

Table 8. Decomposition of overall efficiency between student, school-type, peer and school effect by type of school (Model 3)

| Inefficiency component | Public schools | | | | State-subsidized schools | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Minimum | Maximum | Mean | Std. Dev. | Minimum | Maximum |
| Overall efficiency, OE ($\alpha''$) | 1.2017* | 0.2356 | 0.7227 | 8.6560 | 1.1350* | 0.1673 | 0.7973 | 2.2838 |
| Student Effect, $STE_3$ ($\alpha'$) | 1.1357* | 0.1808 | 0.7736 | 5.9110 | 1.1123* | 0.1400 | 0.9004 | 2.2638 |
| School Effect, $SCE_3$ ($\alpha_2/\alpha'$) | 1.0348* | 0.1156 | 0.7817 | 4.0816 | 1.0225* | 0.0908 | 0.8182 | 2.0260 |
| School-type Effect, $SCT_3$($\alpha_1/\alpha_2$) | 1.0015* | 0.0093 | 0.9604 | 1.0591 | 0.9899* | 0.0170 | 0.8440 | 1.3668 |
| Peer Effect, $PEE_3$ ($\alpha''/\alpha_1$) | 1.0363* | 0.1158 | 0.7735 | 4.1379 | 1.0090* | 0.0131 | 0.9754 | 1.1527 |

*Test statistics significant at 1% level. (non-parametric Mann–Whitney test)

## 5. CONCLUDING REMARKS

In this paper we assess the performance of Spanish students in PISA 2009 using data at student level with the aim of finding divergences between public and state subsidized private schools. Given the uncertain about the specification of the production technology in education, we employ a nonparametric approach. In particular, this study represents the first attempt to measure the efficiency of Spanish students in secondary schools by combining the use of the recently developed order-*m* approach with the metafrontier approach. The former method allows us to estimate robust measures of efficiency, while the latter makes it possible to decompose the effect of students, schools and the effect attributed to the type of school (differences between public and private schools).

The main conclusion that can be drawn from our analysis is that state subsidized private schools are more efficient, although the estimated inefficiency attributable to students is similar in both public and private schools. Actually, the final decomposition of inefficiency allows us to detect that the effect attributable to the school type is almost inexistent, while peer effect and school effect have a greater impact on results, especially in the subsample of public schools.

This result implies that a significant proportion of inefficiency in public schools depends on the characteristics of students enrolled. Those divergences can be explained because students are not randomly distributed between both types of schools, since students with a lower socioeconomic status are prone to be enrolled in public schools due to the higher costs that would entail to attend state subsidized schools[9].

The results obtained in this study must be interpreted cautiously, since we use a restrictive notion of efficiency based on the relationship between the academic results obtained by students and their socioeconomic background and only consider one variable representing the environment in the school (average of socioeconomic background as a proxy of peer effect). Given that we have not considered any input of the school, it implies we assume that the allocation of resources is the same in all schools, which may be difficult to believe when we are comparing public and state subsidized private centres.

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[9] Crespo *et al* (2013) point out that educational materials, extra-curricular activities, school bus and lunch are usually more expensive in private schools (around 30% higher). Moreover, in most private schools, parents are required to pay a fee to improve school facilities or to offer some extra-curricular activities while this fee does not exist in public schoools.

Figure 1. Metafrontier illustration (decomposition of student and school effect)

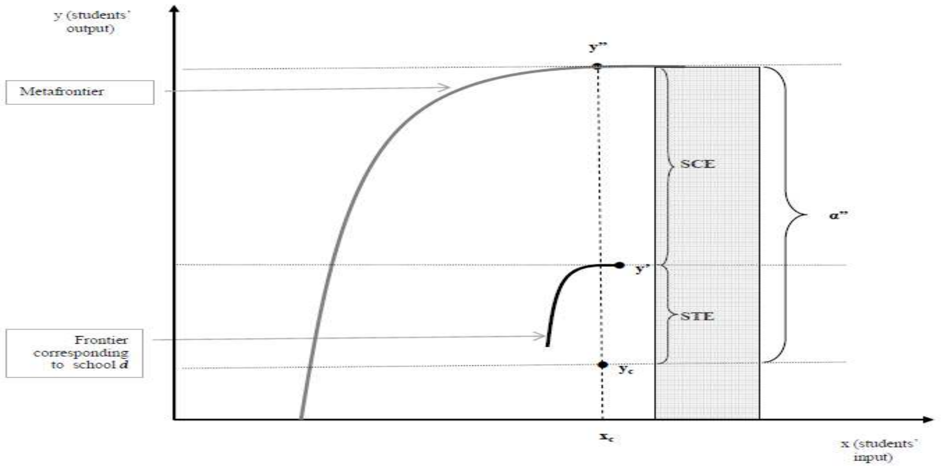



Figure 2. Metafrontier illustration (decomposition of student, school and school-type)

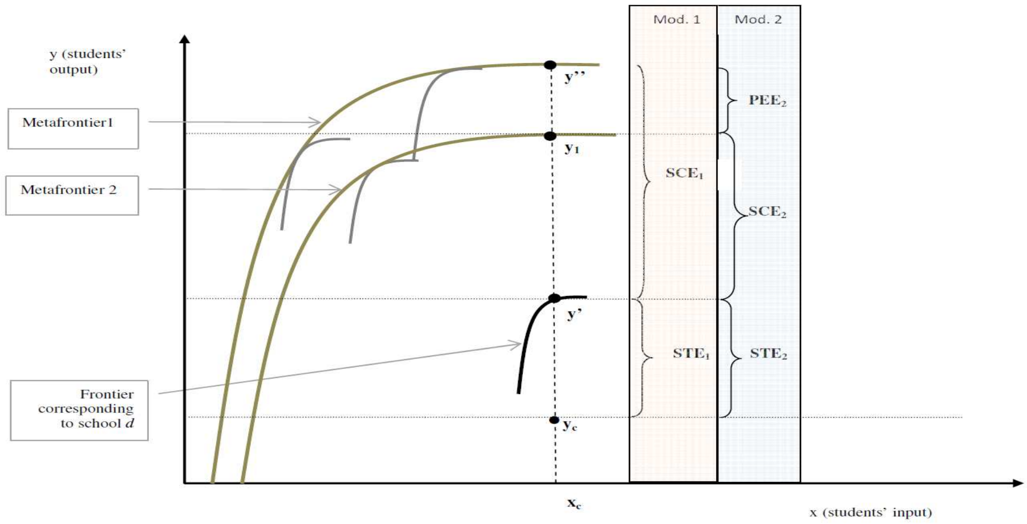
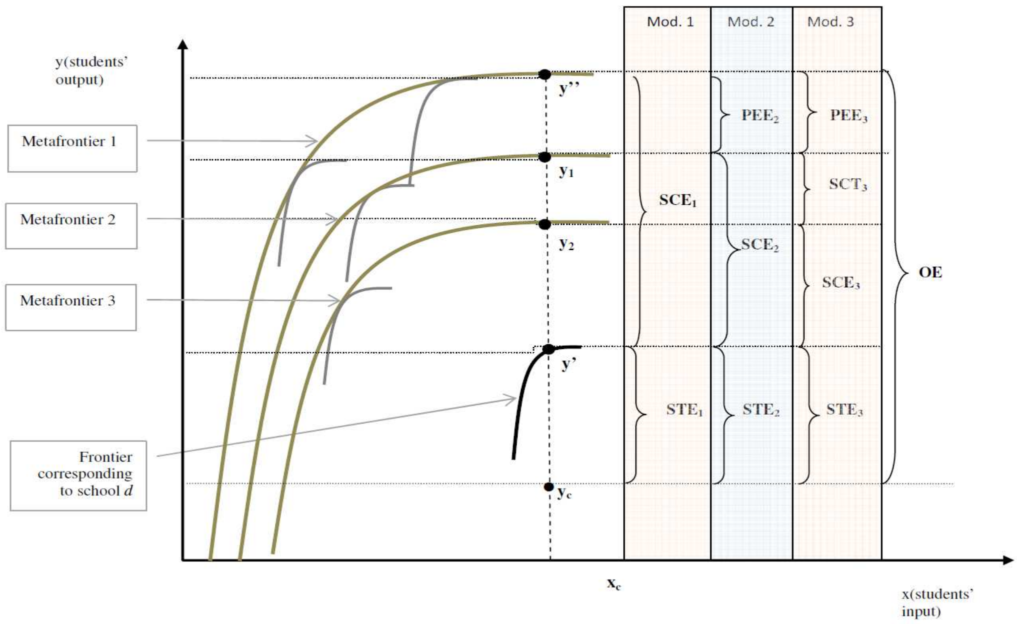



Figure 3. Metafrontier illustration (decomposition of student, school and peer effect)



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