Mathematical Proof of the Breakdown of Capitalism

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Abstract

The existence proof of general equilibrium, which is based on subjective-behavioral axioms, is replaced by the existence proof of a final turning point, which is based on objective-structural axioms. The final turning point is characterized by an irreversible switch from profits to losses for the business sector as a whole and marks the beginning of the breakdown of the monetary economy. This has nothing to do with any market failures or irrationalities. The final turning point can be preceded by an arbitrary number of temporary profit/loss reversals and is in full accordance with the households' optimal intertemporal consumption plans.

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Keywords new framework of concepts; structure-centric; axiom set; consumption economy; Profit Law; simulation; market clearing; budget balancing; final turning point; existence proof

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1 Putting the math right[1]

> The only way to arrive at coherent languages is to set up axiomatic systems implicitly defining the basic concepts. (Schmiechen, 2009, p. 344)
>
> But set theory is not the right mathematical tool because it is too general. Consequently, theorems and proofs in this approach are inordinately unwieldy. (Hestens, quoted in Schmiechen, 2009, p. 368)

Standard economics rests on behavioral assumptions that are formally expressed as axioms (Debreu, 1959; Arrow and Hahn, 1991; McKenzie, 2008). Axioms are indispensable to build up a theory that epitomizes formal and material consistency. The fatal flaw of the standard approach is that human behavior does not lend itself to axiomatization.

> For instance, economists bend their research toward axiomatic theories that are almost embarrassing in their pre-scientific naiveté. Consider utility theory, for instance, which is now taking a drubbing at the hands of experimental psychology and neurophysiology. A scientific orientation would free us of such vestigial dogmas. (Dorman, 2008, p. 170)

Conceptual consequence demands to discard the subjective-behavioral axioms and to take objective-structural axioms as the formal point of departure.

> The great contradiction revealed is as follows: one of the theories greatest strength – its claim to deduce significant results from very general hypotheses about the behavior of economic agents – turns out to be its greatest weakness. (Ingrao and Israel, 1990, p. 364)

The consensus is that general equilibrium theory has failed on all counts (Ackerman and Nadal, 2004). It has not failed because of axiomatization but because of choosing the wrong axioms.

In order to put the math right Section 2 provides the new formal foundations with the set of four structural axioms. These represent the pure consumption economy as the most elementary economic configuration. In Section 3 money, profit, retained profit and saving is defined. With all necessary elements in their proper places it is then possible, in Section 4, to simulate the development of the household sector's debt from the zero starting point to the zero endpoint with the final turning point in between. Thereby, the household sector's credit expansion and contraction runs in parallel with the business sector's profit and loss. Section 5 concludes.

[1] The paper's title is a homage to Nicholas Georgescu-Roegen (1960).

2 Axioms

> Formal axiomatic systems must be interpreted in some domain ... to become an empirical science. (Boylan and O'Gorman, 1995, p. 198)

Contrary to the common sense of methodological individualism, the formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of the real and nominal variables that constitutes the monetary economy.

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N. Nothing is implied at this stage about who owns the shares.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment, no foreign trade, and no government. Disaggregation comes later.

The period values of the axiomatic variables are formally connected by the familiar growth equation, which is added as the 4th axiom.

$$\begin{gathered} Z_t = Z_{t-1}\left(1+\dddot{Z}_t\right) \\ \text{with} \quad Z \leftarrow W, L, D, N, R, P, X, \ldots \end{gathered} \tag{4}$$

The path of the representative variable Z_t is then determined by the initial value Z_0 and the rates of change $\ddot{\ddot{Z}}_t$ for each period:

$$Z_t = Z_0\,(1+\ddot{\ddot{Z}}_1)\,(1+\ddot{\ddot{Z}}_2)\ldots(1+\ddot{\ddot{Z}}_t) = Z_0\prod_{t=1}^{t}(1+\ddot{\ddot{Z}}_t)\,. \tag{5}$$

For a start it is assumed that the elementary axiomatic variables vary at random. This produces an evolving economy. The respective probability distributions of the change rates are given in general form by:

$$\begin{array}{c} Pr\left\{l_W \le \ddot{\ddot{W}}_t \le u_W\right\} \\ Pr\left\{l_L \le \ddot{\ddot{L}}_t \le u_L\right\} \\ Pr\left\{l_D \le \ddot{\ddot{D}}_t \le u_D\right\} \\ Pr\left\{l_N \le \ddot{\ddot{N}}_t \le u_N\right\} \\ Pr\left\{l_R \le \ddot{\ddot{R}}_t \le u_R\right\} \\ Pr\left\{l_P \le \ddot{\ddot{P}}_t \le u_P\right\} \\ Pr\left\{l_X \le \ddot{\ddot{X}}_t \le u_X\right\} \end{array} \tag{6}$$

The four axioms combined with (6) constitute a simulation. For the actual simulation the random variates for each period are taken from the worksheet random number generator and are then appropriately adapted. The assumed probability distributions can at any time be replaced by distributions that have been observed over a reasonable time span. There is, though, no need at this early stage to discus the merits and demerits of different probability distributions. It is, of course, also possible to switch to a completely deterministic rate of change for any variable and any period. The structural formalism does not require a preliminary decision between determinism and indeterminism – and therefore no ontological rigmarole.

The upper (u) and lower (l) boundaries of the respective intervals are, for the time being, symmetrical around zero. This produces an evolving economy that over a longer time span neither grows nor shrinks. The drifting or stationary economy is a limiting case of the growing economy.

The four axioms and the random distributions produce at every run an outcome like that shown in Figure 1 which is the archetype of the monetary economy.

Note well that the consumption economy is *not* heading towards a definite state that has any resemblance with what conventional economists imagine as equilibrium. This is as it should be because it is methodologically illegitimate to put an assumption like equilibrium into the premises. This lapse is known since antiquity as petitio principii (Mill, 2006, pp. 819-827). From the methodological standpoint standard economics can be characterized as the synthesis of inept axiomatization, petitio principii and the fallacy of composition.

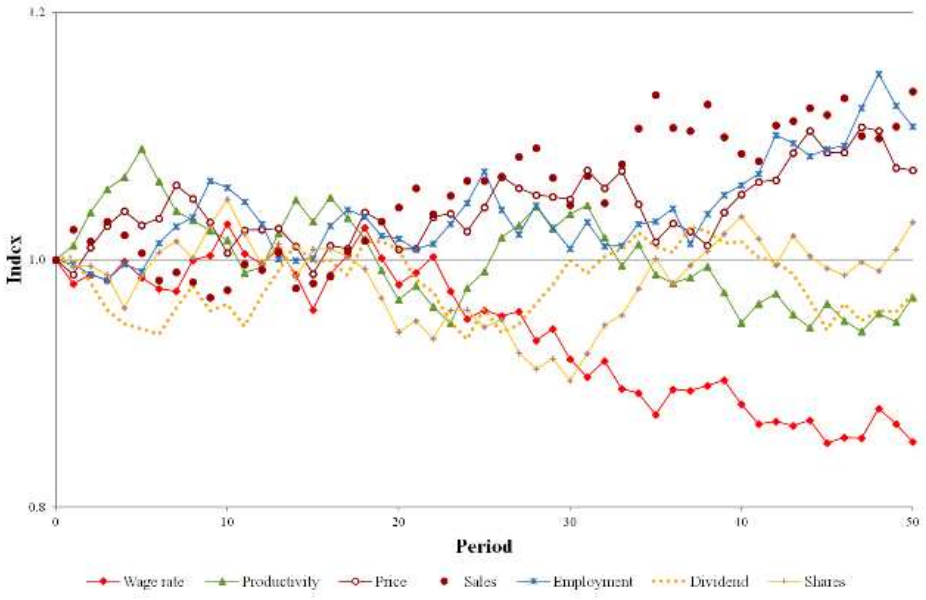


Figure 1: The evolving consumption economy consists initially of entirely independent random paths of the seven elementary axiomatic variables (shown here) and the paths of composed variables

A simulation is a mathematical object just like a system of equations – with the decisive advantage that change and chance can be formally represented in a natural manner. With a system of equations one is inescapably locked in Walras's trap of a deterministic simultaneous equilibrium. No such thing exists. Therefore, the structural axiomatic simulation is the proper tool for economic analysis. Supply-demand-equilibrium or its set theoretical counterpart is a formal no go.

The economic content of the four axioms is plain. One point to mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. This distinction is crucial as will presently become clear.

A question that time and again arises with axiomatization is: am I forced to accept any axioms as self-evident? No, provided you know of a superior set of axioms, otherwise emphatically yes because analysis and discussion require a common ground and are pointless without clearly stated premises.

> Whether an axiom is or is not valid can be ascertained either through direct experimentation or by verification through the result of observations, or, if such a thing is impossible, the correctness of the axiom can be judged through the indirect method of verifying the laws which proceed from the axiom by observation or experimentation. (If the axiom is deemed to be incorrect it must be modified or instead a correct axiom must be found.) (Morishima, 1984, p. 53)

The only alternative to an axiomatic approach is a better axiomatic approach.

3 Definitions

> Skillful use of definitions enables the scientist to extend his deductive analysis to the remotest stages of implication, such as otherwise would be far beyond his mental reach. (Leontief, 1937, p. 342)

3.1 Income categories

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (7) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{7}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

Given the paths of the elementary variables, the development of the composed variables is also determined. From the random paths of employment L and wage rate W follows the path of wage income Y_W. Likewise follows from the paths of dividend D and number of shares N the path of distributed profit Y_D. From the 1st axiom then follows the random path of total income Y.

3.2 Ratios

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{8}$$

A sales ratio $\rho_X = 1$ indicates that the quantity bought/sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \tag{9}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to total income Y, in other words, that the household sector's budget is balanced.

3.3 Stock of money

Money follows consistently from the given axiom set. If income is higher than consumption expenditures the household sector's stock of money increases. The change in period t is defined as:

$$\Delta\bar{\mathsf{M}}_{\mathbf{H}} \doteq Y - C \quad |t. \tag{10}$$

The alternative identity sign $\doteq$ indicates that the definition refers to the monetary sphere.

The stock of money $\bar{\mathsf{M}}_{\mathbf{H}}$ at the end of an arbitrary number of periods $\bar{t}$ is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{\mathsf{M}}_{\mathbf{H}t} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{H}t} + \bar{\mathsf{M}}_{\mathbf{H}0}. \tag{11}$$

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta\bar{\mathsf{M}}_{\mathbf{B}} \doteq C - Y \quad |t. \tag{12}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{\mathsf{M}}_{\mathbf{B}t} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{B}t} + \bar{\mathsf{M}}_{\mathbf{B}0}. \tag{13}$$

The development of the stock of money follows without further assumptions from the axioms and is ultimately determined by variations of the elementary variables. Figure 2 shows the interdependencies between the flows and the stock. In the time span of observation the household sector's overdrafts increase.

3.4 Quantity of money

In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions are carried out without costs by the central bank. The stock of money then takes the form of current deposits or current overdrafts. Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (11) the current overdrafts of the business sector are of equal amount according to (13) and vice versa if the business sector owns current deposits. Money and credit are symmetrical; the stock of money of each sector can be either positive or negative. The current assets and liabilities of the central bank are equal by

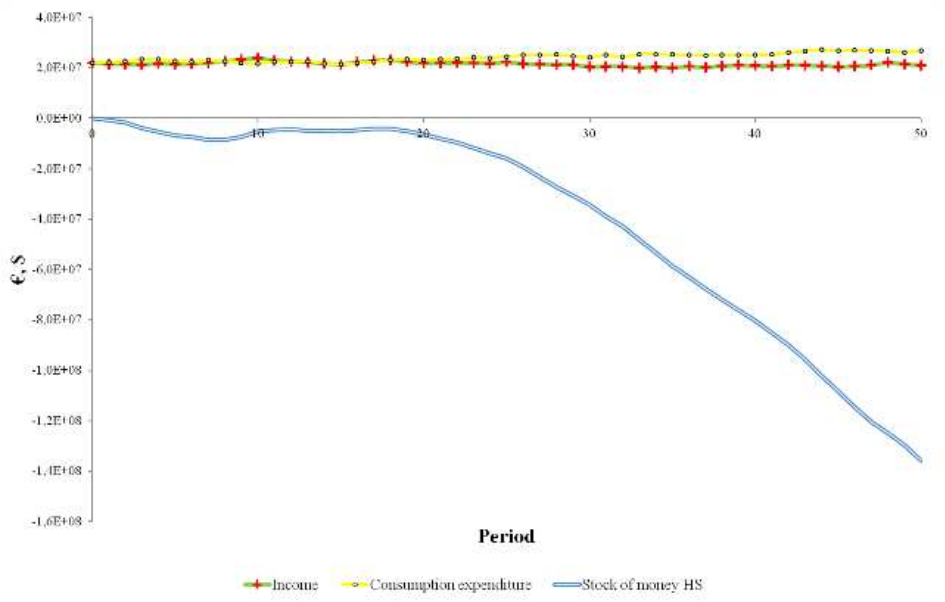


Figure 2: The difference between total income and consumption expenditure in successive periods produces the variations of the households sector's stock of money, which consists here of overdrafts (refers to Figure 1)

construction. From its perspective the quantity of money at the end of an arbitrary number of periods is given by the absolute value either from (11) or (13):

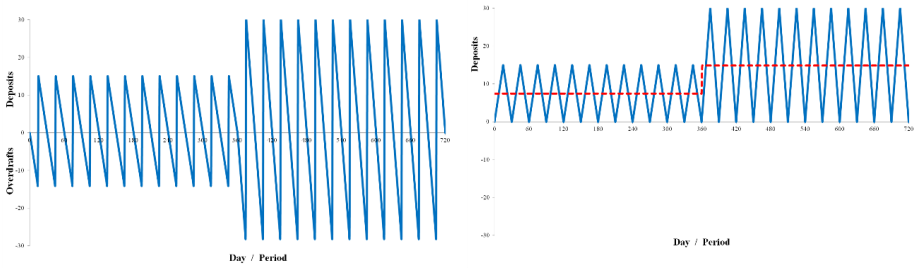
$$\bar{\mathsf{M}}_t \equiv \left| \sum_{t=1}^{t} \Delta \bar{\mathsf{M}}_t \right| \quad \text{with} \quad \bar{\mathsf{M}}_0 = 0. \tag{14}$$

While the stock of money can be either positive or negative the quantity of money is always positive. It is assumed at first that the central bank plays an *accommodative* role and simply supports the autonomous market transactions between the household and the business sector. For the time being, money is the *dependent* variable (for details see 2011a; 2011b).

3.5 Transaction money

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 3a results.

It is assumed that the monthly income $\frac{Y}{12}$ is paid out at mid-month. In the first half of the month the daily spending of $\frac{Y}{360}$ increases the current overdrafts of the households. At mid-month the households change to the positive side and have current deposits of $\frac{Y}{24}$ at their disposal. This amount reduces continuously towards the end of the month. This pattern is exactly repeated over the rest of the year. At



(a) Transaction pattern over two periods **(b)** Average stock of transaction money $\hat{\mathsf{M}}$

Figure 3: Household sector's transaction pattern for different nominal incomes in two periods

the end of each sub-period, and therefore also at the end of the year, both the stock of money and the quantity of money is zero.

In period 2 the wage rate, the dividend and the price is doubled. Since no cash balances are carried forward from one period to the next, there results *no* real balance effect provided the doubling takes place exactly at the beginning of period 2.

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore, the pattern of Figure 3a translates into the average amount of current deposits in Figure 3b. This average stock of transaction money depends on income according to the transaction equation

$$\hat{\mathsf{M}} \equiv \kappa Y \quad |t. \tag{15}$$

For the regular transaction pattern that is here assumed as a idealization the index is $\frac{1}{48}$. Different patterns are characterized by different numerical values of the transaction pattern index. The index is measurable in principle.

By taking (15), (8) and (9) together one gets the explicit transaction equation for the limiting case of market clearing and budget balancing:

$$\begin{gathered} \text{(i)}\ \hat{\mathsf{M}} \equiv \kappa \frac{\rho_X}{\rho_E} RLP \quad \text{(ii)}\ \frac{\hat{\mathsf{M}}}{P} = \kappa O \\ \text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t. \end{gathered} \tag{16}$$

According to (i) the central bank enables the average stock of transaction money to expand or contract with the development of productivity, employment, and price. In other words, the real average stock of transaction money, which is a statistical artifact and not a physical stock, is proportional to output (ii) if the transaction index is given and if the ratios ρ_E and ρ_X are unity. Under these *initial* conditions money

is endogenous and neutral in the structural axiomatic context. Money emerges from *autonomous* market transactions and has three aspects: stock of money ($\bar{\mathsf{M}}_{\mathbf{H}}$, $\bar{\mathsf{M}}_{\mathbf{B}}$), quantity of money (here $\bar{\mathsf{M}} = 0$ at period start and end because of $\rho_E = 1$) and average stock of transaction money (here $\hat{\mathsf{M}} > 0$).

3.6 Profit

Total profit consists of monetary and nonmonetary profit. Here we are at first concerned with monetary profit. Nonmonetary profit is treated at length in (2012).

The business sector's monetary profit/loss in period t is defined with (17) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:

$$Q_m \equiv C - Y_W \quad |t. \tag{17}$$

Because of (3) and (7) this is identical with:

$$Q_m \equiv PX - WL \quad |t. \tag{18}$$

This form is well-known from the theory of the firm. Figure 4 shows how profit develops in the time span of observation.

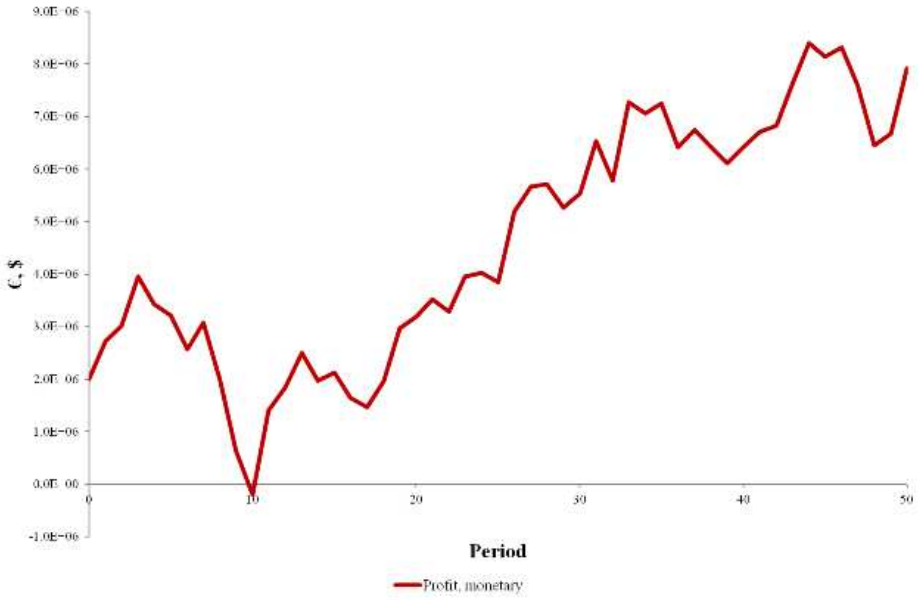


Figure 4: The profit path results from the random paths of the elementary variables price, quantity bought/sold, wage rate and labor input (refers to Figure 1)

The path of monetary profit is *uno actu* determined with the elementary variables. Profit depends on price P, sales X, wage rate W and employment L as defined with (18). The profit path follows from the random variations of four independent elementary variables and the structure of the pure consumption economy which is given with the axiom set.

Formally, the path of profit in Figure 4 is the (discrete) first derivative of the path of the stock of money in Figure 2.

The four structural axioms and the probability distributions (6) constitute the *minimum* of premises. Given the essentials, the simulation delivers the concrete values of all variables for all future periods under the condition that no events beyond the symmetric random changes interfere. There are at the moment *no interdependencies* between the paths of the elementary variables; the evolution of the economy is open and only subject to statistical laws. Should there be any interdependencies, for instance between price and sales or income and saving, then they have to be explicitly added to the formal core.

3.7 The Profit Law

From (17) and (1) follows:

$$Q_m \equiv C - Y + Y_D \quad |t \tag{19}$$

or, using the definitions (8) and (9),

$$Q_m \equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \quad |t. \tag{20}$$

The four equations (17) to (20) are formally equivalent and show profit under different perspectives. The Profit Law (20) tells us that total monetary profit is zero if $\rho_E = 1$ and $\rho_D = 0$. Profit or loss depends on the expenditure and distributed profit ratio and nothing else. Whether the agents maximize profit or not is irrelevant. Whether the allocation of resources is optimal or not is irrelevant. What the myopic agent thinks about profit is irrelevant. What Smith, Walras or Keynes wrote about profit is false (for details see 2013a) and therefore irrelevant. Eq. (20) is, as an objective systemic relation, testable in principle. This, and this alone, is relevant.

3.8 Retained profit

Once profit has come into existence for the first time (that is: logically – a historical account is an entirely different matter) the business sector has the option to distribute or to retain it. This in turn has an effect on profit. This effect is captured by (19) but it is invisible in (17). Both equations, though, are formally equivalent.

Retained profit Q_{re} is defined for the business sector as a whole as the difference between profit and distributed profit in period t:

$$Q_{re} \equiv Q_m - Y_D \quad \Rightarrow \quad Q_{re} \equiv C - Y \quad |t. \tag{21}$$

Retained profit is, due to (19), equal to the difference of consumption expenditures and total income.

3.9 Saving

The household sector's monetary saving is given as the difference of income and consumption expenditures (for nonmonetary saving see 2012):

$$S_m \equiv Y - C \quad |t. \tag{22}$$

In combination with (21) follows:

$$Q_{re} \equiv -S_m \quad |t. \tag{23}$$

Monetary saving and retained profit always move in opposite directions. This is the Special Complementarity. It says that the complementary notion to saving is negative retained profit; positive retained profit is the complementary of dissaving. There is no such thing as an equality of saving and investment in the consumption economy, nor, for that matter, in the investment economy (for details see 2013c).

If distributed profit is zero then follows as a corollary of (23):

$$\begin{aligned} &Q_m = -S_m \\ &\qquad\qquad |t. \\ &\text{if} \quad Y_D = 0 \end{aligned} \tag{24}$$

Profit is zero in the limiting case of zero distributed profit and zero saving. Otherwise profit is equal to dissaving, loss is equal to saving in a given period. To simplify matters for the next Section distributed profit is set to zero, that is eq. (24) holds.

4 The first half of temporal asymmetry

> An axiomatized theory substitutes for an ambiguous economic concept a mathematical object that is subject to entirely definite rules of reasoning. (Debreu, quoted in Ingrao and Israel, 1990, p. 287)

4.1 The market clearing price

From (3), (8), and (9) follows the price as dependent variable:

$$P = \frac{\rho_E}{\rho_X} \frac{W}{R} \left(1 + \frac{Y_D}{Y_W}\right) \quad |t. \tag{25}$$

This is the general structural axiomatic law of supply and demand for the pure consumption economy with one firm. In brief the price equation states that the market clearing price is ultimately determined by the expenditure ratio, unit wage costs, and the income distribution. Note that the quantity of money is *not* among the determinants. This rules the commonplace quantity theory out. The structural axiomatic price formula is testable in principle.

Under the condition of market clearing and zero distributed profit follows:

$$\begin{gathered} P = \rho_E \frac{W}{R} \\ \text{if} \quad \rho_X = 1, Y_D = 0 \quad |t. \end{gathered} \tag{26}$$

The market clearing price depends now alone on the expenditure ratio and unit wage costs. Under the additional conditions of budget balancing follows:

$$\begin{gathered} P = \frac{W}{R} \\ \text{if} \quad \rho_E = 1, \rho_X = 1, Y_D = 0 \quad |t. \end{gathered} \tag{27}$$

The market clearing price is equal to unit wage costs if the expenditure ratio is unity and distributed profit is zero. In this elementary case, profit per unit is zero and by consequence total profit is zero. All changes of the wage rate and the productivity affect the market clearing price in the period under consideration. We refer to this formal property as *conditional* price flexibility because (27) involves no assumption about human behavior, only the purely formal condition $\rho_X = 1$.

With (27) the real wage $\frac{W}{P}$ is *uno actu* given; it is under the enumerated conditions invariably equal to the hourly output R. Hence labor gets the whole product. Since profit is zero at all employment levels it makes no difference from the business sector's perspective whether full employment obtains or not. Under the rule of conditional price flexibility changes of the wage rate do not affect the real wage. This is a systemic property that has nothing at all to do with the notion of money illusion. The real wage is not determined in the labor market and certainly not by supply-demand-equilibrium.

4.2 Employment

Let us assume that the household sector's labor supply increases due to exogenous population growth. What is now needed is a drive on the side of the business sector to expand labor input L, otherwise we are left with growing unemployment. The directed random changes which increase or reduce labor input are made, in a rather straightforward way, dependent on profit:

$$\begin{aligned} \underbrace{\{1,0,-1\}_t}_{\text{direction}} &= sgn\,(Q_{t-1}-0) \\ \ddot{\ddot{L}}_t &= \underbrace{\{1,0,-1\}_t}_{\text{direction}}\,\underbrace{Pr\{0\leq \ddot{\ddot{L}}\leq u\}_t}_{\text{magnitude}}. \end{aligned} \tag{28}$$

The upper half of (28) says that the sign, i.e. the direction of change in period t, depends on whether there was profit or loss in the previous period. In the case of profit the sign is positive, that is, the business sector increases labor input, and vice versa in the case of loss. The lower half combines the direction with a random rate of change. In combination, the two halves define an elementary dependency. No exogenous factors restrict the directed random changes at the moment.

The difference between actual labor input and actual labor supply, i.e. over- or underemployment, is of no consequence. It is alone profit/loss that has any effect on employment. In behavioral terms this means that the business sector expands employment whenever profit is greater zero and vice versa. More is not needed for our present purposes and this simple rule is what (28) conveys. Quantitative constraints or capacity limits can be built into the equation at any time. For a more sophisticated adaptation rule see (2013b, Sec. 6.3).

4.3 Budget balancing in the very, very long run

Hitherto, the expenditure ratio as defined with (9) is a dependent random variable. This is changed now. The expenditure ratio becomes an independent variable. Its random rate of change is given by:

$$\ddot{\ddot{\rho}}_E = Pr\{l\leq \ddot{\ddot{\rho}}_E\leq u\} \quad |t. \tag{29}$$

It is assumed that the upper (u) and lower (l) boundary is symmetrical around zero. The expenditure ratio in each period is given by:

$$\rho_{Et} = 1+\ddot{\ddot{\rho}}_{Et} \tag{30}$$

The expenditure ratio varies in each period randomly around unity. From this results the simple relation between income and consumption expenditure:

$$C_t = \rho_{Et}\, Y_t$$
$$\text{if } \rho_{Et} \text{ independent.} \tag{31}$$

The stochastic consumption function is a corollary of (9) if the expenditure ratio is independent. Yet since this ratio changes in each period according to (30) the relationship between total income and consumption expenditure is not constant over time. Nominal demand C is now an indirect random variable. However, the expenditure ratio is defined such that it is to be expected that aggregate consumption expenditures become equal to aggregate incomes in the course of time.

With (26) the market clearing price has been derived as:

$$P = \rho_E \frac{W}{R}$$
$$\text{if} \quad \rho_X = 1, Y_D = 0 \quad |t. \tag{32}$$

The market clearing price in period t depends on unit wage costs and the expenditure ratio.

From (20) follows as a corollary for monetary profit:

$$Q_m = (\rho_E - 1)\, Y$$
$$\text{if} \quad Y_D = 0 \quad |t. \tag{33}$$

Monetary profit/loss depends also on the expenditure ratio.

Eq. (10), which is reproduced here, finally states that the change of the household sector's stock of money, too, depends on the expenditure ratio:

$$\Delta \bar{\mathrm{M}}_{\mathbf{H}} \doteq (1 - \rho_E)\, Y \quad |t. \tag{34}$$

Price, monetary profit, and the change of the household sector's stock of money are all related via the expenditure ratio. A ratio greater than unity means dissaving and raises the market clearing price, boosts profit, and increases the household sector's stock of overdrafts (or lowers the stock of deposits). The business sector's stock of deposits increases (or the stock of overdrafts decreases) according to (12). The inverse happens if the households save. Price and profit are down and the stock of deposits increases.

According to (30) the expenditure ratio hovers randomly around unity. The probability distribution has been defined such that the expected value of the expenditure ratio is unity, i.e. $E[\rho_E] = 1$. This is the condition for pure stochastic budget balancing. From (32) to (34) then follows:

$$E[P] = \frac{W}{R} \quad E[Q_m] = 0 \quad E\left[\Delta\bar{\mathrm{M}}_{\mathbf{H}}\right] = 0$$
$$\text{if} \quad \rho_X = 1, Y_D = 0 \quad |t. \tag{35}$$

In loose terms this is to say that with an expected expenditure ratio of unity the expected monetary profit is zero. In other words, although profit/loss is a random variable and different from zero in each period the pure consumption economy is a zero profit economy under the conditions of stochastic budget balancing and the absence of profit distribution.

4.4 The final turning point

If the expenditure ratio is a symmetric random variable we will see profit and loss alternating in an irregular fashion. This, however, is not what we observe. The market economy is clearly asymmetric and has produced profits in most of historical time. Since the classicals it has been felt that the sheer existence of profit is a puzzle. To recall, in Walras's general equilibrium profit is zero.

To reproduce the historical permanence of profit it is assumed now that we have only positive random rates of change of the expenditure ratio (i) for a stretch of time and then only the negative ones (ii):

$$\ddot{\ddot{\rho}}_E = Pr\{0 \leq \ddot{\ddot{\rho}}_E \leq u\} \quad \text{(i)}$$
$$\ddot{\ddot{\rho}}_E = Pr\{l \leq \ddot{\ddot{\rho}}_E \leq 0\} \quad \text{(ii)}. \tag{36}$$

Figure 5 shows what happens to the consumption economy if the random expenditure ratio is consistently greater than unity in the first hundred periods and less than unity in the next hundred periods. Eq. (36) simply assorts the random changes and thereby establishes temporal asymmetry of an arbitrary length; it leaves the real world causes of the asymmetry open to interpretation.

An expenditure ratio greater unity means that the households take up credit and exert an additional nominal demand in a given period. This happens when households buy durables like cars or houses on credit. For the business sector as a whole this means an increase of the market clearing price according to (32) and of monetary profit according to (33). The household sector's overdrafts increase according to (34) and the business sector's deposits according to (12).

Whether the overdrafts are replaced by longer term loans like mortgages, or are in any other way securitized is of course important for the asset/liability structure of the banking industry but these details can be left open for the moment. Overdrafts should be taken as a token for *all* types of credit. Mismatches of the types of assets and the types of liabilities – usually the result of bad institutional design –

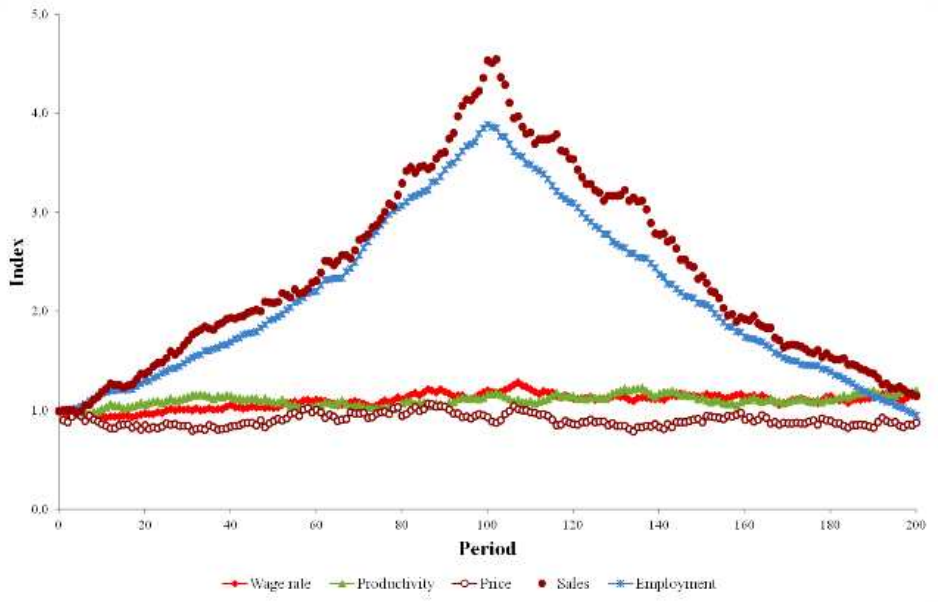


Figure 5: In the first hundred periods employment and output grow because profit is positive, subsequently the economy shrinks because of continuing losses; overall profit/loss depends alone on the expenditure ratio, that is on credit expansion/contraction

can create indigenous problems in the banking industry which, however, do not concern us here (see for example Minsky, 2008). For the time being it is assumed that the banking industry works smoothly and does not create indigenous financial disturbances. Market failure and system failure are entirely different issues.

With a random expenditure ratio greater unity the business sector's profit is varying but always greater than zero and this translates into an employment expansion according to (28). Since productivity is a stationary random variable output grows with employment according to (2). The growth of employment and output lasts for the first hundred periods. In this time span credit expands according to (34) and the quantity of money according to (14).

As can be seen from Figure 5 the process of credit expansion and contraction is neither inflationary nor deflationary. Since wage rate and productivity are both stationary random variables the market clearing price remains flat over the whole time span of observation according to (32).

According to (24) dissaving means profit and saving means loss. With dissaving (that is not compensated among the households themselves) the household sector's credit expands, with saving/redemption credit contracts. Since credit has to be fully repaid it is again zero at the end of the whole process. By consequence, profits and losses cancel out in the process for the business sector as a whole – not, of course, for individual firms.

No matter how long it takes, the household sector's credit expansion must be reversed some day. The final turning point is fatal for the economy. At this point profit for the business sector as a whole turns into loss and the economy – slower or faster – breaks down. The fall of employment is accompanied by a fall of the quantity of money according to (14) and (10); but, to be sure, the central bank is *not* the causative factor as in the monetarist rationalization of the Great Depression. Occasional debt-deflations exemplify the characteristics of the final turn.

A reduction of the wage rate cannot eliminate loss because it is $\rho_E < 1$ that makes the loss. By consequence, the most flexible wage cutting cannot turn the economy around; only an increase of the expenditure ratio could. This does not happen if the household sector aims at full redemption. Note that full redemption is not an accident but a constitutive part of the households' optimal consumption plans. All boils down to budget balancing over the very long haul. A balanced budget is the economic analogon to a physical conservation law.

There may occur numerous temporary credit contractions in the course of time if the expenditure ratio is not consistently greater unity. Overcoming these recessions does not mean that the final turning point vanishes. The existence of this point is guaranteed by the fact that credit has to be eventually redeemed.

In intuitive geometrical terms: two points A and B are either connected by a straight line (on an Euclidean plane) or by a curved line. In the latter case the curve must have a final turning point. The case with profit/loss equal to zero in each single period corresponds to the straight line. This case in turn corresponds to the Walrasian equilibrium, which in the final analysis corresponds to nothing in the real world.

The existence proof of general equilibrium, which is based on indefensible subjective-behavioral axioms, is replaced by the existence proof of a final turning point, which is based on objective-structural axioms.

To summarize it with a metaphor: the elementary consumption economy exclusive of profit distribution is a zero-sum game with time as the nth player. It is neither productive nature nor human effort nor greed which brings profit into existence. It is temporal asymmetry that creates this optical illusions. This, though, is forever beyond the comprehension of those who look at the market economy through the metaphor of a simultaneous behavioral equilibrium.

A change of perspective requires a paradigm shift. In formal terms that means a change of axioms.

4.5 Extensions

Since the pure consumption economy is the most elementary economic configuration there can be only analytical extensions. The first is to take distributed profit into account which has been set to zero in the foregoing analysis in order to keep the focus on the main point.

Profit is, in addition to the household sector's period deficit, i.e. $\rho_E > 1$, and in addition to profit distribution, i.e. $\rho_D > 0$, positively affected by a public budget deficit, by the configuration $I > S$ (for details see 2011c), or by a surplus of exports over imports when we split the world economy into regional economies and consider each in isolation.

All these extensions shift the final turning point in time and from one region to another but do not eliminate it. In the case of the pure consumption economy it is the *growth* of the household sector's debt that keeps the turning point well out of sight.

Under the historical perspective it were the overspending American consumers, the reckless Greek government and its peers (all listed in Reinhart and Rogoff, 2009, p. 24), and the real capacity creating investors in Asia and elsewhere that have expanded credit, boosted profit, and thereby kept the world economy going during the last decades until 2007. Whatever their myopic intentions, objectively they have successfully nudged the final turning point farther into the future.

5 Conclusion

> It is difficult to contemplate the evolution of the economic science over the last hundred years without reaching the conclusion that its mathematization was a rather hurried job. (Georgescu-Roegen, 1979, p. 271)

The existence proof of general equilibrium, which is based on indefensible subjective-behavioral axioms, is replaced by the existence proof of a final turning point, which is based on objective-structural axioms.

The set of four structural axioms constitutes the evolving consumption economy.

In the consumption economy, the final turning point is reached as soon as the household sector starts to redeem *all* hitherto accumulated debt, as it is supposed to do eventually. Because this turns profits for the business sector as a whole into losses the final turning point marks the beginning of the breakdown of the monetary economy. Extensions of the elementary economic configuration shift the final turning point in time and from one region to another but do not unmake it. Growth can keep the final turning point at bay for a long time. We are still in the expansive phase of the grand credit cycle. Inductive extrapolations are unwarranted.

Equilibrium is a nonentity. Under the secular perspective, the monetary economy is heading towards the final turning point.

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